TONY BROWN

CREATING AND KNOWING MATHEMATICS THROUGH LANGUAGE AND EXPERIENCE

ABSTRACT. The radical constructivist assertion that the student constructs his or her own knowledge as opposed to receiving it 'ready made' echoes the classical debate as to whether the human subject constitutes the world or is constituted by it. This paper shows how the philosophical traditions of post-structuralism and hermeneutic phenomenology offer approaches to effacing this dichotomy and how this forces a re-assertion of the teacher's role in the student's constructing of mathematical knowledge. It is also shown how hermeneutic phenomenology provides an opportunity to ground constructivist mathematical thinking in the material qualities of the world.

1. INTRODUCTION

Platonic notions of mathematical knowledge that see mathematics as having 'a real, objective existence in some ideal realm' (Ernest 1991, p.29), untarnished by human intervention, have contributed to the particularly high academic prestige it enjoys. In this perspective, mathematics has an order and consistency disturbed only by the frontiers of research revealing yet further uncovered domain. This image, however, despite its aesthetic appeal, has been challenged by assertions about the 'fallibility' of mathematical knowledge (e.g. Ernest, 1991, pp. 15-20). It also has limited usefulness in debates concerned with the acquisition by humans of this knowledge. The platonist view has been largely responsible for the tradition of seeing mathematics as a body of knowledge to be discovered or encountered by the student. This body, if not defined by the Gods, is put in place by some collection of experts and so is determined outside the consciousness of any individual learner. This view of mathematics tends to be associated with metaphors such as 'delivering', 'receiving' and 'ready made', being used in relation to teaching, learning and content.

Radical constructivism (e.g Von Glasersfeld, 1991) offers a view antithetical to Platonism. According to this view the failure of more traditional mathematics teaching is a consequence of people believing that knowledge can be transferred ready made. The task for the radical constructivist teacher is to set tasks and to create an environment conducive to the student constructing his or her own meanings. The opposition between radical constructivist and platonist views on learning echoes the classic debate which questions whether the human subject constitutes the world in which he operates or is constituted by it. It is precisely this dichotomy that has been effaced by post-structuralist and hermeneutic writings. This paper considers the radical constructivist perspective in the light of these two modern philosophical traditions.

In the next section I will demonstrate how the radical constructivist position is stressing just one side of this dichotomy. Further I shall argue that these writers are downplaying the role of the teacher in asserting that the child's learning takes place through making

constructions. I will suggest that the teacher inevitably contributes some degree of 'ready madeness' to any learning situation he or she sets up in the structuring of any activity.

In the following two sections I will outline aspects of post-structuralism and hermeneutic phenomenology and show how they offer alternative accounts of the philosophical context within which radical constructivism resides[1]. I will discuss how post-structuralism places primacy on the fitting of linguistic and symbolic forms to experience and follow Walkerdine (1988) in showing how this offers a productive view of how mathematical learning takes place. Here the human subject and the world in which he or she operates, are seen as being part of each other. In this way the question as to whether the subject constitutes the world or vice versa is transcended. I then discuss how hermeneutic phenomenology tackles this point by offering the possibility of a view of mathematical knowledge where hard distinctions between subjective and objective knowledge are removed. I will show how this provides an opportunity to introduce an ontology (i.e. a theory of how things are) that complements radical constructivist principles. This is achieved by placing more emphasis on the notion of 'mathematical activity' than on 'mathematics' and by seeing the elements of 'objective knowledge' as being activated by an individual learner classifying the phenomena he or she sees. I then further develop my suggestion that the mathematical expressions of a student within a classroom are not so much 'constructions' in their own right but rather elements in a dialogue between teacher and learner consequential to the teacher framing a certain structure.

2. THE RADICAL CONSTRUCTIVIST IDEOLOGY: THE STRESSING OF THE CONSTITUTIVE

Radical constructivist writings have given a great deal of attention to how children learn. Very often this is accompanied by suggestions as to how teaching might facilitate this view on learning. For example, Steffe (1991, p.178) quotes Sinclair who suggests that from a constructivist point of view ‘the essential way of knowing the real world is not directly through our senses, but first and foremost through our material or mental actions’. Such a description of the acquisition of mathematical knowledge, whilst, presumably, being applicable to all learning, activates definite implications for teaching style. It is in the light of this description that Steffe suggests teacher's goals consistent with such a description of learning, including for example; 'to learn how to communicate mathematically with students, to learn how to organise possible mathematical environments, to learn how to foster reflection and abstraction in the context of goal directed activity, to learn how to encourage students to communicate mathematically among themselves.'

It seems to me there is a need for care in moving from epistemologies to recommendations for practice. Captured in this particular way, the teacher's role within mathematics lessons is consistent with the emphasis being on the student constructing meaning. In the debate as to whether the cognizing subject is constituted or constitutive, the radical constructivist view emphasises the latter resulting in a ideologically selective

account where both the environmental effects on the constructions students make and the teacher's role as environmental manager contained within these effects are insufficiently described. Such environmental management, however, is evident throughout the descriptions offered of constructivist teaching styles. This arises, firstly, in framing the mathematical activity in language and, secondly, in setting up the physical and social frameworks within which this happens.

I suggest the framing of the mathematical tasks and the accommodating environmental design in lessons described by radical constructivist writers is such that the 'ready madeness' which they apparently wish to reject, is already highly developed before the student is invited to engage in constructive activity. The mathematical components within any curriculum are culturally defined resulting in the building blocks of any construction being in place before the child joins in. By providing such a frame these writers are offering different aspects of the task to that which might be offered by more traditionalist writers. Even in such a traditional teaching regime the learning would, in the eyes of a constructivist, be the result of the student making constructions[2]. The constructivist teacher then, may be seen as emphasising the constructive component of the child's learning that would arise in any style of teaching.

