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Consensus Reaching Mechanism for Classification-Oriented Group Decision Making Under Unpredictable Uncertainty Based on Weighted Average Evidential Fusion Rule

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Group decision-making (GDM) is common in practice. When a GDM problem only requires classifying alternatives instead of producing the ranking of alternatives for a preferred choice, it raises an important question about how to design a consensus-reaching process (CRP). Moreover, under uncertain circumstances, a distributed preference relation (DPR) can not only express superior and inferior degrees on discrete linguistic terms, but can also deal with ignorance properly. How to generate DPR based on multiple attributes and unpredictable uncertainty is also an interesting issue. In this paper, the generation of DPRs on multiple attributes considering unpredictable situations is first discussed. Then, the weighted average evidential fusion rule is proposed to aggregate the DPRs on multiple attributes and experts. The proposed evidential fusion rule satisfies all basic properties of an information combination rule. It is also compared with the evidential reasoning algorithm to illustrate its validity. As for the group consensus, the consensus measure is first defined on the classification result of each expert. Consensus identification and adjustment rules are then proposed, where both opinion leader and social network analysis are considered. An illustrative case study is finally presented to demonstrate the effectiveness and advantages of the proposed approach.

Keywords: Group decision making; distributed preference relation; consensus reaching process; classification; weighted average evidential fusion rule.

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1. Introduction

Group decision making (GDM)[1–3] refers to a group of experts or decision makers (DMs) who participate together online or offline to make decisions such as ranking or selection of alternatives. When the number of participants is large, it is called large-scale group decision making (LSGDM).[4,5] Multiple attributes are usually considered in GDM problems, which is termed as multiple-attribute group decision making (MAGDM).[6–9] For example, in the life-cycle sustainability assessment of a complex product, the dimensions of the environment, economy and society are always considered.[10] Furthermore, GDM has been applied in many fields, such as construction industry,[11] supply chain management,[12] site selection,[13] among other fields.

A crucial aspect of GDM is the consensus-reaching process (CRP). Traditional CRP methods primarily adopt cardinal or ordinal approaches, respectively gauging consensus by examining numerical discrepancies among experts' assessments[2,5,14] or by comparing the different ranking orders of alternatives provided by each expert.[15,16] In both cases, various strategies, such as minimum adjustment[17,19] and minimum cost adjustment,[18] are implemented to reconcile conflicting evaluations. Additionally, some models incorporate social network analysis to leverage trust relationships among DMs,[2,5,9] while others consider experts' self-confidence.[10,20] However, these cardinal or ordinal models do not sufficiently address the growing number of classification-oriented decision problems, where alternatives are categorized into specific classes without establishing a complete ranking. Real-world illustrations include scholarship evaluations (e.g., classifying students into first, second, or third prize), postgraduate admission decisions ("admit" versus "not admit"), and course assessments involving categories like excellent, good, satisfactory or unsatisfactory. In such instances, the focal question is how to secure a classification result that most participants endorse, despite potential discrepancies in knowledge, expertise, and cognitive biases.[21] Recent research suggests that classification-based consensus is practically valuable yet still underexplored. A number of studies have begun to focus on classification-based approaches in GDM, including the ordinal classification consensus framework for recruitment,[22] consensus-based classification methods within a social network,[23] and heterogeneous preference modeling for supplier assessment.[24] Additionally, advanced techniques like linguistic distribution assessments and quantum probability theory have been introduced to capture contradictory or uncertain expert behavior, aiming to yield more realistic classification outcomes.[25] Nevertheless, a robust CRP dedicated explicitly to classification remains insufficiently covered in the literature, highlighting the timeliness and importance of developing a comprehensive classification-oriented approach.

Preference elicitation is the premise of GDM. Traditional preference schemes include utility value,[26] fuzzy information,[27,28] linguistic-based assessment,[29–31] either in direct or pairwise comparison formats. Linguistic-based methods are particularly effective as they allow individuals to articulate their perspectives intuitively. Distribution linguistic preference relation (DLPR),[29] an efficient pairwise comparison framework, employs linguistic term sets to capture uncertainties. Building on this, distributed preference relations (DPR)[32] and probabilistic linguistic preference relations (PLPR)[30] were introduced. DPR is developed from the evidential reasoning (ER) approach, and it has the advantage of characterizing preferred, nonpreferred, uncertain and indifferent degrees on the comparison of alternatives with linguistic evaluation grades simultaneously. Consistency measure and adjustment are the key issues in the research of the pairwise comparison-based decision-making problem.[33–36] While obtaining DPRs from experts is a fundamental step, challenges emerge in cases of incomplete information. For example, multi-stage consistency optimization algorithms have been proposed to address incomplete probabilistic linguistic preference relations (InPLPRs) by estimating missing data and refining consistency.[37] These algorithms employ mathematical optimization to minimize information distortion while achieving acceptable consistency thresholds. Nevertheless, the elicitation of DPRs from experts is fundamental in GDM, especially when multiple attributes should be considered jointly. In Ref. 32, each expert provides an accurate evaluation grade on the comparison of two alternatives, and then the frequency of evaluation grade is assessed by all experts is assumed to be the belief degree. Attributes and consensus are not considered. In Ref. 9, DPRs are derived from the attribute level on each cluster instead of the whole group. It is also computed by the frequency of experts' assessments of an evaluation grade in the comparison of two alternatives on an attribute. However, in real decision-making problems, uncertain circumstances and unpredictable future states are common. The probabilities of future states can either be estimated or not, which is determined by the knowledge of the individual and the complexity of the decision scenario. Thus, it is important to study how to acquire the DPRs on attributes in the consideration of unexpected uncertainty.

When the DPRs on all attributes are generated, how to combine the distributions on multiple attributes and experts is a key issue. Traditional fusion rules for aggregating distributions under uncertainty include Dempster's combination rule,[38] Yager's rule,[39] Dubois and Prade's rule,[40] PCR5 rule,[41] and the ER approach.[42] In recent years, some other combination rules have been developed in the evidence structure.[43,44] Each of the above rules has its advantages and disadvantages. The ER approach is a probabilistic reasoning rule, where the weight of evidence is treated before the orthogonal sum is implemented. However, when it is used in MAGDM, some irrational results may occur. First, if the original distributions on all attributes or by all the experts are identical, the fused distribution may be different with the original DPRs. Second, when the distributions of each piece of evidence are the same, the combined distribution will change with the change of evidence weight. Third, the aggregated distribution may be different with the number of evidence grows even though all pieces of evidence are identical. Hence, it is critical to provide a new evidence fusion rule for MAGDM to deal with the above-mentioned problems.

Given these gaps, we derive the following research questions:

(RQ1:) How can we systematically generate DPRs under unpredictable or partially known conditions so that experts are not overburdened with complex computations?
(RQ2:) Which fusion mechanism can reliably combine multiple-attribute DPRs while circumventing irrational results generated by the ER approach?
(RQ3:) How can we redefine consensus for classification-oriented group decision problems and develop an effective CRP that accounts for social network relations (e.g., trust) and opinion leaders?

Addressing these questions allows us to frame the main contributions of this paper:

(1) The generation method of DPRs on attribute considering both certain and unpredictable situations is provided. It tackles the problem of deriving original assessments when there are multiple uncertain states. It also simplifies the decision-making process by unburdening DMs from processing too much information.
(2) Weighted average evidential fusion rule is proposed for the aggregation of independent attribute assessments in the form of DPRs. It is proven theoretically that the fusion rule satisfies the basic properties of a combination algorithm. The fusion rule also resolves some irrational results generated by the ER approach.
(3) CRP for the classification-oriented GDM is presented. The consensus measure based on category is first defined. Then, consensus identification and adjustment rules are given, where opinion leader and social network analysis are both considered.

The remainder of this paper is organized as follows. Section 2 is a brief introduction about preference relation, social network analysis and ER approach. In Sec. 3, the generation of DPRs based on multiple attributes and unpredictable uncertainty is discussed. Then, the weighted average evidential fusion rule is proposed. Section 4 presents the CRP in the situation of the classification-oriented problem. In Sec. 5, a case study of scholarship evaluation is given to demonstrate the effectiveness of the proposed method. This paper is concluded in Sec. 6.

2. Preliminaries

In this section, some basic concepts are reviewed. Linguistic-based preference relations are first presented. Then, social network analysis and the ER approach are briefly introduced.

2.1. *Preference relation*

In a GDM problem, let $E = \{e_k | k = 1, \ldots, K\}$ be the set of experts, $X = \{x_i | i = 1, \ldots, M\}(M \geq 2)$ are M alternatives to be classified considering L attributes $A = \{a_l | l = 1, \ldots, L\}$. The set of evaluation grades to be used is $H = \{H_n | n = 1, \ldots, N\}$. The relative importance of K experts is signified by $R = \{R_k | k = 1, \ldots, K\}$ which satisfies $0 \leq R_k \leq 1$ $(k = 1, \ldots, K)$ and $\sum_{k=1}^{K} R_k = 1$. The weights of L attributes are denoted by $W = \{w_l | l = 1, \ldots, L\}$, and $0 \leq w_l \leq 1(l = 1, \ldots, L)$, $\sum_{l=1}^{L} w_l = 1$. The number of alternatives is relatively large in the classification-oriented GDM problem.

Definition 1 (Distribution linguistic preference relation (Ref. 29)). Suppose $X = \{x_1, x_2, \ldots, x_M\}$ is the set of alternatives, X is pairwise compared by using the linguistic term set $L = \{l_0, l_1, \ldots, l_g\}$ with odd cardinality. $l_t(t = 0, 1, \ldots, g)$ indicates a possible value for a linguistic variable. $m_{ij} = \{(l_t, \rho_{t,ij}), t = 0, 1, \ldots, g\}$ denotes a linguistic distribution assessment of L on the comparison of x_i over $x_j(i, j = 1, \ldots, M)$, where $\rho_{t,ij}$ signifies the symbolic proportion of l_t with $\rho_{t,ij} \geq 0$ and $\sum_{t=0}^{g} \rho_{t,ij} = 1$. The distribution linguistic preference relation (DLPR) is then represented by $M = (m_{ij})_{M \times M}$.

DLPR uses distribution assessments to construct linguistic-based pairwise comparisons. Afterwards, some concepts such as DPR[32,45] and PLPR[30] are introduced.

Definition 2 (Distributed preference relation (Ref. 32)). Suppose alternative set $X = \{x_1, x_2, \ldots, x_M\}$ are pairwise compared with evaluation grades $H = \{H_1, H_2, \ldots, H_N\}$, where N is an odd number. H_1 indicates absolutely inferior, while H_N signifies absolutely superior. Specifically, $H_{(N+1)/2}$ represents the degree of indifference. The inferior degree decreases from H_1 to $H_{(N-1)/2}$, and the superior degree increases from $H_{(N+3)/2}$ to H_N. Then the DPR matrix given by the kth expert $e_k(k = 1, 2, \ldots, K)$ is defined as $D^k \subset X \times X, D^k = (d_{ij}^k)_{M \times M}$, where

$$d_{ij}^k = \{(H_n, d_{ij}^k(H_n)),\ n = 1, \ldots, N; (H, d_{ij}^k(H))\}. \tag{1}$$

$d_{ij}^k(H_n)$ and $d_{ij}^k(H)$ $(i, j = 1, 2, \ldots, M)$ denote the belief degree that alternative x_i is preferred or nonpreferred to x_j on grade H_n and ignorance by e_k, respectively. $d_{ij}^k(H_n) \geq 0$, $d_{ij}^k(H) = d_{ji}^k(H) = 1 - \sum_{n=1}^{N} d_{ij}^k(H_n)$. d_{ij}^k is said to be incomplete if $d_{ij}^k(H) > 0$ or $\sum_{n=1}^{N} d_{ij}^k(H_n) < 1$. If $d_{ij}^k(H_n) > 0$, H_n is called a focal element. If $d_{ij}^k(H) = 0$, it is a complete assessment.

The style of DPR is similar with DLPR. As for DPR, the fusion of different distributions is conducted by the ER approach. To reduce individuals' burden of assessment information expression, DMs are required to give comparisons between adjacent alternatives $d_{i,i+1}^k(i = 1, \ldots, M - 1)$ instead of any pair of alternatives. DPRs between adjacent alternatives should then be transformed into its corresponding score values to induce the consistency values of nonadjacent pairs.

Definition 3 (Score value (Ref. 45)). Let $s(H_n)$ be the score value of evaluation grade $H_n(n = 1, \ldots, N)$ such that $s(H_1) = -1$, $s(H_{\frac{N+1}{2}}) = 0$ and $s(H_N) = 1$. $s(H_n) < s(H_{n+1})(n = 1, \cdots, N-1)$ and $s(H_n) = -s(H_{N-n+1})(n = 1, \ldots, \frac{N-1}{2})$ are to ensure $s(H_n)$ a nondecreasing and symmetric function. The score value transformed from d_{ij}^k is an interval number denoted as $S_{ij}^k = [S_{ij}^{k-}, S_{ij}^{k+}]$ because $d_{ij}^k(H)$ can be assigned to any grade, where

$$S_{ij}^{k-} = \sum_{n=1}^{N} d_{ij}^k(H_n) \cdot s(H_n) + d_{ij}^k(H) \cdot s(H_1), \tag{2}$$

$$S_{ij}^{k+} = \sum_{n=1}^{N} d_{ij}^k(H_n) \cdot s(H_n) + d_{ij}^k(H) \cdot s(H_N). \tag{3}$$

Then the DPR matrix $D^k = (d_{ij}^k)_{M\times M}$ can be converted into a score matrix denoted by $S = (S_{ij})_{M\times M}$, and $S_{ij}^{k-} + S_{ji}^{k+} = 0, S_{ij}^{k+} + S_{ji}^{k-} = 0$, $\forall\, i, j \in \{1, \ldots, M\}$.

Definition 4 (Consistency of DPR (Ref. 32)). Suppose $f : [-1, 1] \times [-1, 1] \to [-1, 1]$ is a function to generate the score value of nonadjacent alternative pair $S_{ih}^k(|i - h| > 1)$, S is defined to be consistent if the following two conditions are satisfied:

$$f(S_{ij}^{k-}, S_{jh}^{k-}) + S_{hi}^{k+} = 0, \quad \forall\, i, j, k \in \{1, \ldots, M\}, \tag{4}$$

$$f(S_{ij}^{k+}, S_{jh}^{k+}) + S_{hi}^{k-} = 0, \quad \forall\, i, j, k \in \{1, \ldots, M\}, \tag{5}$$

where f signifies any function that satisfies Eqs. (4) and (5), which can be applied to construct a consistent score matrix. For instance, one can use the following function $g(y, z) = \frac{y+z-(1+b)\cdot yz}{1-b\cdot yz}$ where b is a parameter usually set within the interval $(-\infty, 1)$. The specific process for determining an appropriate value of b can be found in Ref. 32.

2.2. *Social network analysis*

Social network analysis (SNA) is the main method to investigate interactive relationships among individuals in the GDM problem. The relationships can be decomposed into several dimensions such as economic, cooperative, actional, and so on. The importance of the individual, the prestige, the conflict of interest and group consensus are all influenced by social relationships, including trust relations and contact frequency.

In SNA, three elements are usually involved: (1) set of individuals, (2) the relationship between pairs of individuals, and (3) the individual criteria.[54] It can be represented by three notational schemes as follows. (a) Graph theoretic: it is depicted by a directed graph, where nodes denote individuals and arcs signify directed relationships between individuals. (b) Sociometric matrix: it is applied for representing social network provided that the relationships between pairs of individuals are bidirectional. (c) Algebraic: it uses the representation $e_k R e_h$ to show relationships, which is more flexible for analysis.

Definition 5 (Trust & distrust relationship (Ref. 46)). Suppose $E = \{e_k | k = 1, \ldots, K\}$ is the set of experts in social network. t_{kh} and d_{kh} indicate trust and distrust degree from e_k to e_h. Then the two-tuple trust/distrust relationship from e_k to e_h $(k, h \in \{1, 2, \ldots, K\}, k \neq h)$ is denoted as:

$$\lambda_{kh} = (t_{kh}, d_{kh}), \tag{6}$$

where $0 \leq t_{kh} \leq 1$, $0 \leq d_{kh} \leq 1$ and $0 \leq t_{kh} + d_{kh} \leq 1$.

For the convenience of CRP, Definition 5 is further transformed into a trust score in the form of a numerical number:

$$ts_{kh} = \frac{t_{kh} - d_{kh} + 1}{2}, \tag{7}$$

where $ts_{kh} \in [0, 1]$. Trust score matrix $\mathrm{TS} = [ts_{kh}]_{K \times K}$ is then generated from trust relationship matrix $\mathrm{TR} = [\lambda_{kh}]_{K \times K}$.

2.3. *Evidential reasoning approach*

The ER approach consists of the recursive[42] and analytical[47] algorithms to fuse the assessments on multiple sources. It is then developed to the ER rule[48] to cope with more complex decision situations where both weight and reliability of evidence are considered. Compared with other evidence combination rules,[40–43] the ER rule is a probabilistic information fusion algorithm that satisfies basic axioms. The analytical ER algorithm to combine the DPRs from K experts $d_{ij}^k (k = 1, 2, \ldots, K)$ is presented as follows:

$$m_{ij}^k(H_n) = R_k \cdot d_{ij}^k(H_n), \tag{8}$$

$$\bar{m}_{ij}^k(H) = 1 - R_k, \quad \tilde{m}_{ij}^k(H) = R_k \cdot d_{ij}^k(H), \tag{9}$$

$$m_{ij}^k(H) = \bar{m}_{ij}^k(H) + \tilde{m}_{ij}^k(H), \tag{10}$$

$$m_{ij}(H_n) = \eta \left[\prod_{k=1}^{K} (m_{ij}^k(H_n) + \tilde{m}_{ij}^k(H) + \bar{m}_{ij}^k(H)) - \prod_{k=1}^{K} (\tilde{m}_{ij}^k(H) + \bar{m}_{ij}^k(H)) \right], \tag{11}$$

$$\tilde{m}_{ij}(H) = \eta \left[\prod_{k=1}^{K} (\tilde{m}_{ij}^k(H) + \bar{m}_{ij}^k(H)) - \prod_{k=1}^{K} \bar{m}_{ij}^k(H) \right], \tag{12}$$

$$\bar{m}_{ij}(H) = \eta\left[\prod_{k=1}^{K} \bar{m}_{ij}^{k}(H)\right], \tag{13}$$

$$\eta = \left[\sum_{n=1}^{N}\prod_{k=1}^{K}(m_{ij}^{k}(H_n) + \tilde{m}_{ij}^{k}(H) + \bar{m}_{ij}^{k}(H)) - (N-1)\prod_{k=1}^{K}(\tilde{m}_{ij}^{k}(H) + \bar{m}_{ij}^{k}(H))\right]^{-1}, \tag{14}$$

$$d_{ij}(H_n) = \frac{m_{ij}(H_n)}{1-\bar{m}_{ij}(H)}, \quad d_{ij}(H) = \frac{\tilde{m}_{ij}(H)}{1-\bar{m}_{ij}(H)}, \quad (k = 1,\ldots,K;\ i,j = 1,\ldots,M), \tag{15}$$

$m_{ij}^{k}(H_n)$ is the basic probability assignment (BPA) of x_{ij} being assessed to H_n by e_k, $m_{ij}^{k}(H)$ indicates the remaining BPA unallocated to any evaluation grade after all the N grades have been distributed. $m_{ij}(H_n)$ denotes the aggregated BPA of x_{ij} to H_n by all the K experts. $m_{ij}^{k}(H)$ is decomposed into two segments. $\bar{m}_{ij}^{k}(H)$ is derived by the negation of expert e_k's weight, while $\tilde{m}_{ij}^{k}(H)$ is determined by the global ignorance of d_{ij}^{k}.

