Perspectivism and Wicked Problems - Patterns in the Discovery Process of Leibniz, Bohr, and Turing

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Perspectivism and Wicked Problems - Patterns in the Discovery Process of Leibniz, Bohr, and Turing

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Abstract

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Abstract

This research investigates fundamental patterns in how transformative scientific knowledge emerges and becomes established, examining three pivotal cases: Leibniz's development of calculus, Bohr's formulation of complementarity, and Turing's conceptualisation of computation and artificial intelligence. The study introduces the *o-é-c model* (*Ouverture Ontologique - Épistémè Socialisante - Connaissance Éclairante*) to explain how individual insights become integrated into collective knowledge through structured phases of development.

Drawing on Minsky's conception of knowledge as mental models for problem-solving and Longino's social epistemology, the research demonstrates how wicked problems—those that resist formulation within existing frameworks—catalyse the creation of new knowledge domains. The model reveals how transformative frameworks emerge through three distinct phases: initial conceptual breakthrough, social validation and refinement, and systematic integration into established knowledge.

Analysis of the historical cases reveals remarkable consistency in how new knowledge frameworks develop, despite vast differences in field and context. Each case demonstrates how periods of social upheaval created conditions conducive to fundamental reconceptualisation, how specific mechanisms of social validation shaped the development of new ideas, and how pedagogical tools proved crucial for knowledge transmission.

The thesis makes several original contributions to philosophy of science: it bridges the gap between individual and social accounts of knowledge creation, provides specific mechanisms for how revolutionary insights become established knowledge, and demonstrates the crucial role of pedagogical development in knowledge transmission. The model helps resolve traditional tensions between revolutionary and evolutionary accounts of scientific progress while offering practical insights for addressing contemporary challenges in knowledge creation.

Keywords: *philosophy of science, epistemology, scientific discovery, knowledge creation, wicked problems, social epistemology, scientific progress, paradigm shifts, scientific revolution, perspectivism, history of science, Leibniz, Bohr, Turing, complementarity, calculus, artificial intelligence, pedagogical development, knowledge transmission.*

Academic Search Terms: *knowledge creation theory, social epistemology, scientific discovery process, wicked problems theory, history of science methodology, scientific revolution theory, perspectivism epistemology, knowledge transmission mechanisms, paradigm shift analysis, scientific progress models.*

To Giulia

Everything works and is better with you and because of you

To Steve

Since day 1, you are my constant source of encouragement

Preface & Acknowledgements

March 13 2020 changed everything.

That day, the Administration had told us not to come back to campus on precaution against the spread of a virus. My twin children were in the 9th grade and still in classes that day until a gun threat triggered a lock-down and someone got shot in the school. They were sent home and a further message asked everyone in the state to go on lock down by the pandemic threat. I knew it was not a glitch in the system, the world would not be the same. We planned education going forward: how do we reach out and integrate students living and working remotely? In the following weeks we became fluent in ZOOM and living in lockdown allowed time to think.

Until then and for months I had been drafting a long paper on patterns in Intellectual History stemming from a question that permeated my work: *How do you solve a problem that you have never seen before?* Philosophy became existential while we faced the Covid threat to the world. This was a *super wicked* societal problem, and a second question became cardinal in my thinking: *Is solving problems an individual or a social endeavour?* During a virtual meeting with my father-in-law, he encouraged me to go deeper and to go large and the thought of a PhD in Philosophy of Science was born. My wife smiled and said: “Go for it,” and I began my hunt for an intellectual home for my PhD Thesis until I was accepted at Manchester Metropolitan University several months later with the encouragement of my first Principal Supervisor, Professor Lloyd Strickland.

The most influential people and the most influential books come to your life when you least expect it. And here was my first lesson and the preliminary answers to my questions above: *you ask for help,* and *you cannot do it alone.* And thus, I must acknowledge here those that agreed to help and provided support.

At Dominican University of California, where I have been faculty for 19 years, I found the backing and endorsement of Dr. Mojgan Behmand and Dr. Randall Hall that were fundamental in my acceptance to the Doctoral Program at MMU. I owe them my gratitude for believing I would get to this point.

Dr. Francoise Lepage and Dr. Denise Lucy have been absolutely critical in their multidimensional support to me over the years of my programme particularly on my research trips in 2022 and 2023, but also in multiple ways. The intellectual roots of my thesis include a book that Dr. Lepage gave me years ago: *The Structure of Scientific Revolutions* by Thomas Kuhn, when I began to explore the idea of additions to knowledge.

At Manchester Metropolitan University I owe thanks beyond what I can express to Professor Lloyd Strickland for accepting me as his student and for the manifold ways in which he has been crucial to my PhD program: his extraordinary generosity, professionalism, and mentorship have helped me grow and understand PhD level work in Philosophy, as well as about Leibniz, and the world, organisations, and people working on his legacy. I began to understand some of the immense body of work from Leibniz thanks to Professor Strickland. The majority of the Leibniz texts translations I used in this thesis are his and show here with his permission. Professor Strickland's academic rigor is behind the thoroughness I have aimed at, while my shortcomings are mine alone. He opened doors for me that I could not have been able to do without his help. My gratitude also goes to my second Principal Supervisor, Dr. Christopher Thomas. I also want to recognise Dr. Thomas in his generosity, helpfulness, professionalism and collegiality in hitting the ground running to help me finish my doctoral programme at MMU in a timely and positive manner. I am truly thankful.

Leibniz. At the Gottfried Wilhelm Leibniz Library – Lower Saxony State Library, the Leibniz Archive staff has been supportive, welcoming, and helpful, and in particular I am grateful to Prof. Dr. Michael Kempe and Dr. Charlotte Wahl for their assistance with suggestions at different stages. Their encouragement led me to submit and later present a paper in the International Congress on Leibniz in 2023 in Hannover that is at the heart of Chapter 3 in this thesis. I am particularly indebted to Dr. Siegmund Probst for his invaluable, ever-present unfailing and consistent help, suggestions, tips, access to the Leibniz Manuscripts, discussions on the ideas of the thesis and the historical context of Leibniz life and his work. Dr. Probst was there with encouragement in moments of self-doubt and in moments of excitement with new insights in my project. He is a role-model in his über-generosity with his unparalleled knowledge. Dr. Probst helped me understand Leibniz intellectual milieu and introduced me to critical work from Professor Emily Grosholz, and particularly in 2023, to Dr. Jürgen Renn's *The Evolution of Knowledge: Rethinking Science for the Anthropocene*. While I was working at the Leibniz Archive, Dr. Probst guided me through the safe room where Leibniz's manuscripts are held and showed me some of the lovely preserved handwritten work Leibniz did, moving me to tears of deep emotion, and I now remember it when I teach my Calculus courses at my university. I cannot think about Leibniz work in Mathematics without thinking about Dr. Probst and everything he has done for me and in the academic world. Working at the Leibniz Archive was a productive and thought generating joy. Every day.

Bohr. Since I was studying for my first University degree, back in the early 1980s, I have been interested in the development of Atomic Physics and Quantum Mechanics, and that of course meant exposure to and admiration for the work and ideas of Niels Bohr. From the first day that I walked into the Niels Bohr Institutet at Københavns Universitet (KU) in its famous address at Blegdamsvej 17 in Copenhagen, I felt I was in a heady, stimulating, exhilarating, and privileged opportunity to work in the birthplace of some of the most significant ideas in the History of

Science. In a vivid example of the atmosphere of peer relations and intellectual discussions that permeate the Institute since its creation and influence by Niels Bohr in the 1920s, on day 1 after a warm reception I was gently and firmly encouraged to present and discuss at length my research and thesis, which I did over the next few days with Dr. Christian Joas, Director, Niels Bohr Archive, Københavns Universitet (University of Copenhagen), and Dr. Richard Staley, Hans Rausing Lecturer and Reader in History and Philosophy of Science, University of Cambridge, and Professor at the Department of Science Education, Københavns Universitet (KU). In the middle of one of our conversations, Dr. Joas asked "your work reminds me of the *Forman's Thesis*, are you familiar with it?" "No," I answered, and I was introduced to Paul Forman's work. I felt welcomed, encouraged and prompted to continue in my line of research, and the *Forman's Thesis* is now essential in the roots of my work. I also was immersed in archive work and understanding the context of Bohr's personal history through the conversations and help from Robert Sunderland, Head Archivist, Niels Bohr Archive, Københavns Universitet. The access granted to the original papers, manuscripts, and lecture recordings of Niels Bohr resulted in invaluable insights in the development and evolution of his ideas. The thrill of standing in the classroom where Bohr, Heisenberg, Pauli and others discussed the ideas presented in this thesis and surrounded by the welcoming and stimulating environment that received me was a humbling thrill that I will not forget.

Turing. His biography by Hodges came to my hands years ago and he became my intellectual hero. I worked in the same place where he discussed mechanical intelligence and information theory with Shannon. I teach some of his concepts in some of my classes, and when I introduce Philosophy to my students, I read for them the first pages of his 1950 paper in *MIND*. His life inspires me and saddens me in equal measures. My children are both part of the LGBTQ+ community and I cried with my son together at the end of the "The Imitation Game" movie. When I was accepted at MMU I knew I would go to Manchester where he lived and worked, and years later I finally sat at the side of his statue a couple of days after his birthday in 2022. AI was exploding at that time, but for me, my PhD work had brought me to Manchester and to Turing's memorial. I thought about my work. I thought about my children and my wife. I had journeyed through a long life that took me to that place and this thesis, and from there, to this point.

My thesis has been a transformational and affirming personal journey that began with the nudge of my father-in-law, Steve Welch, and became emboldened by his encouragement. It also has deeper and older roots, and it could not have been done without my wife Giulia. Our countless discussions of ideas, stories and intellectual dialogue permeate my thoughts. The patience, understanding, unfailing belief in me are beacons that illuminate and guide my path. Her companionship and love fuels and colours everything. Truly, I could not have reached this point in my thesis and my life without her, and so, this thesis is dedicated to her, Giulia Welch, the love of my life, and to Steve Welch, my father-in-law.

Ernesto Angel Diaz
Spring 2025.

Introduction

This thesis investigates fundamental patterns in how transformative scientific knowledge emerges and becomes established, focusing on three pivotal cases: Leibniz's development of calculus, Bohr's formulation of complementarity, and Turing's conceptualisation of computation and artificial intelligence. At its core are two essential questions that have persistently challenged researchers and philosophers: *How do you solve a problem that you have never seen before?* and *Is solving problems an individual or a social endeavour?* To address these questions, this research introduces the *o-é-c model* (*Ouverture Ontologique - Épistémè Socialisante - Connaissance Éclairante*), which describes the journey from individual insight to collective knowledge. The model demonstrates how *wicked problems*—those that resist formulation within existing frameworks—catalyse the creation of new knowledge domains through three distinct phases: initial conceptual breakthrough, social validation and refinement, and systematic integration into established knowledge.

The theoretical foundations of the *o-é-c model* draw upon and synthesise multiple philosophical traditions. From Minsky, it adopts a conception of knowledge as mental models that prove their worth through practical problem-solving capability. From Longino's social epistemology, it incorporates understanding of how knowledge claims are validated through critical discourse within scientific communities. The model extends beyond Hanson's emphasis on individual discovery by showing how initial insights must be developed through social processes, incorporates Forman's view of the influence of culture and sociological factors in the development of knowledge, and builds upon Kuhn's notion of paradigm shifts while providing more specific mechanisms for how revolutionary ideas become integrated into established knowledge.

The *o-é-c model* consists of three interconnected phases, each representing a distinct aspect of knowledge creation. The *Ouverture Ontologique* phase focuses on individual creativity and conceptual innovation, characterised by intellectual leaps that combine existing ideas with novel concepts in response to wicked problems. This phase acknowledges the crucial role of individual insight while recognising how such breakthroughs are shaped by historical and cultural contexts, as emphasised in Forman's work. The *Épistémè Socialisante* phase emphasises how scientific understanding emerges through sustained critical discourse and practical applications. This phase draws on Longino's insights about the social nature of knowledge validation while providing specific mechanisms through which individual insights become collective understanding. The role of pedagogical tools and textbooks proves crucial here, demonstrating how new ideas gain traction through systematic presentation and teaching. The *Connaissance Éclairante* phase represents the integration of new frameworks into the broader intellectual landscape. This phase often generates applications and insights beyond original domains, demonstrating how truly transformative ideas continue to evolve through interaction with diverse fields and problems.

The relationship between individual discovery and social validation emerges as a central theme throughout the research. Rather than treating these as opposing forces, the *o-é-c model* shows how they work together in knowledge creation. Individual insights provide the spark of innovation, but these must be developed and refined through social processes to become established knowledge. Methodologically, the research combines historical analysis with philosophical argumentation while maintaining attention to practical mechanisms of knowledge transmission. Primary sources related to each case study are examined both for their content and their reception, showing how new ideas gained acceptance through specific social and institutional processes. This approach bridges traditional divides between different conceptions in philosophy of science, combining attention to historical context with systematic analysis of knowledge development. The thesis makes several original contributions to philosophy of science: it bridges the gap between individual and social accounts of knowledge creation, provides specific mechanisms for how revolutionary insights become established knowledge, and demonstrates the crucial role of pedagogical development in knowledge transmission. These findings have significant implications for understanding how transformative knowledge emerges in an era of accelerating technological change and increasingly complex global challenges.

Chapter 1 - Methodology and Thesis Framework

Preamble: the *o-é-c model* and the thesis methodology

At the root of this thesis are two questions that I have grappled with for over three decades in engineering, research, and teaching: *How do you solve a problem that you have never seen before?* and *Is solving problems an individual or a social endeavour?* These questions have profound implications for understanding scientific progress. When Leibniz developed calculus, when Bohr grappled with quantum mechanics, when Turing conceptualised artificial intelligence - each faced problems that couldn't be solved within existing frameworks. Yet their individual insights somehow became part of our collective knowledge. Understanding this process is crucial for addressing today's complex challenges. This thesis proposes that the additions to the body of knowledge that represent the creation and ultimate acceptance of new fields often follow a similar process that is both an individual and social endeavour, it begins with trying to address problems that are not well defined or understood with existing theories and models, and develops and evolves in an inseparable way from the interactions among members of the epistemic communities that hold the knowledge and with the environment where they are embedded.

The model presented in this thesis identifies three distinct phases in the development and integration of new fields of knowledge:

Ouverture Ontologique (Ontological Opening - Invention Phase)

This initial phase encompasses the historical environment, existing paradigm, identification of a wicked problem, intellectual breakthrough, and emergence of competing frameworks.

Épistémè Socialisante (Socialising Episteme - Understanding Phase)

This phase involves the gestalt shift in understanding, gathering of support for the new paradigm, expansion through pedagogy, and development of textbooks and applications.

Connaissance Éclairante (Enlightening Knowledge - Adoption Phase)

This final phase represents the integration into the established body of knowledge.

The structure of the *o-é-c process* model itself embodies this perspectivist approach. Each phase represents not just a stage but a distinct perspective on knowledge creation, each necessary for understanding how transformative ideas emerge and become established. The model shows how these perspectives interact and build upon each other. The individual perspective of the creator seeing problems in new ways combines with the social perspective of the scientific community, leading to new shared perspectives that transform our collective understanding. This perspectivist framework helps explain both successes and failures in knowledge creation - success often depends on how effectively these different perspectives can be integrated into a coherent whole.

The *o-é-c process* model builds upon and extends key theoretical frameworks from earlier thinkers in the philosophy of science. Each of these thinkers provides crucial insights that our model integrates and develops further. This engagement demonstrates both the model's theoretical foundations and its original contributions to our understanding of scientific progress.
First, it provides a structured framework for understanding how individual insights become collective knowledge - something not fully addressed by any of the theorists individually or any earlier frameworks.
Second, the emphasis on *wicked problems* as catalysts for knowledge creation offers a new perspective on what drives scientific advancement.
Third, the three-phase structure of the *o-é-c model* allows for both revolutionary and evolutionary aspects of scientific change, resolving some tensions between these earlier frameworks.
Fourth, the model demonstrates the role of pedagogical tools and textbooks[1] as decisive elements in knowledge dissemination and it adds a practical dimension often overlooked in more theoretical accounts.

My research methodology integrates three complementary analytical approaches. By combining historical analysis with philosophical investigation and cross-case pattern recognition, we can

[1] Where *textbooks* are defined not as physical objects but as pedagogical and epistemic entities.

understand not just what happened in these transformative moments of science, but how and why they succeeded in creating new fields of knowledge. This integrated approach allows us to identify patterns that might be missed by examining any single dimension.

The Historical Analysis: the depth goes beyond historical accounts to examine the social and cultural context. Taking inspiration from Paul Forman's (1937-) work on quantum mechanics in the Weimar Republic, my historical analysis goes beyond documenting sequences of events. Instead, I examine how each breakthrough emerged within its specific historical context. For instance, in studying Bohr's development of complementarity, I analysed not just his scientific papers, but his discussions with Einstein, institutional documents from Copenhagen, and the broader cultural environment of post-WWI physics. This deep contextual analysis reveals how new ideas emerge and gain acceptance.

The Philosophical Investigation: The philosophical dimension of my methodology draws on multiple traditions. I engage with Thomas Kuhn's work (1922-1996) on paradigm shifts, gestalt shifts and the progress of science, Helen Longino's (1944-) social epistemology, Russell Hanson's (1924-1967) philosophy of discovery, and Paul Forman's (1937-) historical account of sociological influence on science content and development. However, I extend beyond these frameworks by focusing specifically on how new fields of knowledge emerge. This philosophical lens helps us understand not just the historical facts, but the underlying patterns of knowledge creation and establishment.

Cross-case pattern recognition is used both as an analytical lens to highlight the components of the model, as well as a comb to identify further cross-case commonalities while searching for success and failure criteria within each phase.

What makes this methodology particularly useful is how it integrates these different analytical approaches through the *o-é-c process* model. This model serves not just as a theoretical framework but as an analytical tool. It allows us to map historical events, identify patterns

across different scientific fields, and understand the common elements that lead to successful establishment of new knowledge. For example, when I examine Turing's work on computation, I can see how his individual insights progressed through social validation to become established knowledge, following a pattern similar to what I observe with Leibniz and Bohr. The case studies fit all phases of the model, and each one best exemplifies one of the phases:
Gottfried Leibniz (1646-1716) and the development of calculus and mathematical analysis, which is an exemplar of the *ouverture ontologique*.
Niels Bohr (1885-1962) and the concept of complementarity and the Copenhagen interpretation of quantum mechanics (QM), which is an exemplar of the *épistémè socialisante*.
Alan Turing (1912-1954) contributions to computer science (CS) and artificial intelligence (AI), which is an exemplar of the *connaissance éclairante*.

Analysing these cases together reveals common patterns in how transformative knowledge emerges and becomes established. Each shows the importance of systematic framework development, social validation processes, and pedagogical tools. Moreover, each demonstrates how truly transformative ideas continue generating new insights and applications well beyond their initial conception. These patterns validate the *o-é-c process* model while providing insights for understanding current scientific challenges.

Antecedents – Knowledge

Perspectivism and Wicked Problems is an attempt to understand epistemological patterns in the history and philosophy of science. By epistemological patterns I mean repeated processes that resulted in additions to the body of knowledge. Knowledge creating activities such as observation and cognition were part of natural philosophy before the early modern period, and of science afterwards the eighteenth century. The teleology of those activities is presented by Longino this way:

> The purpose of scientific inquiry is not only to describe and catalog, or even explain, that which is present in everyday experience, but to facilitate prediction, intervention,

> control, or other forms of action on and among the objects of nature (Longino, 2002c, p. 124)

I will go further than that idea to argue that there are knowledge creation activities such as intellectual debate, pedagogical discourse, teaching and textbook creation among others, some of which are Longino's *critical discursive interactions* (Longino, 2002c, p. 129). I will also argue that there are intellectual inquiries not restricted to scientific inquiries such as modelling and representation in a number of fields (e.g. astrophysics, theoretical biology, computer science, applied mathematics, behavioural sciences) seeking to produce knowledge that not only facilitates prediction, intervention, control, and other forms of action over objects of nature, but also to produce epistemic frameworks and entities that do not reflect the observable and instead allow to explore the not-yet observable or generate the same forms of action and control in previously un-encountered or foreseeable situations. I therefore need to start by defining *knowledge* in a way that will allow us to identify the end result of this inquiry, and for this I will borrow from the concept of knowledge developed in the book *The Society of Mind* by Marvin Lee Minsky (1927–2016) as it permeates this thesis, and it is applicable to its essence:

> '*Jack knows about **A**' means that there is a 'model' **M** of **A** inside Jack's head… Jack considers **M** to be a good model of **A** to the extent that he finds **M** useful for answering questions about **A…*** This provides us with a simple explanation of what we mean by knowledge: *Jack's knowledge about **A** is simply whichever mental models, processes, or agencies Jack's other agencies can use to answer a questions about **A*** [Minsky's emphasis] (Minsky, 1988, p. 303).

Minsky's concept applies to the thesis in two ways. This model is useful in answering questions about additions to knowledge. The second way in which Minsky's working definition applies to the thesis is in describing the invention that occurs during the first phase of said common patterns: a framework that systematised the understanding and description of a type of problems that could not be addressed with existing theories and methods.

The second phase of the common patterns found in intellectual history is the result of the socialisation of cognition, and here I borrow from the work of Helen Longino in her book *The*

Fate of Knowledge (Longino, 2002d). Longino defines *knowledge* and its cognates as honorific terms that designate success (Longino, 2002b, p. 97) that needs to be understood as socialisation of cognition, and in my thesis I use some of her ideas where I propose to treat the exposition and utilisation of novel frameworks (instead of Longino's activity of observation) and cognition as dialogical activities that consist of interactions between different voices (Longino, 2002b, p. 99). It is this socialisation of cognition and social practices what yields the adoptions in the body of knowledge. While Longino's objective in the book *The Fate of Knowledge* is an "exploratory defence of the philosophical claim that knowledge, especially scientific knowledge, is social" (Longino, 2002d, p. 203), my goal is an attempt to understand the process of additions to the body of knowledge that can be called new fields and have occurred in a number of cases in the history and philosophy of science[2]. I propose that those additions stem from a type of inventions that are triggered by addressing a class of problems called *wicked problems*. In the next section I will begin by introducing the types of problems and situations that describe the intellectual enquiries mentioned before.