This can perhaps be best illustrated by an example. Kaput (1991 p.63) cites the example of the problem of finding the general formula for the sum of the first n consecutive integers. He suggests a few methods including the representation of numerical tables and Cuisenaire rods as representations prior to formulation in algebraic notation, which he has used in 'both didactic teacher-centred lessons as well as in laboratory classes'. His account is of a highly structured presentation targeting a particular conclusion. Clearly, such an approach results in the students constructing meaning but within a very specific framework. An alternative formulation presented by Billington and Evans (1987) broadens the scope of the activity. Here the problem is posed in terms of counting the number of handshakes when all of the people in a room each shake hands with each other. The task in each of these presentations may be seen as leading to the result:

Sum of first n consecutive integers = $n(n+1)/2$

However, there are a variety of routes there, each providing a different context for the production of the final result, and thus a different meaning for the statement. Kaput targets the activity at the production of this result and towards this builds a tight framework which restricts possibilities for the introduction of descriptive language by the student within which mathematical expressions can arise. Billington and Evans seem rather more concerned with the journey there and place emphasis on other mathematical aspects of the activity such as: processing information, making predictions, symbolising, tabulating, finding and investigating patterns, seeing connections, generalising, establishing a proof. In my own work with students on initial teacher education courses I have set the problem as a group task concerned with creating an illustrative poster to describe the nature of the problem to a friend who did not witness the actual handshaking. In all of these situations the task for the student is to make sense of the task and make statements in respect of it. The final algebraic relationship is a reductive outcome of this

activity whose meaning depends on the nature of the reduction and the experience of making this reduction.

For the teacher setting up such a task there is a need to decide on the style of posing the task and the structuring of the task this creates for the student tackling it. To maximise the achievement of a particular result requires a higher degree of structuring on the part of the teacher and a lower contribution in terms of framing the problem by the student. The formulation and solution of any problem can be seen as a joint action by teacher and student, composed of (from the student perspective) a 'ready made' part contributed by the teacher and a student construction made in respect of it. However, the resulting space provides components and offers a frame for what follows and so on. Similarly, any student construction is, in a sense, 'ready-made' for the teacher to work with. This is an issue raised by Wheatley (1992). Such a view moves away from a mechanistic cause and effect model in the teacher-student relationship.

I suggest that the constitutive argument of the radical constructivist view can be resisted by an alternative argument assuming the opposite side of the same debate. This argument asserts the existence of the environment, and the teacher's constructions within this, acting on the student's developing understanding. This apparent opposition, however, can be transcended. Post-structuralism and hermeneutics offer alternative ways of achieving this.

3. POST-STRUCTURALISM

Post-structuralism has not featured prominently in writings on mathematics education but has instead asserted an affinity towards the humanities and the broader field of the social sciences. Utilising as it does, Saussurian linguistics[3], it has emerged from its roots in structuralism to establish itself as an independent movement. Its major exponents have been Derrida and Foucault. Lacan's psychoanalytical work offers a significant, but in some ways separate strand. Walkerdine (1984, 1988) has shown how such an approach can offer powerful ways of describing the learning of mathematics by young children.

Structuralism, as a movement, has its origins in Levi-Strauss's work on the structure of myths in primitive societies. By collecting and analysing the myths of a particular society he identified common threads which emerged, providing some sense of an essence of the society to non-members. In this way, Levi Strauss argued that it was possible to locate and describe some objective structure underlying the mode in which the society operates. Such a model, however, suggesting a fixed relation between object and meaning, tends to underplay the differences between individual perspectives of this. More recent post-structuralist models have sought to accommodate the possibility of personal interpretations of this structure. In so doing, however, they undermine the very existence of the structure. This shift from seeing some objective underlying structure to seeing a structure dependent on the perspective of the observer offers a paradigm for considering mathematical activity. Instead of seeing mathematical activity as containing elements with fixed objective meaning we move to seeing it as a situation featuring social usage of

symbols. Here any meaning is dependent on the way in which the individual user intends it and on how this is interpreted by others.

The theoretical framework used in analysing these issues owes much to Saussurian linguistics which also lends itself readily to the more specific context of mathematical learning. According to Saussure (1966, pp. 65-70), working at the turn of the century, the 'signifier' is the word or image and the 'signified' is the associated concept, which, as a pair, form what he calls a 'sign'. However, a third element is implied, although not explicitly identified by Saussure; namely, the 'referent' or object itself[4].

We can perhaps illustrate it more clearly by reference to an example. The word 'dog' (the signifier) we associate with a certain concept (the signified) and on occasions we might encounter a 'real' dog (the referent) with which we can associate this sign. Even in this simple case, however, there are two sorts of arbitrariness present. Firstly, the word 'dog' is quite arbitrary and does in fact change according to the language being spoken. French people use the word 'chien'. Secondly, the category 'dog', is no more than an arbitrary grouping according to certain selected characteristics somewhere on the continuum starting at 'animal', passing through 'mammal', 'canine', 'spaniel' to 'Fido'. The sign 'dog' can be seen as being associated with a 'real' referent. However, not all referents are real in this sense. The signifier 'learning' for example, gives rise to signifieds and referents which are rather more problematic. Mathematical terms offer similar difficulties.

Saussure was more concerned with the differences between signs rather than with their association with the material world. As such, signs were not seen as having meanings in themselves but rather derived meaning from their relation to other signs. The meaning of a word is thus dependent on the way in which it is used in a sentence. He saw the sign as unstable in the sense that the signifier and signified can move in relation to each other. For example, the signifier 'omnibus' has become 'bus' whilst the signified is in some sense unchanged, i.e. a vehicle capable of carrying a number of people. Similarly, pronunciation might change both geographically and chronologically. Conversely, the phenomena signified by the signifier 'Pythagoras' has shifted through time[5]. Meanwhile, the relationship of the complete sign with the referential field is arbitrary until it becomes a convention through socially consistent usage. Barthes sees certain signifying practices as becoming 'naturalised' in to what he calls myths. Such myths become embedded in ideological descriptions, creating reality for those who hold that ideological position. This is discussed by Coward and Ellis (1977, chap.3). School mathematics, for example, features a particular way of classifying mathematical activity in to components and procedures that themselves become 'mathematics' in the eyes of the clients.