$d_{ij}(H_n)$ signifies the fused belief degree of x_{ij} be assessed to H_n, and $d_{ij}(H)$ denotes the aggregated belief degree of global ignorance. $\sum_{n=1}^{N} d_{ij}(H_n) + d_{ij}(H) = 1$. After the fusion of K DPRs, the distributed assessment of x_{ij} on the general level is represented below:

$$d_{ij} = \{(H_n, d_{ij}(H_n)),\ n = 1,\ldots,N; (H, d_{ij}(H))\}. \tag{16}$$

Compared with the recursive algorithm, the analytical algorithm only needs to be conducted once when combining multiple pieces of evidence. In a multiple attribute decision-making problem, each evidence is assumed to be a piece of evidence. In the GDM problem, the assessment of an expert is considered as a piece of evidence. Each piece of evidence is required to be independent of others before the fusion procedure.

3. DPRs Based on Multiple Attributes and Unpredictable Uncertainty

In this section, the generation of DPRs on attributes considering unpredictable situations is first discussed. Then, a new weighted average evidential fusion rule is proposed to combine multiple pieces of evidence for MAGDM.

3.1. *Collective DPR with no uncertainty*

When an individual compares two alternatives on an attribute, he/she may use linguistic evaluation grades such as ‘better than,’ ‘worse than,’ ‘extremely superior to,' and so on. If the assessment is certain, a sole evaluation grade can be used. If there exists some uncertain matter, more than one evaluation grade should be used to capture all the situations in the future. The question is how to combine these assessments on different attributes into a collective one.

Suppose alternative $x_m (m = 1, \ldots, M)$ is evaluated on L attributes $a_l (l = 1, \ldots, L)$ by expert e_k. w_l is the relative weight of a_l and $\sum_{l=1}^{L} w_l = 1$. N discrete and exhaustive evaluation grades $H_n (n = 1, \ldots, N)$ are used to compare pairs of alternatives. A definite or certain assessment is denoted by $d_{ij}^{l,k} = H_n$, which implies that the preference between x_i and x_j on a_l by e_k is H_n. Table 1 shows the comparison of x_{ij} on L attributes with no uncertainty.

Definition 6 (Collective DPR based on multiple attributes). Suppose the comparison between x_i and x_j on any attribute a_l by e_k is a certain assessment such that

$$d_{ij}^{l,k} = H_{ij}^{l,k} = H_n = \{(H_n, 1)\} \quad (l = 1, \ldots, L;\ n = 1, \ldots, N). \tag{17}$$

$|H_{ij}^{l,k} | H_{ij}^{l,k} = H_n|$ denotes the number of attributes been assessed to H_n when comparing x_i and x_j by e_k. Then the collective DPR on the comparison of x_i over x_j can be denoted by Eq. (1), where

$$d_{ij}^{k}(H_n) = \frac{|H_{ij}^{l,k} | H_{ij}^{l,k} = H_n|}{L}. \tag{18}$$

It is obvious that $\sum_{n=1}^{N} |H_{ij}^{l,k} | H_{ij}^{l,k} = H_n| = L$ if all the L attributes are assessed definitely. So we have $d_{ij}^{k} = \left\{ \left(H_n, \frac{|H_{ij}^{l,k} | H_{ij}^{l,k} = H_n|}{L} \right), n = 1, \ldots, N \right\}$. The belief degree in this case is calculated by the frequency of the assessment. Nevertheless, it is generated on the assumption that the weights of attributes are evenly allocated such that $w_l = 1/L$. Let $\{a_l | d_{ij}^{l,k} = H_n;\ l = 1, \ldots, L\}$ be the set of attributes that e_k evaluates to H_n on the comparison of x_i over x_j. If the condition of $w_l = 1/L$ is not satisfied, we will have the following:

$$d_{ij}^{k}(H_n) = \sum_{\substack{l \in \{1, \ldots, L\} \\ d_{ij}^{l,k} = H_n}} w_l. \tag{19}$$

Obviously, Eq. (18) is a special case of Eq. (19).

Table 1. Comparison of x_{ij} on multiple attributes in certain circumstance.

Weight		w_1	...	w_l	...	w_L
Attribute		a_1	...	a_l	...	a_L
Evaluation grade	e_k on x_{ij}	$H_{ij}^{1,k}$	...	$H_{ij}^{l,k}$	...	$H_{ij}^{L,k}$
DPR on attribute level		$d_{ij}^{1,k}$	...	$d_{ij}^{l,k}$	...	$d_{ij}^{L,k}$

If e_k does not provide any information on the attribute a_l, then $d_{ij}^{l,k} = H$. So the DPR generated in this situation will be incomplete. Suppose $\{a_l | d_{ij}^{l,k} = H;\ l = 1, \dots, L\}$ is the set of attributes that e_k does not give any assessment on the comparison between x_i over x_j. Then the total ignorance is calculated as:

$$d_{ij}^k(H) = \sum_{\substack{l \in \{1,\dots,L\} \\ d_{ij}^{l,k} = H}} w_l. \tag{20}$$

Example 1. Suppose five attributes $a_l(l = 1, \dots, 5)$ are involved in comparing x_i over x_j by e^k. The weights of attributes are $W = \{0.2, 0.3, 0.2, 0.15, 0.15\}$. The assessments on the five attributes are shown in Table 2. If the weights of five attributes are set equally, the DPR is $d_{ij}^k = \{H_2, 0.4;\ H_3, 0.2;\ H_5, 0.2;\ H, 0.2\}$.

Different experts may assign different attribute weights in a decision-making problem. However, in a GDM problem, the attribute weights may be pre-determined by the moderator in advance.

3.2. *Generation of DPR on attributes considering unpredictable situations*

The assessment of alternatives on an attribute may be uncertain because the future is unpredictable. For example, in a scholarship evaluation problem, two students are compared on 'the publication of paper' (attribute a_l). Student A has submitted a manuscript to an outstanding journal, and the status is major revision and under second-round review. Student B has published a paper in an ordinary journal that has attained the graduate standard. How should we judge these two students when considering the publication of paper? Since the manuscript submitted by student A may be either accepted (state s_1^l) or rejected (state s_2^l), the judgment is not a unique evaluation grade anymore. If it is accepted, student A will be superior to student B on a_l; otherwise, student A will be inferior to student B on a_l if it is rejected. Instead, at least two evaluation grades should be assigned to this attribute with different probabilities.

Suppose the set of state for attribute a_l is $S^l = \{s_1^l, s_2^l, \dots, s_{nl}^l\}$, where n_l is the number of states for assessing a_l. Table 3 shows the unpredictable situations for multiple-attribute comparison.

Table 2. The assessments on the five attributes to compare x_i over x_j.

Weight	0.2	0.3	0.2	0.15	0.15
Attribute	a_1	a_2	a_3	a_4	a_5
Assessment ($d_{ij,l}^k$)	H_2	H_2	H_5	H_3	None
DPR	$d_{ij}^k = \{H_2, 0.5;\ H_3, 0.15;\ H_5, 0.2;\ H, 0.15\}$				

Table 3. Unpredictable situations for multiple-attribute comparison of x_i over x_j.

Attribute		a_1				$\dots$	a_l				$\dots$	a_L			
State		s_1^1	s_2^1		$s_{n_1}^1$		s_1^l	s_2^l	$\dots$	$s_{n_l}^l$		s_1^L	s_2^L	$\dots$	$s_{n_L}^L$
Probability	e_k on x_{ij}	$p_1^{1,k}$	$p_2^{1,k}$	$\dots$	$p_{n_1}^{1,k}$	$\dots$	$p_1^{l,k}$	$p_2^{l,k}$	$\dots$	$p_{n_l}^{l,k}$	$\dots$	$p_1^{L,k}$	$p_2^{L,k}$	$\dots$	$p_{n_L}^{L,k}$
Evaluation grade		$H_{1,ij}^{1,k}$	$H_{2,ij}^{1,k}$	$\dots$	$H_{n_1,ij}^{1,k}$		$H_{1,ij}^{l,k}$	$H_{2,ij}^{l,k}$	$\dots$	$H_{n_l,ij}^{l,k}$		$H_{1,ij}^{L,k}$	$H_{2,ij}^{L,k}$	$\dots$	$H_{n_L,ij}^{L,k}$
DPR on attribute level		$d_{ij}^{1,k}$				$\dots$	$d_{ij}^{l,k}$				$\dots$	$d_{ij}^{L,k}$			

$p_t^{l,k}(t = 1, \ldots, n_l)$ indicates the probability that expert e_k assumes s_t^l may occur. Obviously, $\sum_{t=1}^{n_l} p_t^{l,k} = 1$. The worst situation is that there is not any information for e_k to estimate $p_t^{l,k}$ on a_l. In this case, the risk decision-making degenerates to an uncertain decision-making problem. $H_{t,ij}^{l,k} \in \{H_n | n = 1, \ldots, N\}$ denotes the preference degree of x_i over x_j on a_l given by e_k if s_t^l occurs in the future. As mentioned above, not all the attributes have uncertain states. Because there are discrepancies among the risk attitudes of experts, the probability $p_t^{l,k}$ for the same attribute by different experts may be divergent.

Definition 7 (DPR on attribute considering uncertain states). Suppose the comparison of x_i over x_j on a_l in state $s_t^l(t \in \{1, \ldots, n_l\})$ by e_k is $H_{t,ij}^{l,k} = H_n$. Then the belief degree of x_{ij} on a_l to H_n by e_k denoted as $d_{ij}^{l,k}(H_n)$ is computed as

$$d_{ij}^{l,k}(H_n) = \sum_{t \in \{1,\ldots,n_l\}} \{p_t^{l,k} | H_{t,ij}^{l,k} = H_n\}. \tag{21}$$

Similarly, the belief degree of total ignorance is computed as:

$$d_{ij}^{l,k}(H) = \sum_{t \in \{1,\ldots,n_l\}} \{p_t^{l,k} | H_{t,ij}^{l,k} = H\}. \tag{22}$$

Then the DPR on a_l when comparing x_i over x_j by e_k is generated as:

$$d_{ij}^{l,k} = \{(H_n, d_{ij}^{l,k}(H_n)),\ n = 1, \ldots, N; (H, d_{ij}^{l,k}(H))\}. \tag{23}$$

Equation (17) is a special case of Eq. (23) where $d_{ij}^{l,k}(H_n) = 1$ and $\forall m = 1, \ldots, N, m \neq n, d_{ij}^{l,k}(H_m) = 0$. If $H_{t,ij}^{l,k} = H$, e_k does not give any assessment in the state of s_t^l.

Theorem 1. *If $d_{ij}^{l,k}(H_n)$ and $d_{ij}^{l,k}(H)$ are generated by Definition 7, then we have $0 \leq d_{ij}^{l,k}(H_n) \leq 1$, $0 \leq d_{ij}^{l,k}(H) \leq 1$.*

Proof of Theorem 1.

From Definition 7, $d_{ij}^{l,k}(H_n) = \sum_{t \in \{1,\ldots,n_l\}} \{p_t^{l,k} | H_{t,ij}^{l,k} = H_n\}$.

Since $\sum_{t=1}^{n_l} p_t^{l,k} = 1$,

$$\max d_{ij}^{l,k}(H_n) = 1 \quad \text{iff } \forall\, t = 1, \ldots, n_l, H_{t,ij}^{l,k} = H_n,$$

$$\min d_{ij}^{l,k}(H_n) = 0 \quad \text{iff } \forall\, t = 1, \ldots, n_l, H_{t,ij}^{l,k} \neq H_n.$$

Similarly

$$\max d_{ij,l}^{k}(H) = 1 \quad \text{iff } \forall\, t = 1, \ldots, n_l, H_{t,ij}^{l,k} = H$$

$$\min d_{ij,l}^{k}(H) = 0 \quad \text{iff } \forall\, t = 1, \ldots, n_l, H_{t,ij}^{l,k} \neq H. \qquad \square$$

Theorem 2. *If $d_{ij}^{l,k}(H_n)$ and $d_{ij}^{l,k}(H)$ are generated by Definition 7, then we have $\sum_{n=1}^{N} d_{ij}^{l,k}(H_n) + d_{ij}^{l,k}(H) = 1$.*

Proof of Theorem 2.

From Definition 7, $d_{ij}^{l,k}(H_n) = \sum_{t \in \{1,\ldots,n_l\}} \{p_t^{l,k} | H_{t,ij}^{l,k} = H_n\}$.

Since $\sum_{t=1}^{n_l} p_t^{l,k} = 1$,

$$\begin{gathered} H_{1,ij}^{l,k} \in \{H_1, H_2, \ldots, H_N, H\}, \\ H_{2,ij}^{l,k} \in \{H_1, H_2, \ldots, H_N, H\}, \\ ,\ldots, \\ H_{n_l,ij}^{l,k} \in \{H_1, H_2, \ldots, H_N, H\}. \end{gathered}$$

So $\sum_{n=1}^{N} d_{ij}^{l,k}(H_n) + d_{ij}^{l,k}(H) = 1$. □

Remark 1. If there is not any state been assessed to H, then the generated DPR on attribute a_l is complete, i.e., $d_{ij}^{l,k} = \{(H_n, d_{ij}^{l,k}(H_n)), n = 1, \ldots, N\}$ and $\sum_{n=1}^{N} d_{ij}^{l,k}(H_n) = 1$. Otherwise, it is incomplete provided that at least one state is assessed to H, i.e., $d_{ij}^{l,k}(H) > 0$.

Example 2. Two students i and j are compared for the scholarship. The performances of their achievements are provided. One of the achievements is 'the publication of paper'. Student i's manuscript is under review in a well-known journal, while student $j's$ manuscript has been accepted in an ordinary journal. So there are two states for student $i's$ manuscript: accept and reject. Table 4 shows the probabilities and evaluation grades of two states provided by e_k and $e_{k'}$. The evaluation grades are set as $H = \{H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8, H_9\} = \{$*absolutely worse, worse, moderately worse, slightly worse, indifferent, slightly better, moderately better, better, absolutely better*$\}$. The score values of evaluation grades are set as $S = \{s(H_1), s(H_2), s(H_3), s(H_4), s(H_5), s(H_6), s(H_7), s(H_8), s(H_9)\} = \{-1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 1\}$.

It can be seen that $e_{k'}$ is more positive about the acceptance of the manuscript by student i than e_k, i.e., $p_1^{l,k'} > p_1^{l,k}$. $e_{k'}$ thinks student i is better than (H_8)

Algorithm 1. Generation of DPR on attribute considering uncertain states.

Input: alternative x_i and x_j, attribute a_l, evaluation grades
$H = \{H_n | n = 1, \ldots, N\}$

Output: DPR of x_{ij} on a_l by e_k denoted as $d_{ij}^{l,k}$

Step 1: Expert e_k provides the set of states on a_l denoted as $\{s_1^l, s_2^l, \ldots, s_{n_l}^l\}$

Step 2: e_k estimates the probability of $s_t^l (t = 1, \ldots, n_l)$ denoted as $p_t^{l,k}$ based on risk attitude

Step 3: Calculate the belief degree of $d_{ij}^{l,k}(H_n)$ by Eq. (21)

Step 4: Calculate the belief degree of $d_{ij}^{l,k}(H)$ by Eq. (22)

Step 5: Generate the DPR of x_{ij} on a_l from e_k by Eq. (23).

Table 4. The probabilities and evaluation grades of states provided by two experts.

Attribute		a_l(Publication of paper)		DPR
State		s_1^l (Accept)	s_2^l (Reject)	
Probability	e_k on x_{ij}	$p_1^{l,k} = 0.2$	$p_2^{l,k} = 0.8$	$d_{ij}^{l,k} = \{H_2, 0.8; H_7, 0.2\}$
Assessment		$H_{1,ij}^{l,k} = H_7$	$H_{2,ij}^{l,k} = H_2$	
Probability	$e_{k'}$ on x_{ij}	$p_1^{l,k'} = 0.6$	$p_2^{l,k'} = 0.4$	$d_{ij}^{l,k'} = \{H_3, 0.4; H_8, 0.6\}$
Assessment		$H_{1,ij}^{l,k'} = H_8$	$H_{2,ij}^{l,k'} = H_3$	

student j if student $i's$ manuscript is accepted. Otherwise, student i is moderately worse (H_3) than student j provided that it is rejected. Comparatively, e_k is more pessimistic and gives H_7 (moderately better) on s_1^l (Accept) and H_2 (worse) on s_2^l (Reject) on the comparison of student i over j. Based on Algorithm 1, we have $d_{ij}^{l,k} = \{H_2, 0.8;\ H_7, 0.2\}$, $d_{ij}^{l,k'} = \{H_3, 0.4;\ H_8, 0.6\}$.

3.3. *Generation of collective DPR under multiple attributes*

When the DPR on each attribute is generated based on unpredictable situations, the next task is to aggregate them to a general distribution. Next, we will propose the weighted average evidential fusion rule for MAGDM.

3.3.1. *The weighted average evidential fusion rule*

Suppose the DPR of alternative x_i over x_j on attribute a_l is represented by $d_{ij}^{l,k} = \{(H_1, d_{ij}^{l,k}(H_1)), \ldots, (H_N, d_{ij}^{l,k}(H_N)); (H, d_{ij}^{l,k}(H))\}$. $0 \le d_{ij}^{l,k}(H_n) \le 1$, $0 \le d_{ij}^{l,k}(H) \le 1$, $\sum_{n=1}^{N} d_{ij}^{l,k}(H_n) + d_{ij}^{l,k}(H) = 1$. Table 5 shows the DPRs of L attributes by DM e_k. The DPRs on L attributes should be combined into a comprehensive one represented as d_{ij}^k.

The ER algorithm is an orthogonal sum approach to fuse multiple pieces of evidence. Here, each attribute is assumed to be a piece of evidence. The combination of $d_{ij}^{l,k}$ by the ER algorithm is similar with the process in Sec. 2.3.

Definition 8 (Weighted average evidential fusion rule). Suppose the DPR of x_{ij} on attribute a_l is $d_{ij}^{l,k} = \{(H_n, d_{ij}^{l,k}(H_n)), n = 1, \ldots, N; (H, d_{ij}^{l,k}(H))\}$. Then the collective DPR of x_{ij} been assessed to H_n and total ignorance denoted by $d_{ij}^k(H_n)$ and $d_{ij}^k(H)$ can be generated as

$$d_{ij}^k(H_n) = \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H_n), \tag{24}$$

$$d_{ij}^k(H) = \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H). \tag{25}$$

Table 5. Comparison of x_{ij} on multiple attributes in uncertain circumstances.

Weight Attribute		w_1 a_1	... ...	w_l a_l	... ...	w_L a_L
Evaluation grade	e_k on x_{ij}	$(H_1, d_{ij}^{1,k}(H_1))$		$(H_1, d_{ij}^{l,k}(H_1))$		$(H_1, d_{ij}^{L,k}(H_1))$
		...	...	...	...	...
		$(H_N, d_{ij}^{1,k}(H_N))$		$(H_N, d_{ij}^{l,k}(H_N))$		$(H_N, d_{ij}^{L,k}(H_N))$
		$;(H, d_{ij}^{1,k}(H))$		$;(H, d_{ij}^{l,k}(H))$		$;(H, d_{ij}^{L,k}(H))$
DPR on attribute level		$d_{ij}^{1,k}$	...	$d_{ij}^{l,k}$	...	$d_{ij}^{L,k}$
DPR on alternative pair level		$d_{ij}^k = \{(H_n, d_{ij}^k(H_n)), n = 1, \ldots, N; (H, d_{ij}^k(H))\}$				

Then we have $d_{ij}^k = d_{ij}^{1,k} \oplus \cdots \oplus d_{ij}^{l,k} \oplus \cdots \oplus d_{ij}^{L,k} = \{(H_n, d_{ij}^k(H_n)), n = 1, \ldots, N; (H, d_{ij}^k(H))\}$. If the assessments of x_{ij} on all L attributes are crisp evaluation grades, Eqs. (24) and (25) degenerate to Eqs. (19) and (20).