Antecedents – Wicked Problems

Wicked problems are a class of problems that are characterised by the difficulty of understanding or solving them using existing methods, models and values (Rittel and Webber, 1973, pp. 158–167). They are multidisciplinary in nature and defy a unique description. Implementing solutions creates additional complications. In their description of these type of problems, Horst Rittel (1930-1990) and Melvin Webber (1920-2006) explain how typical assumptions of widespread consensus on goals, causal theory sufficiently developed to permit prediction, and effective instrumental knowledge are no longer applicable and therefore it is not possible to solve these problems with traditional approaches. Two of the most important characteristics of *wicked problems* are that there is no definitive formulation of a *wicked problem*: "The information needed to understand the problem depends upon one's idea for

[2] Examples in science are: analysis, quantum mechanics and electrodynamics, information theory, computer science, cognitive science, and artificial intelligence.

solving it." (Rittel and Webber, 1973, p. 161). The other one is that *wicked problems* have no stopping rule:

> Because [according to the characteristic presented earlier] the process of solving the problem is identical with the process of understanding its nature, because there are no criteria for sufficient understanding and because there are no ends to the causal chains that link interacting open systems (Rittel and Webber, 1973, p. 162).

Rittel and Webber argue that traditional approaches to problem solving, based on the *paradigm* of science and engineering, are inadequate for dealing with *wicked problems*:

> The classical paradigm of science and engineering, the paradigm that has underlain modern professionalism, is not applicable to the problems of open societal systems. (Rittel and Webber, 1973, p. 160).

Contemporary challenges increasingly manifest characteristics of what Rittel and Webber termed *wicked problems*, with some evolving into what the literature now identifies as "super wicked problems" - challenges that combine traditional wicked problem characteristics with urgent temporal constraints and the absence of central authority for resolution. Three exemplars particularly illuminate the theoretical structure of such problems:

Climate change represents the paradigmatic super wicked problem, demonstrating how multiple complex systems (atmospheric, oceanic, social, and economic) interact in ways that resist traditional problem-solving approaches. Its analysis reveals how wicked problems generate cascading effects across seemingly unrelated domains, challenging conventional disciplinary boundaries.

The COVID-19 pandemic exemplifies how wicked problems manifest in real-time crisis scenarios, revealing the complex interplay between scientific knowledge, public health policy, economic imperatives, and social behaviour. This case demonstrates how proposed solutions often generate new problems, creating feedback loops that complicate intervention strategies.

The ethical challenges posed by artificial intelligence development illustrate how technological advancement can generate novel categories of wicked problems. These challenges combine technical complexity with fundamental philosophical questions about consciousness, agency, and human value systems, demonstrating how wicked problems can emerge at the intersection of scientific capability and ethical frameworks.

These examples illuminate key theoretical aspects of wicked problems: their resistance to definitive formulation, the interconnectedness of proposed solutions with other complex systems, and their tendency to reveal fundamental limitations in existing knowledge frameworks. In this thesis I will expand the application of the term *wicked* to include situations and challenges that present similar characteristics to those of the original definition by Rittle and Webber. In spite of the seemingly helplessness in attempting to address *wicked problems*, there is a number of institutions and thinkers addressing them that follow the principle of *scientia in servitio societatis* (knowledge in the service of society), one example of which is Bell Laboratories (widely known as Bell Labs), the Research institution to which I dedicated more than a decade of my life and work as a *Member of Technical Staff*[3]*(MTS)* developing the first mobile communication systems with theoretical roots in the work of Alan Turing, Claude Shannon (1916-2001), and their colleagues around Turing's time collaborating half a century earlier and afterwards at the very same institution. Those collaborative efforts and others in higher learning institutions, and the conceptual roots developed from them were fundamental in additions to the body of knowledge that became a number of fields including computer science, artificial intelligence, and theory of communication. Bell Labs had a long-standing culture immersed and intermixed in science and technology, and even as far back as the 1940s and 1950s there were questions in that culture regarding problem solving, discovery, and invention.

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[3] An MTS at Bell Laboratories was a technical position that played a crucial role in the organisation's remarkable history of innovation (Nokia Bell-Labs Timeline). Unlike many industrial research facilities, Bell Labs established a technical career track that allowed scientists and engineers to advance professionally without moving into management roles. The position typically required an advanced degree or equivalent experience in fields like physics, mathematics, computer science, or engineering. What made the MTS role special was its combination of freedom and responsibility. These researchers were given significant autonomy to pursue long-term, fundamental research while still maintaining connections to practical applications that could benefit the telecommunications industry. Some of the most significant technological breakthroughs of the 20th century came from MTS researchers at Bell Labs, including :
The transistor, developed by MTS members William Shockley, John Bardeen, and Walter Brattain
The Unix operating system, created by MTS members Ken Thompson and Dennis Ritchie
The C programming language, developed by MTS member Dennis Ritchie
Information theory, pioneered by MTS member Claude Shannon
The position came with unique cultural aspects as well. MTS members often worked in an egalitarian environment where ideas mattered more than titles. They were expected to collaborate across disciplines, attend regular technical seminars, and contribute to the intellectual life of the institution. This culture of open discussion and cross-pollination of ideas was central to Bell Labs' success, shaping my intellectual growth and it is a great influence in the framework of this thesis.

In that context and regarding problem solving, a question emerges: is discovery a term corresponding with science while invention a term associated with technology? Myron Atkin (1927-2022) and Robert Karplus (1927-1990) proposed a definition of invention vs discovery while working on a programme for the teaching of science in Berkeley in which inventions correspond to the introduction of new concepts while discoveries correspond to the subsequent verification or extension of the concepts' usefulness:

> … in the development of a new concept, it is useful to distinguish the original introduction of a new concept, which can be called invention, from the subsequent verification or extension of the concept's usefulness, which can be called discovery (Atkin and Karplus, 1962, p. 45).

This suggests that the context of problem solving is not only important but also part of the process or framework in which knowledge expands, and I will adopt this definition of invention to describe the introduction of new concepts in novel frameworks in the model presented in the next chapter. In the next section, I will present some antecedents regarding the solution of problems.

Antecedents – Solving Problems

To begin addressing the question of *How do you solve a problem that you have never seen before?* I look first at Aristotle's (384-322 BCE) ideas on solving problems, and I find a starting point in Aristotle's *Metaphysics Book 3* dictum for understanding problems before trying to solve them. It is worth reading Aristotle's argument:

> We must, with a view to the science which we are seeking, first recount the subjects that should be first discussed. These include both the opinions that some have held on to certain points, and any points besides these that happen to have been overlooked. For those who wish to get clear of difficulties, it is advantageous to state the difficulties well; for the subsequent free play of thought implies the solution of the previous difficulties… (Aristotle, 1995c, p. 1572 "Metaphysics III B 995 24–30")

Aristotle directs us to describe, understand the definition, description and approaches used to address problems: what has been done before by others, attempted solutions and encountered failures, lacunae that need to be filled and interpretations are all part of the approach to a problem in which we attempt a novel solution. This implies a familiarisation with both the existing body of knowledge and the current models and methods to describe and understand the problem in a way that resembles the description of *paradigms* introduced by Kuhn in *The Structure of Scientific Revolutions* (Kuhn, 2012) as we will describe later in this chapter. From here we can suggest that while there are internal tools that we bring to bear such as heuristics, intuition, creativity, and knowledge of existing ideas associated with similar efforts in the environment we are immersed with, we can also look at the creation of new ones outside of existing frameworks.

Aristotle's Metaphysics book is often called the "aporetic" book due to the use of aporiae (from aporia, in Greek, απορία, meaning equality of opposing arguments) in presenting the solution to problems, that is to say, the manner in which Aristotle examines the impasse among arguments in a dialectic approach before proceeding to propose a solution. Cleary favours the idea that Aristotle's dialectic method is an inductive way towards the principles understood in a Platonic fashion as he dedicates much attention to questions regarding methods for a variety of inquiries (Cleary, 1995, p. 99). Perhaps the dialectic method will illuminate a path to address the second question regarding individual vs social endeavours to solve problems, as we will see later by looking at some ideas that Longino has developed on social epistemology in her book *The Fate of Knowledge* (Longino, 2002d). Longino's work addresses how the social dimension of scientific knowledge emphasises the importance of community in shaping scientific consensus and the content of scientific theories. It is worth reading her words here in that respect:

> Applying the feminist insight that we are all in relations of interdependence, I have suggested that scientific knowledge is constructed not by individuals applying a method, but by individuals in interaction with one another in ways that modify their observations, theories and hypotheses, and patterns of reasoning. Scientific method, thus, includes not only the complex of activities that constitute hypothesis testing through comparison with experimental data, in principle an activity of individuals. It involves equally centrally the subjection of hypotheses and the background assumptions

> in light of which they seem to be supported by data to varieties of conceptual and evidential criticism (Longino, 1991, p. 670).

In this thesis I will describe a model that applies to the creation and adoption to the Body of Knowledge of what could be called *new fields*. Another guiding concept for this thesis is that of the evolution of knowledge. Jürgen Renn (1956-) presents a novel perspective on the development of human knowledge in his book *The Evolution of Knowledge: Rethinking Science for the Anthropocene* (Renn, 2020) drawing from a wide range of disciplines, including the history and philosophy of science, cognitive science, and anthropology. Renn argues for a co-evolutionary view of knowledge and society, emphasising the role of human cognitive capacities, material culture, and social institutions in shaping the trajectory of knowledge development. One of the central concepts in Renn's work is the notion of *knowledge systems*, which he defines as

> Knowledge amalgamated by the connectivity of its elements within their mental, material, and social dimensions. Knowledge systems are typically part of the shared knowledge of a community (Renn, 2020, p. 427).

This concept highlights the interdependence of knowledge production and social contexts, as well as the cumulative and transformative nature of knowledge development over time. He suggests that the evolution of knowledge is characterised by periods of stability and incremental change, punctuated by transformative episodes that fundamentally reshape the structure and content of knowledge systems. The evolution of knowledge that I describe in this thesis, while having connections and parallels with the work from Jürgen Renn in his book *The Evolution of Knowledge: Rethinking Science for the Anthropocene* (Renn, 2020) also differs from it in some substantial points as I will describe later in this chapter.

Antecedents – Discovery

The conceptual foundations of scientific discovery merit careful philosophical analysis, despite historical tendencies to relegate discovery to sociology or psychology. Building on Hanson's seminal work on patterns of discovery and his argument that discovery deserves philosophical attention (Hanson, 1967, p. 322), this thesis proposes a novel theoretical framework for understanding how new knowledge domains emerge.

Hanson emphasised the theory-ladenness of observation and distinguished between reasons for proposing versus accepting hypotheses (Hanson, 1958, p. 1074). Here is my point of departure from Hanson. I think there is space to propose a view of scientific discovery based on a *discovery-what* that does not apply to a discovered object but to a descriptive invention, a framework, a novel conceptual approach to problems that previously defy clear definition and explanations from existing theories. I argue that we must expand this analysis to include these *descriptive inventions* - conceptual frameworks that address previously intractable problems by creating new theoretical spaces. This approach departs from traditional discovery analysis in three crucial ways: First, it moves beyond Hanson's *quadripartite taxonomy*[4] of discoveries (Hanson, 1967, p. 326) to examine how new conceptual frameworks emerge specifically in response to wicked problems. Second, it avoids R.G. Collingwood's (1889-1943) criticised "substantialist" approach (Collingwood, 2005, pp. 42–45) by examining the dynamic interplay between theoretical innovation and historical context. Third, it synthesises Massimi's diachronic and synchronic perspectivism (Massimi, 2017, p. 2) to analyse how new knowledge domains become established through both historical development and concurrent theoretical competition.

This theoretical innovation provides a framework for understanding how fundamentally new knowledge emerges not through discovery of pre-existing entities but through the invention and social validation of new conceptual architectures. As Massimi argues, scientific knowledge is inherently perspectival, constrained by historically defined models and interpretative frameworks (Massimi, 2017, p. 6). This thesis extends this insight by examining how new perspectives emerge and become integrated into the body of scientific knowledge. I will now present some additional antecedents to the ideas developed in this project regarding knowledge as a social endeavour that is inherent to its evolution.

[4] Hanson makes a distinction between discovering an individual object (an X), discovering a universal process or entity-type (X), discovering that a certain state of affairs obtains (that X), and discovering an entity while misidentifying it (an X as a Y).

Antecedents – The Sociology of Knowledge

In the next chapter I will introduce a model that builds upon, and in some cases extends or differs in part from the work of several intellectuals in the History and Philosophy of Science, and notably from that of Norwood Russell Hanson, Thomas Kuhn, Paul Forman, Helen Longino, and Michela Massimi, where they significantly challenged the traditional view of science as a purely rational, objective enterprise, and their emphasis on historicity and sociology recognised the interplay of historical, social, and cultural factors in shaping scientific knowledge. However, they focus on different aspects of this relationship: Kuhn on the structure of scientific revolutions and the role of *paradigms*, Forman on the influence of specific historical and cultural contexts, and Longino on the importance of diversity and critical discourse within the scientific community. Massimi's work on perspectival realism, attention to historical and cultural context, emphasis on pluralism and inclusivity, and commitment to scientific objectivity offer a nuanced and integrative approach to understanding the social and cultural dimensions of scientific knowledge production. I will introduce here some important concepts that they have developed, as well as where I use, expand or differ, and suggest that the sociology of knowledge should be a multifaceted endeavour, attentive to both the internal and external factors that shape the production of scientific and technological knowledge.

Kuhn is best known for his book *The Structure of Scientific Revolutions* (Kuhn, 2012), in which he challenged the traditional view of science as a linear, cumulative process. According to Kuhn, a *paradigm* is a framework of theories, methods, and standards for what constitutes legitimate contributions to a field (Kuhn, 2012, p. 11). In this view, the sociology of knowledge would focus on how these *paradigms* are constructed, maintained, and challenged within scientific communities and how they influence the production of scientific knowledge. He introduced the concept of *paradigm shifts* (Kuhn, 2012, p. 68) arguing that scientific progress occurs through occasional evolutionary breaks from established ways of thinking, rather than a gradual accumulation of knowledge.

Another important idea from Kuhn's *The Structure of Scientific Revolutions* for this thesis is the concept of a gestalt switch (Kuhn, 2012, p. 85) which plays a crucial role in explaining how scientists transition between different *paradigms*. Kuhn borrows this idea from psychology, where a gestalt switch refers to a sudden perceptual shift that allows one to see the same information in a completely new way. In the context of scientific revolutions, Kuhn argues that scientists experience a similar sudden shift in perception when moving from one *paradigm* to another. When scientists undergo a gestalt switch, they begin to see the same phenomena through a new theoretical lens, often leading to reinterpretations of existing data and the discovery of new problems and solutions.

T. Kuhn emphasised the role of the scientific community in shaping scientific knowledge, suggesting *paradigms* are held together by shared beliefs, values, and methods among scientists that guide scientific research (Kuhn, 2012, pp. 23–24). The sociology of knowledge, when viewed through the lens of Thomas Kuhn's perspective in *The Structure of Scientific Revolutions* includes the arguments that scientific knowledge is not a linear accumulation of facts but is shaped by the dominant *paradigms* held by scientific communities. I build from Kuhn's concept of scientific *paradigms* as essential for understanding the intellectual environment in the century in which the additions to knowledge develop. As an example, during the seventeenth century, the predominant *paradigm* shifted from Aristotelian natural philosophy, which emphasised qualitative descriptions of nature and adherence to ancient authorities, to the mechanistic framework. This new *paradigm* was characterised by mathematical descriptions of nature, as seen in the works of Galileo Galilei (1564-1662), Johannes Kepler (1571-1630), and Sir Isaac Newton (1643-1727). Mathematical work had its own *paradigm* that changed after Leibniz's development of a successful framework in infinitesimal calculus and evolved to become the field of analysis as I will show in Chapter 3. The mechanistic *paradigm* emphasised empirical evidence and experimentation, leading to the gradual establishment of new norms and standards in scientific inquiry. I depart from Kuhn explicitly in what triggers the *paradigm* shift and from adjudicating the existence of the new addition to the brilliance of one person, even though the inventions have an origin in the work of a specific philosopher as they start as a more successful framework among alternate versions or attempts and become new fields after expanding and evolving through the sociological phase.

P. Forman is known for his work on the history and sociology of physics, particularly in the context of Germany during the Weimar Republic and Nazi era. In his paper "Weimar Culture, Causality, and Quantum Theory, *1918-1927* (Forman, 1971), Forman argued that the cultural and political climate of Weimar Germany influenced the development of quantum mechanics. He suggested that the acceptance of *acausality* and indeterminacy in quantum mechanics was, in part, a response to the societal upheavals and intellectual currents of the time. Paul Forman's perspective in "The Primacy of Science in Modernity, of Technology in Postmodernity, and of Ideology in the History of Technology" (Forman, 2007) emphasises the historical and cultural context in which scientific and technological knowledge is produced. He argues that the primacy given to science in modernity and to technology in postmodernity reflects broader cultural and ideological shifts (Forman, 2007, pp. 68–69). Forman's work suggests that the sociology of knowledge should be attentive to the historical and cultural specificity of knowledge production and how it is influenced by factors external to the scientific community.

I use Forman's example of how cultural and political pressure changed the content and views of science in the Weimar Republic as a simile to explain the pressures existing in the historical environment and extend it so I can infer a similar situation when looking at the effects of cultural environment in the challenging of existing ideas that facilitated the development and defence of a framework that was not acceptable among the current *paradigm* of each case. As an example, I will point out in the chapter on Leibniz and calculus (building upon other sources) how the religious and philosophical upheavals of the time that begun in the sixteenth century (such as the Protestant Reformation and its challenge to Catholic dogma) encouraged a move towards empirical and mechanistic explanations of the natural world, which were seen as more objective. This cultural shift could be interpreted as facilitating the acceptance of new scientific ideas that were less reliant on religious or philosophical authority. I expand from Forman in that

I try to show in the three cases covered in this thesis the existence of analogue situations to the one he studied in the influence on the development of Quantum Mechanics that the Weimar Republic had.

H. Longino (1944-) is a prominent feminist philosopher of science who has focused on the role of social and cultural factors in shaping scientific knowledge. In her book *Science as Social Knowledge* (Longino, 1990), Longino argued that scientific objectivity is not achieved through individual impartiality but through the collective critical engagement of diverse perspectives. She emphasises the importance of diverse perspectives and backgrounds in the scientific community, as they can help to identify and challenge biases and assumptions in scientific research. Helen Longino's perspective in *The Fate of Knowledge* (Longino, 2002d) emphasises the role of social and cultural factors in shaping scientific knowledge but also argues for the importance of critical discourse within scientific communities.

I am inspired by Longino's work to extend it in that I explain the sociological elements of the proposed model as essential to understand the development and adoptions of new fields. I do not think I am departing dramatically from her views, just developing examples of how her views play out. Helen Longino's work on the social dimensions of scientific knowledge emphasises the importance of community in shaping scientific consensus and the content of scientific theories. Applying her perspective to the seventeenth century in a later chapter, and again building form the work of other sources, I present the example of the formation of scientific societies, like the *Royal Society* in England and the *Accademia dei Lincei* in Italy, and how they created new social networks that fostered critical discussions and the exchange of ideas. I also use the example of the development and growth of newspapers and journals in the same century and their relationship with their subscribers as catalytic in developing public opinion. These communities, which will play an important role in the intellectual landscape of the century, played crucial roles in challenging old *paradigms* and establishing new ones incorporating diverse viewpoints, which were essential for the robust development of scientific theories that fostered the scientific revolution.

The relationship between the *o-é-c model* and Kuhn's theory of paradigm shifts merits careful examination, as both frameworks seek to explain transformative changes in scientific understanding, albeit through different mechanisms. While Kuhn emphasises revolutionary breaks with existing paradigms triggered by accumulated anomalies, the *o-é-c model* proposes a more structured process through which new knowledge frameworks emerge and become established. This difference reflects a fundamental distinction in how the two approaches conceptualise scientific change.

Kuhn's model suggests that paradigm shifts occur when existing frameworks can no longer accommodate mounting contradictory evidence, leading to crisis and eventual revolution. In contrast, the *o-é-c model* proposes that transformative knowledge often emerges through deliberate engagement with wicked problems that resist formulation within existing frameworks. This distinction is crucial—where Kuhn sees scientific revolution as a response to failure, the *o-é-c model* identifies it as an active process of creating new conceptual spaces to address previously inarticulable challenges. Furthermore, while Kuhn emphasises the incommensurability between successive paradigms, the *o-é-c model* demonstrates how new frameworks can emerge alongside existing ones through structured processes of social validation and integration. This is particularly evident in my case studies: Leibniz's calculus developed alongside existing mathematical frameworks before gradually becoming dominant; Bohr's complementarity principle emerged through sustained dialogue with classical physics rather than its wholesale rejection; and Turing's computational framework expanded existing conceptions of mechanical processes rather than replacing them entirely.

The *o-é-c model* also differs from Kuhn's framework in its treatment of the social aspects of scientific change. Where Kuhn describes scientific communities primarily as validators of existing paradigms, the *o-é-c model* shows how social processes actively shape the development and refinement of new frameworks through the *Épistémè Socialisante* phase. This emphasis on structured social processes helps explain how revolutionary insights become integrated into established knowledge without requiring the complete overthrow of existing frameworks that Kuhn's model suggests. Perhaps most significantly, the *o-é-c model* provides a more detailed account of how new knowledge becomes established through systematic pedagogical development and institutional embedding. While Kuhn acknowledges the role of textbooks and education in maintaining paradigms, the *o-é-c model* shows how these elements actively contribute to the transformation and evolution of scientific understanding. These distinctions should not obscure important areas of agreement between the two frameworks. Both recognise the crucial role of scientific communities in knowledge development and the importance of conceptual frameworks in shaping scientific understanding. However, the *o-é-c model*'s more nuanced account of how transformative knowledge emerges and becomes established offers valuable insights that complement and extend Kuhn's fundamental contributions to our understanding of scientific change.