Mathematics, characterised as a language, necessarily comprises some system of signification but the relationship between the signified and the referent requires some unfolding. Mathematical phenomena do not have a real existence and any meaning is derived purely through relations perceived between these phenomena. Lacan has offered a new reading of Saussure where the signifier has primacy over the signified and a more fundamental instability between them is asserted[6]. Essentially, Lacan asserts that notation as printed on a page or held in a spoken word has more stability than that to

which it refers. In perceiving some phenomena I can capture this in a description employing symbols. This symbolisation however, once fitted, and held in a material form as notation on a page, affects the way in which the phenomena is dealt with subsequently and also mediates any subsequent change in this symbolisation.

Walkerdine has pointed out that Piaget's reading of Saussure resulted in him using the terms 'signifier' and 'signified' but in a very different way to Saussure himself and in a radically different way to the later meaning asserted by Lacan. She says (Walkerdine 1988 p.3):

'For Piaget the relationship of signifier to signified is one of representation; the semiotic function

'consists in the ability to represent something (a signified something: object, event, conceptual scheme, etc.) by means of a signifier which is differentiated and which serves only a representative purpose. (Piaget, quoted in Gruber and Voneche 1977:489)"

Such a view is more akin to the older form of structuralism (and Piaget's (1971) reading of it) with its implied fixed relations between object and meaning, independent of any individual observer. Piaget's work is widely acknowledged as underpinning constructivism but Lerman (1989) argues that radical constructivism asserts itself as a distinct movement to Piaget's genetic epistemology by affirming its commitment to the notion that coming to know 'does not discover an independent, preexisting world outside the mind of the knower'. Von Glasersfeld (Watzlawick 1984, p.25), in addressing this issue, suggests that 'Piaget's position is somewhat ambiguous.' In post-structuralism, and in particular, in Lacan's version of this, words are not mere labels, but rather the tools with which reality is constructed and held in place.

Any field of mathematical symbolisation is a consequence of the mathematical qualities of the world being perceived in some way and then being classified within the categories of a language. 'Square', for example, is a socially conventional signifier associated with the concept 'square' but we never actually have a real square. Consider the commands:

Draw a four sided regular polygon in pencil.
Write REPEAT 4 [FD 100 RT 90] in LOGO.

The signified 'square' is evoked by each of these but the referred to 'square', as defined in geometry, is not physically present and never can be. It can only be imagined. The referent here might be seen as the Lacanian 'objet petit a'; that is the 'lack' encountered after stripping away the various layers of description (e.g. Zizek, 1989, p.95, Brown et al, 1993). Such a post-structuralist position would emphasise the play of meaning held in the various descriptions and would be disinclined to see meaning produced outside of these descriptions. Squareness is a quality that may be seen as being within the physical world but it only comes into being retroactively as part of a human naming (signifying) process. The name 'square' is merely an arbitrary label given to a repeatable idea seen as being worthy of having a name to itself. However, whilst the notion may not have a physical

reality, pursuit of it can govern actions. Althusser's suggests that something is real if people say it is and act as if it is (Hirst and Wooley, 1982, p.134). 'Square' is a human construct that shapes our way of describing the world and our acting within it.

The generation of statements made in respect of mathematical activity is, necessarily, a human classifying process concerned with the selection and combination of signs. Walkerdine (1988) describes these as 'discursive practices' concerned with the building of categories and the 'production' of meaning. In this way, mathematical writing can be seen as comprising statements, associated by the writer with what they see as circumscribed mathematical ideas. This process of producing such writing is concerned with associating signifiers with signifieds where both the arbitrary elements identified by Saussure are present, but where social convention necessarily exerts some influence. Walkerdine suggests that developing mathematical understanding can be seen as the initiation of students into the socially conventional ways of associating linguistic and symbolic forms with phenomena seen as mathematical[7]. This might be seen as learning the local myths. The ontological qualities of these very phenomena, however, is uncertain. Lacan would see the symbolic expressions themselves as becoming the material reality of these phenomena.

The fitting of a signifier, however, does have a holding effect on the signified resulting in a materially stable notation being associated with a conceptualised phenomena, subject to contextual and chronological changes. To provide an example I shall refer to a lesson where ten year old children were investigating the interior angles of polygons. Having done a few examples they had concluded that for three sided shapes the total was 180o, for four sided shapes, 360o and for six sided shapes, 720o. On this basis they made the hypothesis:

'For five sided shapes the total degrees is 540o'

This expression was being used to hold a hypothesised relationship. They then proceeded to make five sided shapes out of wooden 'pattern blocks'. Each time an appropriate shape was made they attempted to sum the angles, employing a variety of techniques for measuring individual angles. However, they made numerous slips in measuring angles and in counting the total, reaching many totals that were not 540. However, the holding effect of the hypothesis was such that they were prepared to reject other results and re-check by other methods. After an hour they had three examples of five sided shapes where they felt the total was 540o. They saw this as adequate proof of the hypothesis and when asked the total for ten sided shapes were quickly able to give a convincing figure. This was followed by the production of a ten sided shape (two touching hexagons) with the angles marked and adding to the expected total.

I would suggest that the expression, whilst materially stable, was associated with an idea whose meaning shifted during the course of this activity. The expression was initially simply the sentence that filled the gap in the sequence 180, 360, ..., 720. It then proceeded to be the story associated with the three examples of five sided shapes. In the

light of this it was seen as sufficient confirmation of the pattern to justify projecting forward to more sophisticated shapes.

Another important aspect of the use of expressions is the self-referential qualities they possess. The fitting of any expression is dependent in the first place on the position and perspective of the subject making an association. The children in the example above are selecting the key expressions which for them epitomise the situation they are observing. They are mapping out their enquiry in a very personal way. Their quest may be to fit an ever more exact form but the very fitting of the form materially affects the context of which the phenomena is part, thus affecting the position and perspective of the subject fitting this form. In this way other expressions may emerge as holding the essence of their work.