Definition 8 is the weighted average of DPR on all attributes, it is an algorithm of partial compensation. If an evaluation grade is supported by an attribute, it will be proportionally supported in the fused DPR. If w_l is not standardized such that $\sum_{l=1}^{L} w_l \neq 1$, it should be normalized as:

$$w'_l = \frac{w_l}{\sum_{l=1}^{L} w_l}(l = 1, 2, \ldots, L). \tag{26}$$

w_l in Definition 8 is then replaced by w'_l.

Theorem 3. *If the collective DPR of x_{ij} considering multiple attributes are calculated by Eqs.* (24) *and* (25), *and $\sum_{l=1}^{L} w_l = 1$, then $\sum_{n=1}^{N} d_{ij}^k(H_n) + d_{ij}^k(H) = 1$.*

Proof of Theorem 3.

$$\begin{aligned}
\sum_{n=1}^{N} d_{ij}^k(H_n) + d_{ij}^k(H) &= \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H_1) + \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H_2) \\
&\quad + \cdots + \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H_N) + \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H) \\
&= \left(\sum_{n=1}^{N} w_1 \cdot d_{ij}^{1,k}(H_n) + w_1 \cdot d_{ij}^k(H)\right) \\
&\quad + \left(\sum_{n=1}^{N} w_2 \cdot d_{ij}^{2,k}(H_n) + w_2 \cdot d_{ij}^k(H)\right)
\end{aligned}$$

$$
\begin{aligned}
&+\cdots+\left(\sum_{n=1}^{N} w_L \cdot d_{ij}^{L,k}(H_n) + w_L \cdot d_{ij}^{k}(H)\right)\\
&= w_1\left(\sum_{n=1}^{N} d_{ij}^{1,k}(H_n) + d_{ij}^{k}(H)\right)\\
&\quad + w_2 \cdot \left(\sum_{n=1}^{N} d_{ij}^{2,k}(H_n) + d_{ij}^{k}(H)\right)\\
&\quad + \cdots + w_L \cdot \left(\sum_{n=1}^{N} d_{ij}^{L,k}(H_n) + d_{ij}^{k}(H)\right)
\end{aligned}
$$

Since $\sum_{n=1}^{N} d_{ij}^{l,k}(H_n) + d_{ij}^{k}(H) = 1$ for $l = 1, 2, \ldots, L$.

So we have

$$\sum_{n=1}^{N} d_{ij}^{k}(H_n) + d_{ij}^{k}(H) = w_1 + w_2 + \cdots + w_L = 1. \qquad \square$$

Theorem 4. *The fusion algorithm of multiple DPRs proposed in Definition* 8 *is a linear combination method. It satisfies commutativity and associativity.*

Proof of Theorem 4.

(a) *Commutativity*: $d_{ij}^{1,k} \oplus d_{ij}^{2,k} = d_{ij}^{2,k} \oplus d_{ij}^{1,k}$

Let $d_{ij}^{1,k} \oplus d_{ij}^{2,k} = d_{ij}^{k} = \{(H_n, d_{ij}^{k}(H_n)), n = 1, \ldots, N;\ (H, d_{ij}^{k}(H))\}$,

$$d_{ij}^{2,k} \oplus d_{ij}^{1,k} = d_{ij}^{k'} = \{(H_n, d_{ij}^{k'}(H_n)), n = 1, \ldots, N;\ (H, d_{ij}^{k'}(H))\}.$$

According to Definition 8,

$$
\begin{aligned}
d_{ij}^{k}(H_n) &= w_1 \cdot d_{ij}^{1,k}(H_n) + w_2 \cdot d_{ij}^{2,k}(H_n),\\
d_{ij}^{k}(H) &= w_1 \cdot d_{ij}^{1,k}(H) + w_2 \cdot d_{ij}^{2,k}(H)\\
d_{ij}^{k'}(H_n) &= w_2 \cdot d_{ij}^{2,k}(H_n) + w_1 \cdot d_{ij}^{1,k}(H_n),\\
d_{ij}^{k'}(H) &= w_2 \cdot d_{ij}^{2,k}(H) + w_1 \cdot d_{ij}^{1,k}(H),
\end{aligned}
$$

It is obvious that $d_{ij}^{k}(H_n) = d_{ij}^{k'}(H_n)$, $d_{ij}^{k}(H) = d_{ij}^{k'}(H)$.

So we have $d_{ij}^{1,k} \oplus d_{ij}^{2,k} = d_{ij}^{2,k} \oplus d_{ij}^{1,k}$.

(b) *Associativity*: $(d_{ij}^{1,k} \oplus d_{ij}^{2,k}) \oplus d_{ij}^{3,k} = d_{ij}^{1,k} \oplus (d_{ij}^{2,k} \oplus d_{ij}^{3,k})$

Let

$d_{ij}^{1,k} \oplus d_{ij}^{2,k} = d_{ij}^{(12),k} = \{(H_n, d_{ij}^{(12),k}(H_n)), n = 1, \ldots, N;\ (H, d_{ij}^{(12),k}(H))\}$.

From Definition 8, we have

$$d_{ij}^{(12),k}(H_n) = w_1 \cdot d_{ij}^{1,k}(H_n) + w_2 \cdot d_{ij}^{2,k}(H_n),$$

$$d_{ij}^{(12),k}(H) = w_1 \cdot d_{ij}^{1,k}(H) + w_2 \cdot d_{ij}^{2,k}(H).$$

Let $(d_{ij}^{1,k} \oplus d_{ij}^{2,k}) \oplus d_{ij}^{3,k} = d_{ij}^{(12),k} \oplus d_{ij}^{3,k} = d_{ij}^{(12)3,k}$

$$= \{(H_n, d_{ij}^{(12)3,k}(H_n)), n = 1, \ldots, N;\ (H, d_{ij}^{(12)3,k}(H))\}.$$

From Definition 8, we have

$$\begin{aligned} d_{ij}^{(12)3,k}(H_n) &= d_{ij}^{(12),k}(H_n) + w_3 \cdot d_{ij}^{3,k}(H_n) \\ &= w_1 \cdot d_{ij}^{1,k}(H_n) + w_2 \cdot d_{ij}^{2,k}(H_n) + w_3 \cdot d_{ij}^{3,k}(H_n), \\ d_{ij}^{(12)3,k}(H) &= d_{ij}^{(12),k}(H) + w_3 \cdot d_{ij}^{3,k}(H) \\ &= w_1 \cdot d_{ij}^{1,k}(H) + w_2 \cdot d_{ij}^{2,k}(H) + w_3 \cdot d_{ij}^{3,k}(H). \end{aligned}$$

Let $d_{ij}^{2,k} \oplus d_{ij}^{3,k} = d_{ij}^{(23),k} = \{(H_n, d_{ij}^{(23),k}(H_n)), n = 1, \ldots, N;\ (H, d_{ij}^{(23),k}(H))\}$.

From Definition 8, we have

$$d_{ij}^{(23),k}(H_n) = w_2 \cdot d_{ij}^{2,k}(H_n) + w_3 \cdot d_{ij}^{3,k}(H_n),$$

$$d_{ij}^{(23),k}(H) = w_2 \cdot d_{ij}^{2,k}(H) + w_3 \cdot d_{ij}^{3,k}(H),$$

Let $d_{ij}^{1,k} \oplus (d_{ij}^{2,k} \oplus d_{ij}^{3,k}) = d_{ij}^{1,k} \oplus d_{ij}^{(23),k} = d_{ij}^{1(23),k}$

$$= \{(H_n, d_{ij}^{1(23),k}(H_n)), n = 1, \ldots, N;\ (H, d_{ij}^{1(23),k}(H))\}.$$

From Definition 8, we have

$$\begin{aligned} d_{ij}^{1(23),k}(H_n) &= w_1 \cdot d_{ij}^{1,k}(H_n) + d_{ij}^{(23),k}(H_n) \\ &= w_1 \cdot d_{ij}^{1,k}(H_n) + w_2 \cdot d_{ij}^{2,k}(H_n) + w_3 \cdot d_{ij}^{3,k}(H_n), \\ d_{ij}^{1(23),k}(H) &= w_1 \cdot d_{ij}^{1,k}(H) + d_{ij}^{(23),k}(H) \\ &= w_1 \cdot d_{ij}^{1,k}(H) + w_2 \cdot d_{ij}^{2,k}(H) + w_3 \cdot d_{ij}^{3,k}(H). \end{aligned}$$

So $(d_{ij}^{1,k} \oplus d_{ij}^{2,k}) \oplus d_{ij}^{3,k} = d_{ij}^{1,k} \oplus (d_{ij}^{2,k} \oplus d_{ij}^{3,k})$. □

Although commutativity and associativity are simple and basic, some combination methods don't satisfy these properties. If associativity is not satisfied, the fusion of different information sources will not be reliable. In Ref. 45, the authors said that a rational fusion algorithm should satisfy four basic axioms: independency, consensus, completeness, and incompleteness. Here, we will prove that the proposed weighted average evidential fusion algorithm satisfies the four axioms.

Independency axiom: If $\forall\, l = 1, 2, \ldots, L$ $d_{ij}^{l,k}(H_n) = 0$, then $d_{ij}^{k}(H_n) = 0$.

Proof. If $\forall\, l = 1, 2, \ldots, L$ $d_{ij}^{l,k}(H_n) = 0$, then from Definition 8, we have

$$d_{ij}^{k}(H_n) = \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H_n) = \sum_{l=1}^{L} w_l \cdot 0 = 0. \qquad \square$$

Independency axiom ensures that the aggregated belief degree to H_n should be 0 if none of the attributes is assessed on it.

Consensus axiom: If $\forall\, l = 1, 2, \ldots, L$ $d_{ij}^{l,k} = (H_n, 1)$, then $d_{ij}^{k} = (H_n, 1)$.

Proof. If $\forall\, l = 1, 2, \ldots, L$ $d_{ij}^{l,k} = H_n$, i.e.,

$$d_{ij}^{l,k} = (H_n, d_{ij}^{l,k}(H_n) = 1;\; H_m, d_{ij}^{l,k}(H_m) = 0, m = 1, \ldots, N, m \neq n).$$

Then from Definition 8, we have

$$d_{ij}^{k}(H_n) = \sum_{l=1}^{L} w_l \cdot 1 = 1,$$

$$d_{ij}^{k}(H_m) = \sum_{l=1}^{L} w_l \cdot 0 = 0 \;(m = 1, 2, \ldots, n-1, n+1, \ldots, N).$$

So $d_{ij}^{k} = (H_n, d_{ij}^{k}(H_n) = 1;\; H_m, d_{ij}^{k}(H_m) = 0, m = 1, \ldots, N, m \neq n)$.

Consensus axiom indicates that the fused belief degree on H_n ought to be 1 provided that all L attributes are assessed to H_n.

Completeness axiom: If all attributes are completely assessed to a subset in $H = \{H_1, H_2, \ldots, H_N\}$ denoted as $\{H_{n_1}, H_{n_2}, \ldots, H_{n_p}\} \subseteq H$, then the fused DPR is completely assessed to the same subset $\{H_{n_1}, H_{n_2}, \ldots, H_{n_p}\}$. That is to say: (1) For $\forall\, H_n \in \{H_{n_1}, H_{n_2}, \ldots, H_{n_p}\}$, there is at least one attribute $a_l (l = 1, 2, \ldots, L)$ that $d_{ij}^{l,k}(H_n) > 0$, and $\forall\, H_m \notin \{H_{n_1}, H_{n_2}, \ldots, H_{n_p}\}$, $d_{ij}^{l,k}(H_m) = 0$ for $\forall\, a_l$, then $d_{ij}^{k}(H_n) > 0 (H_n \in \{H_{n_1}, H_{n_2}, \ldots, H_{n_p}\})$, $d_{ij}^{k}(H_m) = 0 (H_m \notin \{H_{n_1}, H_{n_2}, \ldots, H_{n_p}\})$; (2) If $\forall\, l = 1, 2, \ldots, L$ $d_{ij}^{l,k}(H) = 0$, then $d_{ij}^{k}(H) = 0$.

Proof. (1) From Definition 8, for $H_n \in \{H_{n_1}, H_{n_2}, \ldots, H_{n_p}\}$

$$\begin{aligned} d_{ij}^{k}(H_n) &= \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H_n) \\ &= w_1 \cdot d_{ij}^{1,k}(H_n) + \cdots + w_l \cdot d_{ij}^{l,k}(H_n) + \cdots + w_L \cdot d_{ij}^{L,k}(H_n). \end{aligned}$$

Since $\exists\, l = 1, 2, \ldots, L d_{ij}^{l,k}(H_n) > 0$, and $w_l > 0$, we have

$$w_l \cdot d_{ij}^{l,k}(H_n) > 0.$$

Hence, $d_{ij}^{k}(H_n) > 0$.

Similarly, for $H_m \notin \{H_{n_1}, H_{n_2}, \ldots, H_{n_p}\}$

$$\begin{aligned} d_{ij}^k(H_m) &= \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H_m) \\ &= w_1 \cdot d_{ij}^{1,k}(H_m) + \cdots + w_l \cdot d_{ij}^{l,k}(H_m) + \cdots + w_L \cdot d_{ij}^{L,k}(H_m). \end{aligned}$$

Since $\forall\, l = 1, 2, \ldots, L d_{ij}^{l,k}(H_m) = 0$, we have

$$w_l \cdot d_{ij}^{l,k}(H_m) = 0.$$

Hence, $d_{ij}^k(H_m) = 0$.

Thus, $d_{ij}^k = (H_n, d_{ij}^k(H_n) > 0(n \in \{n_1, n_2, \ldots, n_p\});\ H_m, d_{ij}^k(H_m) = 0(n \notin \{n_1, n_2, \ldots, n_p\}))$.

(2) If $\forall\, l = 1, 2, \ldots, L d_{ij}^{l,k}(H) = 0$.

Then from Definition 8,

$$\begin{aligned} d_{ij}^k(H) &= \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H) \\ &= w_1 \cdot d_{ij}^{1,k}(H) + \cdots + w_l \cdot d_{ij}^{l,k}(H) + \cdots + w_L \cdot d_{ij}^{L,k}(H) = 0. \end{aligned}$$

So, $d_{ij}^k = (H_n, d_{ij}^k(H_n) > 0(n \in \{n_1, n_2, \ldots, n_p\});\ H_m, d_{ij}^k(H_m) = 0(n \notin \{n_1, n_2, \ldots, n_p\});\ H, 0)$.

The completeness axiom contains two implications. If an attribute is assessed to H_n with positive belief degree, then the combined belief degree to H_n will be positive too. So any information source cannot be neglected by the proposed fusion algorithm unless the weight of it is 0. Besides, if all the attributes are assessed with no ignorance, i.e., $\forall\, l = 1, 2, \ldots, L \sum_{n=1}^{N} d_{ij}^{l,k}(H_n) = 1$, then the combined DPR will also be complete, i.e., $\sum_{n=1}^{N} d_{ij}^k(H_n) = 1$.

Incompleteness axiom : If $\exists\, l = 1, 2, \ldots, L d_{ij}^{l,k}(H) > 0$, then $d_{ij}^k(H) > 0$.

Proof. From Definition 8, we have

$$\begin{aligned} d_{ij}^k(H) &= \sum_{l=1}^{L} w_l \cdot d_{ij}^{l,k}(H) \\ &= w_1 \cdot d_{ij}^{1,k}(H) + \cdots + w_l \cdot d_{ij}^{l,k}(H) + \cdots + w_L \cdot d_{ij}^{L,k}(H). \end{aligned}$$

Since $\exists\, l = 1, 2, \ldots, L\ d_{ij}^{l,k}(H) > 0$, and $w_l > 0$, so we have

$$w_l \cdot d_{ij}^{l,k}(H) > 0.$$

Hence, $d_{ij}^k(H) > 0$.

Incompleteness axiom ensures that the combined ignorance is positive if any attribute is assessed to be incomplete.

Example 3. Suppose three attributes $a_l(l = 1, 2, 3)$ are involved in comparing x_i over x_j by e^k. The weight vector of attributes is $W = \{0.2, 0.5, 0.3\}$, and the set

Table 6. The assessments of the three attributes to compare x_i over x_j.

Weight	0.2	0.5	0.3
Attribute	a_1	a_2	a_3
Assessment ($d_{ij}^{l,k}$)	$\{H_1, 0.3; H_2, 0.1; H_3, 0.5; H, 0.1\}$	$\{H_3, 0.6; H_4, 0.2; H_5, 0.2\}$	$\{H_4, 0.7; H_5, 0.3\}$
DPR	$d_{ij}^k = \{H_1, 0.06; H_2, 0.02; H_3, 0.4; H_4, 0.31; H_5, 0.19; H, 0.02\}$		

of evaluation grades is $H = \{H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8, H_9\}$. The assessments on the three attributes are shown in Table 6.

We get the fused DPR as $d_{ij}^k = \{H_1, 0.06;\ H_2, 0.02;\ H_3, 0.4;\ H_4, 0.31;\ H_5, 0.19;\ H, 0.02\}$ from Eqs. (24) and (25). If the weights of three attributes are set equally such that $w_1 = w_2 = w_3 = 1/3$, then the combined DPR is $d_{ij}^k = \{H_1, 0.1; H_2, 0.0333; H_3, 0.3667; H_4, 0.3; H_5, 0.1667; H, 0.0333\}$.

3.3.2. *Comparison with the ER approach*

The ER approach is also used to calculate the combined belief degree in Example 3. The result, which is shown in Fig. 1, is similar to the above proposed method.

Example 4. Suppose the belief degrees (BDs) of two attributes are given in Table 7. Their initial weights are 0.1 and 0.9. We change the weights of attributes steadily from $w_1 = 0.1$ to $w_1 = 0.9$. The fused DPR by the proposed method and ER approach from the two attributes is shown in Figs. 2(a) and 2(b).

Compared with the ER approach, the combined uncertainty is larger by the proposed method. It is rational because the uncertainty of a_2 is 0.95, which will enhance the fused uncertainty although the uncertainty of a_1 is small.

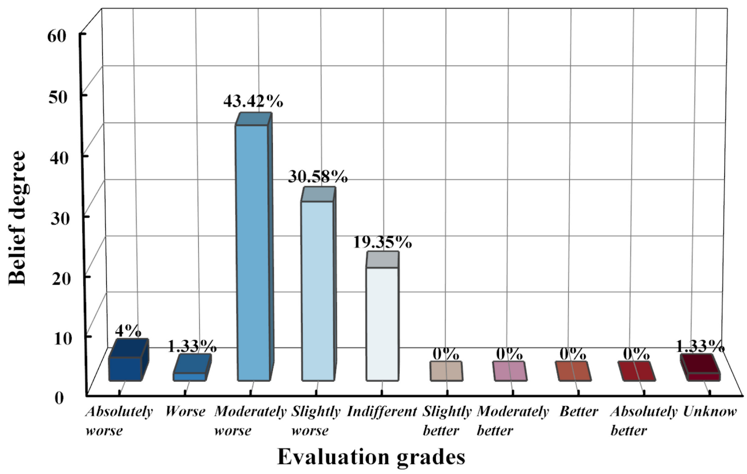
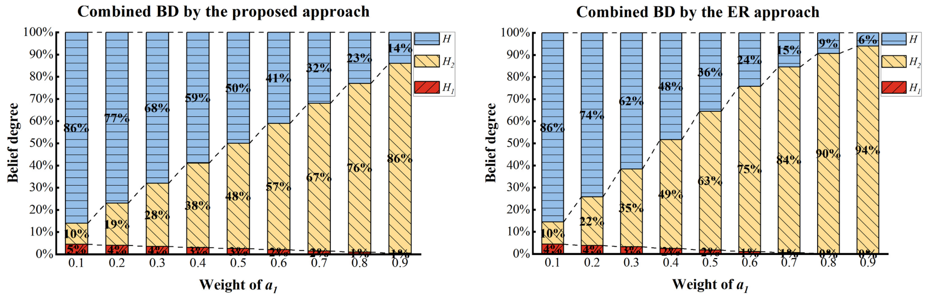


Fig. 1. The combined belief degree generated by the ER algorithm.