M. Massimi is known for her work on *Perspectivism* (Massimi, 2017) and *Perspectival Realism* (Massimi, 2022) which argues that scientific knowledge is always situated within a particular perspective or framework:

> Thus, I take perspectivism to be first and foremost an epistemic view about the nature of our scientific knowledge. It is not intended to be a metaphysical view about scientific facts being perspectival; or natural kinds being relative to scientific perspectives." (Massimi, 2017, p. 1).

In this thesis I borrow from her concept of perspectivism to present an epistemic view that scientific knowledge is situated historically, culturally, intellectually, semantically, and scientifically (within a scientific *paradigm*). This idea resonates with Kuhn's notion of *paradigms*, as he emphasises the role of shared theoretical and methodological commitments in shaping scientific knowledge. Like Forman, Massimi is attentive to the historical and cultural context in which scientific knowledge is produced, with examples such as her paper “Realism, perspectivism, and disagreement in science” (Massimi, 2021) where she presents the idea of perspectival disagreement in science, and the question on how to keep ontological realism (meaning that entities postulated by our best scientific theories such as electrons, genes, gravitational fields actually exist in the world, independent of our minds or theories) when faced with perspectival disagreement between scientists (Massimi, 2021, p. 6115). Massimi is committed to a form of scientific realism and the idea that science can make progress toward objective truth, that aligns her with Longino's view that objectivity can be achieved through critical discourse and engagement with diverse perspectives:

> Perspectivism is not a backward-looking reflection on the practice of science and its being historically and culturally situated. It is also, first and foremost, a forward-looking commitment to engage across scientific perspectives and retain knowledge claims that have served us well across multiple perspectives. It is a perfect illustration of how scientific knowledge grows from a perspectival point of view: how epistemic communities come to agree, and how successful our perspectival data-to-phenomena inferences can be in delivering knowledge 'from within' the boundaries of what is historically conceivable at any point in time (Massimi, 2021, p. 6139).

I, however, am not centrally concerned with the realism debate but rather with developing an account of knowledge that integrates the social and rational, and in doing so, the work in this thesis offers a novel perspective on the realism/anti-realism debate through the *o-é-c model*'s emphasis on how transformative knowledge frameworks emerge and become established. Rather than taking a stark position on either side, my research suggests a more nuanced understanding of how scientific frameworks relate to reality. The *Ouverture Ontologique* phase illuminates an interesting aspect of the realism debate. When Leibniz created infinitesimals, Bohr developed complementarity, or Turing conceptualised abstract machines, they weren't simply discovering pre-existing entities. Rather, they were creating new conceptual frameworks for understanding reality. This might seem to support anti-realism, but there's a crucial nuance: these frameworks proved successful precisely because they captured something real about the world, even if that reality couldn't be directly accessed through previous conceptual schemes.

The *Épistémè Socialisante* phase shows how scientific frameworks become validated not just through correspondence with reality, but through social processes of refinement and testing. However, unlike strong social constructivist positions, my model suggests that these social processes serve to refine and validate frameworks that genuinely capture aspects of reality. The success of these frameworks in predicting and explaining phenomena suggests they're tracking real features of the world, even if our access to those features is mediated by our conceptual schemes. Consider Bohr's *complementarity*: it emerged from attempting to understand quantum phenomena that couldn't be adequately described using classical concepts. The framework isn't simply 'discovering' reality in a straightforward realist sense, nor is it merely a social construction. Instead, it provides a way of understanding real phenomena that transcend our classical intuitions. The framework's success in predicting experimental results suggests it captures something real, even if that reality can't be pictured in classical terms.

The Connaissance Éclairante addresses the fact that these frameworks continue generating new insights across multiple domains and it suggests they're capturing genuine features of reality. When Turing's computational framework proves useful in understanding cognition, or when Leibniz's calculus illuminates physical processes, this suggests these frameworks are not merely convenient fictions but are tracking real patterns in the world. This thesis suggests moving beyond the traditional realism/anti-realism dichotomy. Instead of asking whether our scientific frameworks directly represent reality, we might ask how new conceptual frameworks enable us to understand and interact with reality in increasingly sophisticated ways. The *o-é-c model* shows how successful scientific frameworks emerge through a process that combines individual insight, social validation, and practical application - suggesting that scientific progress involves developing better ways of conceptualising and engaging with reality rather than simply discovering or constructing it.

While Massimi focuses on the historically and culturally situated nature of scientific perspectives, Longino emphasises the socially interactive practices of observation and reasoning that constitute a community's knowledge-productive practices, and I incorporate both views in the construction of the epistemic knowledge of additions to knowledge. I agree with both Massimi and Longino in their treating of scientific reasoning as a social, interactive process involving critical discursive interactions within a community, rather than an algorithmic process carried out by individual minds. As Longino states:

> The critical dimension of cognition is a social dimension, requiring the participation of multiple points of view to ensure that the hypotheses accepted by a community do not represent someone's idiosyncratic interpretation of observational or experimental data (Longino, 2002b, p. 106).

Antecedents – Situational analysis in earlier frameworks on the evolution of knowledge

This thesis emerged from my search for patterns in Intellectual History and my belief that commonalities exist in how new knowledge is generated and evolves. While I engage with ideas from prominent History and Philosophy of Science scholars like Hanson, Kuhn, Forman, Minsky, Longino, and Massimi throughout different chapters, their work was not the primary catalyst, nor did my framework–the *o-é-c model*–emerge from analysis of deficiencies or lacunas in earlier bodies of work from specific philosophers or philosophical traditions. Rather, my multidisciplinary experience at Bell Labs, combined with my broader academic background across various fields, led me to develop the o-é-c model. Before presenting this model in detail and applying it to three case studies, I will briefly contextualise how my framework relates to existing theories about knowledge creation, transfer, and dissemination.

Immanuel Kant (1724-1804)

Key Resonances with Kantian Philosophy:

Synthetic Structure: The *o-é-c model* shares with Kant's work an interest in how we synthesise new knowledge. Just as Kant describes how understanding emerges through the interaction of intuitions and concepts, the o-é-c model shows how new knowledge frameworks emerge through the interaction of individual insight and social processes. Both approaches recognise that knowledge creation involves active structuring rather than passive reception.

Conditions of Possibility: The *Ouverture Ontologique* phase echoes Kant's interest in the conditions that make knowledge possible. Just as Kant investigated the necessary conditions for experience and understanding, this phase examines how new conceptual frameworks emerge to make previously intractable problems comprehensible.

Categories of Understanding: Like Kant's categories of understanding, the *o-é-c model* suggests that certain structural elements must be in place for knowledge to develop. However, where

Kant saw these as fixed and universal, the model shows how new frameworks can emerge to reshape our categories of understanding.

Key Extensions and Modifications:

Historical Contingency: Unlike Kant's transcendental approach, the *o-é-c model* emphasises the historical and social contingency of knowledge frameworks. Where Kant sought universal structures of understanding, this model shows how new ways of knowing emerge through specific historical processes.

Social Dimension: The *Épistémè Socialisante* phase represents a significant departure from Kant's more individualistic epistemology. The model shows how knowledge is validated and refined through social processes, an aspect largely absent from Kant's account.

Dynamic Development: While Kant presented a static structure of understanding, the *o-é-c model* demonstrates how knowledge frameworks evolve through time. The model shows how new categories of understanding can emerge through the interaction of individual insight and social validation.

Important Contrasts:

Nature of Knowledge: Where Kant maintained a sharp distinction between phenomena (things as they appear) and noumena (things in themselves), the *o-é-c model* suggests a more fluid relationship between our frameworks of understanding and what they help us comprehend. The model shows how new frameworks can reveal previously inaccessible aspects of reality.

Role of Experience: The model presents a more complex view of how experience relates to knowledge than Kant's rigid distinction between a priori and a posteriori knowledge. It shows how new frameworks emerge through dynamic interaction between theoretical insight and practical application.

Validation of Knowledge: Unlike Kant's emphasis on logical necessity, the o-é-c model emphasises how knowledge frameworks prove themselves through practical effectiveness and social validation. This represents a more pragmatic approach to understanding how knowledge becomes established.

Contemporary Relevance:

It suggests how we might preserve Kant's insights about the structured nature of understanding while acknowledging the historical and social dimensions of knowledge creation. It helps bridge the gap between transcendental approaches to epistemology and more historically oriented accounts of scientific development. It provides a framework for understanding how new categories of understanding can emerge to address previously intractable problems.

Georg William Friedrich Hegel (1770-1831) and Post-Hegelian Thought

Key Resonances with Hegelian Thought:

The *o-é-c model* shares with Hegel's account the fundamental recognition that knowledge development is both dialectical and social. The progression through *Ouverture Ontologique* to *Épistémè Socialisante* to *Connaissance Éclairante* mirrors, in some ways, the Hegelian dialectical movement of thesis-antithesis-synthesis. Like Hegel, I see knowledge creation as necessarily involving both individual and collective dimensions.

My emphasis on how new frameworks emerge through addressing *wicked problems* parallels Hegel's notion of how consciousness develops through encountering contradictions in existing ways of thinking. As I will describe later in the case studies, the tension between continuity and discreteness in all three cases (Leibniz, Bohr, and Turing) exemplifies this kind of dialectical development, and this will be shown as well in the cross-case analysis in chapter 6.

Key Modifications and Departures:

The *o-é-c model* significantly modifies the Hegelian framework in several important ways:

Historical Contingency: Where Hegel saw the development of knowledge as following a necessary logical progression, my analysis emphasises the role of historical contingency and cultural context. In the case studies I will discuss how post-war environments facilitated paradigm shifts suggest a more contextual understanding than Hegel's logical determinism. This pattern will appear in the cross-case analysis as well.

Social Validation: The *Épistémè Socialisante* phase gives more specific attention to the concrete mechanisms of social validation than Hegel's more abstract notion of Spirit (Geist). I make an

emphasis on textbooks, pedagogical tools, and institutional networks providing a more detailed account of how knowledge actually becomes established.
Multiple Frameworks: Unlike Hegel's single progressive narrative, the *o-é-c model* allows for competing frameworks to coexist and even complement each other (as in the discussion of wave-particle duality in chapter 4). This aligns more with post-Hegelian pluralistic approaches.

Relationship to Post-Hegelian Thought:
My emphasis on how frameworks prove themselves through practical application and expansion beyond their original domains shows affinity with pragmatist philosophers like Peirce and James, who emphasised the practical consequences of ideas. The *o-é-c model* strongly resonates with Peirce's account of scientific inquiry, particularly his emphasis on how communities of investigators gradually work toward truth. The way I describe the *Épistémè Socialisante* phase, with its emphasis on practical validation and pedagogical development, aligns with pragmatist insistence that knowledge claims must prove themselves through practical consequences. My attention to how frameworks expand beyond their original domains particularly echoes Peirce's view of how scientific concepts grow and develop. However, I go beyond traditional pragmatism by providing a more structured account of how revolutionary insights become established knowledge. Where pragmatists sometimes struggled to explain scientific revolutions, the *o-é-c model* shows how radical breaks can be integrated into a broader process of knowledge development.

Novel Contributions and Practical Mechanisms:
The *o-é-c model* provides a more nuanced account of how individual insights interact with social processes, avoiding both individualistic and purely social deterministic extremes. I pay attention to specific mechanisms of knowledge transmission (like textbooks and pedagogical tools) that provide concrete detail often missing from philosophical accounts. My emphasis on how fundamentally difficult-to-formulate problems (*wicked problems*) drive knowledge creation offers a novel perspective on scientific progress.

My thesis represents not so much a rejection of Hegelian and post-Hegelian approaches as a sophisticated synthesis that incorporates their insights while adding more concrete mechanisms and contemporary understanding of scientific practice. The *o-é-c model* preserves the dialectical and social aspects of knowledge development that Hegel emphasised while providing a more nuanced and practically applicable framework for understanding how transformative knowledge emerges and becomes established.

Other Philosophical Traditions

Structuralism and Post-Structuralism:

The *o-é-c model* shares parallels with structuralist approaches to understanding knowledge systems. Like structuralists, I recognise that knowledge exists within frameworks of meaning that shape how we understand problems and solutions. However, I go beyond traditional structuralism by showing how these frameworks can be transformed through systematic processes. The post-structuralist emphasis on how meaning and knowledge are constantly being negotiated and renegotiated aligns with my description of how new frameworks evolve beyond their original domains.

Hermeneutic Tradition:

The way I describe how new frameworks emerge and become integrated into existing knowledge resembles Gadamer's account of how understanding develops through dialogue between different perspectives. My emphasis on pedagogical tools and transmission of knowledge aligns with hermeneutic concerns about how understanding is passed between generations.

Analytical Philosophy:

While analytical philosophy often focuses on logical analysis of scientific knowledge, the *o-é-c model* incorporates aspects of this tradition while extending beyond it. My attention to systematic framework development shows influence from analytical approaches, but I combine this with attention to historical and social factors that analytical philosophers sometimes overlook. Wittgenstein's emphasis on how knowledge is embedded in "forms of life" and language games connects with my attention to how new frameworks must be integrated into existing practices and institutions and permeates knowledge through semantic adoption.

American Naturalism:

The naturalistic tradition, particularly Dewey's work on inquiry and education, strongly resonates with the *o-é-c model*. Like Dewey, I see knowledge development as an active process of problem-solving that involves both individual insight and social validation. My emphasis on pedagogical tools particularly aligns with Dewey's educational philosophy.

Husserl's Phenomenological Approach:

Edmund Husserl (1859-1938) developed phenomenology as a method for examining how things appear to consciousness, emphasising the need to "bracket" our usual assumptions to see phenomena freshly. This connects to my concept of *Ouverture Ontologique* - the phase where existing frameworks must be transcended to see problems in new ways. Like Husserl's method, the *o-é-c model* recognises the need to step outside established ways of thinking to achieve genuine breakthroughs.

Contemporary Social Epistemology:

A persistent challenge in philosophy of science is understanding the relationship between scientific objectivity and social processes. Some approaches sometimes treat social factors as external to "real" science, while others can seem to reduce science to social construction. The *o-é-c model* makes significant contributions to contemporary social epistemology by showing specific mechanisms through which social processes contribute to knowledge creation. I extend beyond both individualistic and purely social accounts to show how individual insights and social validation processes work together. The *o-é-c model* offers a more nuanced view by showing how social processes are integral to knowledge development without undermining scientific objectivity. The *Épistémè Socialisante* phase shows how social validation processes strengthen rather than weaken scientific knowledge.

Frankfurt School and Instrumental Reason:

The Frankfurt School philosophers (including Adorno and Horkheimer) developed a critique of how modern rationality had become primarily "instrumental" - focused on efficient means to achieve ends without questioning the ends themselves. They worried that this approach reduced knowledge to purely technical problem-solving, losing sight of broader human values and purposes. My thesis engages with this tradition by showing how truly transformative knowledge involves more than just technical solutions - it requires new conceptual frameworks and ways of thinking that can address *wicked problems* that resist purely instrumental approaches.

Habermas's Theory of Knowledge Interests:

Jürgen Habermas (1929-) proposed that knowledge is shaped by three fundamental human interests: technical control (empirical-analytic sciences), practical understanding (historical-hermeneutic sciences), and emancipation (critical sciences). This theory suggests that different types of knowledge serve different human needs and purposes. The *o-é-c model* aligns with this by showing how new knowledge frameworks emerge not just from technical problems but from deeper conceptual challenges that require new ways of understanding and thinking.

Critical Theory Connection:

This broader philosophical movement (which includes the Frankfurt School) examines how knowledge and reason are shaped by social and historical conditions, with an emphasis on how knowledge can serve human emancipation. Critical theorists argue that knowledge isn't neutral but is always embedded in social contexts and power relations. My thesis acknowledges this social embedding of knowledge while showing how it can still lead to genuine advances through structured processes of validation and integration. My attention to institutional and social factors in knowledge creation resonates with Frankfurt School approaches, though without their specific critique of instrumental reason. My emphasis on historical and social context shows some alignment with Frankfurt School approaches, particularly Habermas's theory of knowledge interests. Like Habermas, I recognise that knowledge development isn't purely objective but is shaped by social and institutional factors. However, the *o-é-c model* avoids the more radical critique of scientific rationality found in some critical theory, instead showing how social processes can contribute positively to knowledge validation.

Michel Foucault (1926-1984)

Continuities with Foucault:

Historical Contingency: Like Foucault, the *o-é-c model* recognises that knowledge frameworks are historically contingent. My analysis of how post-war environments facilitated paradigm shifts (in Leibniz, Bohr, and Turing's cases) aligns with Foucault's emphasis on how historical conditions shape what counts as knowledge.

Power-Knowledge Relations: The *o-é-c model* implicitly acknowledges what Foucault called power-knowledge relations through my attention to institutional structures, pedagogical tools, and validation mechanisms. The *Épistémè Socialisante* phase particularly shows how knowledge becomes established through institutional networks and power structures.

Discursive Practices: My emphasis on how new frameworks must develop systematic ways of speaking about and conceptualising problems aligns with Foucault's interest in discursive practices. The *o-é-c model*'s description of how new fields establish their terminology and methods parallels his analysis of discourse formation.

Key Modifications and Extensions:

Agency and Innovation: Where Foucault tends to emphasise systemic constraints on knowledge, the *o-é-c model* gives more attention to how individual insights and innovations can transform knowledge systems. The *Ouverture Ontologique* phase particularly shows how new ways of thinking can emerge and challenge existing frameworks.

Structured Progress: Unlike Foucault's emphasis on epistemological breaks and discontinuities, the *o-é-c model* shows how revolutionary insights can be systematically integrated into established knowledge through structured processes.

Practical Mechanisms: The *o-é-c model* goes beyond Foucault's historical analysis to identify specific mechanisms through which knowledge becomes established. The attention to textbooks, pedagogical tools, and institutional networks provides concrete detail that Foucault's more abstract analysis sometimes lacks.

Critical Contributions:

Bridging Individual and Social: The *o-é-c model* provides what Foucault's approach often lacks - a clear account of how individual insights become integrated into broader epistemic frameworks. The three-phase structure shows how personal discoveries become social knowledge.

Wicked Problems: My emphasis on how wicked problems drive knowledge creation offers a different perspective than Foucault's focus on power relations. This suggests how genuine cognitive challenges, not just social forces, shape knowledge development.

Progressive Development: While maintaining Foucault's insights about historical contingency, the *o-é-c model* suggests how knowledge can progressively develop through structured processes of validation and integration. This offers a more optimistic view of knowledge development than Foucault's sometimes sceptical stance.

Minsky's Cognitive Approach to Knowledge

The Cognitive-Social Gap in Knowledge Theory: Minsky's theory excellently describes how individuals build mental models to understand and solve problems. His "society of mind" approach shows how different cognitive processes work together to create understanding. However, his account does not fully explain how these individual mental models become shared scientific knowledge. On the other hand, purely social theories of knowledge sometimes neglect the cognitive processes that generate initial insights.

How the *O-É-C Model* Bridges this Gap:

The *Ouverture Ontologique* Phase: This phase aligns with Minsky's cognitive approach by showing how individuals develop new mental models when confronting *wicked problems*. For example, when Turing developed his concept of computability, he was building what Minsky would call a mental model to address the *Entscheidungsproblem*. The *o-é-c model* shows how this individual cognitive process is the first step in knowledge creation, but not the whole story. The *Épistémè Socialisante* Phase: This is where the *o-é-c model* begins to bridge to the social dimension. I show how individual mental models must be transformed through what Longino calls "critical discursive interactions." For instance, when Bohr's initial ideas about quantum phenomena evolved through discussions with Einstein and others, we see individual cognitive models being refined through social processes. The *Connaissance Éclairante* Phase: This final phase shows how individually conceived and socially refined knowledge becomes part of our collective understanding. When these ideas appear in textbooks and become teaching tools, they've completed the journey from individual mental model to shared knowledge framework. The *o-é-c mode*l shows that knowledge creation isn't simply either cognitive or social - it's both, working in a structured sequence. Individual cognitive breakthroughs must be transformed through social processes to become established knowledge. This bridging function is particularly important for understanding current challenges in knowledge creation. For example, in artificial intelligence development: Individual researchers develop new algorithmic approaches (cognitive); these must be validated through peer review and testing (social); and successful approaches become part of the standard AI toolkit (established knowledge)

Jürgen Renn and *The Evolution of Knowledge*

Fundamental Alignments

Both my thesis and Renn's work in his book *The Evolution of Knowledge: Rethinking Science for the Anthropocene* (Renn, 2020) recognise knowledge development as a complex interplay between individual innovation and social processes. Renn's emphasis on the "shared knowledge economy" parallels aspects of the *Épistémè Socialisante* phase in the *o-é-c model*. Both frameworks acknowledge that knowledge evolution involves more than just accumulation of facts—it requires fundamental transformations in how we understand and approach problems. The recognition of historical contingency appears in both works. Renn discusses how knowledge development depends on specific historical conditions and technological capabilities, similar to how the *o-é-c model* identifies the importance of historical context in enabling new conceptual frameworks to emerge. Both approaches see knowledge evolution as neither purely contingent nor purely deterministic.

Key Theoretical Contributions

Renn focuses on what he calls "knowledge systems" and their evolution through history, examining how different forms of knowledge interact and transform over time. The *o-é-c model*, while acknowledging systemic aspects, provides a more specific account of how individual breakthroughs become integrated into collective knowledge through structured phases. Where Renn emphasises long-term evolutionary processes, the *o-é-c model* offers particular insight into transformative moments when new fields of knowledge emerge. This complementary focus allows the o-é-c model to illuminate specific mechanisms that Renn's broader historical approach sometimes overlooks.

Methodological Approaches

Renn's work draws extensively on historical case studies across multiple civilisations and epochs, providing a grand narrative of knowledge evolution. The *o-é-c model*, while also using historical cases, focuses more intensively on specific transformative moments in modern scientific development. This different scale of analysis leads to complementary insights. The *o-é-c model's* attention to specific mechanisms of knowledge validation and transmission provides detail that complements Renn's broader evolutionary framework. Where Renn describes general patterns of knowledge development, the *o-é-c model* shows precisely how new frameworks become established through social and institutional processes.

Treatment of Innovation

Both approaches recognise the importance of what Renn calls "mental models" in knowledge development. However, the *o-é-c model's* concept of *Ouverture Ontologique* provides a more specific account of how fundamentally new ways of thinking emerge from confronting *wicked problems*. This offers a mechanism for innovation that adds detail to Renn's evolutionary account. Renn emphasises how new knowledge often emerges from the interaction between different knowledge systems. The *o-é-c model* complements this by showing how specific frameworks develop through structured phases of validation and integration.