The subject's attempts to fit symbolic forms to particular phenomena are also deeply rooted in his or her perspective of how the phenomena relates to him or her. The child learns to position him or her self in the world before he or she has learnt to speak. However, there remains a residue of this pre-linguistic state as the child captures ever more of his or her world in language. In this way the subject perceives part of him or her self in fitting any symbolic form to phenomena which are necessarily seen from an individual perspective. The act of fitting any form has a reflexive dimension where, simultaneously, the form signifies both the perspective of the subject and the meaning of the phenomena to him or her. In this way the quest to find the most appropriate form is in part concerned with self description. However, the final description of this subject-in-context is always in the future since any attempt at closure always affects that described. Zizek (1989, chap.6) argues that this thwarted attempt at reaching some concluding position is always in hand with a failure on the part of the subject to constitute him or her self (in language). The story told by the children about their work on angles is never finished, since as their work proceeds the meaning of the expressions they use forever shift as the perceived context for them changes.

Similarly, any form of mathematical instruction introduced by a teacher conceals a socially conventional way of making associations between symbolic forms and phenomena seen as mathematical. Walkerdine (1984), following Foucault's work, has argued that much that went under the banner 'child-centredness' was to do with covert regulation, by asserting conventional forms of signification as 'natural'. This is also certainly true of the 'neo-child centrists' among the constructivists. Within a child-centred philosophy the child is engaged in constructing meanings and so in the development of signifying practices. Expressions are being fitted to phenomena towards producing meaning. However, this constructing arises in an environment where certain conventional discourses prevail and the task becomes to fit inherited forms in a conventional way as represented by the teacher or published scheme. In this way, the constructions are controlled but in an environment described as if the child is responsible controlling the agenda.

When the constructivists talk of student's constructing they underplay the fact that this constructing is being done in an inherited language associated with a conventional way of

using it. The ready-madeness they wish to reject is necessarily implicit in the language teacher and child use in communicating with each other. Also, in framing any activity for the child, the teacher is selecting the particular domain of discourse that further contextualises and thus conditions any terminology used. Conventional mathematical discourse has classified mathematical phenomena in a particular way that is arbitrary in both of the ways identified by Saussure. The same is true of the procedures employed in respect of these phenomena insofar as there are conventional ways of tackling certain sorts of tasks (e.g. the decomposition method for subtraction problems). In this way learning mathematics is akin to learning a language, but not so much as a system and structure but rather as it is realised in individual everyday acts.

Walkerdine (1988) takes account of this more in her alternative (post-structuralist) reading of how children learn mathematics by placing more emphasis on the actual fitting of words and symbols to experience. Here meaning is produced as the student develops signifying practices. Brown (1990) provides an example of this with an investigational task where a variety of alternative forms (e.g. drawings on squared paper, models made out of plastic squares, algebraic symbolisation) are fitted by student to various lattice arrangements. He suggests that meaning is produced through the student making and combining metaphoric associations (e.g. between a drawing on squared paper and its plastic equivalent) and metonymic associations (e.g. between two plastic models). The sense that the student makes of this situation is held in the stories he or she tells about it.

In summary, post-structuralism moves away from seeing the world as something constructed by the individual human subject by recognising that this is always done in an inherited language. The chief consequence of this is that a style of structuring is already implicit in the conventional ways of describing the world that teacher and child find themselves obliged to use in facilitating communication between them. This is not only true of the phenomena seen as being components of mathematics but also of the procedures applied to these. This is not inconsistent with radical constructivism but places a firmer emphasis on language and on the role of the teacher as representative of the dominant discourse. It also provides a clearer connection with the much broader field of post-structuralism as it has been developed in respect of other areas within the social sciences.

Whilst post-structuralist writers do offer views on materiality (see for example, Coward and Ellis, 1977, p.127) I shall not pursue these within this paper. I will however, now address how writers in hermeneutic phenomenology tackle this issue and so provide a possibility of an ontology consistent with radical constructivism.

4. HERMENEUTIC PHENOMENOLOGY

Radical constructivism (e.g. Von Glasersfeld, 1991) explicitly avoids entering into theories about how things are. According to this view there may or may not be some independently existing reality, but whatever the case the cognizing subject does not discover such a reality directly. Cognizing is seen as an adaptive process and the subject

is in search of a viable but forever renewable model, generated through his or her perceiving of and acting in the world. Critics (e.g. Kilpatrick, 1987) have identified a weakness here and suggest that there is a need for ontological issues to be addressed more explicitly in setting up an epistemological framework, towards making a firmer connection with reality. I will argue here that by using notions of objectivity derived from phenomenology, a theory of how things are can be introduced without undermining the constructivist theory of how we know (e.g. Noddings 1990, Von Glasersfeld, 1990).

Post-structuralism asserts that the meaning of any situation is held in the play of accounts offered in respect of it, where there is no truth outside that constructed in these accounts. Hermeneutics, the theory and practice of interpretation, however, holds on to the notion that there might be an ultimate truth but that this cannot be accessed directly. This truth is always conditioned by the process through which it is seen. Hermeneutics thus readily lends itself to the disciplines within the human sciences, which in general, 'deal with the world of meaningful objects and actions (as opposed to physical objects and events in themselves' (Culler, 1976), where the human subject is assumed to have a particular position and perspective rather than some God-like overview. The phenomenological underpinning implicit in such a view of the human sciences has been highly influential in the development of modern hermeneutic thinking. Here I shall outline how phenomenology and its development within hermeneutics can offer a useful framework for considering mathematical learning.