Table 7. The BDs of two attributes.

w_l	a_l	H_1	H_2	H
0.1	a_1	0	0.95	0.05
0.9	a_2	0.05	0	0.95



(a) (b)

Fig. 2. (a) The combined BD by the proposed approach. (b) The combined BD by the ER approach.

Example 5. Suppose there are 100 attributes been assigned with the same belief distributions shown in Table 8. The weights of these attributes are equally allocated such that $w_l = 0.01(l = 1, 2, \ldots, 100)$.

The fused BD from these 100 attributes by the proposed method is $\{H_0, 0.6; H_1, 0.4\}$, which is identical to each of the original BD. The fused BD by the ER algorithm is shown in Fig. 3. As the number of attributes increases, the belief degree on H_0 also increases steadily. It is controversial because the combined BD from identical attributes should be unchanged, while the ER algorithm enlarges the larger focal element.

Axiom 1. The combined BD should be identical to the original BDs if all the original BDs are identical. That is to say, if $\forall\, l \neq l', l, l' = 1, 2, \ldots, L, d_{ij}^{l,k} = d_{ij}^{l'}, k$, then $d_{ij}^{l} = d_{ij}^{l,k}$.

Example 6. Let a_1 and a_2 be two attributes assigned with the same BD as shown in Table 9. The weights of two attributes are changed from 0 to 1 and satisfy $w_1 + w_2 = 1$.

Table 8. The BDs of 100 attributes.

w_1	a_l	H_0	H_1
0.01	a_1	0.6	0.4
0.01	a_2	0.6	0.4
	......		
0.01	a_{100}	0.6	0.4

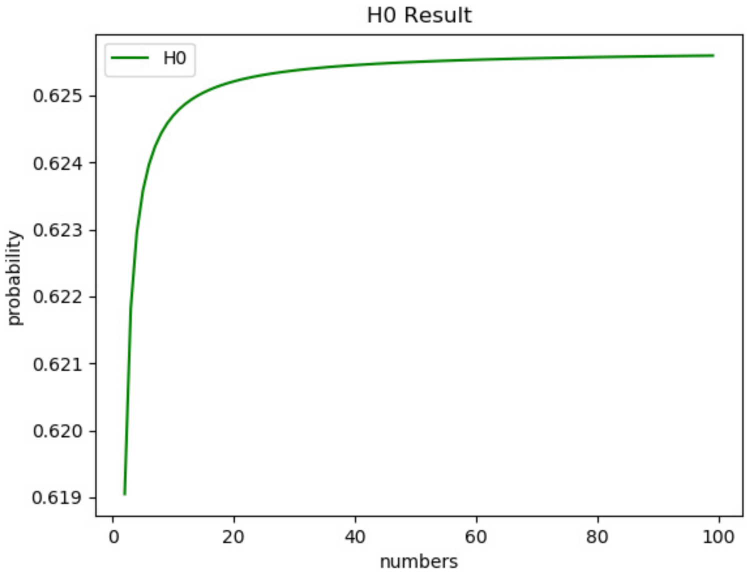


Fig. 3. The results of combining 100 attributes by the ER algorithm.

Table 9. The belief degrees of 2 attributes.

w_l	a_l	H_0	H_2
0–1	a_1	0.6	0.4
1–0	a_2	0.6	0.4

The combined BD by ER algorithm is shown in Fig. 4. Obviously, the combined BD changes with the weights of the two attributes. And it reaches the maximum value when $w_1 = w_2 = 0.5$. So it does not satisfy Axiom 1. It is irrational because the two attributes are identical although they are assigned with changed weights. The combined BD by the ER algorithm is $\{H_1, 0.6; H_2, 0.4\}$ only when $w_1 = 1$, $w_2 = 0$ or $w_1 = 0$, $w_2 = 1$. The fused belief degree by the proposed method is still identical with a_1 and a_2 no matter how the weights of two attributes change.

The generated combined DPR matrix from e_k denoted as $D^k = (d_{ij}^k)_{M\times M}$ is presented in Table 10. Here, only adjacent alternative pairs are compared to release the burden on experts in providing assessment information. So the pairwise comparisons of alternatives are absolutely consistent, and there is no need to make adjustments to inconsistent assessments originating from the large number of alternatives.

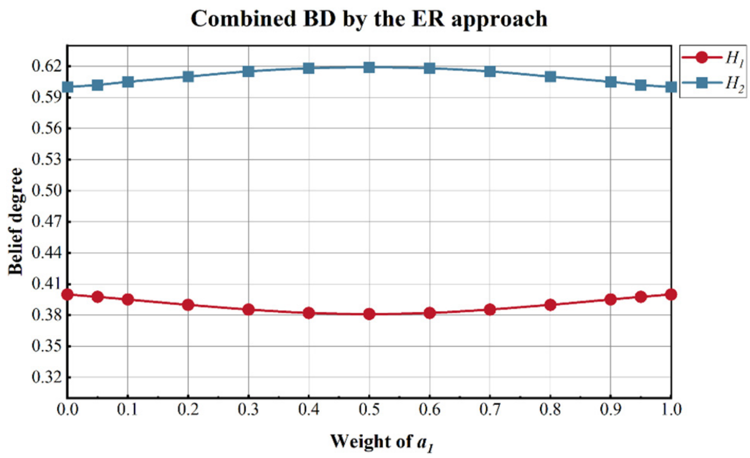


Fig. 4. The combined BD of two attributes by the ER algorithm.

Table 10. DPR D^k provided by expert e_k.

DPR	x_1	x_2	x_3	...	x_{M-1}	x_M
x_1	$\left\{\left(H_{\frac{N+1}{2}},1\right)\right\}$	d_{12}^k	\	\	\	\
x_2	d_{21}^k	$\left\{\left(H_{\frac{N+1}{2}},1\right)\right\}$	d_{23}^k	...	\	\
x_3	\	d_{32}^k	$\left\{\left(H_{\frac{N+1}{2}},1\right)\right\}$	...	\	\
...	\	\	...	...	...	...
x_M-1	\	\	\	...	$\left\{\left(H_{\frac{N+1}{2}},1\right)\right\}$	$d_{(M-1)M}^k$
x_M	\	\	\	\	$d_{M(M-1)}^k$	$\left\{\left(H_{\frac{N+1}{2}},1\right)\right\}$

4. CRP Based on Classification-Oriented GDM

CRP based on classification-oriented GDM is different from traditional ranking-oriented GDM. In a ranking-oriented GDM problem, the consensus measure is usually computed by the original cardinal assessment values derived from experts. Two methods are commonly used. One is to measure the similarity between the opinions of each pair of experts.[5,9] The other one is to quantify the similarity between each expert's judgment and the aggregated group opinion by a fusion algorithm.[14] Some research also uses ordinal preference value to calculate the consensus degree.[16] No matter which method is applied, it cannot be directly used to measure the consensus

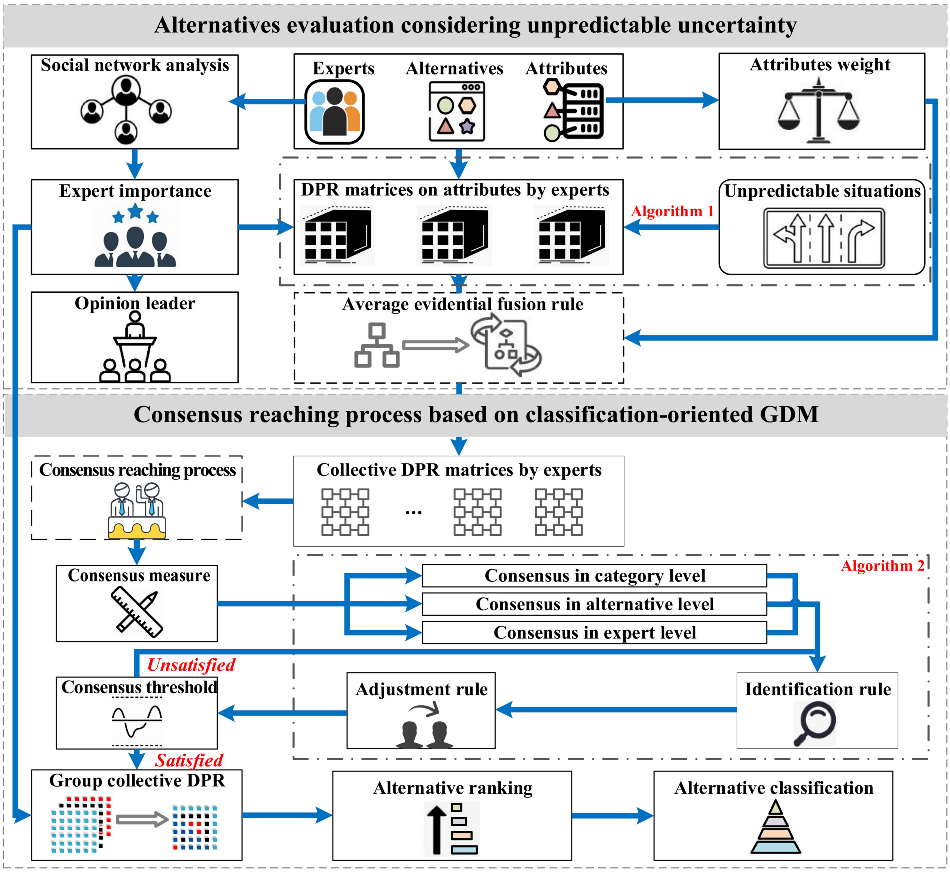


Fig. 5. The methods applied in the whole decision-making process.

degree of the group in a classification-oriented GDM problem. The whole decision process is depicted in Fig. 5.

4.1. *Consensus measure and identification*

4.1.1. *Consensus measure*

In a classification-oriented GDM problem, consensus is usually reached based on the ranking order generated from the original DPRs of experts. So the consensus status may not be measured directly by the DPRs of experts. The DPR of each expert should be converted to the ranking of alternatives.

Definition 9 (Possibility degree (Ref. 32)). Suppose the interval score value matrix transformed from D^k is calculated by Definition 3 and denoted as $S^k = (S^k_{ij})_{M\times M} = (S^k_{ij} \in [S^{k-}_{ij}, S^{k+}_{ij}])_{M\times M}$. It can be used to make a ranking order between alternative x_i and x_j. The possibility degree of $S_{ij}^k \succ S_{ji}^k$ is calculated as

$$p(S_{ij}^k \succ S_{ji}^k) = \left\{\begin{array}{cc} 1, & S_{ij}^{k-} > S_{ji}^{k+}, \\ 1 - \frac{2(S_{ij}^{k-})^2}{(S_{ij}^{k+} - S_{ij}^{k-})^2}, & S_{ji}^{k-} \le S_{ij}^{k-} \le S_{ji}^{k+} \le S_{ij}^{k+}, \\ \frac{2(S_{ij}^{k+})^2}{(S_{ij}^{k+} - S_{ij}^{k-})^2}, & S_{ij}^{k-} \le S_{ji}^{k-} \le S_{ij}^{k+} \le S_{ji}^{k+}, \\ 0, & S_{ij}^{k+} \le S_{ji}^{k-}. \end{array}\right\} \tag{27}$$

(1) $p(S_{ij}^k \succ S_{ji}^k) = 1$, then x_i is absolutely superior to x_j.
(2) $p(S_{ij}^k \succ S_{ji}^k) = 0$, then x_i is absolutely inferior to x_j.
(3) $p(S_{ij}^k \succ S_{ji}^k) = 0.5$, then x_i equals to x_j and $s_{ij}^{t-} + s_{ij}^{t+} = 0$.
(4) $p(S_{ij}^k \succ S_{ji}^k) > 0.5$, then x_i is superior to x_j.
(5) $p(S_{ij}^k \succ S_{ji}^k) < 0.5$, then x_i is inferior to x_j.

The final ranking order of M alternatives is $X^k(\succ^p) = {}'x_{k_1} \succ \ldots \succ x_{k_m} \succ \ldots \succ x'_{k_M}$, where $x_{k_m}(m = 1, \ldots, M)$ denotes the mth preferred alternative by e_k.

Suppose the set of categories to be classified is $C = \{C_1, \ldots, C_q, \ldots, C_Q\}$. $Q(Q < M)$ is the number of categories, and $C_q \succ C_{q+1}(q \in \{1, 2, \ldots, Q-1\})$. It means that the alternatives in category C_q is absolutely superior to that in C_{q+1}. So we also have $C_q \succ C_{q+num}(num = 1, \ldots, Q-q)$. Here, the number of alternatives in each category are determined in advance. Let n_q be the number of alternatives been assigned in category C_q. So we have $C_q = \{x_{q_1}, x_{q_2}, \ldots, x_{q_{n_q}}\}$, and $\sum_{q=1}^{Q} n_q = M$. In other words, alternative x_{k_m} is classified in category C_q by expert e_k, which is denoted as $C_q^{k_m}$ where $\{C_q^{k_m} | x_{k_m} \in C_q^k = \{x_{q_1}^k, x_{q_2}^k, \ldots, x_{q_{n_q}}^k\}\}$. Usually, the number of alternatives in a higher-level category is less than the lower-level category, i.e., $n_q < n_{q+1}(q = 1, \ldots, Q-1)$. But it is not a necessary condition.

Definition 10 (Dissimilarity on category between two experts). Suppose the number of different alternatives been classified in category C_q by expert e_k and $e_{k'}$ is $|C_q^k - C_q^{k'}|$. Then the Dissimilarity between the opinions of e_k and $e_{k'}$ on category C_q is defined as

$$\mathrm{Diss}_q^{k,k'} = \frac{|C_q^k - C_q^{k'}|}{2n_q}. \tag{28}$$

The value of $\mathrm{Diss}_q^{k,k'}$ is between 0 and 1. In the case of $C_q \succ C_{q+\mathrm{num}}$ and $n_q < n_{q+1}$, $\mathrm{Diss}_q^{k,k'}$ is more likely to be larger than $\mathrm{Diss}_{q+\mathrm{num}}^{k,k'}$.

Definition 11 (Dissimilarity between two experts). Let $|C_q^k - C_q^{k'}|$ be the number of different alternatives been classified in category C_q by expert e_k and $e_{k'}$. Then the Dissimilarity between the opinions of e_k and $e_{k'}$ on all categories is defined as

$$\mathrm{Diss}^{k,k'} = \frac{\sum_{q=1}^{Q} \frac{|C_q^k - C_q^{k'}|}{2n_q}}{Q}. \tag{29}$$

Properties:

(1) $0 \leq \mathrm{Diss}^{k,k'} \leq 1$;
(2) $\mathrm{Diss}^{k,k'} = \mathrm{Diss}^{k',k}$;
(3) $\mathrm{Diss}^{k,k'} = 0$ iff $\forall\, q \in \{1, 2, \ldots, Q, |C_q^k - C_q^{k'}| = 0$;
(4) $\mathrm{Diss}^{k,k'} = 1$ iff $\forall\, q \in \{1, 2, \ldots, Q, |C_q^k - C_q^{k'}| = 2n_q$.

Property (3) indicates that if the alternatives have been classified in any category by two experts are identical, there is no Dissimilarity between their opinions. Contrarily, from Property (4), if there are no alternatives classified in the same category, the Dissimilarity between the opinions of two experts is the largest. In a real decision-making problem, it will not happen unless there is a great conflict of interest between two experts (Table 11).

Example 7. Suppose 9 alternatives $\{x_1, x_2, \ldots, x_9\}$ are assessed by three experts $\{e_1, e_2, e_3\}$. They are to be classified into three categories $\{C_1, C_2, C_3\}$. The number of alternatives in a category increases with the subscript of the category such that $n_1 = 2$, $n_2 = 3$, $n_4 = 4$. The classifications by the three experts are shown in Table 11.

By Definition 10, we can get $\mathrm{Diss}_1^{1,2} = 0.5$, $\mathrm{Diss}_2^{1,2} = 0.333$, $\mathrm{Diss}_3^{1,2} = 0.25$, $\mathrm{Diss}_1^{1,3} = 1$, $\mathrm{Diss}_2^{1,3} = 1$, $\mathrm{Diss}_3^{1,3} = 0.25$, $\mathrm{Diss}_1^{2,3} = 0.5$, $\mathrm{Diss}_2^{2,3} = 0.667$, $\mathrm{Diss}_3^{1,2} = 0.25$. Then from Definition 11, we have $\mathrm{Diss}^{1,2} = 0.361$, $\mathrm{Diss}^{1,3} = 0.75$, $\mathrm{Diss}^{2,3} = 0.472$. The Dissimilarity between e_1 and e_3 is the largest.

Definition 12 (Category disparity of alternative). Suppose $\mathrm{dis}^{C}_{m_{|e_k - e_{k'}|}}$ is the category disparity that x_m be assigned by e_k and $e_{k'}$. Hence, $\mathrm{dis}_{m_{|e_k - e_{k'}|}}{}^{C}$ equals to the Dissimilarity of subscript that x_m be classified to C_q and $C_{q'}$ by e_k and $e_{k'}$.

Table 11. The classification of alternatives to categories by three experts.

Category Expert	e_1	e_2	e_3
C_1	x_1	x_1	x_4
	x_2	x_4	x_5
C_2	x_3	x_6	x_1
	x_4	x_3	x_2
	x_5	x_5	x_6
C_3	x_6	x_7	x_3
	x_7	x_2	x_7
	x_8	x_8	x_9
	x_9	x_9	x_8

If e_k and $e_{k'}$ assign x_m to the same category, then $\mathrm{dis}^C_{m_{|e_k-e_{k'}|}} = 0$. Hence, the disparity that x_m be assigned to category by all the K experts is calculated as

$$\mathrm{dis}^C_m = \frac{2}{K(K-1)} \sum_{k=1}^{K-1} \sum_{k'=k+1}^{K} \mathrm{dis}^C_{m_{|e_k-e_{k'}|}}. \tag{30}$$

Remark 2. It is clear that $\mathrm{dis}^C_{m_{|e_k-e_{k'}|}} = |q-q'|(q,q' \in \{1,2,\ldots,Q\})$. The minimum value of $\mathrm{dis}^C_{m_{|e_k-e_{k'}|}}$ is 0, and the maximum value of $\mathrm{dis}^C_{m_{|e_k-e_{k'}|}}$ is $Q-1$.

In Example 7, $\mathrm{dis}^C_{1_{|e_1-e_2|}} = 0$, $\mathrm{dis}^C_{1_{|e_1-e_3|}} = 1$, $\mathrm{dis}^C_{1_{|e_2-e_3|}} = 1$, so we have $\mathrm{dis}^C_1 = 0.667$. The category disparities of 9 alternatives in Example 7 are depicted in Fig. 6.

Definition 13 (Dissimilarity of expert). Let $\mathrm{Diss}^{k,k'}$ be the Dissimilarity between experts e_k and $e_{k'}$, then the classification Dissimilarity of expert e_k's opinion with others can be calculated as:

$$\mathrm{Diss}^k = \frac{1}{K-1} \sum_{k'=1,k'\neq k}^{K} \mathrm{Diss}^{k,k'}. \tag{31}$$

Definition 13 measures the average Dissimilarity of expert e_k with other $K-1$ experts on the classification of alternatives. The larger the value of Diss^k, the more conflict of e_k's judgment. In Example 7, we have $\mathrm{Diss}^1 = 0.556$, $\mathrm{Diss}^2 = 0.417$, $\mathrm{Diss}^3 = 0.611$.