Social Dimensions

Renn's concept of the "shared knowledge economy" provides valuable context for understanding the social processes described in the *o-é-c model*. Where Renn focuses on broad social and economic conditions, the *o-é-c model* shows specific mechanisms through which social processes validate and integrate new knowledge. Both approaches recognise the crucial role of transmission mechanisms—what Renn calls "representation structures" and what the *o-é-c model* examines through attention to pedagogical tools and institutional networks. However, they analyse these at different scales and with different emphases.

Contemporary Relevance

Both frameworks offer insight into current challenges in knowledge development. Renn's emphasis on sustainable knowledge economies complements the *o-é-c model's* attention to how new frameworks emerge to address complex contemporary problems. Together, they suggest how we might better understand and facilitate knowledge development in an era of rapid technological change. The o-é-c model's specific attention to *wicked problems* adds valuable detail to Renn's broader evolutionary framework when considering contemporary challenges. Where Renn provides historical context for understanding current developments, the *o-é-c model* offers specific guidance about how new knowledge frameworks might emerge and become established.

Synthesis and Extension

Rather than contradicting Renn's work, the *o-é-c model* provides complementary insights at a different scale of analysis. Renn's broad evolutionary framework helps explain the historical conditions that make transformative developments possible, while the *o-é-c model* shows specific mechanisms through which these developments occur.

Next Steps

In the next chapters I will argue that the inventions of novel frameworks triggered by *wicked problems* include scientific representations and evolve towards their eventual acceptance in the body of knowledge. Those additions become new fields that are successful insofar as they conform sufficiently to enable users to profitably interact with their represented domains, that is, to address both original questions regarding a wicked problem or situation, and new applications that were unforeseen before. Starting from somewhat different places, I will aim to develop situated accounts of scientific knowledge that move beyond traditional dichotomies between the rational and social. They converge on viewing scientific knowledge as perspectival and representational success as a matter of conformation between representations and an independent reality, enabling successful practices in the understanding of wicked-like problems and situations. The process by which the individual inventions become accepted fields of knowledge in epistemic communities is a pattern that I shall describe in Chapter 2, and it can be found in the History and Philosophy of Science, as I will show in the analysis of three case studies presented in Chapters 3, 4, and 5.

Chapter 2 - The *o-é-c model* for addition of new fields to knowledge

This chapter introduces a novel model for understanding how new fields of knowledge emerge and become integrated into the broader intellectual landscape. At its core, this model—termed *the o-é-c process*—seeks to bridge the gap between individual insight and collective knowledge creation, offering a fresh perspective on the evolution of scientific and philosophical thought. The additions to the Body of Knowledge that follow this process stem from both individual philosophical insight and collective scientific understanding. This thesis proposes a three-phase process termed the *o-é-c model* (*Ouverture Ontologique - Épistémè Socialisante - Connaissance Éclairante*), reflecting the progression from ontological innovation through social epistemology to established knowledge.

The *Ouverture Ontologique* (Ontological Opening - Invention Phase) phase represents the initial creative act where new conceptual domains are established to address *wicked problems*. The term draws from computer science ontology - formal knowledge representation systems - while the French *ouverture* captures both the opening and the promise of new possibilities. *Ouverture Ontologique* reflects the creation of new conceptual spaces and entities. The 'ontological' aspect emphasises how these breakthroughs often require creating entirely new categories of understanding. For instance, when Leibniz developed calculus, he was not just solving mathematical problems but creating new mathematical objects.

The *Épistémè Socialisante* (Socialising Episteme - Understanding Phase) name draws deliberately on Foucault's concept of épistémè as "the total set of relations that unite... the discursive practices that give rise to epistemological figures" (Foucault, 2009, p. 211). The socialising aspect, drawing on Longino's social epistemology, while emphasising the social processes through which knowledge frameworks become validated and refined. The term *socialisante* specifically highlights the active, ongoing nature of this process. We see this clearly in Bohr's case, where complementarity developed through sustained dialogue within the physics community.

For the final phase I chose the name *Connaissance Éclairante* (Enlightening Knowledge - Adoption Phase) to capture how established knowledge illuminates previously unapproachable questions. This stage manifests when the new conceptual structures become powerful explanatory tools across multiple domains, demonstrating their broader epistemological value. *Connaissance* specifically refers to knowledge that has become part of our shared understanding, while *Éclairante* emphasises its power to illuminate new areas of inquiry. Turing's work exemplifies this - his ideas about computation continue generating new insights across multiple fields.

These terms connect to important philosophical traditions while offering precise meanings that English alternatives might miss. French philosophical terminology often captures nuances about knowledge and understanding that are harder to express in English. Additionally, using French terms helps distinguish these specific technical concepts from their everyday English counterparts, much as we use Latin terms in biology or Greek in philosophy and mathematics. Together, these terms describe a process where new conceptual spaces are opened (*Ouverture*), developed through social processes (*Socialisante*), and ultimately become illuminating knowledge (*Éclairante*). The terminology reflects both the distinct character of each phase and their interconnected nature in the overall process of knowledge creation.

The *o-é-c process* outcome is not validated by empirical data since its purpose is not to identify or understand an event or entity but to provide a perspectival method to understand the creation of new ontological spaces in intellectual revolutions and expansions that we can describe as novel fields of knowledge. Structurally it is represented by the following outline and diagrammatic model:

<u>*The o-é-c model for New Fields of Knowledge*</u>

Ouverture Ontologique: Invention Phase
Historical Environment
Existing Paradigm
Wicked Problem

Intellectual leap
Alternate Views - Competing frameworks
Épistémè Socialisante: Understanding Phase
Gestalt Shift
Support for new paradigm
Expansion and Pedagogy
Textbooks and new applications
Connaissance Éclairante: Adoption to Body of Knowledge Phase
Becoming Knowledge

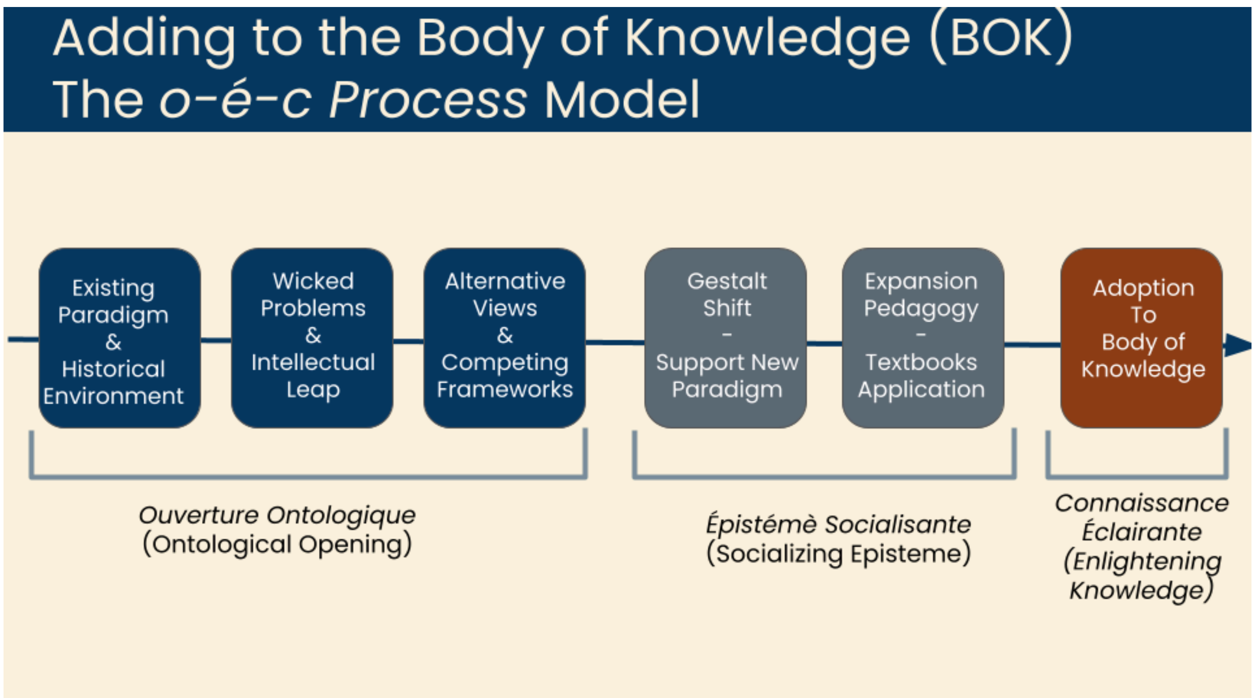


Figure 1: The *o-é-c model* block diagram

I will now describe in more detail each one of the phases and their components, providing examples that illustrate them and are taken from intellectual history.

Ouverture Ontologique - Ontological Opening

In this phase, an intellectual immersed in a current paradigm works on a *wicked-like* problem or situation with an approach that is philosophical in nature, and multidisciplinary in its application, and to do so needs to describe the situation in a way that is not consistent with the current accepted models. I define *paradigm* following Thomas Kuhn's conception of paradigm as a set of methods, values, and standards for success that defines a particular field and group of professionals (Kuhn, 2012, p. 11).

Historical Environment; the sociological atmosphere

The historical and sociological environment proves fundamental to understanding the emergence of scientific innovations. This thesis draws on three key historiographical perspectives to analyse this relationship. Collingwood's insight that historical understanding requires reconstructing past thought processes through the historian's mind (Collingwood, 2005, pp. 7–13) provides a methodological foundation for examining how new knowledge emerges from specific historical contexts. Kuhn's analysis of scientific development demonstrates how competing valid approaches can coexist, with their resolution dependent not solely on empirical evidence but also on broader historical and social factors. As he argues, while empirical constraints bound scientific beliefs, their final acceptance involves elements of historical contingency and social context (Kuhn, 1994, p. 4). Forman's study of quantum mechanics' development in Weimar Germany provides compelling evidence for how the cultural environment shapes fundamental scientific concepts. His demonstration that German physicists' abandonment of causality preceded its mathematical justification, driven by "social-intellectual pressure exerted upon the physicists as members of the German academic community" (Forman, 1971, p. 110), exemplifies how social contexts can influence even the most fundamental scientific concepts. These perspectives collectively demonstrate that scientific innovations emerge not in isolation but through complex interactions between individual insight and sociocultural context—a framework essential for understanding the *ouverture ontologique* phase of knowledge creation.

An example of the role of historicity can be seen in the so-called *Forman's Second Thesis*. Forman wrote a paper in 2007 regarding the shift in funding from science to technology (Forman, 2007). He argues that there has been a historical shift from the primacy of science in society and culture during the modern period to the primacy of technology in the postmodern period that occurred in the 1980s. This transition reflects broader changes in societal values, economic structures, and cultural priorities. (Forman, 2007, p. 2). Forman examines technology's role in society beyond technical achievements, emphasising its ideological context. In postmodernity, technology transitions from a mere scientific tool to a societal force that shapes culture and human identity: "Technology is not autonomous; rather, it is impelled by choices made in the context of circumstances in ambient realms, very often in the context of disputes over political power" (Forman, 2007, p. 67). This shift represents a broader transition from modern rationality and universal truths to postmodern subjectivity and socially constructed knowledge.

A second example of the role of historicity can be found in Freudenthal's and McLaughlin's analysis (Freudenthal and McLaughlin, 2009a) of two fundamentally important works in the history of science: *The Social and Economic Roots of Newton's Principia* (Hessen, 2009) published in 1931, and *The Social Foundations of the Mechanistic Philosophy and Manufacture* (Grossmann, 2009) published in 1935. Hessen and Grossman independently developed a view of science using Marx's concept of labour as a root for their work, in which Freudenthal and McLaughlin identify the so-called "Hessen-Grossmann theses'' that describe the interrelations between economic and technology development and the emergence of science in the early modern period. The first thesis proposes that the abstractions necessary for the development of theoretical mechanics and physics stem from the development of machine technology and the study of how to improve it, making technology an engine for the advancement of science (Hessen, 2009, p. 53). The second thesis proposes the converse statement, this is to say that some areas of science like thermodynamics did not develop in the seventeenth-century since scientists could not use existing technology such as steam engines as a base to develop abstraction models (Hessen, 2009, p. 74). The third thesis proposes that science is affected and constrained by social factors such as ideological and religious belief or existing political and philosophical theories (Freudenthal and McLaughlin, 2009b, p. 3). One additional work has influential views on how the development of knowledge is socially situated: Ludwik Fleck's communications with the *Vienna Circle* regarding essential relations between scientific knowledge and social mechanisms like education and tradition. In 1933, in a letter to Moritz Schlick, one of the founders of the *Vienna Circle*, Fleck proposed the idea that science is a collective enterprise while arguing that all knowledge is socially conditioned. In his letter, he criticised traditional epistemology that held that all knowledge originates from sensation, and he argued that the variety of knowledge stems from textbooks. He further added that historical development of knowledge shows exceptional common aspects (Renn, 2022, pp. 53–55; Engler and Renn, 2016, p. 140; Fleck, Trenn and Bradley, 1981; Fleck, 1933).

I find many of the arguments in my thesis echo some of Fleck's work, which remained mostly unknown until the 1960s when Kuhn's work and that of others with similar arguments prompted the vision of science situated in history and sociology, finally dethroning logical positivism as the main view in philosophy of science. Some other philosophers that contributed to that paradigm shift were Paul Feyerabend with his critique of scientific method and his book *Against Method* (1975), Imre Lakatos proposing *The Methodology of Scientific Research programmes* (1978), Larry Laudan on the rationality of science in his work *Science and Values* (1984), and Michael Polanyi with the role of personal knowledge in *Personal Knowledge: Towards a Post-Critical Philosophy* (1958).

Although I do not want to imply causality, it can be seen how different aspects of the historicity and sociology of the environment surrounding the emergence of new fields follow periods of great instability and questioning of the status quo like those after wars. There are many examples in intellectual history, as well as those used in this chapter, showing this pattern, and here are some of those examples: the Vietnam and civil rights era and the return to philosophy in physics with Bell's theorem and quantum entanglement becoming precursors to quantum cryptography as shown in David Kaiser's work (Kaiser, 2012), post-reformation period and the development of the Jesuit Education system that systematised education at a global scale for the first time as shown in John O'Malley's work (O'Malley *et al.*, 1999) (O'Malley *et al.*, 2000), Napoleonic wars and Great Britain's conflicts in the pre Darwin period as precursors of challenges to a static view of Natural History (Stott, 2012), post-world war II period for Claude Shannon's theory of information (Soni and Goodman, 2017), and Richard Feynman's Diagrams (Kaiser, 2005). The post-war periods facilitate the challenging of established ideas and paradigms that can no longer be justified in view of the conflicts and their aftermath. Proposing new ideas in such an environment can be done with lower risk of ridicule, attacks of disfranchisement by intellectual and professional circles, and in some cases, follow a cultural pressure for change as Forman showed was the case of the Weimar republic and the scientific shift towards acausality (Forman, 1971). The historical environment and the intellectual paradigm form the context in which some philosophers work on problems that cannot be understood with existing models and descriptions, the so-called *wicked problems*. I am now going to describe the intellectual paradigms and their role in the process.

Existing Paradigm; working within the accepted view

Kuhn's analysis of scientific development emphasises how intellectual frameworks shape what questions scientists pursue and how they approach them. While Kuhn argues that normal science focuses on solvable puzzles within established paradigms (Kuhn, 2012, pp. 36–37) , the history of major scientific developments suggests a more complex dynamic where researchers also pursue questions that challenge paradigmatic boundaries.

The pre-Darwinian era provides an illuminating example of this interplay between paradigm and innovation. The dominant framework, grounded in pre-existence theory and supported by Swammerdam's and Leeuwenhoek's empirical studies, viewed species as fixed entities unfolding from divinely established forms. As Sloan notes, Malebranche developed this into a comprehensive theistic mechanism that shaped the intellectual environment Darwin would later transform (Sloan, 2024b, p. 2.2 Mechanism, Pre-existence Theory and Species Fixity). This paradigm structured both scientific practice (through Linnaeus's taxonomic work) and theoretical possibilities (by restricting concepts of species change). However, emerging geological theories from Hutton and Lyell, particularly uniformitarianism, began creating conceptual space for evolutionary thinking by demonstrating how gradual processes could produce profound change. Darwin's subsequent theoretical innovation thus emerged not in isolation but through critical engagement with existing frameworks, reinterpreting earlier transformist discussions within a new conceptual architecture (Sloan, 2024b, p. 5. Summary and Conclusion).

Wicked Problems

Natural philosophy encounters fundamental problems that resist definitive solutions while catalysing intellectual innovation. These *wicked-like* problems share key characteristics: they resist complete formulation, involve multiple stakeholders with competing perspectives, and generate solutions that are better-or-worse rather than true-or-false. Several classical examples illustrate this pattern. The mind-body problem exemplifies these characteristics, challenging fundamental assumptions about consciousness and physical reality. Despite frameworks ranging from Cartesian dualism to contemporary materialism, each proposed solution generates new philosophical and ethical questions about free will and personal identity. Similarly, Hume's problem of induction reveals deep uncertainties in the foundations of scientific knowledge itself. The nature of light provides a particularly illuminating example of how wicked problems drive scientific and philosophical development. The historical tension between particle and wave theories, exemplified in the Newton-Huygens debate, demonstrates how such problems resist simple resolution. Newton's methodological response to Huygens illustrates this complexity:

> It seems to me that M. Hugens takes an improper way of examining the nature of colours... Perhaps he would sooner satisfy himself by resolving light into colours as far as may be done by Art, and then by examining the properties of those colours apart... (Newton, 1673, p. 31).

This exchange reveals how *wicked problems* demand thinking beyond existing paradigms, often requiring what this thesis terms an "intellectual leap" - a fundamental reconceptualisation that opens new theoretical possibilities while acknowledging the inherent complexity of the problem space.

Intellectual Leap

This thesis extends and reframes Kuhn's concept of paradigm shifts to analyse how fundamentally new knowledge frameworks emerge. While Kuhn views intellectual leaps primarily as responses to accumulated anomalies within existing paradigms (Kuhn, 2012, pp. 19–20), and Forman emphasises external sociocultural pressures as drivers of scientific transformation (Forman, 1971, p. 7), I propose a distinct mechanism for conceptual innovation.

I argue that intellectual leaps can arise specifically from confrontation with wicked-like situations where existing models prove fundamentally inadequate. These leaps differ from both Kuhnian paradigm shifts and Forman's sociologically-driven transformations in three crucial ways: First, they emerge not from the need to resolve anomalies within a paradigm but from the recognition that entirely new epistemic frameworks are required. Second, rather than being driven by external pressures toward intellectual conformity, these leaps often occur in opposition to such pressures. Third, they frequently transcend disciplinary boundaries, creating new conceptual spaces that enable novel approaches to previously intractable problems. This perspective aligns with Renn's observation that "there is no general scheme according to which a scientific transformation takes place" (Renn, 2020, p. 20), while offering a specific mechanism for how fundamentally new knowledge frameworks can emerge. This theoretical innovation provides a more nuanced understanding of how transformative scientific insights develop, particularly in response to complex, multidimensional challenges that resist traditional disciplinary approaches. Jürgen Renn introduces the term “epistemic islands” to describe how a set of insights, results or activities sometimes develops partially separated from the exploration of a system of knowledge and loosely connected to it by unusual applications, unfamiliar mental models, or audacious hypothesis, and in some cases, the epistemic islands can develop their own systematicity (Renn, 2012, p. 83). Renn posits that some transformations of systems of knowledge are the product of catalytic processes triggered by external and internal changes and emerge by the formation of epistemic islands somewhat decoupled from the core of the pre-existing system of knowledge. Such islands may then act as matrices or seedbeds for building a new system (Renn, 2012, pp. 118–119).

I argue that in some cases, the intellectual leap occurs not from triggering changes within existing systems of knowledge but from the exploration of a new *ontological space*. This *space* develops from addressing *wicked problems* with a multidisciplinary approach needing the creation of new frameworks that combine existing ideas with intuition and *a priori* thoughts, expands the existing understanding through effective use of methods, symbols, notations and descriptions that can be adapted and used by others. The systematisation of these *spaces* occurs in their invention directly as the result of the need to describe them and continue in their development as new epistemic entities. To illustrate this description, I will use the example of Claude Shannon's work. Shannon's breakthrough emerged from addressing a wicked-like situation in electrical circuit design, which previously relied on trial and error. While working at MIT on a mechanical differential analyser that required complete reconfiguration for each new problem (Soni and Goodman, 2017, pp. 32–33), Shannon recognised that relay operations could be abstracted into binary states - open or closed. Drawing on his undergraduate exposure to Boolean algebra (Soni and Goodman, 2017, pp. 35–36), he made the transformative connection between mathematical logic and circuit design. As articulated in his master's thesis, Shannon demonstrated that circuit configurations could be represented through equations analogous to logical propositions:

> … any circuit is represented by a set of equations, the terms of said equations corresponding to the various relays and switches in the circuit... This calculus is shown to be exactly analogous to the calculus of propositions used in the symbolic study of logic (Shannon, 1993a, p. 471).

This insight transformed circuit design from intuitive craftsmanship to a precise science (Soni and Goodman, 2017, p. 39), establishing a framework applicable across different physical implementations.

Shannon created an ontological space disconnected from existing systems of knowledge that was systematised since its creation and that combined existing ideas with intuition and *a priori* thoughts expanding the existing understanding of different fields through effective use of methods, symbols, notations and descriptions that can be adapted and used by others. Shannon systematised the framework and provided a pedagogical method of transmitting its use and applications through a clear description of the symbolic rules and graphical depictions and with illustrative practical examples unrelated to the original wicked-like situation that gave origin to his insight and invention. One such example is the design of an *electrical combination lock* (Shannon, 1993a, p. 493), and another is an *electrical calculator using base two* (Shannon, 1993a, p. 494). This is a successful model according to the definition of knowledge adopted from Minsky in Chapter 1.