Action and meaning

In his 'Philosophical Investigations' Wittgenstein (1958, p.20) suggests that the meaning of a word might be seen as its usage in language and is thus dependent on both situation and time. This offers an alternative to seeing words as having inherent meaning and a key to analysing expressive activity as action. To say a sentence is to perform an action. The meaning of a sentence, seen as an action, can then be seen as being related to the description of its effect, in a social situation (Thompson 1981, p.126). Ricoeur (1981, p.210), a leading exponent of modern hermeneutics, sees this effect, the way the action leaves its mark, as the 'objectivity' of the action. He follows Husserl, the founder of modern phenomenology, in seeing objectivity and subjectivity in a more complementary relationship with each other.

If the production of any mathematical expression can be seen as an action, the meaning of such an expression is necessarily subject to an interpretation that transcends any meaning in the expression itself. We need to look at how the expression is being used by someone in a particular context. A distinction needs to be drawn between 'mathematics' (as might be perceived in a Platonic sense) and 'activity seen as mathematical'. In doing this, the stating of any mathematical expression, verbally or in writing, can be seen as an action subject to interpretation. The meaning of this action, would therefore go beyond that which would be found in a purely literal or symbolic investigation and cannot be separated from its agent or the context in which it arises.

To unfold this idea Ricoeur (1981, p.199) concurs with Austin (1962) in suggesting that the effect of a sentence might be described at three levels; a) the locutionary effect, (the literal meaning), b) the illocutionary effect, (what is construed through tone of voice, perceived context etc.) and c) the perlocutionary effect, (the action done by the sentence). Ricoeur suggests these levels form a hierarchy according the degree of interpretation needed. The locutionary meaning can be checked in a dictionary. To describe the perlocutionary meaning, however, requires the subject to have experience of living in an appropriate language using community and to be accustomed to fitting words to given situations in a conventional way.

Husserl's phenomenology

Ricoeur (1966) has done substantial work in developing Husserl's writing on phenomenology and offers an alternative approach to tackling the constitutive/constituted debate. He does this by seeing subject and object as part of each other; the individual is always part of what he or she sees. For example, in handling some practical mathematics apparatus I am finding out about myself. The apparatus is only meaningful insofar as it resists and guides my actions. The apparatus and my body become unified in any action.

In asserting subject and object as part of each other Ricoeur softens yet maintains the distinction between them. He speaks of any action as having reciprocal 'voluntary' and 'involuntary' components. The voluntary component gives rise to the involuntary component which has no independent meaning but rather can be seen as the immediate context, or the resistance, which gives the voluntary component its meaning. The involuntary shapes itself around the voluntary act. This implies a (hermeneutic) process where the subject voluntarily acts in the world he or she supposes it to be, but this in turn gives rise to (involuntary) resistances which are always at some distance from those anticipated. However, in order to act the subject suspends doubt whilst acting as if his or her reading is correct.

In his detailed discussion of Ricoeur's work, Thompson (1981, p.128) identifies this aspect of his work as dealing with the constitutive/constituted dichotomy.

'For Ricoeur's attempt to understand the reciprocity of the voluntary and the involuntary is a systematic attack on the dualism of an autonomous self-consciousness exiled from an objective world which it regards as an other. Ricoeur pursues this attempt through a detailed demonstration of how each moment of the will comprises both a voluntary and involuntary aspect, the ultimate unity of which remains an unattainable ideal.'

A key notion in Husserl's phenomenology is that of 'intentionality'. In seeking to clarify this notion Schutz (1962) asserts:

'There is no such thing as thought, fear, fantasy, remembrance as such; every thought is thought *of* , every fear is fear *of*, every remembrance is remembrance *of,* the object that is thought, feared, remembered.'

Similarly, Ricoeur asserts that a consciousness is always a consciousness of something. This is not to say that the subject is conscious of a discreet object which it sees as the other, but rather

'the basic datum of experience at its most immediate level is the intentional unity of subject and object from which both the concept of a pure subject and of a pure object are subsequently derived by reflexive consciousness' (Ricoeur, 1966, p.xiii).

My understanding of this is that I might talk about the situation I see myself in, as if it were independent of me, after experiencing myself as part of it. I experience myself 'acting' through time but I am unable to talk about this as it happens. In my subsequent descriptions however, I can speak of 'actions', facts after the event, which can be classified in language after reflection. It is in such a description, made in hindsight, that I am able to describe myself as if I am separate to the situation I am in.

Zizek in his lengthy discussion of Lacan's psychoanalytic work (1989, pp 11-14) locates a similar notion. Referring to Freud's work on dreams he distinguishes between the dream, the memory of the dream and the recounting of the dream in words. The meaning of the dream cannot be captured as it happens, but rather some retroactive categorisation is necessary to prepare it for description in language. I am very much a part of the dream as it happens and I need to reflect subsequently in order to make sense of it. This reflective process results in myself, and the world of which I am part, being described in language, providing an orientation to the world I have experienced through my senses. As another example, Zizek (1991, p.100) identifies the Marxist notion of 'class struggle' as a 'structuring principle' around which we can orientate social phenomena we have experienced through historical processes. In mathematical activity concerned with making sense of certain situations we are confronted with a similar task of introducing structures around which we can orientate our thinking. By introducing successive linguistic and symbolic overlays the various aspects of our work within an activity can be examined through the 'stressing and ignoring process' implied, without which 'we cannot see anything' (Gattegno, 1971, p.11).

Ricoeur uses Husserl's notion of 'bracketing' where the existence of objects, and relations between them are assumed and fixed for the time being so that consciousness is directed towards 'phenomena', that is, objects having certain meanings to an individual person at a given time. According to Ricoeur (1966, p.xiii-xiv),

'(T)here is no consciousness unless it is consciousness of an object-and, conversely, an object presents itself as an object only for a consciousness.... by imposing the phenomenological brackets we transform the contents of experience from a physical world of objects into a world of phenomena, that is objects as meanings presenting themselves to a consciousness'.

Coward and Ellis (1977, p.132), in discussing Husserl's work, suggest

'.. phenomenology disputes the so-called 'natural attitude', the existence of the external real world. It is not concerned with the spatio-temporal existence of things, such concerns are simply bracketed out: if it is real to consciousness, then it is real.'[8]

Objectivity

Gadamer (1962, p.220) quotes Husserl who asserts that such phenomenological developments displace traditional notions of objectivity where it is seen as the antithesis of subjectivity.