Here, the importance of an expert is calculated based on SNA. Firstly, trust relationships between all pairs of experts are obtained as

$$\mathrm{TR} = [\lambda_{kh}]_{K\times K} = \begin{bmatrix} - & \lambda_{12} & \ldots & \lambda_1 K \\ \lambda_{21} & - & \ldots & \lambda_2 K \\ \ldots & \ldots & \ldots & \ldots \\ \lambda_{K1} & \lambda_{K2} & \ldots & \lambda_{KK} \end{bmatrix}. \tag{32}$$

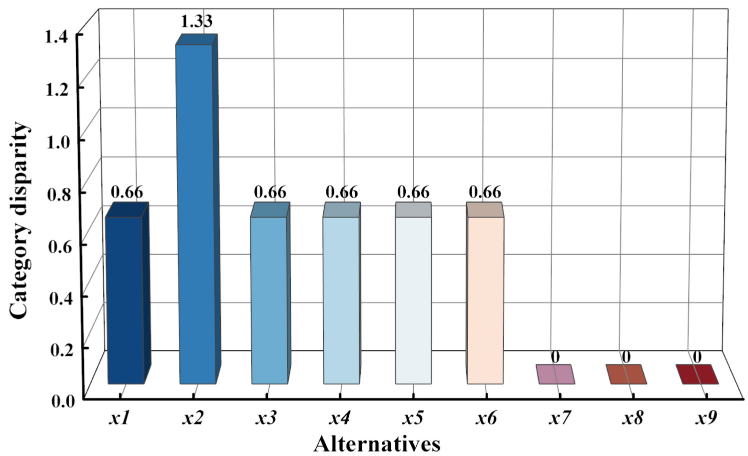


Fig. 6. Category disparity of alternatives in Example 7.

Since the number of experts is not large, we assume that there is no missing value in TR. Then Eq. (32) is used to calculate the trust score matrix as

$$\mathrm{TS} = [ts_{kh}]_{K\times K} = \begin{bmatrix} — & ts_{12} & \dots & ts_1K \\ ts_{21} & — & \dots & ts_2K \\ \dots & \dots & \dots & \dots \\ ts_{K1} & ts_{K2} & \dots & ts_{KK} \end{bmatrix}. \tag{33}$$

The normalized importance of e_k is computed as

$$R_k = \frac{\sum_{h=1,h\neq k}^{K} ts_{kh}}{\sum_{k=1}^{K}\sum_{h=1,h\neq k}^{K} ts_{kh}}. \tag{34}$$

Obviously, $\sum_{k=1}^{K} R_k = 1$.

Definition 14 (Dissimilarity of the group). Let Diss^k be the Dissimilarity of expert e_k. The relative importance of expert e_k is R_k which satisfies $0 \leq R_K \leq 1 (k = 1, \dots, K)$ and $\sum_{k=1}^{K} R_K = 1$. Then the classification Dissimilarity of the whole group can be computed as

$$\mathrm{Diss}^G = \sum_{k=1}^{K} R_k \cdot \mathrm{Diss}^k. \tag{35}$$

Specifically, when the weights of all the K experts are equal, we have $\mathrm{Diss}^G = \frac{1}{K}\sum_{k=1}^{K} \mathrm{Diss}^k$.

Let δ be the consensus threshold. If $\mathrm{Diss}^G \leq \delta$, the group consensus is satisfied. Otherwise, CRP should be implemented among experts with one round or several rounds. In practice, the value of δ should be chosen based on the specific decision-making environment. For instance, if the classification outcome has high stakes, such as safety assessments, medical diagnoses, or major financial investments, DMs may set a lower δ to minimize conflicts. This ensures stricter convergence to a unanimous viewpoint. On the other hand, when the environment tolerates moderate disagreement (e.g., exploratory research or preliminary market studies), a higher δ may be acceptable. Additionally, historical data or pilot testing can be used to determine an empirical threshold: if consistent group decisions were achieved in previous similar tasks with a certain conflict level, then empirical baseline can guide the choice of δ. In all cases, the moderator or decision facilitators should balance efficiency (fewer rounds of consensus adjustment) against the desired accuracy or risk level to ensure the final classification meets practical requirements.

Remark 3. When the consensus is measured based on classification, consensus status is attained if the consensus based on ordinal ranking is reached. That is to say, classification-based consensus is a necessary condition of ordinal ranking-based consensus.

Remark 3 is easy to be interpreted. If all the experts give similar ranking orders of alternatives after CRP, it's obvious that the classification on them by different experts also reaches a high consensus level. On the contrary, if classification-based consensus status is satisfied, we cannot conclude that the ranking orders of alternatives provided by experts attain a high consensus level. The reason lies in that the ranking orders of alternatives in the same classification may be divergent by different experts. But this does not affect the final results.

In the assessment by experts, an alternative may be directly classified into a category without comparison to other ones. An excellent alternative may be directly assigned to the best category, while an alternative with very bad performance will be automatically eliminated from all the categories.

4.1.2. *Consensus identification*

If $\mathrm{Diss}^G > \delta$, group consensus is not satisfied, and the adjusted element should be identified. The importance of expert ought to be taken into account in the identification process. The higher importance of an expert, the less intension he/she will adjust the opinion. Additionally, the importance of expert also determines opinion leader, which directs the adjustment extent for identified expert. Two kinds of identification route may be carried out as follows.

(1) *Expert identification*

(a) Identification in the expert level

Identification rule: Let Diss^k be the Dissimilarity of expert e_k. The relative importance of expert e_k is R_k. $\mathrm{Diss}^{k^*} = \max_{k=1,\dots,K} \mathrm{Diss}^k$, if $R_{k^*} \neq \max_{k=1,\dots,K}\{R_k\}$, then e_{k^*} will adjust his/her opinion to some extent according to adjustment rule in Sec. 4.2. If $R_{k^*} = \max_{k=1,\dots,K}\{R_k\}$, e_{k^*} maybe an opinion leader (OL) although his/her opinion contradicts with other experts. Then $\mathrm{Diss}^{k^{**}} = \max_{k=1,\dots,K^*-1,k^*+1\dots,k} \mathrm{Diss}^k$, and $e_{k^{**}}$ will adjust his/her opinion.

(b) Identification in the expert pair level

Identification rule: $\mathrm{Diss}^{k^*,k'} = \max_{k=1,\dots,K k\neq k^*} \mathrm{Diss}^{k^*,k}$, then the Dissimilarity between e_{k^*} and $e_{k'}$ is the largest respect to e_{k^*}.

(c) Identification in the category level

Identification rule: $\mathrm{Diss}_{q^*}^{k^*,k'} = \max_{q=1,\dots,Q} \mathrm{Diss}_q^{k^*,k'}$, then the category to be adjusted respect to e_{k^*} and $e_{k'}$ is identified as C_{q^*}.

(2) *Category identification*

If the Dissimilarity on one category is the largest, there may be conflict among the opinions of experts. Hence, the consensus adjustment on this category should be implemented first. So we should define the Dissimilarity on category.

Definition 15 (Dissimilarity of expert on category). Suppose the Dissimilarity on category C_q between e_k and $e_{k'}$ is $\text{Diss}_q^{k,k'}$. Then the Dissimilarity of expert e_k with other experts on C_q is computed as:

$$\text{Diss}_q^k = \frac{1}{K-1} \sum_{k'=1, k' \neq k}^{K} \text{Diss}_q^{k,k'}. \tag{36}$$

Definition 16 (Dissimilarity on category among all experts). Let $\text{Diss}_q^{k,k'}$ be the Dissimilarity on category C_q between e_k and $e_{k'}$. Then the Dissimilarity among the opinions of all the K experts on category C_q is defined as:

$$\text{Diss}_q = \frac{2}{K(K-1)} \sum_{k=1}^{K-1} \sum_{k'=k+1}^{K} \text{Diss}_q^{k,k'}. \tag{37}$$

Diss_q can also be calculated as $\text{Diss}_q = \frac{1}{K} \sum_{k=1}^{K} \text{Diss}_q^k$.

(a) *Identification in the category level*

Identification rule: Let $\text{Diss}_q^* = \max_{q=1,\ldots,Q} \text{Diss}_q$. Diss_q^* represents the largest Dissimilarity among all the Q categories. Then $\{C_q | \text{Diss}_q = \text{Diss}_q^*\}$ should be selected as the category to be modified on its included alternatives by different experts.

(b) *Identification in the alternative level*

Anyway, if one alternative is assigned to nonconsecutive categories by two experts, i.e., $|q - q'| > 1$, the conflict among experts cannot be neglected. Select the alternative that has the largest category disparity such that $\text{dis}_{m^*}^C = \max_{m=1,\ldots,M} \text{dis}_m^C$. Then x_{m^*} is the alternative that should be conduct consensus adjustment first.

(c) *Identification in the expert level*

Calculate $\max_{k,k'=1,\ldots,k} \text{dis}_{m^*|e_k - e_{k'}|}^C$ which represents the largest disparity between two experts associated with x_{m^*}. Here, $\text{dis}_{m^*|e_k - e_{k'}|}^C \leq Q - 1$.

In Example 7, the largest category disparity of alternative is $\text{dis}_{2^*C} = 1.333$, and the largest disparity between experts is $\text{dis}_{2^*|e_1 - e_2|}^C = 2$.

4.2. *Consensus adjustment*

When the identified element is confirmed, consensus adjustment will be implemented. Some assumptions are given as follows:

(1) The number of alternatives in each category is fixed, and $n_q < n_{q+1} (q \in \{1, 2, \ldots, Q-1\})$;
(2) The scale of GDM is not large, i.e., not more than 20 persons;
(3) Multiple attributes are considered in GDM, and the standard and weight of each attribute are provided by the moderator;

(4) Two situations may occur: (a) Some alternatives with low performances may be finally eliminated. The remaining alternatives are then classified; or (b) There is no eliminated alternative in the classification process.

Remark 4. As for assumption (1), $n_q < n_{q+1}$ may not always occur. For example, the science index citation (SCI) classification satisfies $n_q = n_{q+1}$ because the proportions of Q1, Q2, Q3 and Q4 are 25%. In the scholarship evaluation, $n_q < n_{q+1}$ is usually satisfied and the number in each category is fixed. For assumption (2), we limit our discussion to groups with a relatively small number of DMs to ensure manageability of the CRP and feasibility in data handling. In many institutional committees or award evaluations, it is common for the number of panelists to be below 20. This assumption simplifies certain computational steps and reduces the complexity of communication or negotiation overhead during the consensus adjustment. With respect to assumption (3), in real-world decision-making tasks, alternatives are usually assessed against multiple criteria. For example, in subject assessments of universities — such as those sponsored by the China Academic Degrees & Graduate Education Development Center (CDGDC) — the performance of a subject (or department) might hinge on students' achievements, faculty publications, research output, and other qualitative characteristics. These attributes often come with given standards and weights assigned by a moderator or organizing body to reflect their relative importance. By providing a fixed set of attribute weights in advance, we maintain consistency and transparency during the evaluation, ensuring that DMs focus on applying the criteria rather than debating them. With regard to assumption (4), the scholarship reviewing, science & technology award assessment belong to case (a). Some students or science & technology achievements will not be awarded after discussion or reviewing by a group of experts. The subject assessment of university corresponds to case (b). All the alternative subjects submitted by universities will be classified with one of the following category: A+, A (2–5%), A − (5–10%), B + (10–20%), B (20–30%), B − (30–40%), C + (40–50%), C (50–60%), C − (60–70%), D. And there is no fee in the reviewing process.

Expert with the maximum weight is assumed to be OL such that $e_{OL} = \{e_k | R_k = \max_{k'=1,\ldots,K}\{R_{k'}\}\}$. R_k is assigned by the moderator in advance. In consensus adjustment procedure, the alternative with the largest classification disparity between two experts should be adjusted to the direction of e_{OL}. That is to say, $\{C_{q^*}^{k_{m^*}} | x_{k_{m^*}} \in C_{q^*}^k = \{x_{q_1^*}^k, x_{q_2^*}^k, \ldots, x_{q_{n_{q^*}}^*}^k\}\}$ will be modified to be equal with $\{C_{q^*}^{k_{m^*}^{OL}} | x_{k_{m^*}^{OL}} \in C_{q^*}^{k^{OL}} = \{x_{q_1^*}^{k^{OL}}, x_{q_2^*}^{k^{OL}}, \ldots, x_{q_{n_{q^*}}^*}^{k^{OL}}\}\}$. When the group has attained consensus, the final classification C_q^m of x_m could be calculated by the weighted average of the subscripts of $C_q^{k_m}$ $(k = 1, 2, \ldots, K, q = 1, 2, \ldots Q, m = 1, 2, \ldots, M)$

$$\left\{C_q^m | x_m \in C_q, q = \left[\sum_{k=1}^{K} R_k q_k\right]\right\}. \tag{38}$$

Algorithm 2. Consensus identification and adjustment rule.

Input: DPRs of alternative x_i over x_j on attribute a_l by $e_k(k = 1, \ldots, K)$: $d_{ij}^{l,k} = \{(H_n, d_{ij}^{l,k}(H_n)), n = 1, \ldots, N; (H, d_{ij}^{l,k}(H))\}$, trust relationships between pair of experts $\mathrm{TR} = [\lambda_{kh}]_{K\times K}$,$\{n_1, n_2, \ldots, n_Q\}$ where $\sum_{q=1}^{Q} n_q = M$.

Output: Final alternative classification under satisfactory consensus such that $\mathrm{Diss}^G \leq \delta$.

Step 1: Obtain the DPR of expert e_k on attribute a_l by Algorithm 1 denoted as $d_{ij}^{l,k}$

Step 2: Estimate attribute weights vector as $W = \{w_l | l = 1, \ldots, L\}$

Step 3: Generate $d_{ij}^{k} = \{(H_n, d_{ij}^{k}(H_n)), n = 1, \ldots, N; (H, d_{ij}^{k}(H))\}$ by Eqs. (24) and (25)

Step 4: Calculate the score value of d_{ij}^{k} by Eqs. (2) and (3)

Step 5: Using Eq. (27) to generate alternative ranking and the classification by $e_k(k = 1, \ldots, K) : X^k(\succ^p) = {}'x_{k_1} \succ x_{k_2} \succ \ldots \succ x'_{k_M}$, $C_q^k = \{x_{q_1}^k, x_{q_2}^k, \ldots, x_{q_{n_q}}^k\}(q = 1, \ldots, Q)$

Step 6: Using Eqs. (32)–(34) to compute the importance of experts as $R = \{R_k | k = 1, \ldots, K\}$

Step 7: Using Eqs. (28), (29), (31), (35) to calculate Diss^G
If $\mathrm{Diss}^G \leq \delta$, then go to Step 10
Otherwise, go to Step 8

Step 8: Using Eq. (30) to calculate ${\mathrm{dis}_m}^C$, compute ${\mathrm{dis}_{m^*}}^C = \max_{m=1,\ldots,M} {\mathrm{dis}_m}^C$ and $\max_{k,k=1,\ldots,K} \mathrm{dis}_{m^*|e_k - e_{k'}|}^C$

Step 9: If $R_k > R_{k'}$, then adjust $\{{C_{q^*}^{k'}}^{m^*} | x_{k'_{m^*}} \in C_{q^*}^{k'} = \{x_{q_1^*}^{k'}, x_{q_2^*}^{k'} \ldots, x_{q_{n_{q^*}}^*}^{k'}\}\}$ to be equal with $\{{C_{q^*}^{k^{OL}}}^{m^*} | x_{k_{m^*}^{OL}} \in C_{q^*}^{k^{OL}} = \{x_{q_1^*}^{k^{OL}}, x_{q_2^*}^{k^{OL}} \ldots, x_{q_{n_{q^*}}^*}^{k^{OL}}\}\}$.
Return to Step 7

Step 10: Using Eq. (38) to generate the final classification of all alternatives $C = \{C_1, C_2, \ldots, C_Q\}$ according to $\{n_1, n_2, \ldots, n_Q\}$ such that $x_{c_1}, x_{c_2}, \ldots, x_{c_{n_1}} \in C_1$, $x_{c_{n_1+1}} x_{c_{n_1+2}}, \ldots, x_{c_{n_2}} \in C_2, \ldots, x_{c_{n_{Q-1}+1}}$, $x_{c_{n_{Q-1}+2}}, \ldots, x_{c_{n_Q}} \in C_Q$.

5. Case Study

In this section, a case study is presented to illustrate the effectiveness of the proposed approach. Then, comparative analysis is conducted to demonstrate the advantage of the classification-oriented CRP.

5.1. *Case description*

Scholarship evaluation is common in universities for undergraduate and postgraduate students. It is mainly implemented based on subjective and objective evaluation.

Usually, four categories are classified, i.e., first-class scholarship (10%)/second-class scholarship (30%)/third-class scholarship (50%)/disqualified (10%). Several criteria are always considered in the evaluation process, such as morality, research ability and social work. Morality refers to the effort of study, moral value, positivity and so on. It is assessed with qualified and unqualified. If a student is unqualified on the first criterion, he/she will be canceled the opportunity for applying the scholarship. Research ability includes the attributes of 'award,' 'publication of paper,' 'project participation,' 'authorized patent.' Social work reflects the public actions a student has taken part in, which consists of 'scientific service,' 'public work,' 'laboratory management' and 'other social practical actions.'

Several experts are usually invited to participate in the evaluation. Group consensus is necessary because experts may have opinion divergence. They only need to give the grade level of each student in the final decision instead of exact ranking order or score values of students. Unpredictable uncertainties often occur in the evaluation process. For example, one student has submitted a manuscript to a popular journal and the status is under second-round review. The paper may be either accepted or rejected in the future. Another student has submitted a manuscript to an ordinary journal and was accepted. Due to the unpredictable uncertainties, each expert will probably provide DPR on the attribute of paper publication, and experts may have different judgments when comparing the two students on the attribute.

5.2. *Model solving and analysis*

5.2.1. *Calculation process*

In this case, 5 experts ($E = \{e_1, \ldots, e_5\}$) are invited to evaluate third year 17 postgraduate students ($X = \{x_1, \ldots, x_{17}\}$) for the scholarship application. Each student is required to report 5 min first, and then questions are asked by experts within 5 min. The linguistic grades are set as $H = \{H_1, H_2, H_3, H_4, H_5, H_6, H_7\} =$ {absolutely worse, worse, slightly worse, indifferent, slightly better, better, absolutely better}. After consultation, the scores of grades are set as $S =$

Table 12. Predefined parameters.

Implications	Symbol	Value
The set of experts	$E = \{e_1, \ldots, e_K\}$	$K = 5$
The set of alternatives	$X = \{x_1, x_2, \ldots, x_M\}$	$M = 17$
The set of attributes	$A = \{a_1, \ldots, a_L\}$	$L = 4$
Linguistic evaluation grades	$H = \{H_1, \ldots, H_N\}$	$N = 7$
The scores of grades	$S = \{s(H_1), \ldots, s(H_N)\}$	$S = \{-1, -0.7, -0.3, 0, 0.3, 0.7, 1\}$
The set of categories	$C = \{c_1, \ldots, c_Q\}$	$Q = 4$
The number of each category	$n_q(q = 1, \ldots, Q)$	$n_1 = 2$, $n_2 = 5$, $n_3 = 8$, $n_4 = 2$
The weights of attributes	$W = \{w_l \mid l = 1, \ldots, L\}$	$w_1 = 0.5$, $w_2 = 0.1$, $w_3 = 0.3$, $w_4 = 0.1$
Consensus threshold	δ	$\delta = 0.2$

$\{s(H_1), s(H_2), s(H_3), s(H_4), s(H_5), s(H_6), s(H_7)\} = \{-1, -0.7, -0.3, 0, 0.3, 0.7, 1\}$. All the experts should finally reach consensus and each student is then divided into one of 4 levels ($C = \{c_1, \ldots, c_4\}$). The set of categories C stands for 4 different scholarship application results and the number of each class will be defined as $n_q(q = 1, \ldots, 4)$ which satisfies required proportional distribution. In addition, Table 12 shows the details of predefined parameters.

Table 13 shows the information about 17 students from four attributes ($A = \{a_1, \ldots, a_4\}$) which reflect the research ability of them. The weights of attributes are determined in advance.

Step 1. Obtain the DPRs of experts on attributes.