An intellectual leap of even more influence came from Claude Shannon's work in information theory and its applications can be seen as grappling with a *wicked-like* problem or situation related to the nature of *information*, and the limits of signal transmission in the realm of communication and information processing while he was working at Bell Labs. This *wicked-like situation* involved several interconnected and complex challenges. Before Shannon, there was no formal, quantitative way to measure information. Shannon had to grapple with the complex task of precisely defining and quantifying the concept of *information.* This involved abstracting the notion of *information* from its existing semantic and contextual meanings and developing a mathematical framework to measure it in terms of probabilities and uncertainties. Communication was understood in qualitative terms, and the concept of information was vague and multifaceted.

Shannon's work addressed the *wicked-like* challenge of dealing with noise and interference in communication systems. Noise is inherent in any real-world communication channel, and its effects are complex, unpredictable, and can distort or corrupt the transmitted information in non-linear ways. Claude Shannon's theory of information, introduced in his landmark paper “A Mathematical Theory of Communication” (Shannon, 1948), represented a profound intellectual leap in understanding how information could be quantified, transmitted, and encoded efficiently. Shannon introduced the concept that information could be quantified in terms of bits (binary digits) (Shannon, 1948, p. 380). This was a revolutionary idea that allowed for the measurement of information in a way that was independent of the content or meaning of the information itself. It enabled a mathematical analysis of communication systems, focusing on the transmission and processing of information as a physical phenomenon (Shannon, 1948).

During the war years Shannon began to work on the properties of systems for the transmission of intelligence and on the possibility of machines learning to think, a topic that was present in the daily teatime meetings that he had with Alan Turing at the cafeteria of Bell Labs in 1943 (Gleick, 2011, p. 204). They were also both thinking about the probability of occurrence of messages in communications. Shannon thought about the structure of language and working on ciphering systems he found persistent patterns and he had to understand language in a way that was not done before. Shannon understood that language has a statistical structure and redundancy so he suggested the measure of the amount of text that can be reduced in length without losing any information. He created a framework that abstracted the idea of the message from its physical details. Shannon explains the fundamentals thus:

> In any branch of applied mathematics, the vague and ambitious concepts of a physical problem are given a more refined and idealised meaning. In information theory, one of the basic notions is that of the amount of information associated with a given situation. 'Information' here, although related to everyday meaning of the word, should not be confused with it. In everyday usage, information usually implies something about the semantic content of the message. For the purposes of communication theory, the 'meaning' of a message is generally irrelevant; what is significant is the difficulty in transmitting the message from one point to the other (Shannon, 1993b, p. 173).

Shannon's seminal contribution to information theory lay in his application of thermodynamic entropy principles to quantify message uncertainty, thereby establishing theoretical boundaries for information compression and transmission (Shannon, 1993b, pp. 22–25). His interdisciplinary synthesis across mathematics, engineering, computer science, and physics yielded a unified theoretical framework whose impact transcended disciplinary boundaries. Shannon's conceptualisation of *information* as uncertainty reduction and *bits* as fundamental units achieved remarkable transdisciplinary resonance, influencing fields from molecular genetics to organisational theory, while demonstrating the critical importance of continuous theoretical refinement through cross-disciplinary collaboration.

The inventions that this element of the *o-é-c model* describes, occur in a fertile intellectual environment in which similar frameworks arise to compete for acceptance. This is the subject of the next section.

Alternate Views - Competing frameworks

Intellectual leaps lead to the exploration of alternative frameworks and entities that are used to address and understand problems and *wicked-like situations*. This process is not smooth or purely rational, as Kuhn highlighted by describing the role of subjective, psychological, and sociological factors in the acceptance of a new paradigm (Kuhn, 2012, pp. 77–91). Scientists' commitment to existing paradigms and the revolutionary nature of paradigm shifts mean that these intellectual leaps often involve significant debate, resistance, and only gradually achieve consensus within the scientific community. In Kuhn’s view the decision to abandon a paradigm and adopt another is simultaneous, and at the same time, the decision is associated with the contrast between the two paradigms and between the paradigms and nature (Kuhn, 2012, p. 78).

In the context of this thesis, the existence of a fertile environment of ideas together with sociological factors foster the emergence of alternate frameworks that compete to expand into similar ontological spaces, and as we will see later, the adoption of one in particular is due in part to the systematisation incorporated from the beginning, as well as sociological and pedagogical factors that we will explore in the section on the *Épistémè Socialisante*. While the process of choosing one framework over another is sometimes acrimonious like the cases of Leibniz and Newton with the Calculus or with Richard Feynman and Julian Schwinger on methods for quantum electrodynamics; this is not always the case like in the examples of Darwin and Wallace on natural selection, Einstein and Hilbert on relativity, or Turing and Church on computability. As an example of this element of the *o-é-c process* I will describe the case of the ideas on natural selection.

Charles Darwin's landmark work, *On the Origin of Species* (Darwin, 1859), directly addressed one of the most profound and *wicked-like* problems in the history of natural philosophy and biology: the origin and diversity of life on Earth. The question of how life in its myriad forms came into being could be seen as a *wicked problem*. Darwin's evolutionary theory exemplifies the interdisciplinary nature characteristic of complex scientific frameworks, integrating insights from geological temporality, Malthusian economic principles, selective breeding practices, and biogeographical distribution patterns. This multifaceted approach illustrates the inherent complexity of addressing wicked problems in scientific inquiry. As Sloan articulates, the philosophical implications of evolutionary theory continue to generate scholarly discourse across multiple domains: questions of natural teleology, ethical considerations, the intersection of evolutionary naturalism with religious thought, and humanity's position within the broader organic world. Particularly contentious areas of ongoing philosophical debate include Darwin's theoretical framework regarding human cognitive capabilities, moral attributes, and his application of sexual selection theory to explain human gender dynamics—the latter remaining a significant focus of feminist scholarly critique. (Sloan, 2024a, p. 5. Summary and Conclusion).

It is often accepted that Wallace and Darwin reached similar conclusions independently, highlighting the parallel development of evolutionary ideas, and therefore they are co-credited for the theory of natural selection In his recollections, Wallace described how during a feverish bout of malaria, and while thinking about Thomas Malthus's *An Essay on the Principle of Population* (Malthus, 1798), he had an insight to ask why, within populations, some die and some live. His realisation was that

> In every generation the inferior would inevitably be killed off and the superior would remain—that is, *the fittest would survive*.… The more I thought over it the more I became convinced that I had at length found the long-sought-for law of nature that solved the *problem of the origin of species [my italics]* (Wallace, 1905, p. 362).

During his four years in isolation in the jungle, Wallace had independently come up with the same arguments as Darwin. He wrote his conclusions and ideas to Darwin in an essay signed in February 1858 from Ternate in the Malay Archipelago (Wallace, 1858).

Darwin picked up a package addressed to him one morning in June 1858 wondering who could be reaching out to him from a remote island between Celebes and New Guinea when he recognised Alfred Russell Wallace's handwriting. Reading Wallace's letter had a cataclysmic impact: "All of Darwin's main ideas were repeated. To Darwin's agitated mind these ideas seemed to hang together in Wallace's essay far better than they did in his own unpublished writings" (Browne, 2002, p. 14). Darwin reached out to Lyell and Joseph Hooker for advice, who afterwards presented papers to the Linnaean Society to ask for a judgement on the primacy of discovery. The verdict reached was that both men needed to be congratulated and that Darwin had first recorded the idea and had it witnessed by others in an unpublished essay in 1844 (Darwin, 1858, pp. 46–50) (Stott, 2012, pp. 265–266). Wallace and Darwin were exemplars of gentlemanly behaviour, and with the influence of Lyell and Hooker, Wallace's essay (Wallace, 1858, pp. 53–62) and a short account of Darwin's own findings (Darwin, 1858, pp. 50–53) was submitted for publication along with a double announcement and reading at the meeting of the Linnean Society in London on 1 July 1858, although neither Darwin nor Wallace was present at the meeting.

Prompted by these events, Darwin's book was written as a compressed version of the manuscript he had, and *On the Origin of Species* was published in London on 24 November 1859 (Darwin, 1859) (Darwin, 2011). The intertwining ideas of both men continued:

> In one of his copies of *On the Origin of Species*, Alfred Russell Wallace crosses out 'natural selection' and writes 'survival of the fittest' next to it. Wallace always felt that 'selection' inappropriately imported anthropomorphic notions of Nature choosing purposefully between variants into natural history (Lennox and Pence, 2024, p. 2.3.2 The Nature, Power and Scope of Selection).

There is great focus and attention on the issue of primacy of invention or discovery in Intellectual History, however, what eventually becomes part of the Body of knowledge is not necessarily contingent on that primacy, but on other aspects that are sociological in nature. This is what constitutes the second phase of the *o-é-c process,* as I will describe next.

Épistémè-Socialisante - Socialising Episteme:

Knowledge beyond an individual's capability to solve problems is developed, maintained, transmitted and evolved within epistemic communities. In science and its practice, the reliability of knowledge claims stems from the theoretical tools and resources available within its community used to justify those claims, making knowledge perspectival as it is historically and culturally situated by the boundaries of those communities. However, sanctioning the reliability of knowledge claims and their justification is the product of the intersection of the outcomes of different epistemic communities. As Massimi says:

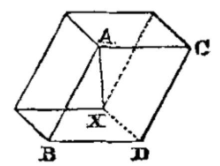
> It is the ability of this plurality of communities to compare data, to enter in dialogue, and to make relevant and appropriate inferences from data to the phenomena in question that ultimately allows for increasingly reliable knowledge about the phenomenon of, for example, global warming (Massimi, 2023, p. 6).

The *Épistémè Socialisante* phase represents the social dimension of knowledge expansion beyond traditional scientific discovery frameworks. While Schindler (Schindler, 2015) defends Kuhn's view that scientific discoveries require both observation (discovery-*that*) and conceptualisation (discovery-*what*), this thesis argues for a broader understanding that incorporates invention, modelling, and social validation processes.

This phase involves creating new ontological spaces with novel frameworks and semantic entities that open previously unconsidered possibilities. As these spaces are explored, their implications continually develop through social discourse, teaching, and practical applications. The resulting knowledge propagates through collective understanding and becomes integrated into the broader body of knowledge, providing new perspectives on human activity. This social transformation begins with a gestalt shift in perception when working within these new spaces.

Gestalt shift

Norwood Hanson Russell introduces his ideas on the patterns of discovery (Hanson, 1985, pp. 4–30) by explaining that perception is composed of two inseparable aspects of observation; *seeing* and *interpreting,* and he gives the example of a three dimensional cube rendered in a two dimensional figure to illustrate how we can see the figure as going inside of the plane and down to the right, we can interpret the lines differently to see the figure going outside of the plane and upwards to the left. That optical illusion is called the *Necker cube,* and it was first introduced as a rhomboid by the Swiss crystallographer Louis Albert Necker (1786-1861) (Necker, 1832, p. 366) and reproduced here.



Under certain circumstances we see some things differently without this being due to interpretation. To exemplify this, he uses reversible figures from textbooks on Gestalt psychology. The example of the figure reproduced here can be seen as a young woman "à la Toulouse-Lautrec" or as an old Parisian woman (Hanson, 1985, p. 11). The idea that we might see different things is expanded to include the notion that what we see is modified and moulded by our prior training: while looking at the sky at dawn, Tycho Brahe, following Ptolemy and Aristotle, saw a mobile sun in the sky where the Earth was fixed, while Johannes Kepler saw the Sun as fixed with the Earth moving.

> If seeing different things involves having different knowledge and theories about *x*, then perhaps the sense in which they see the same thing involves their sharing knowledge and theories about *x* (Hanson, 1985, p. 18).

Thomas Kuhn expands on those ideas and proposes that

> During scientific revolutions scientists see new and different things when looking with familiar instruments in places they have looked before. It is rather as if the professional community has been suddenly transported to another planet where familiar objects are seen in a different light and are joined by unfamiliar ones as well (Kuhn, 2012, p. 111).

While Kuhn maintains that paradigm shifts require scientists to be retrained to see a new gestalt, he argues this shift is irreversible, unlike traditional gestalt experiments where subjects can alternate between perspectives. Within established paradigms, Kuhn (Kuhn, 2012, p. 115) asserts that interpretive activity can only articulate, not correct the paradigm, with breakthroughs emerging from individual genius responding to crisis through sudden insight (Kuhn, 2012, p. 122). However, I propose that there is a different possibility for the advent of a gestalt shift which can be due to sociological causes, and Renn offers an alternative social perspective on gestalt shifts. He introduces the concept of Epistemic Communities as groups dedicated to maintaining and advancing shared knowledge bodies (Renn, 2022, p. 428). Rather than attributing paradigm shifts to individual genius, Renn argues these transformations emerge from community efforts through the interaction of social networks and knowledge systems (Renn, 2022, p. 314).

In the additions of knowledge that follow the *o-é-c process*, a scientist's invention stems from working on a wicked-like problem or situation that cannot be understood from within an existing paradigm, and in order to address it they invent a systematised framework and entities from the beginning, requiring a gestalt shift not because the paradigm from which the thinker was working, but because it did not exist before. While I agree with Renn in his view of a new paradigm as the product of a social process, I will add that the origin is a thinker that lifts existing ideas and combines them with insights and *a priori* concepts to create something new and different. Operating in the new space requires a gestalt shift to accept the semiotic and semantic entities that form the framework, and it is the social process of the discussion, clarification, and refinement of the system among researchers operating within the new space that allows its further development, expansion and the implementation of new applications.

One example of this shift is the development and adoption of the Feynman diagrams. Richard Feynman's groundbreaking work in quantum electrodynamics (QED), particularly his 1949 paper "Space-Time Approach to Quantum Electrodynamics" (Feynman, 1949) and his invention of *Feynman diagrams*, addressed a quintessential wicked problem in theoretical physics:

understanding and calculating the behaviour of subatomic particles and their interactions. Quantum mechanics presents a wicked problem because it defies classical intuitions, with particles exhibiting both wave-like properties and probabilistic behaviour. In the context of QED, the phenomena of superposition and entanglement emerge as fundamental yet paradoxical concepts from a classical physics perspective.

In QED, particles exist in a superposition of multiple quantum states until observed or measured. An electron, for instance, can simultaneously occupy multiple positions or energy states—a direct contradiction to everyday experience. When measured, this superposition collapses to a specific state, with the probability determined by the particle's wavefunction. Entanglement further complicates this picture, as particles become correlated such that their quantum states cannot be described independently, even across large distances. This apparent violation of locality manifests when measuring one particle instantaneously affects its entangled partner, regardless of separation. QED's computational challenges were particularly daunting before Feynman's contributions. Calculations often yielded problematic infinities, especially when computing quantities like electron self-energy. These infinities arose from virtual particles—ephemeral entities emerging from vacuum fluctuations. Feynman's approach, alongside work by Schwinger and Tomonaga, introduced renormalisation as a solution. This technique, while mathematically sound, replaced unobservable quantities with their observable counterparts, effectively managing the infinities. Though the resulting predictions achieve remarkable experimental accuracy, the philosophical implications of renormalisation remain debated, spurring ongoing research into more fundamental theories. The significance of QED as a *wicked problem* lies in its successful merger of quantum mechanics' probabilistic nature with special relativity's continuous, causal character. This reconciliation exemplifies the interdisciplinary complexity of wicked problems while raising profound questions about determinism, realism, and the nature of physical law.

Feynman's most famous contribution, his *diagrams*, illustrates his approach to this *wicked problem*. Instead of getting lost in mathematical complexity, he developed a visual, intuitive language for particle interactions. Each diagram represents a term in the calculation, allowing physicists to organise and interpret the complex web of interactions. This visual tool transformed a mathematically daunting problem into one that could be approached with physical intuition.

Feynman diagrams represent particles as lines that travel through space and time, with their interactions depicted as vertices where these lines meet. Electrons, positrons, photons, and other particles are all represented within this framework, making it a universal language for particle physics. The figure included here is the first published Feynman Diagram from his 1949 paper on Quantum Electrodynamics: “we wish to describe electrodynamics as a delayed interaction between particles… the results of this interaction can be expressed very simply” (Feynman, 1949, p. 776). This visual and calculational simplification allowed physicists to more intuitively understand and calculate the probabilities of various quantum events. This was not the case from the beginning. During the 1950s and 1960s they were interpreted in a number of ways: “They were drawn differently and mustered in different fashions to varying calculational, and ultimately ontological, ends.” (Kaiser, 2005, pp. 4–5). By the end of the 1960s they were beginning to be used by some scientists in gravitational physics.

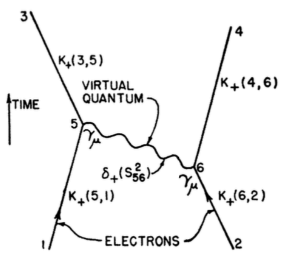


Fig. 1. The fundamental interaction Eq. (4). Exchange of one quantum between two electrons.

Feynman’s invention of the *diagrams*, and the systematisation that he provided for them gave semantic and semiotic value for the physicists in a way that became a gestalt shift for their perception and understanding. *Feynman diagrams* are more than a computational convenience; they represent a paradigmatic shift in the way the fundamental nature of reality is conceptualised and understood in physics aligning more closely with human intuition. This shift was not merely methodological but conceptual, altering the very language physicists used to describe the quantum world. It moved the focus from abstract mathematical formalisms to more tangible, visual representations, thus changing the way physicists thought about and approached problems in quantum mechanics. The adoption of *Feynman diagrams* became a cornerstone in the development of the Standard Model of particle physics, contributing to the theoretical underpinnings of the strong, weak, and electromagnetic forces in this new ontological space.

There are positions within science studies that advocate for either the use of scientific illustrations as pictures of reality or as social constructions. Bruno Latour and Steve Woolgar's ideas on the important role of *inscriptions* in scientific works generated case studies on a variety of representational case studies ranging from the computer-enhanced images of quasars to the Renaissance botanical woodcuts.

> These discussions from art history and science provide important insights into *Feynman diagrams*' tenacity during the 1950s and 1960s even as the original meaning, theoretical embeddings, and associations with other calculational practices came and went (Kaiser, 2005, pp. 361–362).

At the end of his book on *Drawing Theories Apart*, Professor David Kaiser from MIT beautifully writes: "Feynman diagrams do not occur in nature, and theoretical physicists are not born, they are made. During the middle decades of the twentieth century, both were fashioned as part of the same pedagogical process." (Kaiser, 2005, p. 387)

Support for new paradigm

The claim that knowledge is social, and in agreement with Longino's social epistemological perspective, by social I mean it is interactive in the epistemic community that accepts it, is not enough to justify its validity, in particular when its justification cannot be adjudicated to a binary truth value of either true or false. It is necessary to show that the social dimension of cognition is capable of correction and protection of vulnerability due to epistemically undermining possibilities such as inappropriate exercise of authority or shared hidden biases in the community that holds the knowledge. That capability stems from the knowledge producing activities in discursive interactions that become both constructive and justificatory (Longino, 2002a, p. 205). The gestalt shift generated in this phase of the process gains support though those discursive interactions as we will see in the next examples.

“Science is rooted in conversations” (Heisenberg, 1971, p. xvii). At the end of his life, Werner Heisenberg wanted to recount the development of quantum physics over a period of 50 years prior, and to do so, he chose to tell that history in his book *Physics and Beyond - Encounters and Conversations* (Heisenberg, 1971) by recounting the way he experienced and participated in it, through interactions, conversations, and discussions with others. Renn connects with this idea thus

> Knowledge has not only mental, but also social, and material dimensions. It can be passed on from individual to individual and across generations with the help of ‘external representations’ such as writing or symbol systems which are part of the material culture of a society (Renn, 2020, p. 9).

The testing of the framework and learning of its use and entities begins to expand through interaction with other members of the epistemic community, and this happens in, and is affected by the historical and cultural perspective of that community. To illustrate an example of this element of the process I will rely on the work of David Kaiser regarding the dispersion of the Feynman Diagrams in post-war physics in his book *Drawing Theories Apart* (Kaiser, 2005).

After World War II, every aspect of the work of physicists, including the methods of training, the communications of results and techniques, and the decisions of what to study and by what means. The paradigm of Physics was embedded in the political and social environment, and in the US, the cold war and McCarthyism affected the scientists' civil liberties and patterns of thought, the political differences between older and new generations in educational environments, and the relations and competing politics between research groups. (Kaiser, 2005, pp. 5–7).

While Feynman's work made QED calculations tractable and agreed remarkably well with experiments, it did not *solve* quantum physics. Instead, it opened new avenues of inquiry.

Questions about quantum gravity, the standard model's many parameters, and interpretations of quantum mechanics remain active *wicked problems* in physics, as Feynman explains in the conclusions of his seminal paper on QED introducing the *diagrams*:

> Calculations are very easily carried out in this way to lowest order in g^1 for the various theories for nucleon interaction, scattering of mesons by nucleons, meson production by nuclear collisions and by gamma-rays, nuclear magnetic moments, neutron electron scattering , etc. However, no good agreement with experiment results, when these are available, is obtained. Probably all of the formulations are incorrect. An uncertainty arises since the calculations are only to first order in g^1, and are not valid if g^3/hc is large (Feynman, 1949, p. 784).

The Feynman diagrams transcended their role as mere algorithmic tools to become a central framework in physics, requiring extensive training and collaborative practice to master—exemplifying Heisenberg's observations about scientific knowledge transmission. This collaborative nature was evident in 1947 when Enrico Fermi, despite his expertise and close wartime collaboration with Hans Bethe, required face-to-face interaction to fully grasp and expand Bethe's work (Kaiser, 2005, pp. 12–13). As Longino argued, such informal personal communications proved essential for knowledge transfer, particularly in training new theorists. Rather than representing Kuhn's concept of incommensurability, this phenomenon demonstrated the importance of discursive interactions in building epistemic community support.

The dissemination of Feynman diagrams relied heavily on personal networks. Feynman's direct engagement through tutorials at Cornell and his willingness to assist other researchers contributed significantly to their adoption. The rise of postdoctoral training programs, particularly under Freeman Dyson at the Institute for Advanced Study, further accelerated their spread (Kaiser, 2005, pp. 43–59). However, notable resistance emerged, exemplified by Julian Schwinger at Harvard, who viewed the diagrams as mere "pedagogy, not physics." (Kaiser, 2005, p. 104) This tension paralleled the historical divide between Newton's fluxions and Leibniz's calculus, with Feynman's more accessible approach ultimately prevailing through broader community acceptance and discourse, much like Leibniz's notation. Similarly, it is Feynman's system, the one that is now used extensively.