'The naivete of talk about 'objectivity' which completely ignores experiencing, knowing subjectivity, subjectivity which performs real concrete achievements, the naivete of the scientist concerned with nature, with the world in general, who is blind to the fact that all truths that he acquires as objective, and the objective worlds itself that is the substratum in his formulation, is his own life construct that has grown within him, is of course, no longer possible when life comes on the scene'

Phenomenological views offer the opportunity for providing a clearer ontological grounding for action. Whilst the independent existence of the material world, prior to any classification and outside of individual consciousnesses is not denied, the world of material objects existing, in a fixed way, independently of an individual consciousness, is denied. My ability to distinguish shapes, colours, smells, textures, objects is dependent on my senses, is culturally conditioned, and emerges through time. The material is only describable within linguistic categories to an individual consciousness and any description is the result of an interpretation. In particular, the meaning of an action, that is, its objectivity, is related to how it is described in retrospect. The voluntary action brings in to play involuntary resistances that only have any meaning in subsequent descriptions of their effect. Meaning is dependent on the categories we introduce and objectivity is a function of describable traits.[9] As an example of this Brown (in press) uses particular traits identified by Ricoeur to analyse the status of a transcript recording some children working together in a mathematics lesson

Objectivity within hermeneutic phenomenology emerges through an interplay between the experiencing of the world and making statements in respect of it. Whilst my experiences lead to me describing them in a particular way, these descriptions I offer condition the way in which I subsequently experience things. My experiences are mediated by language. Here objectification is a consequence of a reflexive process where notions, and relations between them, are bracketed (i.e seen as phenomena and assumed for the time being), and through this bracketing, regulate the way in which the world is described. In a sense the objective world can only be seen through successive linguistic overlays; or rather, particular time-dependent partitionings. The way in which I experience the world governs the way I talk about it. The way in which I talk about the world, however, now informs the way in which I see it in the future. This circularity is an embodiment of the two arcs of the hermeneutic circle, from understanding to explanation (i.e. from understanding the world through a process of categorising it, to making

constructions in language in respect of it) and vice versa (Ricoeur, 1981, pp.210-221).This sort of recognition underlies hermeneutic understanding which has been discussed in relation to mathematical activity by Brown (1991). Seen in this way 'objectivity' is not synonymous with 'intersubjectivity', which appears to be the case in certain constructivist writings.

The hermeneutics of classroom experience

In a field not noted for its simplicity Gallagher (1992, pp.34-39) provides a remarkably lucid account of how hermeneutics offers a powerful way of considering classroom experience. He suggests that writers in hermeneutics have generally been too concerned with text interpretation and this has blinded them from the general applicability of the approach within the field of education. He identifies classroom and play experience as being similarly valid objects for hermeneutic analysis. For Ricoeur (1981) text interpretation is not about re-joining the author and the meaning of the text is more dependent on what the reader brings to it. Similarly, Gallagher suggests that learning is not about replicating the ideas of the teacher. Rather it is more appropriate to see the learner's experience as a sequence of interpretations made in respect of the classroom situation. The hermeneutic circle can be seen here as the interplay between these interpretions and taking actions in respect of them.

Any act of constructing by a child in a mathematics lesson can be seen as part of an event that is at the same time modifying the environment for this action. The action, which comprises voluntary and involuntary components, has agent and structure as part of each other. For example, the production of a mathematical expression by a student, a voluntary action, would be complemented by the structure (the source of the involuntary resistances), part of which may have been constructed by the teacher in response. The event thus comprises a shared action seen from different perspectives. For some outside observer looking on, a 'shift of attention' (Mason & Davis, 1990) may be required in switching from seeing the joint action as one of the child constructing, to one of the teacher asserting some framework within which this arises - from constitutiveness to constitutedness.

As an example, I shall describe a recent observation of a seven year old girl working with Dienes blocks. Five children and a teacher were seated around a table covered with a wide variety of base 10 material. After a period of 'free-play' the girl was directed towards counting the number of unit cubes in a 10x10x10 cube. She declared that she knew that on one face there were 100 unit cubes. She then proceeded to count the number of faces and concluded that since there were six faces there must be 600 cubes altogether. The teacher's response was to pick up a 10x10 'flat' and ask how many would be needed to make the big cube. The girl started piling one flat upon another and counted as she did this. After saying 'five hundred' she grinned broadly recognising that the result was not going to be 600. She went on to conclude that there were 1000 cubes. The girl was subsequently able to give two accounts of how to calculate the total yet seemed unable to

reconcile them. She opted for the latter result primarily because her teacher had nudged her there.

Whilst engaging in this activity the girl was making 'constructions' of various sorts. Her work during the free-play interlude seemed to have her engaging in a variety of voluntary tasks defined by herself. The environmental resistances included; the obligation to stay at the table, the suggestion that the materials be used in a certain way, the other children's use of the materials, the framing of the activity consequential to the physical properties of the materials, the verbal guidance of the teacher. These resistances shaped themselves around the voluntary actions of the girl.

The more specific guidance of the teacher, towards counting the unit cubes in the 'block', served to re-orientate the girl's gaze redefining for her the phenomenological structure of her perceptual field. For the girl it seemed the solution was to be found in the 'block' itself as, in the first instance, she made no reference to the other pieces available. She seemed to count those cubes that were visible to her in taking such a perspective and consequently decided that there were 600. However, once the teacher became aware of this and introduced the possibility of counting by using some of the other pieces available her gaze was again re-directed so that the 'flats' were used in a particular way towards constructing the 'block'. Any other potential use of the 'flats', however, was being ignored for now.