According to the data on candidates presented in Table 13 and the answers to questions, 5 experts compared candidate students in pairs to form a preliminary impression. Subsequently, they engaged in thorough discussions and exchanged views on the research ability and academic achievement of each student. Following the discussion, experts independently provided their DPRs $d_{ij}^{l,k}(i = 1, \ldots, 16;\ j = i+1, \ldots, 17;\ k = 1, \ldots, 5)$ on each attribute $a_l(l = 1, \ldots, 4)$. However, comparison of x_{ij} on a_l could exist in certain circumstance or unpredictable situations simultaneously. For instance, when experts considering about the publication of paper (a_1) between student 1 (x_1) and student 2 (x_2). There are 4 states for their manuscripts: x_1 accept & x_2 accept; x_1 Accept & x_2 Reject; x_1 Reject & x_2 Accept; x_1 Reject & x_2 Reject. With discrepancies and risk attitudes of experts, the DPRs will be given based on different probabilities and assessments. Table 14 gives the DPRs of a_1 on x_{12} by experts which considering unpredictable states. Besides, the other similar situations are given in Supplementary Material A. Then, the DPR given by expert e_1 is partly shown in Table 15, and the complete DPRs given by 5 experts are presented in the Supplementary Materials B. As such, Step 1 is completed.

Step 2. Estimated attribute weights vectors.

The attributes weight vectors are defined as $w_1 = 0.5, w_2 = 0.1, w_3 = 0.3, w_4 = 0.1$.

Step 3–5: Generate the collective DPR, score value, alternative ranking and classification by each expert.

The collective DPR d_{ij}^k of each expert $e_k(k = 1, \ldots, K)$ could be calculated by Eqs. (24) and (25), which are partly shown in Table 16 and totally shown in Supplementary Materials C. By using Eqs. (2) and (3), the score values of d_{ij}^k are generated and displayed in Table 17. According to the possibility degrees by Eq. (27), the first-round alternatives ranking and classification could be generated in Table 19.

Step 6: Generation of expert importance

In this step, the experts are invited to evaluate their trust degree t_{kh} and distrust degree d_{kh} between themselves and the others. According to these, trust scores ts_{kh}

Table 13. The information about 17 students.

Student order	Publication of paper (50%) a_1	Project participation (10%) a_2	Patent and case writing (30%) a_3	Award (10%) a_4
1	Submit a manuscript to a SCI journal (JCR Q2, under review R1, first author).	Participate in one national level project and one provincial level project.	Three patents authorized (One 2nd inventor, two 3rd inventor).	Provincial scientific and technology 3rd prize (6th); Internet plus provincial competition 3rd prize (1st)
2	A manuscript accepted by a core Chinese journal (Second author); Submit a manuscript to a SCI journal (JCR Q4, under review R1, second author).	—	—	—
3	Submit a manuscript to an EI Chinese journal (under review, first author)	Participate in one national level project and two corporate projects	One patent accepted (not authorized, 2nd inventor).	Internet plus provincial competition silver prize (6th); First-level scholarship once and third-level scholarship once; "Three good" student once.
4	A manuscript submitted to a core Chinese journal (second-round revision).	Participate in one national level project and two corporate projects	One patent accepted (not authorized, 2nd inventor).	Internet plus provincial competition silver prize (4th); Second-level scholarship once and third-level scholarship once.
5	A manuscript accepted by a SCI journal (JCR Q1, second author).	—	—	—
6	—	Participate in two national level projects and one provincial project	One patent accepted (not authorized, 2nd inventor).	Third-level scholarship once.

Table 13. (*Continued*)

Student order	Publication of paper(50%) a_1	Project participation (10%) a_2	Patent and case writing (30%) a_3	Award (10%) a_4
7	Submit two manuscripts to SCI journals (JCR Q1, second author, one is under review and the other is with editor, second author); A manuscript accepted by a core Chinese journal.	Participate in one university project.	One patent authorized (3rd inventor)	One paper was awarded the second prize in a conference.
8	—	Participate in one national level project.	One patent under actual review (not authorized, 2nd inventor).	Second-level scholarship once.
9	Submit a manuscript to a SCI journal (JCR Q1, under review, second author).	Participate in one national level project.	—	Second-level scholarship once and third-level scholarship once.
10	Submit a manuscript to a SCI journal (JCR Q2, under review, second author).	Participate in one university level project.	One patent authorized (2nd inventor)	Second-level scholarship twice.
11	Submit a manuscript to a SCI journal (JCR Q1, waiting for editorial decision, second author).	Participate in one national level project and one provincial project.	Two patents under actual review (not authorized, one 2nd inventor, one 3rd inventor).	—
12	—	Participate in four projects (level unknown)	—	—

Table 13. (*Continued*)

Student order	Publication of paper(50%) a_1	Project participation (10%) a_2	Patent and case writing (30%) a_3	Award (10%) a_4
13	A manuscript accepted by a conference. (Fourth author)	Participate in one national level project and one provincial project.	Two patents under actual review (not authorized, one 2nd inventor); One patent authorized (4th inventor).	Internet plus university competition third prize (1st).
14	Submit a manuscript to a Chinese core journal (chief-editor editing, corresponding author).	Participate in two corporate projects.	One patent under preliminary review (not authorized, 2nd inventor); Two computer software copyrights.	Internet plus provincial competition silver prize (1st).
15	Submit a manuscript to a Chinese core journal (submitting stage, first author)	Participate in one national level project.	One patent accepted (not authorized, 2nd inventor)	—
16	Submit a manuscript to a SCI journal (under review, second author).	Participate in two national level projects.	One patent accepted (not authorized, 2nd inventor)	—
17	Two manuscripts accepted by SCI journals (JCR Q1, second author); One manuscript accepted by a conference (second author).	Participate in two national level projects, and one provincial project.	One patent authorized (2nd inventor); One patent under actual review (not authorized, 2nd inventor).	Second-level scholarship once.

Table 14. The DPRs of a_1 on x_{12} by experts.

Attribute		a_1 (Publication of paper)				DPR
State		s_1^1 (x_1 Accept & x_2 Accept)	s_1^2 (x_1 Accept & x_2 Reject)	s_1^3 (x_1 Reject & x_2 Accept)	s_1^4 (x_1 Reject & x_2 Reject)	
Probability	e_1 on x_{12}	$p_1^{1,1} = 0.5$	$p_2^{1,1} = 0.2$	$p_3^{1,1} = 0.2$	$p_4^{1,1} = 0.1$	$d_{12}^{1,1} = \{H_3, 0.1; H_4, 0.2; H_5, 0.5; H_6, 0.2\}$
Assessment		$H_{1,12}^{1,1} = H_5$	$H_{2,12}^{1,1} = H_6$	$H_{3,12}^{1,1} = H_4$	$H_{4,12}^{1,1} = H_3$	
Probability	e_2 on x_{12}	$p_1^{1,2} = 0.6$	$p_2^{1,2} = 0.1$	$p_3^{1,2} = 0.2$	$p_4^{1,2} = 0.1$	$d_{12}^{1,2} = \{H_1, 0.2; H_2, 0.1; H_6, 0.7\}$
Assessment		$H_{1,12}^{1,2} = H_6$	$H_{2,12}^{1,2} = H_6$	$H_{3,12}^{1,2} = H_1$	$H_{4,12}^{1,2} = H_2$	
Probability	e_3 on x_{12}	$p_1^{1,3} = 0.3$	$p_2^{1,3} = 0.1$	$p_3^{1,3} = 0.5$	$p_4^{1,3} = 0.1$	$d_{12}^{1,3} = \{H_1, 0.5; H_2, 0.1; H_5, 0.3; H_7, 0.1\}$
Assessment		$H_{1,12}^{1,3} = H_5$	$H_{2,12}^{1,3} = H_7$	$H_{3,12}^{1,3} = H_1$	$H_{4,12}^{1,3} = H_2$	
Probability	e_4 on x_{12}	$p_1^{1,4} = 0.4$	$p_2^{1,4} = 0.1$	$p_3^{1,4} = 0.2$	$p_4^{1,4} = 0.3$	$d_{12}^{1,4} = \{H_2, 0.2; H_3, 0.3; H_5, 0.1, H_6, 0.4\}$
Assessment		$H_{1,12}^{1,4} = H_6$	$H_{2,12}^{1,4} = H_5$	$H_{3,12}^{1,4} = H_2$	$H_{4,12}^{1,4} = H_3$	
Probability	e_5 on x_{12}	$p_1^{1,5} = 0.4$	$p_2^{1,5} = 0.1$	$p_3^{1,5} = 0.1$	$p_4^{1,5} = 0.4$	$d_{12}^{1,5} = \{H_3, 0.4; H_4, 0.1; H_5, 0.5\}$
Assessment		$H_{1,12}^{1,5} = H_5$	$H_{2,12}^{1,5} = H_5$	$H_{3,12}^{1,5} = H_4$	$H_{4,12}^{1,5} = H_3$	

Table 15. DPR matrix D^1 provided by expert e_1.

	a_1	a_2	a_3	a_4
x_{12}	$\{H_3, 0.1; H_4, 0.2; H_5, 0.5, H_6, 0.2\}$	$\{H_7, 1.0\}$	$\{H_7, 1.0\}$	$\{H_7, 1.0\}$
x_{23}	$\{H_5, 0.2; H_6, 0.6; H_7, 0.2\}$	$\{H_1, 1.0\}$	$\{H_1, 1.0\}$	$\{H_1, 1.0\}$
x_{34}	$\{H_2, 0.5; H_3, 0.5\}$	$\{H_4, 1.0\}$	$\{H_1, 0.3; H_4, 0.5; H_7, 0.2\}$	$\{H_3, 0.4; H_4, 0.5; H, 0.1\}$
x_{45}	$\{H_1, 0.8; H_2, 0.2\}$	$\{H_7, 0.8; H, 0.2\}$	$\{H_5, 0.7; H_7, 0.3\}$	$\{H_7, 0.8; H, 0.2\}$
x_{56}	$\{H_7, 1.0\}$	$\{H_1, 0.9; H, 0.1\}$	$\{H_1, 0.1; H_3, 0.9\}$	$\{H_1, 1.0\}$
x_{67}	$\{H_1, 1.0\}$	$\{H_6, 0.8; H, 0.2\}$	$\{H_4, 0.9; H_7, 0.1\}$	$\{H_3, 0.8; H, 0.2\}$
x_{78}	$\{H_7, 1.0\}$	$\{H_3, 0.8; H, 0.2\}$	$\{H_1, 0.3; H_3, 0.7\}$	$\{H_5, 0.7; H, 0.3\}$
x_{89}	$\{H_1, 0.4; H_2, 0.6\}$	$\{H_4, 1.0\}$	$\{H_5, 0.7; H_7, 0.3\}$	$\{H_3, 0.8; H_4, 0.2\}$
x_{910}	$\{H_3, 0.2; H_4, 0.4; H_6, 0.4, H_7, 0.1\}$	$\{H_4, 0.7; H, 0.3\}$	$\{H_4, 1.0\}$	$\{H_5, 0.8; H, 0.2\}$
x_{1011}	$\{H_1, 0.4; H_2, 0.3; H_3, 0.1, H_6, 0.2\}$	$\{H_2, 0.5; H_3, 0.4, H, 0.1\}$	$\{H_1, 0.1; H_2, 0.5; H_3, 0.4\}$	$\{H_7, 1.0\}$
x_{1112}	$\{H_4, 0.2, H_6, 0.8\}$	$\{H_6, 0.5; H, 0.5\}$	$\{H_5, 0.4; H_6, 0.5; H_7, 0.1\}$	$\{H_4, 1.0\}$
x_{1213}	$\{H_1, 1.0\}$	$\{H_3, 0.2; H_5, 0.2, H, 0.6\}$	$\{H_1, 0.7; H_2, 0.3\}$	$\{H_1, 1.0\}$
x_{1314}	$\{H_2, 0.7; H_6, 0.3\}$	$\{H_4, 0.1; H_5, 0.4; H_6, 0.5\}$	$\{H_5, 0.2; H_6, 0.3; H_7, 0.5\}$	$\{H_2, 0.5; H_3, 0.4; H, 0.1\}$
x_{1415}	$\{H_3, 0.2; H_4, 0.1; H_5, 0.3, H_6, 0.4\}$	$\{H_4, 0.3; H_5, 0.6, H, 0.1\}$	$\{H_1, 0.1; H_4, 0.8; H_7, 0.1\}$	$\{H_7, 1.0\}$
x_{1516}	$\{H_2, 0.2; H_3, 0.4; H_4, 0.2, H_6, 0.2\}$	$\{H_3, 0.4; H_4, 0.4, H, 0.2\}$	$\{H_1, 0.1; H_4, 0.8; H_7, 0.1\}$	$\{H_4, 1.0\}$
x_{1617}	$\{H_1, 0.5; H_2, 0.5\}$	$\{H_3, 0.2; H_4, 0.5, H, 0.3\}$	$\{H_1, 0.2; H_2, 0.5; H_3, 0.2, H_4, 0.1\}$	$\{H_1, 1.0\}$

Table 16. The collective DPRs.

	e_1	e_2	e_3	e_4	e_5
x_{12}	$\{H_3, 0.05; H_4, 0.10; H_5, 0.25; H_6, 0.10; H_7, 0.50\}$	$\{H_1, 0.10; H_2, 0.05; H_6, 0.35; H_7, 0.50\}$	$\{H_1, 0.25; H_2, 0.05; H_5, 0.48; H_7, 0.05; H, 0.17\}$	$\{H_2, 0.10; H_3, 0.15; H_5, 0.05; H_6, 0.20; H_7, 0.50\}$	$\{H_3, 0.20; H_4, 0.05; H_5, 0.25; H_7, 0.50\}$
x_{23}	$\{H_1, 0.50; H_5, 0.10; H_6, 0.30; H_7, 0.10\}$	$\{H_1, 0.40; H_5, 0.05; H_6, 0.15; H_7, 0.30; H, 0.10\}$	$\{H_1, 0.34; H_5, 0.20; H_6, 0.25; H_7, 0.05; H, 0.16\}$	$\{H_1, 0.41; H_5, 0.05; H_6, 0.05; H_7, 0.40; H, 0.09\}$	$\{H_1, 0.39; H_5, 0.05; H_6, 0.15; H_7, 0.30; H, 0.11\}$
x_{34}	$\{H_1, 0.09; H_2, 0.25; H_3, 0.29; H_4, 0.25; H_5, 0.05; H_7, 0.06; H, 0.01\}$	$\{H_1, 0.20; H_2, 0.06; H_3, 0.33; H_4, 0.51; H_5, 0.06; H_6, 0.03; H, 0.01\}$	$\{H_1, 0.30; H_2, 0.29; H_4, 0.35; H_7, 0.06\}$	$\{H_1, 0.09; H_2, 0.25; H_3, 0.29; H_4, 0.25; H_5, 0.04; H_7, 0.06; H, 0.02\}$	$\{H_1, 0.24; H_2, 0.35; H_2, 0.03; H_4, 0.10; H_5, 0.05; H_6, 0.18; H_7, 0.03; H, 0.02\}$
x_{45}	$\{H_1, 0.40; H_2, 0.10; H_5, 0.21; H_7, 0.25; H, 0.04\}$	$\{H_1, 0.45; H_2, 0.05; H_6, 0.15; H_7, 0.35\}$	$\{H_1, 0.35; H_2, 0.15; H_5, 0.24; H_7, 0.26\}$	$\{H_1, 0.45; H_2, 0.05; H_4, 0.24; H_7, 0.26\}$	$\{H_1, 0.45; H_2, 0.05; H_5, 0.24; H_7, 0.25; H, 0.01\}$
x_{56}	$\{H_1, 0.22; H_3, 0.27; H_7, 0.50; H, 0.01\}$	$\{H_1, 0.25; H_3, 0.24; H_7, 0.50; H, 0.01\}$	$\{H_1, 0.27; H_4, 0.21; H_7, 0.50; H, 0.02\}$	$\{H_1, 0.22; H_3, 0.27; H_7, 0.50; H, 0.01\}$	$\{H_1, 0.25; H_4, 0.24; H_7, 0.50; H, 0.01\}$
...	...	...	...	...	...
x_{1516}	$\{H_1, 0.03; H_2, 0.10; H_3, 0.24; H_4, 0.48; H_6, 0.10; H_7, 0.03; H, 0.02\}$	$\{H_1, 0.10; H_2, 0.03; H_3, 0.30; H_4, 0.41; H_5, 0.10; H_7, 0.03; H, 0.03\}$	$\{H_2, 0.36; H_3, 0.04; H_4, 0.36; H_5, 0.15; H_6, 0.06; H, 0.03\}$	$\{H_1, 0.03; H_2, 0.20; H_3, 0.25; H_4, 0.38; H_5, 0.10; H_6, 0.03; H, 0.01\}$	$\{H_1, 0.06; H_2, 0.05; H_3, 0.20; H_4, 0.46; H_6, 0.15; H_7, 0.06; H, 0.02\}$
x_{1617}	$\{H_1, 0.41; H_2, 0.40; H_3, 0.08; H_4, 0.08; H, 0.03\}$	$\{H_1, 0.46; H_2, 0.21; H_3, 0.22; H_4, 0.05; H_5, 0.03; H, 0.03\}$	$\{H_1, 0.46; H_2, 0.41; H_3, 0.04; H_4, 0.08; H, 0.01\}$	$\{H_1, 0.39; H_2, 0.39; H_3, 0.14; H_4, 0.06; H, 0.02\}$	$\{H_1, 0.41; H_2, 0.12; H_3, 0.41; H_4, 0.05; H, 0.01\}$

Table 17. The score values.

	e_1	e_2	e_3	e_4	e_5
x_{12}	[0.63, 0.63]	[0.61, 0.61]	[−0.261, 0.079]	[0.54, 0.54]	[0.515, 0.515]
x_{23}	[−0.16,−0.16]	[−0.08, 0.12]	[−0.215, 0.105]	[−0.05, 0.13]	[−0.08, 0.14]
x_{34}	[−0.287, −0.267]	[−0.312, −0.292]	[−0.443, −0.443]	[−0.3, −0.26]	[−0.343, −0.303]
x_{45}	[−0.197,−0.117]	[−0.03, −0.03]	[−0.123, −0.123]	[−0.225, −0.225]	[−0.173, −0.153]
x_{56}	[0.189, 0.209]	[0.168, 0.188]	[0.21, 0.25]	[0.189, 0.209]	[0.24, 0.26]
x_{67}	[−0.478, −0.398]	[−0.353, −0.273]	[−0.398, −0.298]	[−0.464,−0.404]	[−0.355, −0.295]
x_{78}	[0.294, 0.394]	[0.246, 0.406]	[0.331, 0.471]	[0.215, 0.395]	[0.371, 0.511]
x_{89}	[−0.281, −0.281]	[−0.261, −0.241]	[−0.348, −0.308]	[−0.308, −0.308]	[−0.333, −0.313]
x_{910}	[0.155, 0.255]	[0.227, 0.267]	[0.143, 0.203]	[0.233, 0.333]	[0.117, 0.217]
x_{1011}	[−0.378, −0.358]	[−0.359, −0.319]	[−0.343, −0.343]	[−0.313, −0.273]	[−0.359,−0.319]
x_{1112}	[0.436, 0.536]	[0.498, 0.618]	[0.437, 0.537]	[0.53, 0.55]	[0.634, 0.714]
x_{1213}	[−0.933, −0.813]	[−0.914, −0.814]	[−0.892, −0.812]	[−0.948, −0.828]	[−0.923, −0.823]
x_{1314}	[0.021, 0.041]	[0.109, 0.169]	[−0.063, 0.037]	[0.019, 0.079]	[−0.108, −0.068]
x_{1415}	[0.263, 0.283]	[0.247, 0.267]	[0.128, 0.188]	[0.218, 0.238]	[0.126, 0.186]
x_{1516}	[−0.092, −0.052]	[−0.181, −0.121]	[−0.207, −0.147]	[−0.204, −0.184]	[−0.01, 0.03]
x_{1617}	[−0.744,−0.684]	[−0.694, −0.634]	[−0.769, −0.749]	[−0.725, −0.685]	[−0.627, −0.607]

Table 18. The trust scores and importance of experts.

	e_1			e_2			e_3			e_4			e_5		
	t_{kh}	d_{kh}	ts_{kh}	t_{kh}	d_{kh}	ts_{kh}	t_{kh}	d_{kh}	ts_{kh}	t_{kh}	d_{kh}	ts_{kh}	t_{kh}	d_{kh}	ts_{kh}
e_1		—		0.80	0.10	0.85	0.90	0.00	0.95	0.50	0.20	0.65	0.60	0.10	0.75
e_2	0.40	0.50	0.45		—		0.80	0.10	0.85	0.60	0.20	0.70	0.60	0.30	0.65
e_3	0.50	0.40	0.55	0.40	0.40	0.50		—		0.40	0.50	0.45	0.70	0.10	0.80
e_4	0.60	0.20	0.70	0.70	0.10	0.80	0.80	0.10	0.85		—		0.50	0.20	0.65
e_5	0.30	0.40	0.45	0.80	0.10	0.85	0.90	0.10	0.90	0.60	0.00	0.80		—	
R_k		0.152			0.212			0.251			0.184			0.201	

are computed by Eq. (7). Further, using Eqs. (32)–(34) to compute the importance of experts as $R = \{R_k | k = 1, \dots, K\}$. All of the above process are shown in Table 18.