The socialisation of knowledge contributes to its acceptance in a progressively wider epistemic community. The expansion that becomes the third element in the *o-é-c process* is fuelled by pedagogical means as I will explain next with a different example of the creation of a new paradigm: the institutionalisation and standardisation of educational structure and methods by the Society of Jesus in the sixteenth and seventeenth centuries.

Expansion and Pedagogy

After its creation in the sixteenth century, the Society of Jesus began to expand worldwide very rapidly. The Society realised that the high level of education present in its founding members was both an asset for their mission and an attractive characteristic for potential sponsors and benefactors. The consistency and thoroughness of the preparation of its members became a necessity and a priority. Education began as one of the most important aspects of the Society in its origins as a means of moulding the future members of the society into the type of man that could fulfil its mission anywhere that was required. The Jesuit educational system was the first systematic approach to education in the western world and had its origins in the humanistic tradition of the early Renaissance and in the ancient Isocratic ideals of creating men (and women) that would positively influence their society. (Diaz, 2009, p. 10).

The development of the *Ratio Studiorum (system of studies),* formally titled *Ratio atque Institutio Studiorum Societatis Iesu (The System and Organisation of Studies of the Society of Jesus)* (Pavur, 2005) by the Jesuit order in the sixteenth century addressed a complex wicked problem in education, pedagogy, and the Catholic Church's role following the Reformation. This challenge emerged during a period of profound religious and intellectual transformation, as the Protestant Reformation challenged Catholic authority while the Renaissance revival of classical learning and the Age of Discovery expanded European worldviews. At its core, the Jesuits faced the challenge of harmonising traditional Catholic theology with humanistic emphasis on classical texts and critical reasoning. They needed to create a curriculum that embraced

Renaissance humanism's intellectual rigor while maintaining Catholic orthodoxy. This became particularly complex as they established schools across Europe and in missions worldwide, requiring a universal educational system adaptable to diverse cultural contexts from Rome to Beijing.

The *Ratio Studiorum* was the Jesuits' systematic attempt to address this multilayered *wicked problem*. Early in the history of the Society, and following the mandate of the general superior, the Jesuits set into a multinational project that lasted fifty years and collected the current thinking and experience of Jesuits, teachers, and students in their many schools to establish a handbook of what should be taught, how, and in which order, including the roles and responsibilities of faculty, administration, and students. Early drafts were tested, feedback gathered from schools worldwide, and revisions made. This manual was published in its third and last version in 1599 under the name *Ratio atque Institutio Studiorum Societatis Iesu* (Pavur, 2005), Even after its 1599 promulgation, it saw adaptations, showing that such complex educational challenges resist one-time solutions as it is the case of *wicked problems.*

The impact of the *Ratio Studiorum* extended far beyond the Jesuit order, influencing European education for centuries. Yet, true to *wicked problems*, it did not *solve* the challenges of education in a changing world. Instead, it offered a robust framework that could adapt and evolve, shaping how generations of students engaged with the complexities of faith, reason, and cultural change. At the request of the officials of the city of Messina in Italy in 1548, the Society of Jesus opened the first Jesuit School, the Collegio di San Nicolò, with Ignatius selecting 10 priests and scholastics (advanced students) among the best available to him in Rome. This event became “a crucial event in the history of schooling within the Catholic church and in western civilisation” (O’Malley, 2000, pp. 56–57), and it originates a monumental shift to the propagation of a systematic education framework that became central to the mission of the Jesuits. The Jesuits’ educational project became the standard to model, follow, mimic, compare against, and evaluate in education until the eighteenth century and in some places, well into the nineteenth. In the next section we will see a bit more deeply the development of the *Ratio*’s section on Mathematics and its influence, as an example of the next element in the sociological phase of the *o-é-c process* for new fields of knowledge.

Textbooks and New applications

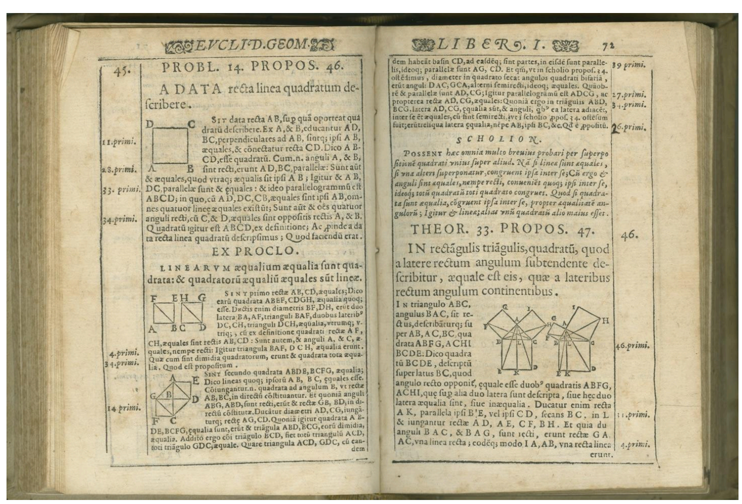
Textbooks, in the context of the *o-é-c model*, serve as more than pedagogical tools; they function as epistemic entities that consolidate and disseminate knowledge. Their significance transcends traditional notions of instructional utility, as they encapsulate the conceptual frameworks, methods, and exemplars of a given field, thereby playing a pivotal role in both the socialisation and institutionalisation of knowledge. During the socialisation of new paradigms, textbooks serve as repositories of collective understanding, reflecting the epistemic consensus reached within an intellectual community. They transform individual insights into shared knowledge by organising complex ideas into accessible and teachable formats. This process is crucial for paradigm support, as it allows the emergent framework to be disseminated across academic and professional contexts. Textbooks facilitate gestalt shifts by presenting new paradigms in ways that challenge and reshape existing cognitive frameworks. By codifying novel insights and aligning them with practical applications, they bridge the gap between theoretical abstraction and real-world relevance.

Once a paradigm gains broad acceptance, textbooks become tools for sustaining and expanding its influence. They ensure that the knowledge is transmitted to subsequent generations, embedding it within the epistemic fabric of the field. In this way, textbooks help solidify the integration of new knowledge into the broader intellectual landscape, enabling its application to transdisciplinary problems. Beyond their role in transmission, textbooks can also be loci of epistemic innovation. By integrating disparate ideas, they often reveal new connections, fostering further advancements. This dual role—as vessels of established knowledge and incubators of novel insights—underscores their indispensability in the evolution of intellectual fields. In this expanded sense, textbooks are not merely instruments of education but active participants in the epistemic process. Their creation, adoption, and evolution mark critical milestones in the transition from individual invention to collective knowledge as we will see next in the example of textbooks in the Jesuit system of education.

After 50 years in international consultation and in the fourth and last version of the *Ratio Studiorum*, the so called *1599 Ratio [system]* (Pavur, 2005) became the official approved version and it was the textbook on how to set up and run the schools. Within the *Ratio* there was the provision of the inclusion of Mathematics in its curriculum, which was in itself an innovation in Education at that time, and the architect of that inclusion was Christoph Clavius. Clavius advocated for the importance of Mathematics in mainstream academia, and through his pedagogical approach, curriculum chosen, and series of textbooks written, had a lasting and powerful legacy that was felt throughout the schools of the Society and beyond. The *Ratio* included the rules for the professors of philosophy, with a few that were specific to moral philosophy and ethics, and then the regulations for the professor of mathematics (Padberg, 2000, pp. 96–97). An example is this extract from the Section 21: "Rules for the Professor of Mathematics", paragraph 239:

> He should teach the physics students Euclid's *Elements* in class for around three quarters of an hour. After having gained some experience with the material for about two months, he should add to this something about geography or the Sphere, or about those things that are usually of interest. And he should do this with Euclid, either on the same day, or on alternate days (Pavur, 2005, pp. 109–110).

Testimonies of the scientific activity developed at the Collegio Romano, the Jesuit flag school in Rome, reveal that teaching brought together a set of disciplines and themes including pure and mixed mathematics. A text from 1566 specifies that mathematics was taught according to this programme: the six books of Euclid, arithmetic, sphere, cosmography, astrology, theory



of the planets, Alphonsine tables, etc., perspective, gnomonics (the art of using and making sundials). The importance of Euclid including the commentary on the first six books fuelled not only Jesuit mathematics teaching but also all other educational structures, secular or religious until the second half of the 17th century (Romano, 1999, p. 82). An example of the ripples of diffusion of the new mathematical approaches developed by the Jesuits is taken from Clavius's *oeuvre*. Clavius wrote a text containing Euclid's works *Euclidis Elementorum Libri XV (Fifteen Books of the Elements of Euclid)* (Clavius, 1574a) as well as comments on it and his own evaluation of Euclid's axioms in textbooks that were reprinted and used in Jesuit schools and Protestan institutions for decades by people such as Leibniz and Descartes.

The figure attached above is an excerpt from the 1574 edition of Clavius' *Elements* showing proposition 47 which is the Pythagorean theorem (Clavius, 1574b, p. 72). Clavius' influence is described by Smolarski when he states

> Clavius' vision also influenced Jesuits after him to become involved in science at the dawn of the scientific age. Influenced by this vision and guided by the Ratio during their studies, many Jesuits became pioneers in scientific fields – so many that thirty-five craters on the moon are named after Jesuit mathematicians, astronomers, and scientists (Smolarski, 2002, p. 455).

The Jesuit mathematicians worked hard at the beginning of the 17th century to develop techniques that would blend singular experiences and events in accredited knowledge regarding the natural world. They derived rules for scientific procedure that stemmed from Aristotelian tradition used by Jesuit natural philosophers and logicians, and in this they were motivated, beginning with Clavius, to enhance the credibility and acceptance of the mathematical disciplines in the academic system (Dear, 1987, p. 173). The influential *Commentarius in Sphaeram Joannis de Sacro Bosco (Commentary on the sphere of John of Sacro Bosco)* (Clavius, 1570) was Clavius's first published work and became the standard reference for generations, influencing Tycho Brahe, Johannes Kepler, and Galileo Galilei. Clavius's astronomical work allowed him to become the main architect in the calendar reform of 1582 that introduced the Gregorian calendar (Jesseph, 2015, p. 124). Another example is that of the Jesuit mathematician Matteo Ricci who studied under Clavius in Rome before 1586 when he used mathematics and astronomical technology preparation to gain credibility in China, initially by translating two of Clavius' mathematical books on Euclid into Chinese, becoming the doorway by which Ricci was able to gain entry into the culture of China while at the same time mathematics was demeaned by philosophers in Italy (Smolarski, 2002, p. 451).

Today, in the vast movement to reconsider the Jesuit contribution to European culture, specialists in the history of science give it a leading place in the development of mathematics, not only within the Company, but also from the point of view of the history of the discipline: Clavius, by his defence of mathematics within the framework of the Jesuit objectives in education, and by his creation of a school of mathematics within the Collegio Romano where the principal scientists of the Company studied, was the main architect of the establishment of a Jesuit policy and of possible successes in mathematical science (Romano, 1999, p. 89). As an example, William A. Wallace O.P. has studied extant documents from lectures at the Collegio Romano between 1559 and 1599 and the Jesuit influences on Galileo's Science, and he concludes that Galileo was not a hypothetico-deductivist as some sources describe him today, but "very much Aristotelian" in his logic, and his knowledge of scientific methodology were behind his claims for "necessary demonstrations" throughout his life, including his 1632 book *Dialogo (Dialogue on the Two Chief World Systems, Ptolemaic and Copernican)* (Galilei, 1632), and his *Discorsi* on 'two new sciences" of 1638 (*Discorsi e dimostrazioni matematiche intorno a due nuove scienze) [Discourses and Mathematical Demonstrations Concerning Two New Sciences]* (Galilei, 1638), and "in that mind-set he was influenced by the Jesuits, in a unique and distinctive way" (Wallace, 2000, p. 315). Wallace further arrives to a second conclusion that the basic proposition at the root of the second science of the *Discorsi*, that is to say,

> … that the motion of falling bodies is uniformly accelerated with respect to time was influenced by a tradition of deriving from a teaching of Domingo the Soto, first proposed at Salamanca around 1551 (Wallace, 2000, p. 315).

The acceptance of new fields of knowledge and intellectual practice becomes the last phase of the *o-é-c process*, as I will describe in the next section.

Connaissance Éclairante - Adoption to Body of Knowledge:

Becoming Knowledge

At what point and how do we know that there is an addition to the Body of Knowledge? For the process described in this thesis, the answer is sociological in nature, and it corresponds with the emergence of a new field of knowledge which rests upon what Renn describes as *epistemic networks*. These networks store, accumulate, transfer, transform, and appropriate knowledge and are constituted by semantic networks, semiotic networks, and social networks (Renn, 2020, p. 423). Justification takes form in the discussion of the ideas and their implementation in solving problems, describing, experiencing, and predicting usable results connecting evidence within the network of collective beliefs in a field. I am not advocating the concept of *strong programs* in the sociology of science as proposed by Bruno Latour, and Steve Woolgar where they argue that *facts* are constructed, constituted, or fabricated by science (Goldman and O'Connor, 2021), instead, I propose that the concept presented in this thesis has more connections with Richard Rorty and the notion of *social justification of belief* (Goldman and O'Connor, 2021) and with Thomas Kuhn's, Paul Forman's and Helen Longino's argument that scientific beliefs must be influenced by social factors.

I believe that we can rethink the concept of challenges to a scientific theory in terms of the discussions and clarifications that help explain and understand the theory. In the course of normal science (in the Kuhnian sense) scientific theories are seldom tested for falsifiability as Karl Popper advocated, and instead are used to solve problems and make predictions; however, in the course of the development of refinements, reinterpretations, or new theory development, ideas are subject to discussion, challenged in their domain and applicability, and questioned for clarification in the course of academic discussion, and it is in this analytical process that they become adopted, improved or abandoned.

In agreement with Longino's social epistemological perspective, I have claimed that knowledge is social in both its generating process, and in its acceptance and utilisation within an epistemic community in controlling and predicting activities. N. R. Hanson stresses the importance of controversy for research science: researchers sometimes appreciate data differently: “It is important to realise… that sorting out differences about data, evidence, observation… may require a comprehensive reappraisal of one’s subject matter” (Hanson, 1985, p. 19).

Those discursive interactions become constructive and justificatory, and in doing so they provide the justification needed to become a knowledge generating activity. It is in the infusion of interdisciplinary work where the new field coalesces into an addition to the Body of Knowledge. It is a good illustration of this idea to study the concept of information rooted in Shannon’s ideas on entropy and the degree of uncertainty in communications. While Shannon's theory was initially focused on telecommunications, its principles have been applied to a wide range of fields, including computer science, cryptography, linguistics, quantum computing, and even genetics.

Summary

This chapter has developed situated accounts of scientific knowledge that transcend traditional dichotomies between rational and social approaches. These accounts converge on understanding scientific knowledge as perspectival, where representational success emerges from the alignment between our representations and independent reality, enabling effective practices in understanding wicked-like problems.

Drawing on Minsky's definition of knowledge—“*Jack’s knowledge about **A** is simply whichever mental models, processes, or agencies Jack’s other agencies can use to answer questions about **A**”* —the subsequent chapters demonstrate how the *o-é-c model* describes significant additions to the Body of Knowledge through three case studies: Gottfried Leibniz's development of Calculus and Analysis (Chapter 3), Niels Bohr's work on Complementarity and Quantum Mechanics (Chapter 4), and Alan Turing's contributions to Computability and Mechanical Intelligence (Chapter 5).

Chapter 3 - Leibniz and the calculus

Having described the process of additions to the Body of Knowledge, this chapter analyses the first of three cases: Gottfried Leibniz's development of calculus and its evolution into the field of Analysis. While this case spans centuries, the focus lies on the invention of the Leibnizian calculus framework in its seventeenth-century context and its subsequent development.

Leibniz's philosophy centred on his quest for a universal science to unify all knowledge. His metaphysical framework, built on the concept of monads as indivisible units of force, aligned with his mathematical work on infinitesimal calculus and sought to reconcile mechanical and spiritual explanations of nature. This work emerged during the rise of modern science and decline of Aristotelian physics. Leibniz's extensive correspondence network—comprising over 20,000 letters with 1,300 correspondents across disciplines—established him as a central figure in his era's intellectual discourse. Though his final years were marked by the priority dispute with Newton over calculus invention, his legacy includes approximately 200,000 manuscript pages spanning philosophy, mathematics, sciences, law, politics, theology, and history (Probst, 2024).

Richard Brown identifies a paradox in Leibniz's mathematical revolution: while its substance emerged from contemporary work on tangent and quadrature problems, its motivation and symbolic form stemmed from non-mathematical projects predating 1672. These ideas, which Leibniz termed the "*Universal Characteristic*" or "*Ars Inveniendi*," reflected his belief in creating a philosophical language to mechanise reasoning processes (Brown, 2012, p. 175). Leibniz confronted the wicked problem of developing a universal method of discovery through his *Ars Characteristica* (*Characteristic Art*) and universal language. His approach represented a gestalt shift from geometric arguments and deductive processes toward mathematical methods based on infinitesimals, introducing novel notation for analysis. This transformation sparked debates between competing formulations, notably between Newton’s and Leibniz's approaches, as exemplified by their supporters Maclaurin and L'Hôpital. The subsequent acceptance and integration of calculus into mainstream science illustrates a broader pattern in the History and Philosophy of Science, where new additions to the Body of Knowledge progress through phases of invention, expansion, and integration.

Ouverture Ontologique - Ontological Opening

Historical Environment; the sociological atmosphere

The seventeenth century was marked by profound change and development in Europe, shaped by fundamental forces that reshaped religious, political, and intellectual landscapes. The century began with religious conflicts, most notably the devastating Thirty Years' War (1618-1648). In contrast to the Renaissance, the Reformation stemmed from deep-rooted devotional fervour that captivated scholars and the masses. Religious zeal permeated various aspects of society, from global expansion to the arts. The separation between Catholics and Protestants was reflected in distinct cultures within their spheres of influence (Davies, 1996, p. 482).

An essential characteristic of the seventeenth century is the growth in religious intolerance and how the zealousness once reserved for the wars against the Muslims was now born from mistrust on other branches of Christianism and gave origin to conflicts all across Europe in the late sixteenth century, and became a driver of the wars in the seventeenth century (Davies, 1996, pp. 484–507). The Thirty Years' War changed the political and religious map of Europe and led to the Peace of Westphalia in 1648, which introduced the concept of state sovereignty by settling the independence of the Dutch Republic from Spain, and the structure of the German-speaking lands, and with all that, it permanently established confessional pluralism. As fundamental and as a transformational result the Peace of Westphalia was, it left many things unresolved that were at the root of fundamental instability and mistrust in greater Europe, with the War of the Spanish Succession (1701-1714) and the Great Northern War sparking until the Peace of Nystad in 1721, the Second Hundred years War between England and France that started in 1689, and the expansion of Russia in 1695. Political environments became uncertain and evolved in different directions, with the beginning of the French civil war in 1648 and its resolution in the reign of Louis XIV in 1661, and the execution of Charles I in England and the subsequent proclamation of the English Republic in 1649 until the restoration of the monarchy in 1660 and the Glorious Revolution in 1688 (Blanning and Cannadine, 2008, pp. i–iii). The period mentioned here extends beyond the Leibinz's years of 1673-1676 when he developed the fundamental notion and algorithmic rules of differential and integral calculus to suggest that the instability of the background historical environment in which the challenges to the *status quo* were present in all areas and might have contributed to the willingness to go against established norms, methods, and problems and present challenging ideas that could risk ridicule and opposition.

Drawing on the example of Paul Forman's analysis of Weimar culture and quantum mechanics, we can understand how seventeenth-century contexts influenced scientific development. This approach suggests that the period's cultural milieu significantly shaped the development and acceptance of new mathematical paradigms. The religious and philosophical upheavals of the time encouraged empirical and mechanistic explanations, which were perceived as more objective than traditional authority-based reasoning. While seventeenth-century natural philosophers maintained connections between scientific and religious thought—as exemplified by Newton's view that natural philosophy encompassed questions about divine attributes—the cultural environment became increasingly receptive to ideas that challenged established intellectual frameworks, including in mathematics (Brooke, 1991, p. 7).

Society began to organise into social groups or states, defined by their functions, legal restrictions, privileges, and corporate institutions (Davies, 1996, p. 516). Absolute monarchies consolidated power, setting the stage for modern nation-states. Social change took hold, and the emergence of the public sphere challenged the authority of the old regime (Blanning and Cannadine, 2008, p. iv). A catalyst was the emergence of newspapers, which mechanised and increased the volume of communication. The relevance of popular culture and access to printed information was integrated with the emergence of science. (Pettegree, 2018, pp. 327–328). The commercial model of newspapers, dependent on consistent readership, profoundly influenced society. Pettegree argues that press freedom played a crucial role in revolutionary events, with the press acting as both agency and cause. The relevance of popular culture and print media emerging in the seventeenth century integrated with the rise of science, as natural philosophers increasingly targeted broader audiences (Bowler and Morus, 2020, p. 406).

These trends coincided with the introduction of scientific journals for disseminating ideas. Elizabeth L Eisenstein (1923-2016) argues that to understand scientific change, one must look beyond popularisation and propaganda as effects of print media (Eisenstein, 2009, p. 454). The first two learned journals, the *Journal des Sçavans* and the *Philosophical Transactions* both started the same year, 1665, shifting from handwritten letters to a more effective way of presenting current research (Eisenstein, 2009, pp. 460–462). This contribution is intrinsic to Helen Longino's concept of social cognition. Learned journals were particularly significant for Leibniz and calculus development, as publications in *Acta Eruditorum* prompted him to publish his methods beginning in 1684. Subsequent discussions and expansion of calculus analysis were intrinsically connected to journal publications by Leibniz and his followers.