The selection of, and the meaning assigned by the girl to, the various pieces was consequential to the framing of them by the teacher. The child was without doubt making constructions but component 'objects' within these constructions were suggested in the teacher's actions which stressed and ignored certain qualities. The pieces were not being seen as 'units', 'flats' and 'blocks' in themselves but as phenomena which had certain meanings in relation to each other, in the eyes of the girl. The successive overt actions of the child and teacher suggested a space for the other to work in. This space, in being interpreted in particular ways, gave rise to intentional actions made in respect of the individually perceived phenomenological field. The interlude can thus be described as a jointly created sequence of actions where both teacher and child construct but in a way responsive to the jointly created environment within which this happens.

In summary, there are two key implications for radical constructivism. Firstly, by introducing such a notion of objectivity an ontological position rooted in the material world can be introduced that at the same time allows for the possibility of an individual constructing meaning. The material existence of the world is fully accepted but it only presents itself according to some particular phenomenology subsequent to being carved up in some time dependent categorisation by an individual. Secondly, acknowledgement can be given to the teacher's contribution to the joint process of constructing, necessarily implicit in any teacher/learner exchange. Any assertion of a 'mathematical' domain is an assertion of a culturally bound form of structuring. In the example above, the task of 'counting the unit cubes in the big cube' is as much a negotiation about the language to be used as a problem employing conventional mathematical terminology. As in the post-structuralist reading, the teacher, in asserting the conventional structuring is demanding

that the child's constructions be made using socially constructed building blocks. The teacher is thus complicit in any construction by the child.

5. CONCLUDING REMARKS

In recognising that mathematics does not have a reality outside of its symbols and the meanings humans bring to these we are faced with a difficulty in framing the precise nature of the mathematical object. When considering issues of learning by individual students, however, we can be more productive if we brush aside notions of mathematics as created by the Gods, or at least by the experts. Radical constructivism moves firmly in this direction. However, as a consequence of placing too much emphasis on the notion of students constructing their own meaning the radical constructivists are underplaying the role of (the inherited) language and the framing implicit in the teacher's setting up of a mathematical activity. Post-structuralism, in many ways, complements radical constructivism but in emphasising the constituted aspect of children's learning insists on a higher profile being given to linguistic issues. Seeing mathematics in this way, social conventions govern mathematical behaviour through having naturalised certain signifying practices. Learning mathematics can then be seen as learning to employ a language where certain conventional forms and procedures are inherited.

In doing this, however, the post-structuralist position radically redefines notions of the human subject. This subject, and the situation of which he or she is part, are both constructed in language, in the sense that they are held in the successive stories told about them. In being defined by these stories there is no stability about who this subject is since any description can always be extended. The subject can never be fully constituted since the closure of such descriptions is always in the future - another story can always be told[10].

By following phenomenological writers we can assert mathematics as that which is done within mathematical activity and so individual acts seen as mathematical can take primacy over an externally defined objective mathematical knowledge. The process of coming to know mathematics can be seen as classifying the experience of this activity into a certain order within language in individual consciousnesses. This presupposes a physical world that cannot be independent in any describable way since any description ties it to the perspective of an individual consciousness. By seeing objectification as the activation of phenomena, through the classifying of perception in language, by an individual, we can introduce an ontology supportive of the constructivists' emphasis on individuals constructing meaning.

By opting for constitutiveness and in so doing asserting an opposition between constitutiveness and constitutedness the radical constructivist position is losing an opportunity for such an ontological grounding. Their ideologically selective description of teaching emphasises those bits consistent with a constructivist view of learning. By asserting constitutiveness insufficient account is being made of the nature of the teacher's task and the way in which it supports learning. I have shown here that by transcending

opposition between constitutiveness and constitutedness a broader account can be given of learning where teacher and learner engage in a dialogue comprising actions which can be simultaneously described as constructions by the teacher and learner that frame the space and provide the building blocks of successive actions by both.

NOTES

1. The separation of these traditions is in some ways artificial since there is much cross fertilisation. However, many writers would assert the distinction for both philosophical and political reasons.

2. This is discussed more fully by Pirie and Kieren (1992).

3. 'Saussurian linguistics' is a rather ambiguous term. Saussure himself did not write a book on the subject, a task he saw as far too complex. The Course in General Linguistics was put together posthumously by his students compiling their lecture notes. Within modern linguistics Saussure is definitely associated with the past and has been superseded by many writers. However, within literary theory and other post-structuralist writings he is referred to more than any other linguist. Saussure's work was specifically targeted at the task of linguistics but subsequent readings have given his work status as a framework for all of the human sciences. Jackson (1992) has suggested that the Saussure who consorts with Derrida and Lacan is not the empirical linguist at all but rather an idealist philosopher of language invented in Paris in mid-century long after the real Saussure passed away.

4. This third element is much more controversial and is discussed in detail elsewhere (e.g. Hodge and Kress (1988), Lacan/Wilden (1968) and Ellis (1989) offer varying perspectives).

5. The name was originally associated with theories about number, but to many people now this name conjures up a particular geometrical relationship (Tahta, 1991). An interesting account of word usage within the social sciences, showing this instability, has been offered by Williams (1983).

6. Lacan speaks of the signified sliding under the signifier and uses the metaphor of 'points de capiton' (upholstery pins) in speaking of nodal points fixing the system of symbolisation in a 'quilting process'. This aspect of Lacan's work is discussed at length by Zizek (1989).

7. These ideas can be traced through a debate continuing through a number of articles, in particular; Corran & Walkerdine (1981), Tahta (1985), Liebeck (1986), Pimm (1986) and in Pimm's (1991) review of Walkerdine (1988).

8. For fuller accounts of Husserl's phenomenology are offered by Schutz (1962, pp. 99-149), Gadamer (1962, pp 214-234) and Pivcevic (1970).

9. Thompson (1981, pp. 139-149) outlines a more developed theory of action.

10. This is a definite move away from the subject implied by the Descartesian aphorism "I think there I am' where the subject is seen as a being in his or her self.

REFERENCES

Austin, J. :1962, *How to do Things with Words*, Oxford University Press.