Step 7–Step 10: CRP based on classification-oriented GDM and generation final classification of all alternatives

In the first round, $\mathrm{Diss}_q^{k,k'}$, $\mathrm{Diss}^{k,k'}$, Diss^k are calculated by Eqs. (28), (29), (31). For example, $\mathrm{Diss}_1^{1,2} = 0, \mathrm{Diss}_2^{1,2} = 0.4, \mathrm{Diss}_3^{1,2} = 0.375, \mathrm{Diss}_4^{1,2} = 0.5, \mathrm{Diss}^{1,2} = 0.319, \mathrm{Diss}^1 = 0.316$. Then, the Dissimilarity of the group is generated by using Eq. (35). Due to $\mathrm{Diss}^G = 0.338 > \delta = 0.2$, using Eq. (30) to calculate dis_m^C such as $\mathrm{dis}_1^C = 1$, $\mathrm{dis}_2^C = 0.4$ and so on. According to $\mathrm{dis}_{m^*}{}^C = \max_{m=1,\dots,M} \mathrm{dis}_m^C$ and $\max_{k,k'=1,\dots,K} \mathrm{dis}_{m^*}{}^C_{|e_k - e_{k'}|}$, m^* could be identified as x_1, e_k and $e_{k'}$ could be e_1 & $e_3/e_2\&e_3/e_3\&e_5$. Because $R_1 < R_3, R_2 < R_3, R_3 > R_5$ and e_3 is opinion leader e_{OL}, then $C_1^{1_1} C_1^{2_1} C_1^{5_1}$ could be adjusted towards $C_3^{3_1}$. Similarly, each iteration could be the same as the above process and totally shown in Table 19. Finally, the classification is $C_1 = \{x_{17}, x_7\}$, $C_2 = \{x_{11}, x_5, x_9, x_{10}, x_{16}\}$, $C_3 = \{x_{15}, x_{13}, x_4, x_3, x_2, x_{14}, x_1, x_6\}$, $C_4 = \{x_8, x_{12}\}$.

5.2.2. *Sensitivity analysis*

Considering a different weight assignment for the attributes, where $w_1 = 0.8$, $w_2 = 0.05$, $w_3 = 0.1$, $w_4 = 0.05$, which places greater emphasis on the publication of paper (a_1). Based on this weight setting, the iteration process is shown in Table 20, which finally generates the classification results as $C_1 = \{x_{17}, x_7\}$, $C_2 = \{x_{11}, x_5, x_9, x_1, x_2\}$, $C_3 = \{x_{10}, x_{16}, x_{15}, x_{13}, x_4, x_3, x_{14}, x_6\}$, $C_4 = \{x_8, x_{12}\}$. Comparatively, the weight setting in Sec. 5.2.1 tends to focus on a comprehensive evaluation of the attributes. In this scenario, Student 10 (x_{10}) and Student 16 (x_{16}) perform average in both paper publication (a_1) and patent authorization (a_3), which are the two heavily weighted attributes. Therefore, they are classified in the second-class scholarship category (C_2). However, the weight setting in the Sec. 5.2.2 leans towards a paper-dominated evaluation, aligning with the current trend of "publish or perish". As a result, Student 1 (x_1) and Student 2 (x_2) outperform Student 10 (x_{10}) and Student 16 (x_{16}) in terms of paper publication and are therefore classified in the second-class scholarship category (C_2). This illustrates that the attribute weight settings in this study should be flexibly adjusted according to the specific application decision context.

5.3. *Comparative analysis*

In this section, comparative analysis is conducted on the proposed method with some existent CRP and information fusion methods.

5.3.1. *Compared with not considering unpredictability*

The methodology described in Sec. 3.2 introduces an approach to GDM, integrating the unpredictability inherent in the future. This method significantly diverges from traditional deterministic models that typically assume a static environment with predictable outcomes. This section, the comparative analysis will delineate the characteristics of incorporating unpredictability into the evaluation process.

On the one hand, the consideration of unpredictability acknowledges the uncertainty of future events, thereby offering a more realistic and flexible framework for decision-making. In contrast, static models fails to adequately capture the complex nature of real-world scenarios, including the unknown result in academic publication issue. For instance, in Table 14, if the experts have not considered four states for a_1, there would be more belief degree distributed towards ignorance. This leads to even more information being lost.

On the other hand, by accounting for multiple potential outcomes and their associated probabilities, this approach facilitates a deeper analysis of alternatives on attributes. And it promotes a more nuanced understanding of risk by identifying and assessing the likelihood of various outcomes.

In sum, the decision to incorporate unpredictability into the evaluation of alternatives represents a trade-off between increased realism and complexity. While this approach can significantly enhance the depth and quality of decision-making by accommodating the dynamic nature of real-world scenarios, it also introduces challenges related to complexity, subjectivity, and communication. Decision-makers must carefully weigh these factors by considering the specific context and requirements of their evaluations to determine the most appropriate methodology.

5.3.2. *Comparison with ER approach for the aggregation of attribute assessments*

To compare the proposed weighted average evidential fusion rule with the ER approach for the aggregation of attribute assessments, we analyze the outcomes by using the numerical case in Sec. 5.2.

First, both methods are capable of effectively aggregating DPRs under certain conditions, thus providing a foundational level of reliability and validity to their assessments. However, as the weights of attributes change, the proposed method tends to yield a larger combined uncertainty compared with the ER approach. This

Table 19. The iteration processes of initial weight setting.

The 1st round						
Expert		e_1	e_2	e_3	e_4	e_5
Alternatives ranking		$x_{17} \succ x_1 \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{10} \succ x_{16} \succ x_{13} \succ x_4 \succ x_3 \succ x_2 \succ x_{14} \succ x_{15} \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_1 \succ x_7 \succ x_{11} \succ x_2 \succ x_{16} \succ x_5 \succ x_9 \succ x_{10} \succ x_{13} \succ x_{14} \succ x_{15} \succ x_4 \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{16} \succ x_4 \succ x_1 \succ x_{10} \succ x_2 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_1 \succ x_{16} \succ x_{15} \succ x_{14} \succ x_{11} \succ x_5 \succ x_2 \succ x_9 \succ x_{10} \succ x_4 \succ x_{13} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_1 \succ x_{11} \succ x_7 \succ x_5 \succ x_{16} \succ x_9 \succ x_2 \succ x_{10} \succ x_{14} \succ x_{15} \succ x_4 \succ x_{13} \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$
Category	C_1	x_{17}, x_1	x_{17}, x_1	x_{17}, x_7	x_{17}, x_7	x_{17}, x_1
	C_2	$x_7, x_{11}, x_5, x_9, x_{10}$	$x_7, x_{11}, x_2, x_{16}, x_5$	$x_{11}, x_5, x_9, x_{16}, x_4$	$x_1, x_{16}, x_{15}, x_{14}, x_{11}$	$x_{11}, x_7, x_5, x_{16}, x_9$
	C_3	$x_{16}, x_{13}, x_4, x_3, x_2, x_{14}, x_{15}, x_6$	$x_9, x_{10}, x_{13}, x_{14}, x_{15}, x_4, x_3, x_8$	$x_1, x_{10}, x_2, x_{14}, x_{13}, x_{15}, x_3, x_6$	$x_5, x_2, x_9, x_{10}, x_4, x_{13}, x_3, x_6$	$x_2, x_{10}, x_{14}, x_{15}, x_4, x_{13}, x_3, x_8$
	C_4	x_8, x_{12}	x_6, x_{12}	x_8, x_{12}	x_8, x_{12}	x_6, x_{12}
Diss^k		0.316	0.334	0.334	0.412	0.294
Diss^G				**0.338**		
The 2nd round						
Expert		e_1	e_2	e_3	e_4	e_5
Alternatives ranking		$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{10} \succ x_{16} \succ x_1 \succ x_{13} \succ x_4 \succ x_3 \succ x_2 \succ x_{14} \succ x_{15} \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_2 \succ x_{16} \succ x_5 \succ x_9 \succ x_1 \succ x_{10} \succ x_{13} \succ x_{14} \succ x_{15} \succ x_4 \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{16} \succ x_4 \succ x_1 \succ x_{10} \succ x_2 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{16} \succ x_4 \succ x_1 \succ x_{10} \succ x_2 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_{11} \succ x_7 \succ x_5 \succ x_{16} \succ x_9 \succ x_2 \succ x_1 \succ x_{10} \succ x_{14} \succ x_{15} \succ x_4 \succ x_{13} \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$
Category	C_1	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7	x_{17}, x_{11}
	C_2	$x_{11}, x_5, x_9, x_{10}, x_{16}$	$x_{11}, x_2, x_{16}, x_5, x_9$	$x_{11}, x_5, x_9, x_{16}, x_4$	$x_1, x_{16}, x_{15}, x_{14}, x_{11}$	$x_7, x_5, x_{16}, x_9, x_2$
	C_3	$x_1, x_{13}, x_4, x_3, x_2, x_{14}, x_{15}, x_6$	$x_1, x_{10}, x_{13}, x_{14}, x_{15}, x_4, x_3, x_8$	$x_1, x_{10}, x_2, x_{14}, x_{13}, x_{15}, x_3, x_6$	$x_5, x_2, x_9, x_{10}, x_4, x_{13}, x_3, x_6$	$x_1, x_{10}, x_{14}, x_{15}, x_4, x_{13}, x_3, x_8$
	C_4	x_8, x_{12}	x_6, x_{12}	x_8, x_{12}	x_8, x_{12}	x_6, x_{12}
Diss^k		0.243	0.262	0.243	0.365	0.393
Diss^G				**0.300**		

Table 19. (*Continued*)

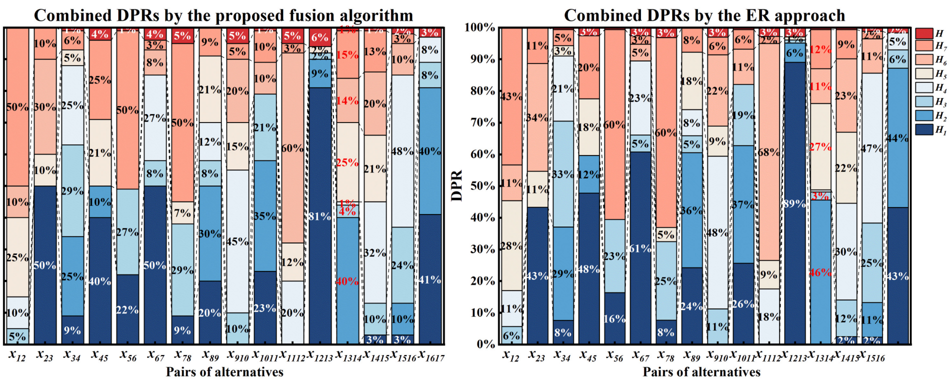
The 3rd round						
Expert		e_1	e_2	e_3	e_4	e_5
Alternatives ranking		$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{10} \succ x_{16} \succ x_1 \succ x_{13} \succ x_4 \succ x_3 \succ x_2 \succ x_{14} \succ x_{15} \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_2 \succ x_{16} \succ x_5 \succ x_9 \succ x_1 \succ x_{10} \succ x_{13} \succ x_{14} \succ x_{15} \succ x_4 \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{16} \succ x_4 \succ x_1 \succ x_{10} \succ x_2 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{16} \succ x_4 \succ x_1 \succ x_{10} \succ x_2 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_{11} \succ x_7 \succ x_5 \succ x_{16} \succ x_9 \succ x_2 \succ x_1 \succ x_{10} \succ x_{14} \succ x_{15} \succ x_4 \succ x_{13} \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$
Category	C_1	x_{17}, x_7	$x_{17}, x_7,$	x_{17}, x_7	x_{17}, x_7	x_{17}, x_{11}
	C_2	$x_{11}, x_5, x_9, x_{10}, x_{16}$	$x_{11}, x_2, x_{16}, x_5, x_9$	$x_{11}, x_5, x_9, x_{16}, x_4$	$x_1, x_{16}, x_{15}, x_{14}, x_{11}$	$x_7, x_5, x_{16}, x_9, x_2$
	C_3	$x_1, x_{13}, x_4, x_3, x_2, x_{14}, x_{15}, x_6$	$x_1, x_{10}, x_{13}, x_{14}, x_{15}, x_4, x_3, x_6$	$x_1, x_{10}, x_2, x_{14}, x_{13}, x_{15}, x_3, x_6$	$x_5, x_2, x_9, x_{10}, x_4, x_{13}, x_3, x_6$	$x_1, x_{10}, x_{14}, x_{15}, x_4, x_{13}, x_3, x_6$
	C_4	x_8, x_{12}	x_8, x_{12}	x_8, x_{12}	x_8, x_{12}	x_8, x_{12}
Diss^k		0.165	0.145	0.165	0.287	0.276
Diss^G				**0.206**		
The 4th round						
Expert		e_1	e_2	e_3	e_4	e_5
Alternatives ranking		$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{10} \succ x_{16} \succ x_1 \succ x_{13} \succ x_4 \succ x_3 \succ x_2 \succ x_{14} \succ x_{15} \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_{10} \succ x_{16} \succ x_5 \succ x_9 \succ x_1 \succ x_2 \succ x_{13} \succ x_{14} \succ x_{15} \succ x_4 \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{16} \succ x_4 \succ x_1 \succ x_{10} \succ x_2 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_{16} \succ x_4 \succ x_1 \succ x_{10} \succ x_2 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_{11} \succ x_7 \succ x_5 \succ x_{16} \succ x_9 \succ x_{10} \succ x_1 \succ x_2 \succ x_{14} \succ x_{15} \succ x_4 \succ x_{13} \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$
Category	C_1	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7	x_{17}, x_{11}
	C_2	$x_{11}, x_5, x_9, x_{10}, x_{16}$	$x_{11}, x_{10}, x_{16}, x_5, x_9$	$x_{11}, x_5, x_9, x_{16}, x_4$	$x_1, x_{16}, x_{15}, x_{14}, x_{11}$	$x_7, x_5, x_{16}, x_9, x_{10}$
	C_3	$x_1, x_{13}, x_4, x_3, x_2, x_{14}, x_{15}, x_6$	$x_1, x_2, x_{13}, x_{14}, x_{15}, x_4, x_3, x_6$	$x_1, x_{10}, x_2, x_{14}, x_{13}, x_{15}, x_3, x_6$	$x_5, x_2, x_9, x_{10}, x_4, x_{13}, x_3, x_6$	$x_1, x_2, x_{14}, x_{15}, x_4, x_{13}, x_3, x_6$
	C_4	x_8, x_{12}	x_8, x_{12}	x_8, x_{12}	x_8, x_{12}	x_8, x_{12}
Diss^k		0.125	0.125	0.165	0.287	0.256
Diss^G				**0.191**		

Table 20. The iteration processes of weight setting under sensitivity analysis.

The 1st round						
Expert		e_1	e_2	e_3	e_4	e_5
Alternatives ranking		$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_1 \succ x_9 \succ x_{10} \succ x_{16} \succ x_4 \succ x_2 \succ x_3 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_2 \succ x_5 \succ x_1 \succ x_{16} \succ x_9 \succ x_{10} \succ x_{14} \succ x_{15} \succ x_{13} \succ x_4 \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_2 \succ x_1 \succ x_{16} \succ x_{10} \succ x_4 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_5 \succ x_1 \succ x_{16} \succ x_{15} \succ x_{14} \succ x_{11} \succ x_2 \succ x_9 \succ x_{10} \succ x_4 \succ x_{13} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_5 \succ x_1 \succ x_{11} \succ x_9 \succ x_2 \succ x_{10} \succ x_{16} \succ x_{14} \succ x_{15} \succ x_4 \succ x_{13} \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$
Category	C_1	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7
	C_2	$x_{11}, x_5, x_1, x_9, x_{10}$	$x_{11}, x_2, x_5, x_1, x_{16}$	$x_{11}, x_5, x_9, x_2, x_1$	$x_5, x_1, x_{16}, x_{15}, x_{14}$	$x_5, x_1, x_{11}, x_9, x_2$
	C_3	$x_{16}, x_4, x_2, x_3, x_{14}, x_{13}, x_{15}, x_6$	$x_9, x_{10}, x_{14}, x_{15}, x_{13}, x_4, x_3, x_8$	$x_{16}, x_{10}, x_4, x_{14}, x_{13}, x_{15}, x_3, x_6$	$x_{11}, x_2, x_9, x_{10}, x_4, x_{13}, x_3, x_6$	$x_{10}, x_{16}, x_{14}, x_{15}, x_4, x_{13}, x_3, x_8$
	C_4	x_8, x_{12}	$x_6 \succ x_{12}$	x_8, x_{12}	x_8, x_{12}	x_6, x_{12}
Diss^k		0.220	0.239	0.180	0.302	0.219
Diss^G				**0.229**		
The 2nd round						
Expert		e_1	e_2	e_3	e_4	e_5
Alternatives ranking		$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_1 \succ x_9 \succ x_2 \succ x_{10} \succ x_{16} \succ x_4 \succ x_3 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_2 \succ x_5 \succ x_1 \succ x_{16} \succ x_9 \succ x_{10} \succ x_{14} \succ x_{15} \succ x_{13} \succ x_4 \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$	$x_{17} \succ x_7 \succ x_{11} \succ x_5 \succ x_9 \succ x_2 \succ x_1 \succ x_{16} \succ x_{10} \succ x_4 \succ x_{14} \succ x_{13} \succ x_{15} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_5 \succ x_1 \succ x_{16} \succ x_{15} \succ x_2 \succ x_{14} \succ x_{11} \succ x_9 \succ x_{10} \succ x_4 \succ x_{13} \succ x_3 \succ x_6 \succ x_8 \succ x_{12}$	$x_{17} \succ x_7 \succ x_5 \succ x_1 \succ x_{11} \succ x_9 \succ x_2 \succ x_{10} \succ x_{16} \succ x_{14} \succ x_{15} \succ x_4 \succ x_{13} \succ x_3 \succ x_8 \succ x_6 \succ x_{12}$
Category	C_1	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7	x_{17}, x_7
	C_2	$x_{11}, x_5, x_1, x_9, x_2$	$x_{11}, x_2, x_5, x_1, x_{16}$	$x_{11}, x_5, x_9, x_2, x_1$	$x_5, x_1, x_{16}, x_{15}, x_2$	$x_5, x_1, x_{11}, x_9, x_2$
	C_3	$x_{10}, x_{16}, x_4, x_3, x_{14}, x_{13}, x_{15}, x_6$	$x_9, x_{10}, x_{14}, x_{15}, x_{13}, x_4, x_3, x_8$	$x_{16}, x_{10}, x_4, x_{14}, x_{13}, x_{15}, x_3, x_6$	$x_{14}, x_{11}, x_9, x_{10}, x_4, x_{13}, x_3, x_6$	$x_{10}, x_{16}, x_{14}, x_{15}, x_4, x_{13}, x_3, x_8$
	C_4	x_8, x_{12}	$x_6 \succ x_{12}$	x_8, x_{12}	$x_8, x_{12},$	x_6, x_{12}
Diss^k		0.139	0.198	0.139	0.220	0.178
Diss^G				**0.174**		

phenomenon is particularly evident when the uncertainty of one attribute is significantly high. The proposed method's sensitivity to attribute uncertainty enables a more nuanced representation of uncertainty in the aggregated result. In addition, the ER approach's tendency to amplify the larger focal element contravenes the axiom that the fused DPRs should remain identical to the original DPRs if all original DPRs are identical. In contrast, the proposed method adheres to this axiom, demonstrating its capability to maintain the integrity of identical attribute assessments across a broad scale. In conclusion, the comparison between the proposed method and the ER approach underscores the former's advantages in managing uncertainty, maintaining consistency in attribute aggregation, and adhering to logical axioms that underpin rational decision-making. These characteristics render the proposed method a compelling alternative for applications requiring nuanced and reliable fusion of attribute assessments.