From Helen Longino's perspective on social dimensions of scientific knowledge, we see the importance of community in shaping scientific consensus and theory content. A key seventeenth-century trend was the move from clerical scholarship domination to scientific society formation, where most eminent intellectuals weren't clergy members. Societies like the *Royal Society* (England, 1660) and *Accademia dei Lincei* (Italy, 1603) created networks fostering critical discussions and idea exchange, crucial for challenging old paradigms and establishing new ones. They facilitated consensus and incorporated diverse viewpoints, essential for robust scientific theory development (Brooke, 1991, pp. 55–56). Leibniz sought to create academic associations for knowledge gathering and development. The Berlin Academy of Sciences, founded in 1700 largely due to Leibniz's efforts, was intended to promote scientific research and contribute to Brandenburg-Prussia's prestige. Leibniz envisioned it advancing physical, mathematical, and practical sciences benefiting the state and economy. The Academy's interdisciplinary culture emphasised science and humanities unity, reflecting Baroque thinkers' polymathic approach. Leibniz believed in applying scientific knowledge to practical problems, influencing the Academy to integrate theoretical research with practical applications. The primary journal, *Miscellanea Berolinensia*, published seven volumes between 1710 and 1744, with the first under Leibniz's stewardship.

Standardisation of education and communications networks spread, and this period saw dramatic scientific thought transformation. The scientific method was refined, increasingly challenging traditional beliefs and intellectual authorities. In Catholic regions, a strong theological component in philosophical conceptions was transmitted through the Jesuit educational network and newly formed academic societies (Diaz, 2009). The Reformation promoted individual scripture interpretation, paralleling the emerging scientific approach favouring direct observation over established authorities. Thinkers like Kepler exemplify this transition, with his planetary motion laws derived from meticulous observations representing a shift towards mechanistic universe understanding. Protestant critique of Catholic authority extended to general scepticism of tradition, mirrored in the scientific sphere. This led to questioning classical texts and the scholastic method. Evidence of Protestant theology providing resources to see experimental science as key to human progress includes French Academy of Sciences data showing a significant Protestant majority among foreign associates (Brooke, 1991, pp. 111–112). However, this view isn't straightforward; astronomical realism was not exclusively Protestant, as exemplified by Galileo's Catholic devotion and embrace of Kepler's arguments. Furthermore, scepticism about scientific realism in astronomy remained fashionable among both radical Protestants and conservative Catholics (Brooke, 1991, p. 93).

The transitory and unstable historical environment of the seventeenth century was conducive to questioning existing paradigms and status quo in every dimension, as boundaries were blurred and exposure to information, beliefs, and new conceptions made it easier to voice new frameworks even at the risk of opposition. As we will see next, understanding and addressing wicked-like problems required approaches that were easier to adopt in the flux of this milieu.

Existing Paradigm; working within the accepted view

The intellectual environment of seventeenth-century mathematics is complex, with roots in Greek, Arabic, and mediaeval sources, incorporating elements of philosophy, religion, mysticism, and literature. This rich tapestry coalesced into the foundation of modern mathematics, and since then, there has been no significant external influence to the European tradition now spread worldwide (Whiteside, 1961, p. 183). This environment provides the context for the paradigmatic character of methods and values in 17th-century mathematics, including antecedents to calculus development resting on two areas: geometry and infinitary arithmetic.

In the sixteenth century, knowledge of Greek mathematics formed the foundation for most mathematicians. Mathematics in the first six decades of the seventeenth century was developing rapidly, influenced by classical Greek mathematics and that of the late sixteenth century. However, Greek mathematics was not heuristic, and its methods did not facilitate replicable problem-solving or attacking new problems.

> It was natural, therefore, to search for other methods which, if they could not live up to the Greek requirement of exactness, were at least able to suggest ideas as to the solution of problems (Pedersen, 1980, p. 11).

We will see later how Leibniz's interest in producing a general problem-solving method played a role in his development of calculus and its later acceptance.

Until that point, numbers were associated with entities reflecting reality. The first conceptions stem from Platonic and Pythagorean literary tradition, where numbers guided speculation in metaphysics, symbolism, and interpretation. The Pythagoreans symbolised the number ten with the τετρακτύς (*tetraktýs*), representing the first four numbers (Kalvesmaki, 2013, p. 325). The *tetraktýs*, defined the perceived world: atoms as indivisible entities (I), lines defined by atom pairs (II), planes as extensions of 3 (III), and the tetrahedron representing volume and three-dimensional space (IV). This raised issues with unity (1), not considered a number but associated with atoms and part of a continuous line:

> The earliest Greek philosophers were noted for asking how many sources or roots —ἀρχαί [the word indicating *at first*]— there were to the universe. Was it monadic, dyadic, polyarchic, or a synthesis? How many levels of reality, if any, existed above the material world? If the universe started out as a monadic unity, how did the second element or level originate? What kind of metaphysical entity were numbers? These questions were discussed by the earliest Greek philosophers, including Plato (Kalvesmaki, 2013, p. 12).

Plato (429? – 347 B.C.E.) considered mathematical sciences paramount in training philosophers and rulers, and over the door of his school read “let no one destitute of geometry enter my doors” (Heath, 1981, p. 284). In his *Republic* he argues for geometry's critical role in developing philosophical knowledge (Plato, 1997, p. 1146 'Republic VII 526e–527b'). This mode of thinking produced a way to support arguments using visceral experience of known objects (Whiteside, 1961, p. 270). In classical works of Greek mathematics such as those from Euclid (~300 BCE), Archimedes (287-212 BCE), and Apollonius of Perga (240-190 BCE), there were two kinds of geometrical propositions: problems and theorems. Solutions involved constructions combined with proof that the constructed figures included the required properties. Greek geometers preferred constructions using circles and straight lines [compass and straightedge], legitimised in Euclid's first three postulates (Euclides, 2010, p. 2) where no other constructions were used throughout the book (Bos, 1984, p. 332).

Another source for the outsised influence of Greek mathematics in the seventeenth century mathematicians can be traced down to the mathematics of Aristotle’s (384–322 B.C.E.) work, while not containing much beyond elementary plane geometry, was influential in illuminating mathematics with the concepts of *continuous* and *infinite* (Heath, 1981, pp. 341–342). These concepts will affect the ideas regarding the infinitary and infinitesimal mathematics in the seventeenth century. In his *Physics* Aristotle explicates *continuous*:

> A thing that is in succession and touches is contiguous. The *continuous* [my italics] is a subdivision of the contiguous: things are called continuous when the touching limits of each become one and the same and are, as the world implies contained in each other: continuity is impossible if these extremities are two. This definition makes it plain that continuity belongs to things that naturally in virtue of their mutual contact form a unity.

> And in whatever way that which holds them together is one, so too will the whole be one, e.g. by a rivet, or glue or contact or organic union (Aristotle, 1995a, p. 384 'Physics V 227a 10–16').

Regarding the *infinite*, he argued that the continuous cannot be constructed from indivisible parts, implying the impossibility of infinite or unlimited (Aristotle, 1995a, p. 354 'Physics III 207b 28–34'). Infinitesimal and infinitary mathematics are essential parts of calculus development and its usefulness. As part of the existing paradigm of seventeenth-century mathematics, it's important to include ideas regarding the Cavalieri-Torricelli theory of indivisibles. Derek Thomas Whiteside (1932-2008) calls the *indivisibles* and the *arithmetick of infinites* [Whiteside's spelling] part of a range of thought within the "17th century achievement in arithmetising the infinite" (Whiteside, 1961, p. 311).

Mediaeval Aristotelian treatments of limit concepts regarding continuous variation and instantaneous speed are at the foundation of Cavalieri's indivisible theories and his justification arguments. In his theory, Bonaventura Cavalieri (1598-1647) uses cross sections of solids and the concept of similarity explain how adjacent figures or lines generate respective solids or figures, as you can see in the following figures from his paper "Patterns of Mathematical Thought in the later Seventeenth Century":

> … the solids are made up of the limit-sums of these cross-sections when the distance between two adjacent cross-sections becomes indefinitely small (Whiteside, 1961, p. 315)

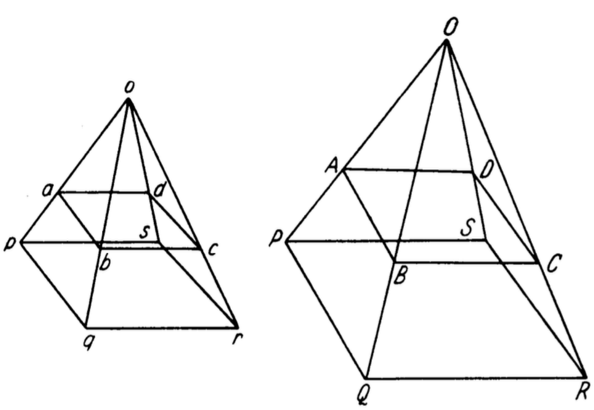


Fig. 63

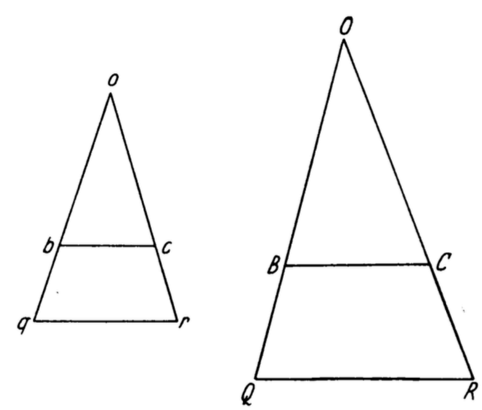


Fig. 64

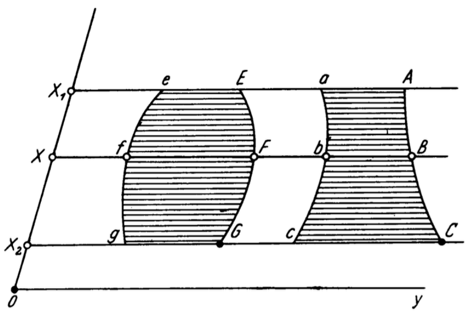


Fig. 65

Cavalieri extrapolates arguments from solids to flat figures, using similar arguments to explain how adjacent line segments can add up to recreate flat figures. These are the bases of Cavalieri's *indivisible* techniques, using the terms *omnia plana* or *omnes lineae* to indicate the limit sums of the respective areas for the reconstructed volumes or line segments for areas (Whiteside, 1961, pp. 315–316). A simplified and more accessible treatment of Cavalieri's methods was developed by Evangelista Torricelli (1608-1647) where he used a standard result from Greek Mathematics to produce an inverted result from Cavalieri's work. These ideas inspired Grégoire de Saint-Vincent (1584-1667) and John Wallis (1616-1703) in their work on the quadrature of the circle (Whiteside, 1961, pp. 318–319), a problem that Leibniz also attacked in his mathematical development.

Leibniz arrived in Paris for the first time in 1672 with minimal foundations in mathematics (C.S. Roero, 2005, p. 48) (Antognazza, 2009, p. 140) describing his own ignorance in mathematics as "superb" (Strickland, 2023, p. 20). There he accessed some of the members of the *Académie des Sciences* and Huygens, who shared work from contemporary mathematical minds like Honoré Fabri (1608-1688), Nicolas Mercator (1620-1687), Blaise Pascal (1623-1662), James Gregory (1638-1675), René Descartes (1596-1650), and John Wallis (1616-1703) (Probst, 2018, pp. 213–214). Their efforts revolved around the works of Euclid, Apollonius of Perga, Archimedes, and Pappus; and Richard Brown argues that

> …they indicated the way Mathematics should be done, what standards it should meet, the general type of problems that were of interest, and what should constitute a precise solution to them (Brown, 2012, p. 19).

Much emphasis was placed on geometric constructions of classical problems like finding areas under curves (squaring) in general, and of the circle in particular. In the late 1500s and early 1600s, some especially talented mathematicians like Françoise Viète (1540-1603) and Christopher Clavius (1538-1612) began transferring algebraic methods to geometric constructions, working within classical Greek mathematics constraints which had no concept of real numbers (Feldhay, 1999, pp. 109–110) (Diaz, 2009, pp. 43–45) (Brown, 2012, pp. 22–24). Numbers represented distances and dimensions, and relations could only be expressed with homogeneous entities. Polynomial use was limited to physical dimensions, and higher-order polynomials were seen as abstractions without real applications (Brown, 2012, pp. 25–27). Algebra progressed from a tool for specific problems to a symbolic language for abstraction affecting geometry applications. Multiplication of two segments was seen as an area and that of three segments as a volume. Given that multiplication was seen as a dimension change, there was no way to represent more than three dimensions graphically (Grosholz, 2007, p. 166).

René Descartes began transforming geometry by using algebra beyond construction aid, introducing non-homogeneous equations, with the cost of adding relativity to some operations like line segment multiplication. His work, however, remained within the existing classical paradigm (Brown, 2012, p. 27). Descartes' publication of the *Discours de la Méthode (Discourse on Method)* (Descartes, 1637), *Meditationes de Prima Philosophia (Meditations on First Philosophy)* (Descartes, 1644a) (Descartes, 2008), and *Principia Philosophiae (Principles of Philosophy)* (Descartes, 1644b). Descartes' publications located mathematics and physics as antecedents to the *order of reason* provided their metaphysical justification and legitimisation, and organised knowledge items within domains and domains within human knowledge (Grosholz, 2007, p. 166). For him, geometry's purpose was to solve problems using the simplest constructions, and he argued that his *Discours de la Méthode* (Descartes, 1637) produced a method for geometry as well (Bos, 1984, p. 338). Descartes was uninterested in general curve study and excluded transcendental curves (a term introduced by Leibniz) as they weren't developed from arithmetical methods. Although committed to Greek geometrical canons of construction and existence, his analytical geometry showed how classical construction means fail to provide a foundation for both geometry and number study (Grosholz, 2007, pp. 231–232).

The mathematical environment of the seventeenth century was characterised by work on specific problems and questions regarding nature and current technology. One example is the study of the tractional motion that occurs when an object is hauled over a plane by a rope or rod attached to it and moving in a straight line. Henk Bos provides a detailed analysis of that work to point out Huygens' motivations and their relation to mathematical problems (H. J. M. Bos, 1993, pp. 5–7). Christiaan Huygens (1629-1695) was particularly interested in studying this physical process, and in doing so he spent considerable time and effort designing precise mechanical implementation. Huygens noted that part of the friction was related to a force of attraction perpendicular to the plane of movement and parallel to a line to the centre of the earth. The graphical representation of those implementations led him to propose a curve that followed the hauled object, and he called it a *Tractoria* that was later accepted as *Tractrix* that you can see in the figure drawn by Bos (H. J. M. Bos, 1993, p. 6) and attached here.

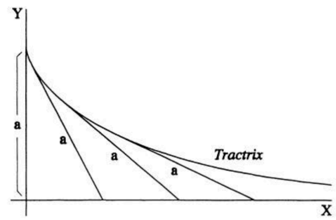


Figure 3.

Two critical aspects in knowledge acceptance in this milieu are the willingness to publish and explicate work, and the systematisation of developed frameworks and entities for replication and further use. These differences will be seen again in the discussion of Leibniz's and Newton's calculus work acceptance. Huygens' limited lasting legacy is attributed to his work being the culmination of a particular style that marked the boundary of the attainable within that style, and his reluctance in publishing results (H. J. M. Bos, 1993, p. 72). Huygens' work resembled that of a technical designer: brilliant steps lacking a larger unifying context in a period where the focus was on complete explanatory systems for natural processes (H. J. M. Bos, 1993, p. 73). He couldn't produce methods and sophisticated frameworks for obtaining his results or expanding to other applications, leaving no students or followers (H. J. M. Bos, 1993, p. 74). We will see a similar situation regarding the difficulty for followers to build on his methods in Newton's invention of calculus.

There is an important social component in the intellectual environment of Mathematics in the seventeenth century: a significant number of mathematicians were not mathematicians by profession, particularly in France, which was the centre of the European mathematical world, with examples like Françoise Viète (1540-1603), Pierre de Fermat (1607-1665), René Descartes, and Blaise Pascal (1623-1662). Descartes, as a former member of the School of Engineering in Leyden (Holland) published his treatises, among them the *Géométrie* (1637) which was translated into Latin by Frans van Schooten jr. (1649), who was teaching at the Engineering School at that time. This translation was reprinted along with commentaries and treatises of his most important students–Jan de Witt, Johan Hudde, and Hendrick van Heuraet–in two volumes (1659/1661) (O'Connor and Robertson, 2009). The collection also contained some results of his most illustrious follower Christiaan Huygens (1629-1695) who dedicated his life to mathematics and physics. Huygens accepted a membership of the *Académie des Sciences* in Paris in 1666 which provided him with a generous stipend. (Pedersen, 1980, p. 12). Besides the desire for recognition of primacy, there was a monetary stimulus as well. Becoming accepted to the newly created scientific societies of the seventeenth century became a source of funds that supported the social trend of mathematical work outside of professional circles, and therefore relevant in stimulating scientific work in the wider intellectual environment. Leibniz was forever looking for ways to secure financial solvency and freedom, and the financial incentive that the societies offered can explain in part Leibniz's efforts to develop work that would earn him the accolades necessary to get accepted and become a fellow in some of these societies, as well as the interest in founding additional ones. Leibniz financial need would become a powerful motivator for his work on a number of fields, and mathematics was not an exception.

At the end of 1673 Leibniz was working on the quadrature problems common to the mathematics of the time and in particular on the then classical problem of finding the area of a circle inscribed in a square of a slide length of 1, a problem Leibniz solved when he

> …discovered the general infinite series for squaring the circle in autumn 1673 (see A VII 4, 749–750; A VII 6, 24, 29–30; A VII 3, 282–288)... at the time Leibniz devised his quadrature formula he was on unpaid leave from his role in Mainz (see A I 1, 349), and his only other income, from working as tutor to the Boineburg family, came to an end

> soon afterwards, on 13 September 1674 (see A I 1, 396). The need to improve his prospects and secure suitable employment was not lost on Leibniz, who entertained various ways of exploiting his work on the quadrature to that end (Strickland, 2023, p. 20).

His work on quadrature would be an essential component in the foundation of the differential and integral calculus that he developed in the mid 1670s, and at the end of October of 1674 he drafted a text that could be used to seed later attempts to communicate his results. The draft was called "The Quadrature of the Circle by a Rational Progression (10 September – October 1674)" [translation by Lloyd Strickland] (Leibniz, 2022a) and it "begins with a short history of the quadrature problem and past attempts to solve it before presenting Leibniz's own solution in terms of the infinite series" (Strickland, 2023, p. 21).

Although he did not include proofs or methods in the essay, I find it likely that in order to add credibility to his arguments in an intellectual environment that cherished classical Greek intellectual heritage he structured the draft following the Aristotelian roots he developed in his intellectual development when "his interest was captured by the 'the Ancients': the greatest writers of the writers of classical Greece and Rome…" (Antognazza, 2009, p. 35). The structure of the text echoes Aristotle's *Metaphysics Book 3* dictum for understanding problems before trying to solve them [Metaphysics III B 995 24-30]:

> We must, with a view to the science which we are seeking, first recount the subjects that should be first discussed. These include both the opinions that some have held on certain points, and any points besides these that happen to have been overlooked. For those who wish to get clear of difficulties, it is advantageous to state the difficulties well; for the subsequent free play of thought implies the solution of the previous difficulties… (Aristotle, 1995c, p. 1572).

Indeed, after having shown that evaluating other infinite series produces finite numbers Leibniz finishes the essay by making the inference that "the series for the quadrature of the circle would likely prove to be summable also" (Strickland, 2023, p. 21):

> … Finally, I believe that those who understand the subject will agree that this is perhaps the first means that has been given to arrive at the geometric quadrature of the circle;

> and that even having got half of the way there, it is very probable, if it will ever be found, it will be by this way (Leibniz, 2022a, p. 2).

He mentioned that he had a result on the quadrature problem in a letter written in 15 July 1674 to Henry Oldenburg (1619–1677), the secretary of the Royal Society, and at the same time, he begun to work on a lengthier and more detailed essay for Huygens which he sent later in the year and was read and returned with enthusiastic positive feedback (Strickland, 2023, p. 22). At the end of 1674 Leibniz was investigating general methods for curves with the goal to publish a couple of papers to announce his candidacy for membership at the *Académie* in France in hopes to secure stable financial support. By the end of 1675 [around the same time he was working on his calculus methods and the notation that eventually became part of it], he crafted a plan to publish his quadrature findings in the *Journal des Sçavans* and he sketched a letter to Jean Paul de La Roque (d.1691), who was the editor of the *Journal* at that time, however, he never sent it, likely because he realised that his paper did not address the target audience of a journal that was intended more for the general reader (Strickland, 2023, pp. 22–23).

He continued to work towards securing a membership at the Royal Academy, however, this did not materialise. Leibniz redoubled his efforts and worked on mathematical methods, and by the end of 1675 he had invented both the differential and the integral calculus, and he began to collaborate with Ehrenfried Walter von Tschirnhaus in October 1675 on a number of topics. In the summer of 1676 he prepared a treatise on arithmetic circle squaring for publication with a proven foundation of curve quadrature using infinitesimal quantities called “De quadratura arithmetica circuli ellipseos et hyperbolae” that was not fully and critically edited until 1993 by E. Knobloch, and Mayer and Probst label the treatise “the most extensive, coherent mathematical text that Leibniz ever wrote” (Leibniz, 2012) (Leibniz, 2014). Financial want was endemic in Leibniz’s life and eventually he left Paris when he was no longer able to support himself financially and, although he tried, he never published his work before leaving, and after a series of misplaced documents, missed connection, and the death of a collaborator who was to oversee the printing, the work remained unpublished (Strickland, 2023, pp. 24–25).

There was another social trend that is important to note. Printing evolved to serve social demands as we saw in the last section, and after 1640 it was difficult to find publishers interested in printing unprofitable mathematical works that were difficult to put in print and contained many errors with formulas and signs. Mathematicians were reluctant to publish their developed methods, preferring instead to publish their results, and in the early 1670s there were few publications dealing with mathematical work: *Transactions* of the *Royal Society*, *Journal des Sçavans* in Paris, and a few Italian magazines with small subscriber circulation (Echeverría, 2023, p. 50). In the case of the *Journal des Sçavans,* the "journal's usual content of short reports of recently-published books, mostly on non-science subjects such as religion, history, and classics" (Strickland, 2023, p. 23) would have been reluctant to publish lengthy text with complicated terminology, equations, and figures.