Billington, J and Evans, P.:1987, 'Levels of knowing 2: the handshake', *Mathematics Teaching*, 120, 12-19.

Brown, T.: 1990, 'Active learning within mathematical investigations', *Mathematics Teaching*, 133, 15-18.

Brown, T.: 1991, 'Hermeneutics and mathematical activity', *Educational Studies in Mathematics*, 22, 475-480, Kluwer, Dordrecht.

Brown, T, Hardy, T and Wilson, D.: 1993, 'Mathematics on Lacan's couch', *For the Learning of Mathematics*, 13,1, FLM, Montreal

Brown, T.: (in press), 'Creating evidence towards making statements about classroom activity', *Research in Education,* Manchester University Press.

Corran, G and Walkerdine, V.: 1981, *The Practice of Reason*, Institute of Education, London

Coward, R and Ellis, J.:1977, *Language and Materialism,* Routledge and Kegan Paul, London.

Culler, J.:1976, *Saussure*, Fontana, London.

Ellis, J.: 1989, *Against Deconstruction*, Princeton University Press Princeton.

Ernest, P.:1991, *The Philosophy of Mathematics Education*, Falmer, Basingstoke.

Gadamer, H.G.: 1962, *Truth and Method,* Sheed and Ward, London.

Gallagher, S.: 1992, *Hermeneutics and Education,* State University of New York Press.

Gattegno, C.: 1971, *What we owe children,* Routledge and Kegan Paul, London.

Gruber, H. and Voneche, J.: 1977, *The Essential Piaget,* Routledge, London.

Hirst, P. and Wooley, P.:1982, *Social Relations and Human Attributes*, Tavistock, London.

Hodge, R and Kress, G.: 1988, *Social Semiotics*, Polity Press, Cambridge.

Jackson, L.:1992, 'Theorists Saussure of themselves', *Times Higher Education Supplement* (September 18), London.

Kaput, J.: 1991, 'Notations and Representations', in Von Glasersfeld, E (Ed), *Radical Constructivism in Mathematics Education*, Kluwer, Dordrecht.

Kilpatrick, J.:1987, 'What constructivism might be in mathematics education', *Proceedings of 11th International Conference for the Psychology of Mathematics Education*. Montreal, VOL.1, PP.3-27.

Lacan, J.:1977, *Ecrits: A Selection*, Tavistock/Routledge, London.

Lacan, J and Wilden, A, .:1968, *Speech and Language in Psychoanalysis*, John Hopkins, Baltimore.

Lerman, S.:1989, 'Constructivism, mathematics and mathematics education', *Educational Studies in Mathematics*, Kluwer, Dordrecht, 20, 211-223.

Liebeck, P.: 1986, 'Reading mathematics', *Mathematics Teaching*, 115.

Mason, J and Davis, J.:1990, 'Cognitive and metacognitive shifts of attention and a methodology for their study' (unpublished manuscript), Open University, Milton Keynes.

Noddings, N, 1990, in Davis, R, Maher, C and Noddings, N.: 'Constructivist views on the teaching and learning of mathematics, Monograph No. 4', *Journal for Research in Mathematics Education*, National Council of Teachers of Mathematics.

Piaget, J.: 1971, *Structuralism*, Routledge, p.12.

Pimm, D.: 1986, 'Beyond reference', *Mathematics Teaching*, 116.

Pimm, D.: 1991, 'Signs of the times', *Educational Studies in Mathematics*, 22, 391-405.

Pirie, S and Kieren, T.: 1992, 'Creating constructivist environments and constructing creative mathematics', in *Educational Studies in Mathematics*, 23, 505-528.

Pivcevic, E.: 1970, *Husserl and Phenomenology,* Hutchinson, London.

Ricoeur, P.:1966 Ed, *Freedom and Nature: The Voluntary and the Involuntary*, North Western University Press, Evanston, (Kohak's translator's introduction).

Ricoeur, P.:1981, *Hermeneutics and the Human Sciences*, Cambridge University Press.

Saussure, F de.:1966 Ed, *A Course in General Linguistics*, McGraw-Hill, New York.

Schutz, A.: 1962, *The Problem of Social Reality,* Martinus Nijhoff, The Hague.

Steffe, L.: 1991, in Von Glasersfeld, E (Ed), *Radical Constructivism in Mathematics Education,* Kluwer, Dordrecht.

Tahta, D.: 1985, 'On notation', *Mathematics Teaching*, 112

Tahta, D.:1991, in Pimm, D and Love, E, *Teaching and Learning School Mathematics*, Hodder and Stoughton, London.

Thompson, J.: 1981, *Critical Hermeneutics*, Cambridge University Press.

Von Glasersfeld, E. : 1990, in Davis, R, Maher, C and Noddings, N.: 'Constructivist views on the teaching and learning of mathematics, Monograph No. 4', *Journal for Research in Mathematics Education*, National Council of Teachers of Mathematics.

Von Glasersfeld, E (Ed).:1991, *Radical Constructivism in Mathematics Education,* Kluwer, Dordrecht.

Walkerdine, V.:1984, 'Developmental psychology and the child centred pedagogy' in J. Henriques et al, *Changing the Subject*, Methuen, London.

Walkerdine, V.:1988, *The Mastery of Reason*, Routledge, London.

Watzlawick, P (Ed).: 1984, *The Invented Reality*, Norton, New York.

Wheatley, G.: 1992, 'The role of reflection in mathematics learning', in *Educational Studies in Mathematics*, 23, p.530.

Williams, R.:1983, *Keywords*, Flamingo, London.

Wittgenstein, L.:1958, *Philosophical Investigations,* Basil Blackwell, Oxford.

Zizek, S.:1989, *The Sublime Object of Ideology,* Verso, London.

Zizek, S.:1991, *For They Know Not What They Do: Enjoyment as a Political Factor,* Verso, London.

Manchester Metropolitan University,
799 Wilmslow Road, Didsbury,
Manchester, M20 8RR, UK