Tables C1 and C2 in Supplementary Materials show the results of the collective DPRs by using the proposed weighted average evidential fusion rule and the ER rule. For a more intuitive comparison, Figs. 7(a) and 7(b) display the combined DPRs of e_1 by the two methods. Given that the relative weight of a_1 is 0.5, which is higher than the other three attributes (0.1, 0.3, 0.1), the ER approach tends to magnify the belief degree of a_1 in almost each focal element. For example, considering the DPRs of x_{1314}, the original DPRs by expert e_1 on the four attributes are as follows: $d_{1314}^{1,1} = \{H_2, 0.7; H_6, 0.3\}$, $d_{1314}^{2,1} = \{H_4, 0.1; H_5, 0.4; H_6, 0.5\}$, $d_{1314}^{3,1} = \{H_5, 0.2; H_6, 0.3; H_7, 0.5\}$, $d_{1314}^{4,1} = \{H_2, 0.5; H_3, 0.4; H, 0.1\}$. And the results by fusing the four attributes through the two methods are depicted in Figs. 7(a) and 7(b). The score interval values on x_{1314} are [0.021, 0.041] by the proposed method and [−0.051, −0.039] by the ER approach. Obviously, the weighted average evidential fusion rule implies that $x_{13} \succ x_{14}$, whereas the ER approach indicates the opposite. Because student 13 has two patents under actual review while student 14 only



(a) (b)

Fig. 7. (a) The combined DPRs of e_1 by the proposed approach. (b) The combined DPRs of e_1 by the ER approach.

has one patent under preliminary review, student 13 is superior than student 14 intuitively.

5.3.3. *Comparison with different CRP methods*

In this subsection, qualitative analysis is undertaken to illustrate the characteristics of the proposed CRP based on classification-oriented GDM with other CRP frameworks. Five pivotal elements are chosen to delineate the differences in feedback mechanism designs: CRP mechanism, preference structure, SNA model, aggregation method, consensus identification, and application (refer to Table 21).

Advantages: (1) As for the generation of individual assessment, apart from directly assessment[4,7,15,17,18] and pairwise assessment[1] without considering uncertain states, the paper introduces a method for generating DPRs on multiple attributes considering both certain and unpredictable situations. It effectively addresses the derivation of original assessments in the presence of multiple uncertain states. (2) Different from the ER approach, the proposed fusion rule for aggregating independent attribute assessments in the form of DPRs is a significant contribution. It not only satisfies the basic properties of a combination algorithm but also resolves some irrational results generated by the ER approach, providing a more reliable aggregation method. (3) The article presents a CRP designed for classification-oriented GDM. By defining a consensus measure based on category and integrating opinion leader together with social network analysis in the identification and adjustment rules, it offers a robust framework for achieving group consensus. At the same time, it avoids frequent modification of the original evaluation matrix, improves the efficiency of reaching consensus, and pays more attention to the results of classification.

Limitations: (1) While the proposed method is theoretically sound, its complexity and the requirement for detailed data (such as the generation of DPRs under unpredictable circumstances and the calculation of consensus measures) might pose challenges for practical application in real-world scenarios where time and resources are limited. (2) The effectiveness of the weighted average evidential fusion rule heavily depends on the accurate assignment of weights to attributes and expert opinions. Misjudgments in weight assignments can lead to unexpected decision-making outcomes. (3) Although an illustrative case study is provided, the application and validation of the proposed approach in a wider range of decision-making contexts and scenarios are not extensively explored. This limits the understanding of its effectiveness across different domains and situations. (4) While the classification-oriented design streamlines adjustments, some LSGDM challenges remain outside our current scope, such as subgroup incompatibility, missing data, or extremely large committees.[49,50] These issues necessitate additional mechanisms like clustering and decentralized feedback to maintain efficiency and consensus integrity in larger groups.

Efficiency: In many traditional consensus models, experts must iteratively refine their detailed pairwise preferences or scores until acceptable consensus threshold is reached. This can become resource-intensive, particularly when the number of alternatives or rounds of feedback is large. By contrast, our classification-oriented approach focuses on targeted adjustments within categories, thus reducing the number of changes needed to achieve consensus. Specifically: (1) The number of feedback rounds often decreases because experts primarily revise the placement of alternatives in categories rather than fine-tuning an entire preference matrix. (2) Operating with category-level consensus can lead to fewer computations per round than global preference adjustments, especially in medium-sized groups.

6. Conclusions

In this paper, the generation of DPR in MAGDM problem under uncertainty is firstly presented. Then, a new weighted average evidence fusion rule is proposed to aggregate the DPRs associated with multiple attributes provided by experts. The fusion rule satisfies basic properties and can overcome some irrational results generated by the ER approach. Classification-oriented group CRP is analyzed subsequently. Identification rule and adjustment rule are proposed based on the consensus measure of category, alternative and expert. The advantage of the proposed group consensus method lies in its easy applicability for real decision problems. The weighted average evidential fusion rule is effective in MAGDM problem where the weights of attributes and the importance of experts are both taken into consideration.

Future research could be done to extend the proposed method to different scenarios, such as classification-oriented LSGDM, which involve complex social network analysis and conflict of interest among individuals. Recent research in opinion dynamics, such as the trust exploration and leadership incubation-based models for social network GDM under a quantum theory perspective,[51] provides an interesting avenue to extend our approach when dealing with large sets of experts. Future work could incorporate these advanced dynamics to estimate or update missing trust relationship and to handle evolving social networks in large-scale environments. Such an integration would also require new algorithms for efficient consensus updating and might benefit from parallel or distributed computing methods. Furthermore, in classification-oriented GDM where each expert only provides original assessment without consensus adjustment process, it is also important to further study how to make a final decision based on these different opinions.

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Table 21. Comparisons of the proposed method with several consensus reaching methods.

CRP mechanism	Reference	Preference structure	SNA	Aggregation method	Consensus identification	Application
Trust chain driven bidirectional feedback mechanism	1	Hesitation fuzzy preference relations (HFPRs)	√	—	Mutual consensus index/Global consensus index	Metaverse virtual community decision-making
Weight penalty mechanism for noncooperative behavior	4	Unbalanced linguistic term sets	√	Symbolic aggregation operation	Euclidean distance among experts	Discipline construction plan selection
Social Trust-Driven Consensus Reaching Model	7	Cardinal assessment	√	Power-weighted average operator	Consensus level and group consensus level	Old urban reconstruction plans selection
Minimum cost consensus with altruism utility constraints	18	Direct assessment	√	—	Psychological distance	Energy-saving target formulation of iron and steel industry
An optimization-based consensus model with bounded confidences	15	Linguistic distribution assessment	—	Collective preference vector	Method for the FMEA team based on the ordinal classes	The problem of crankcase explosion in marine diesel engines
The bounded confidence-based consensus reaching algorithm	17	Multi-granular unbalanced linguistic information	—	Symbolic aggregation operation	Consensus level over the element/ alternative/ individual/ group	Appropriate ERP system selection in the market
A similarity-conformity-based clustering & decentralized feedback mechanism	49	Probabilistic linguistic preference relations	√	Probabilistic linguistic averaging operator	The cohesion degree and subgroup consensus level	Public transportation construction
A large-scale group success likelihood index method problem	50	Fuzzy ratings in LSGDM	—	—	Two optimization models for noncooperative subgroups	Human error probabilities estimation in railway driving process
CRP based on classification-oriented GDM	The proposed method	DPR	√	The weighted average evidential fusion rule	Consensus in category/ alternative/expert level	Scholarship evaluation

Declaration of Interest Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

1. T. Gai, J. Wu, M. Cao, F. Ji, Q. Sun and M. Zhou, Trust chain driven bidirectional feedback mechanism in social network group decision making and its application in metaverse virtual community, *Expert Systems with Applications* **228** (2023) 120369.
2. M. Zhou, Z. Guan, Y. Chen, Z. Zhou, J. Wu and E. Herrera-Viedma, Consistency and consensus reaching process for group decision making based on complete interval distributed preference relations under social network analysis, *Information Fusion* **88** (2022) 126–145.
3. M. Cao, T. Gai, J. Wu, F. Chiclana, Z. Zhang, Y. Dong, E. Herrera -Viedma, F. Herrera, Social network group decision making: Characterization, taxonomy, challenges and future directions from an AI and LLMs perspective, *Information Fusion* **120** (2025) 103107.
4. Q. Sun, F. Chiclana, J. Wu, Y. Liu, C. Liang and E. Herrera-Viedma, Weight penalty mechanism for noncooperative behavior in large-scale group decision making with unbalanced linguistic term sets, *IEEE Transactions on Fuzzy Systems* **31** (2023) 3507–3521.
5. Y.-J. Zhou, M. Zhou, X.-B. Liu, B.-Y. Cheng and E. Herrera-Viedma, Consensus reaching mechanism with parallel dynamic feedback strategy for large-scale group decision making under social network analysis, *Computer & Industrial Engineering* **174** (2022) 108818.
6. K. Kumar and S.-M. Chen, Multiple attribute group decision making based on advanced linguistic intuitionistic fuzzy weighted averaging aggregation operator of linguistic intuitionistic fuzzy numbers, *Information Sciences* **587** (2022) 813–824.
7. P. Liu, Y. Li and P. Wang, Social trust-driven consensus reaching model for multi-attribute group decision making: Exploring social trust network completeness, *IEEE Transactions on Fuzzy Systems* **31** (2023) 3040–3054.
8. Z. Zhang, W. Yu, L. Martínez and Y. Gao, Managing multigranular unbalanced hesitant fuzzy linguistic information in multiattribute large-scale group decision making: A linguistic distribution-based approach, *IEEE Transactions on Fuzzy Systems* **28** (2020) 2875–2889.
9. M. Zhou, Y. Qiao, J. Yang, Y. Zhou, X.-B. Liu and J. Wu, A three-level consensus model for large-scale multi-attribute group decision analysis based on distributed preference relations under social network analysis, *Expert Systems with Applications* **204** (2022) 117603.

10. M. Zhou, Y. Zheng, Y. Chen, B. Cheng, E. Herrera-Viedma and J. Wu, A large-scale group consensus reaching approach considering self-confidence with two-tuple linguistic trust/distrust relationship and its application in life cycle sustainability assessment, *Information Fusion* **94** (2023) 181–199.
11. Z. Chen, J. Chen, Y. Chen, Y. Yang, L. Jin, E. Herrera-Viedma and W. Pedrycz, Large-group failure mode and effects analysis for risk management of angle grinders in the construction industry, *Information Fusion* **97** (2023) 101803.
12. Z. Chen, Z. Zhu, Z. Wang and Y. Tsang, Fairness-aware large-scale collective opinion generation paradigm: A case study of evaluating blockchain adoption barriers in medical supply chain, *Information Sciences* **635** (2023) 257–278.
13. Y. Wang, S. He, D. G. Zamora, X. Pan and L. Martínez, A large scale group three-way decision-based consensus model for site selection of new energy vehicle charging stations, *Expert Systems with Applications* **214** (2023) 119107.
14. M. Zhou, Y.-J. Zhou, X.-B. Liu, J. Wu, H. Fujita and E. Herrera-Viedma, An adaptive two-stage consensus reaching process based on heterogeneous judgments and social relations for large-scale group decision making, *Information Sciences* **644** (2023) 119280.
15. J. Xiao, X. Wang and H. Zhang, Exploring the ordinal classifications of failure modes in the reliability management: An optimization-based consensus model with bounded confidences, *Group Decision and Negotiation* **31** (2022) 49–80.
16. F. Meng, D. Zhao, C. Tan and Z. Li, Ordinal-cardinal consensus analysis for large-scale group decision making with uncertain self-confidence, *Information Fusion* **93** (2023) 344–362.
17. Z. Zhang, Z. Li and Y. Gao, Consensus reaching for group decision making with multi-granular unbalanced linguistic information: A bounded confidence and minimum adjustment-based approach, *Information Fusion* **74** (2021) 96–110.
18. Y. Zhang, X. Chen, W. Pedrycz and Y. Dong, Minimum cost consensus with altruism utility constraints in social network group decision making, *IEEE Transactions on Systems, Man, and Cybernetics: Systems* **53** (2023) 5032–5045.
19. T. Gai, F. Chiclana, W. Jin, M. Zhou and J. Wu, A transformation method of non-cooperative to cooperative behavior by trust propagation in social network group decision making. *IEEE Transactions on Fuzzy Systems* **33** (2025) 2238–2250.
20. L. Li, L. Qiu, X. Liu, Y. Xu and E. Herrera-Viedma, An improved HK model-driven consensus reaching for group decision making under interval-valued fuzzy preference relations with self-confidence, *Computers & Industrial Engineering* **171** (2022) 108438.
21. W. Guo, Z. Gong, W.-G. Zhang and Y. Xu, Minimum cost consensus modeling under dynamic feedback regulation mechanism considering consensus principle and tolerance level, *European Journal of Operational Research* **306** (2023) 1279–1295.
22. H. Zhang, W. Zhu, J. Xiao and H. Liang, Application of a maximum classification consensus approach for construction of a group ordinal classification of applicants in employee recruitment, *Journal of the Operational Research Society* **75** (2024) 742–765.
23. J. Xiao, X. Wang and H. Zhang, Managing classification-based consensus in social network group decision making: An optimization-based approach with minimum information loss, *Information Fusion* **63** (2020) 74–87.
24. Z. Li, Z. Zhang and W. Yu, Consensus reaching for ordinal classification-based group decision making with heterogeneous preference information, *Journal of the Operational Research Society* **75** (2024) 224–245.
25. F. Balderas, E. Fernández, L. Cruz-Reyes, C. Gómez-Santillán and N. Rangel-Valdez, Solving group multi-objective optimization problems by optimizing consensus through multi-criteria ordinal classification, *European Journal of Operational Research* **297** (2022) 1014–1029.

26. Z. Huang, H. Zhang, D. Wang, H. Yu, L. Wang, D. Yu and Y. Peng, Preference-based multi-attribute decision-making method with spherical-Z fuzzy sets for green product design, *Engineering Applications of Artificial Intelligence* **126** (2023) 106767.
27. C. Zhang, W. Lu, Z. Xu and W. Xue, Evaluating power system reform proposals based on the evidential reasoning algorithm with hesitant fuzzy information, *Applied Intelligence* **53** (2023) 26079–26097.
28. Y. Xu, W. Dai, J. Huang, M. Li and E. Herrera-Viedma, Some models to manage additive consistency and derive priority weights from hesitant fuzzy preference relations, *Information Sciences* **586** (2022) 450–467.
29. G. Zhang, Y. Dong and Y. Xu, Consistency and consensus measures for linguistic preference relations based on distribution assessments, *Information Fusion* **17** (2014) 46–55.
30. Y. Zhang, Z. Xu, H. Wang and H. Liao, Consistency-based risk assessment with probabilistic linguistic preference relation, *Applied Soft Computing* **49** (2016) 817–833.
31. Y. Wu, Z. Zhang, G. Kou, H. Zhang, X. Chao, C.-C. Li, Y. Dong and F. Herrera, Distributed linguistic representations in decision making: Taxonomy, key elements and applications, and challenges in data science and explainable artificial intelligence, *Information Fusion* **65** (2021) 165–178.
32. C. Fu, D. Xu and S. Yang, Distributed preference relations for multiple attribute decision analysis, *Journal of the Operational Research Society* **67** (2016) 457–473.
33. M. Zhou, M. Hu, Y. Chen, B. Cheng, J. Wu and E. Herrera-Viedma, Towards achieving consistent opinion fusion in group decision making with complete distributed preference relations, *Knowledge-Based Systems* **236** (2022) 107740.
34. C. Li, Y. Dong, Y. Xu, F. Chiclana, E. Herrera-Viedma and F. Herrera, An overview on managing additive consistency of reciprocal preference relations for consistency-driven decision making and fusion: Taxonomy and future directions, *Information Fusion* **52** (2019) 143–156.
35. Y. Dong, C. Li and F. Herrera, An optimization-based approach to adjusting unbalanced linguistic preference relations to obtain a required consistency level, *Information Sciences* **292** (2015) 27–38.
36. F. Meng, W. Pedrycz and J. Tang, Research on the consistency of additive trapezoidal fuzzy preference relations, *Expert Systems with Applications* **186** (2021) 115837.
37. P. Wang, P. Liu and F. Chiclana, Multi-stage consistency optimization algorithm for decision making with incomplete probabilistic linguistic preference relation, *Information Sciences* **556** (2021) 361–388.
38. A. P. Dempster, Upper and lower probabilities induced by a multivalued mapping, in *Classic Works of the Dempster-Shafer Theory of Belief Functions*, eds. R. R. Yager and L. Liu (Springer, 2008), pp. 57–72.
39. R. R. Yager, On the dempster-shafer framework and new combination rules, *Information Sciences* **41** (1987) 93–137.
40. D. Dubois and H. Prade, La fusion d'informations imprécises, *Traitement Du Signal* **11** (1994) 447–458.
41. F. Smarandache and J. Dezert, Proportional conflict redistribution rules for information fusion, in *Advances and Applications of DSmT for Information Fusion*, eds. F. Smarandache and J. Dezert (American Research Press, 2006), pp. 3–68.
42. J.-B. Yang and D.-L. Xu, On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty, *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans* **32** (2002) 289–304.

43. F. Xiao, GEJS: A generalized evidential divergence measure for multisource information fusion, *IEEE Transactions on Systems, Man, and Cybernetics: Systems* **53** (2023) 2246–2258.
44. F. Xiao, J. Wen and W. Pedrycz, Generalized divergence-based decision making method with an application to pattern classification, *IEEE Transactions on Knowledge and Data Engineering* **35** (2023) 6941–6956.
45. C. Fu, W. Chang, M. Xue and S. Yang, Multiple criteria group decision making with belief distributions and distributed preference relations, *European Journal of Operational Research* **273** (2019) 623–633.
46. J. Wu, R. Xiong and F. Chiclana, Uninorm trust propagation and aggregation methods for group decision making in social network with four tuple information, *Knowledge-Based Systems* **96** (2016) 29–39.
47. J. Yang, Y. Wang, D. Xu and K. S. Chin, The evidential reasoning approach for MADA under both probabilistic and fuzzy uncertainties, *European Journal of Operational Research* **171** (2006) 309–343.
48. J. Yang and D. Xu, Evidential reasoning rule for evidence combination, *Artificial Intelligence* **205** (2013) 1–29.
49. J.-L. Zhou, X.-Y. Chen and H.-W. Wang, A similarity-conformity-based clustering and decentralized feedback mechanism to manage incompatible decision-makers in large-scale group decision-making, *Expert Systems with Applications* **246** (2024) 123209.
50. J.-L. Zhou, Z.-T. Yu and R.-B. Xiao, A large-scale group success likelihood index method to estimate human error probabilities in the railway driving process, *Reliability Engineering & System Safety* **228** (2022) 108809.
51. P. Wang, P. Liu, Y. Li, F. Teng and W. Pedrycz, Trust exploration- and leadership incubation-based opinion dynamics model for social network group decision-making: A quantum theory perspective, *European Journal of Operational Research* **317** (2024) 156–170.