Despite all his work and realisation of the importance of his invention, Leibniz was reluctant to make public his method until he published his first paper on calculus, titled *Nova Methodus pro Maximis et Minimis* in 1684 (Leibniz, 1684b) in the recently created *Acta Eruditorum* (Leibniz, 2011), that was founded in 1682 with support from the Duke of Saxony, however, he did not publish a cornerstone of his theory, later known as the fundamental theorem of calculus, until 1693. The stimulus to finally publish in 1684 was the fear that his friend Tschirnhaus (1651-1708) was about to publish very similar methods and steal the invention primacy from him:

> Ehrenfried Walter von Tschirnhaus, Leibniz' German-born friend and companion of studies in Paris in 1675, was publishing articles on current themes and problems using infinitesimal methods which were very close to those that Leibniz had confided to him during their Parisian stay (*Nova Methodus tangentes curvarum expedite determinandi*, Acta Eruditorum, December 1682, 391- 393; *Nova Methodus determinandi maxima et minima*, Acta Eruditorum, March 1683, 122-124; *Methodus datae figurae, aut quadraturam, aut impossibilitatem ejusdem quadraturae determinandi*, Acta Eruditorum, October 1683, 433-437) (Clara Silvia Roero, 2005, p. 5)

Leibniz was influenced by traditional proof methods and tried to follow rational paradigms by criticising Wallis for using experiments and phenomena to derive laws (Brown, 2012, p. 78). His work on the infinitary series was influenced by Huygens, Mercator, Wallis, Pascal, and others, and this environment forms the antecedents of calculus. However, Leibniz did not simply produce a combination of elements existing in the fertile ground of the intellectual development of mathematics at the time, as he was working on a broader problem. This will be my next point.

Wicked Problems

Thomas Kuhn's concept of scientific paradigms is crucial to understand the significant shifts in knowledge during this century. The seventeenth century witnessed a shift from the Aristotelian natural philosophy paradigm to a mechanistic framework characterised by mathematical descriptions of nature. I argue that for novel fields, the trigger resides in the attempt to conceptualise the problem being addressed and the success of the invention stems from its systematisation and the socialisation of cognition characterising its development and expansion. As we will see next, understanding and addressing wicked-like problems required approaches that were easier to adopt in the flux of this milieu. While natural philosophers and mathematicians like Isaac Newton and Christiaan Huygens were working on specific problems and *results*, Leibniz was interested in developing and publishing general methods for solving problems, a system of analysis and calculus that would allow *discovery*: "*Analysis* concerns heuristic methods employed to solve problems… analysis corresponds to what the tradition called an *art of discovery (ars inveniendi)*" (Mugnai, 2018, p. 179).

For Leibniz, mathematical method development was part of his broader philosophical framework. Emily Grosholz explains:

> For Leibniz, the key to understanding scientific knowledge is to start with the propositional schema 'S is P.' When claims of the form 'S is P' emerge in scientific discourse, the term S typically names a problematic object like π, the tractrix, the solar system, or a fossil. That is, it can be used to refer to something that is well enough understood and stable enough as an existing object to figure in a well-defined problem; and we think we are in a good position to find out more about it. Finding out more about a problematic object is, for Leibniz, a process of analysis, a search for the conditions of intelligibility of a thing, its requisites, as well as a search for the conditions

> of solvability of the problem which involves the thing under scrutiny (Grosholz, 2013, p. 97)

While there was an alternative view of the calculus in Newton's work, Maria Rosa Antognazza (1964-2023) addresses the differing approaches to calculus development:

> … Leibniz's approach to the calculus was very different from Newton's. For the great Cambridge mathematician, his calculus or method of fluxions was basically a brilliant way to solve some difficult mathematical problems. For Leibniz, typically, it was part of a much broader project with implications reaching far beyond mathematics and physics to logic, philosophy, religion, ethics, and politics: the project of *characteristica universalis* (Antognazza, 2009, p. 165)

Leibniz's wicked problem/situation was to produce a method of discovery following Descartes: a philosophical thesis followed by examples. Descartes' *Discours de la Méthode* full title is "Discourse on the Method of Rightly Conducting One's Reason and of Seeking Truth in the Sciences" (Descartes, 1637) and among its purpose and central ideas was the introduction of a new philosophical and scientific method for seeking truth and advancing knowledge. Leibniz's approach to the *wicked problem* was deeply rooted in his philosophical and mathematical worldview. As Phillip Beeley notes, Leibniz believed that "nature has a mathematical core" (Beeley, 1999, p. 125), which informed his search for a universal method of discovery. This belief led Leibniz to develop an approach that could be applied across various scales and levels of complexity, based on what Beeley calls an "infinitely replicated structure" in nature (Beeley, 1999, p. 126). Leibniz's use of analogies, which Beeley describes as "decidedly heuristic" (Beeley, 1999, p. 126), was key to his method. By drawing connections between different phenomena and levels of reality, Leibniz sought to develop general problem-solving techniques that could be applied to a wide range of issues. This approach aligns with the nature of wicked problems, which often require innovative thinking and the ability to see connections across disparate domains.

Crucially, Leibniz was not content with purely theoretical solutions. As Beeley points out, Leibniz sought to "reconcile the phenomena with rational knowledge as represented by mathematics and the mixed sciences" (Beeley, 1999, p. 137). This desire to bridge theory and practice is particularly relevant to addressing wicked problems, which resist purely theoretical solutions and require engagement with real-world complexity. Moreover, Leibniz prioritised applicability over excessive precision. Beeley emphasises that for Leibniz, "applicability, and not excessive calculation to unnecessary degrees of certainty, is the overriding principle as far as mathematics is concerned" (Beeley, 1999, p. 145). This pragmatic approach is well-suited to wicked problems, where perfect solutions are often impossible, and the goal is instead to find workable approaches that can be refined over time.

Arnaud Pelletier describes (Pelletier, 2018) how Leibniz worked all his life on the idea of creating a new form of encyclopaedia that could function as a tool for discovering and demonstrating new knowledge, with the goal of adding to the wisdom or happiness of humankind. Beyond pedagogical value, he wanted to give heuristic and transformative form to his idea of an encyclopaedia and to infuse it with philosophy as a set of demonstrations. He was never able to realise his project, and early on he pinpointed two obstacles: the need for many collaborators, which linked to his project to establish later in his life scientific societies inspired by the *Académie des Sciences* in France and the *Royal Society* in England, and the second was the realisation that such a complex project could only stem from a series of projects at the core of “what Leibniz calls *Scientia Generalis* [*general science*], that is, the science on which the construction of the *characteristic art* is based.” (Mugnai, 2018, p. 178)

> ... determining the concept, the object, and even the purpose of such a *Scientia Generalis* within the Leibnizian corpus proves to be particularly problematic (Pelletier, 2018, p. 164).

One of the three clear and distinct attributes that Leibniz gave to the *Scientia Generalis* was that of being an *ars inveniendi* (art of inventing), in particular in about fifteen texts written between 1679 and 1688 (Pelletier, 2018, p. 165). The following is a definition of the *Scientia Generalis* that Leibniz gives in 1683 (the year before he would publish his paper on differential calculus)]:

> By “general science” I mean the science which contains the principles of all other sciences and the method of using these principles in such a way that someone, even of moderate ability, when venturing into particular things, can understand the most

> difficult things and discover the most beautiful truths and most useful practices, as much as is possible for a human being to do from the data, and this with easy reflection and little experience. Therefore, it must treat the method of thinking well—that is of discovering, judging, controlling the passions, retaining and remembering—as well as the elements of the whole encyclopaedia, and of the investigation of the supreme good, for the sake of which all reflection is undertaken, as wisdom is nothing other than the science of happiness. [Lloyd Strickland's translation from Latin in A VI 4 n.127 with original manuscript in LH IV 7A] (Leibniz, 1999b, p. 532).

Leibniz's concept of *General Science* as outlined in this text represents a pivotal idea in the history of philosophy and scientific methodology. It encapsulates his vision of a universal method for understanding and discovering knowledge across all disciplines. The significance of this text lies in Leibniz's ambitious goal to create a system that would enable individuals of average intelligence to comprehend complex ideas and uncover profound truths through structured thinking and practical experience. This approach emphasises the democratisation of knowledge and the power of systematic reasoning. Leibniz's *General Science* encompasses not only principles and methods for using those principles but also extends to the art of thinking well - including discovery, judgement, emotional control, memory, and the pursuit of encyclopaedic knowledge. Crucially, Leibniz positions the search for the highest good and happiness as the underlying motivation for all intellectual endeavours, effectively linking epistemology with ethics. This holistic view of knowledge acquisition and its ultimate purpose reflects Leibniz's broader philosophical project of reconciling reason, faith, and human flourishing. In the context of my thesis, this text serves to illustrate Leibniz's goal to contribute to the development of scientific methodology, his vision of accessible and practical knowledge, and his integration of cognitive, ethical, and metaphysical concerns in the pursuit of wisdom.

At some point between 1679 and 1681 (as documented by the watermark of the extant copy) and before his publications in the *Acta Eruditorum*, Leibniz created a document called *De Arte in Mathematicis* (Leibniz, 2021, pp. 174–177) regarding some of his ideas on current work in mathematics. The tone of the text seems to indicate that it was meant for publication, however, I am not aware of the essay ever being published or studied anywhere outside of the *Academy*

Edition in 2021. I will quote here *in extenso* [the paragraph breaks are mine] the first part and the end section of the text[5]. This manuscript presents Leibniz's vision for a comprehensive approach to mathematics. He outlines a method for understanding, teaching, and advancing mathematical knowledge, emphasising the power of analytical thinking and the importance of a universal method for problem-solving:

> I have promised to teach the art of mathematics, both for understanding easily what has been handed down by others, and—if no one has handed down what we want—for discovering by our own efforts what is commonly considered as secret or even as insurmountable. Thus, in the future, it will not be necessary to burden the memory with rules (although it is sometimes advisable to retain elegant theorems and useful shortcuts), nor will there be need to meticulously seek various trials, for we can always reach our goal by following a sure and determined path, which is called analysis, though I admit it is not always the shortest...
>
> But this analysis, as I conceive it, is perhaps not yet publicly known. Certainly, it is something very different from algebra, and the rules of algebra itself have been discovered through a kind of higher analysis. Indeed, the majority of the most significant mathematical problems do not depend on algebra, nor can it be said whether they can be reduced to equations of the first, second, third, or any other degree; for they are often of no degree, or of all degrees, or even of uncertain or infinite degree, which is why I call them transcendental. Such are most of the problems of that geometry cultivated by Archimedes, for example, finding a line from a given property of tangents, the quadrature of the circle or hyperbola, the section of an angle in a given ratio, finding logarithms, solving trigonometric problems without tables, and innumerable others which commonly occur in the application of geometry to mechanics and other arts. Therefore, anyone who thinks that algebra can accomplish anything whatsoever either speaks thoughtlessly or has no experience in dealing with problems of greater importance. But those who doubt this will perhaps be convinced when they try out the examples given in this little book.
>
> I would venture to say that the arts taught here have advanced the mathematical sciences immeasurably, and that the difference between our analysis and the publicly

[5] Professor Lloyd Strickland has been kind enough and generous to translate the main body of the text for me in an unpublished document that he transmitted to me via email on 15 August 2024 and it is quoted in its entirety here. I have translated the end section corresponding to a separate page of the original manuscript in Latin using the transcription of the manuscript that was recently made available in the first (current) version of *Mathesis. Transkriptionen und Vorauseditionen mathematischer Schriften für die Leibniz-Akademie-Ausgabe* (the Leibniz Academy Edition) with the state of the work in the text current as of May 2021 (the same source for Professor Strickland's translation: Academy transcription in Mathesis, Version 1, N. 60 p. 174-177).

known algebra of today is far greater than the difference between the algebra of the ancients and the so-called specious algebra of the moderns. Indeed, I assert that it is now finally possible to make good on that long-discussed general problem, that there is no problem that cannot be solved. For I have a sure method by which it can be known whether and in what sense the quadrature of the circle is possible or not, and other similar problems to which no analysis has hitherto reached…

But I return to mathematical discovery, where I confess that I have profited especially from the writings of Archimedes, Viète, and Descartes, and the discoveries of the most eminent geometers: both Gregorys, Huygens and Hudde, Mercator, Newton, Pascal, Ricci, Sluse, Wallis, and Wren. And I attribute so much to these men that I think they could also have achieved what I am presenting here if they had applied their minds to it. Therefore, my address is not to them or to those like them, but to certain boasters who[6] [...] algebra [Professor Strickland's translation] (Leibniz, 2021)

[Part 2, end section]
… Therefore, we will first present numbers and calculation, then figures and the motions by which they are described, and finally the method of reducing figures to calculation, and conversely, constructing calculation through figures. And since all mathematical science consists in these things, it will only remain for us to explain afterwards the use of inventions in the sciences subordinate to arithmetic and geometry; and to treat the method by which mechanical questions and similar ones with concrete matter can be abstracted from there and reduced to problems of pure geometry [my translation[7]] (Leibniz, 2021).

The manuscript *De Arte in Mathematicis* represents a crucial transitional moment in Leibniz's mathematical and philosophical thinking, written during a period when he was developing the foundations of calculus. Its significance can be analysed across several key dimensions: The text reveals Leibniz's vision of a new mathematical methodology that transcends traditional algebraic approaches. His assertion that "analysis...is perhaps not yet publicly known" and is "very different from algebra" signals his recognition that conventional mathematical tools were insufficient for addressing what he termed "transcendental" problems. This awareness presages his development of calculus and indicates his growing understanding of the need for new

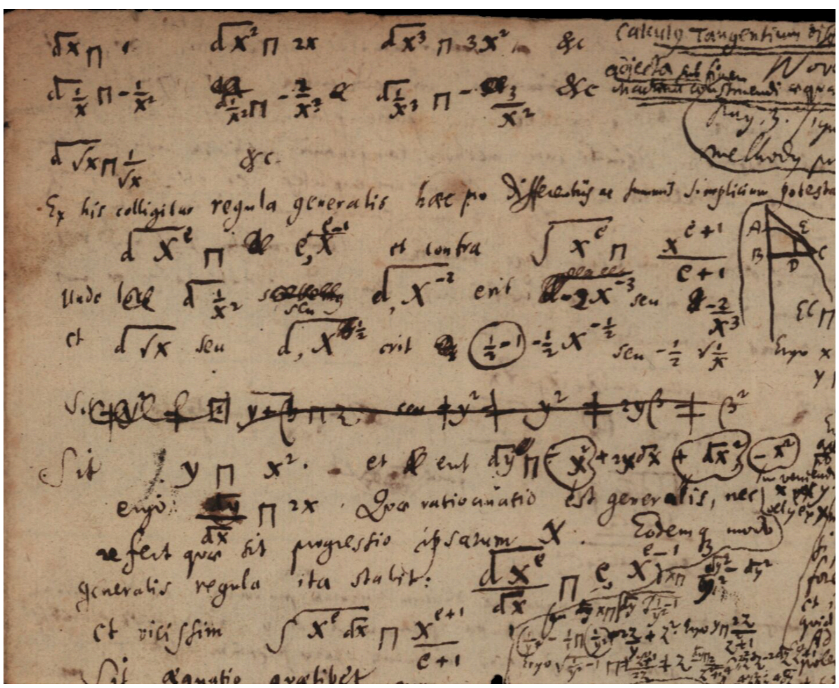
[6] The piece ends abruptly here
[7] All translations are my own except where indicated.

mathematical frameworks to handle infinite series and continuous change. Universal Method and Mathematical Philosophy: Leibniz's emphasis on developing a "sure and determined path" reflects his broader philosophical project of creating a universal method for problem-solving. This ambition connects directly to his work on the *characteristica universalis*, demonstrating how his mathematical innovations were intrinsically linked to his philosophical vision of a universal logical language. The text shows Leibniz grappling with the relationship between symbolic representation and mathematical truth, a theme that would become central to his later philosophical works. The document's acknowledgment of contemporaries like Newton, Huygens, and Pascal provides valuable insight into Leibniz's intellectual network and the collaborative nature of 17th-century mathematical development. His statement that these mathematicians "could also have achieved what I am presenting here" suggests both his respect for their work and his recognition that he was pushing beyond their achievements in significant ways. Particularly noteworthy is Leibniz's articulation of the relationship between different mathematical domains. His discussion of "reducing figures to calculation, and conversely, constructing calculation through figures" demonstrates his understanding of the deep connections between geometric and algebraic approaches. This synthesis would prove crucial in his development of differential calculus, where geometric problems could be solved through analytical methods. The text's emphasis on teaching mathematics without burdening memory with rules represents a significant departure from traditional mathematical pedagogy. This approach reflects Leibniz's broader philosophical commitment to developing systematic methods that could guide discovery rather than merely cataloguing known results. The manuscript's discussion of "transcendental" problems that are "of no degree, or of all degrees, or even of uncertain or infinite degree" reveals Leibniz's growing appreciation for mathematical infinity and continuity. These concepts would later inform his metaphysical theory of monads and his understanding of the relationship between the finite and the infinite.

The significance of this text stems from its helpfulness for understanding Leibniz's intellectual development. Written during the crucial period when he was developing calculus, it provides unique insight into how he conceived of mathematics as both a practical tool and a model for philosophical thinking. His integration of practical problem-solving with broader philosophical concerns about method, infinity, and universal knowledge exemplifies the sophisticated synthesis that characterised his mature thought. This document helps bridge the gap between Leibniz's mathematical work and his philosophical projects and provides valuable context for understanding the conceptual innovations that led to the development of calculus and modern mathematical analysis.

To explicate these thoughts, it can be helpful to illustrate with an exemplar of Leibniz’s work. The following excerpt is from a manuscript called “Calculus Tangentium Differentialis” that Leibniz wrote in November of 1676, where he is working on differentials in tangents and explains the “general rule for the differences and sums of simple powers” (Leibniz, 1676, p. 3r) (Leibniz, 2008a, p. 612):

Leibniz explains here (LH 35 V 12 Bl.3–4, and A VII, 5 N.96 p. 612) using several examples such as $dx^2=2x^1$ and $dx^3=3x^2$ [I substituted the symbol “Π” for the equivalent modern equality symbol “=”] “From these we gather this general rule for the differences and sums of simple powers… it is general, and it does not matter what the progression of x itself is”. In the same way the general rule can be used to solve the inverse operation of integration such as in $dx^e=ex^{e-1}$ and its inverse $\int x^e=(x^{e+1})/(e+1)$. The explanation of the rule and its utilisation and applicability is presented almost exactly the same as in modern College Calculus textbooks, and many of the ideas described earlier in *De Arte in Mathematicis* can be



seen here clearly illustrated with a tone and style that is purposely chosen by Leibniz and carry immense pedagogical value for the reader.

De Arte in Mathematicis provides a window into Leibniz's revolutionary mathematical thinking. It showcases his vision of mathematics as a powerful, universal tool for understanding the world, his innovative approaches to problem-solving, and his integration of mathematical and philosophical thinking. This text is not only influenced by his development in mathematics but also reflects Leibniz's broader philosophical project of creating a universal method of reasoning and understanding. Leibniz's philosophical and mathematical approach laid the groundwork for addressing complex, interconnected problems – what we now call wicked problems. His emphasis on universal methods, practical applicability, and the reconciliation of theory with phenomena anticipates many of the challenges and approaches associated with tackling wicked problems in modern contexts. I will now describe the intellectual leap that resulted in Leibniz's invention of calculus.

Intellectual Leap

Leibniz published *Nova Methodus pro Maximis et Minimis* (Leibniz, 1684b, pp. 466–473) (Leibniz, 1989, pp. 96–117) (Leibniz, 2011, pp. 37–44) his seminal paper of 1684 on differential calculus as part of his goal of developing the art of discovery at the centre of his idea of a *General Science* and the compilation of an encyclopaedia of all knowledge available at the time. His publication came at the period where similar ideas were being developed by others with antecedents in Descartes, Malebranche, and Spinoza, and in particular, Leibniz was worried about Ehrenfried Walter von Tschirnhaus, his friend and companion of studies in Paris in 1675, stealing the primacy of his work. Leibniz was afraid that the ideas he had been developing since his period in Paris the decade before were in danger to be preempted by Tschirnhaus who in August of 1682 had proposed a method for squaring any space contained by a geometric line at the *Académie des Sciences* [Procès-verbaux 9, p. 172r.], and who in 1683, a year after his election to the *Académie des Sciences*, used the *Acta Eruditorum*[8] to publish a paper entitled "Methodus datae figurae, rectis lineis & Curva Geometrica terminatae, aut Quadraturam, aut

[8] the first scientific journal of the German speaking world which Leibniz supported by sending many papers from Hanover

impossibilitatem ejusdem Quadraturae determinandi" [Method of determining either the quadrature or the impossibility of its quadrature of a given figure bounded by straight lines and a geometric curve] (Tschirnhaus, 1683, pp. 433–437). In addition to his likely deeply felt jealousy at Tschirnhaus' election to the *Académie des Sciences* [which Leibniz had not been able to achieve], the diffusion of Tschirnhaus' theorems and methods made Leibniz furious because it presented a similar treatment for quadrature with a method that reminded Leibniz of explanations he had provided around 1679 and he was worried Tschirnhaus' ideas would evolve in the same direction of his calculus. This prompted Leibniz

> ... to publish his theory sooner than he had planned, not his whole calculus at once, but only the differential calculus, and not in extenso but, to secure priority, in a short seven-page paper… A few lines of introduction were followed by a short paragraph stating the familiar rules for differentiating sums, products, and quotients. Then Leibniz turned to extrema, with the earliest known distinction between maximum and minimum, and further to the notions of concavity, convexity, and inflection point (punctum flexus contrarii). The remaining portion of the paper included the power rule, also for negative and fractional exponents; the introduction of new variables (substitution), with the earliest appearance of the colon as a sign for division; Snell's law as an example of a minimum problem; and Beaune's problem. (Kracht and Kreyszig, 1990, p. 24)

What were some of Leibniz's root ideas for the calculus? Trying to contextualise the concepts of infinity and continuum, infinitary series, infinitesimals, and indivisibility. His thinking shifted from deductive reasoning in a continuum characterised by geometric constructions and proofs to inductive, inferential thinking in a *quantised* space. This is how the linking of ideas on infinitary descriptions, conceptions, and processes connects to the need to operate with indivisibles and infinitesimals. They correspond to ontological conceptions that are necessary to describe and understand. This did not occur in a vacuum, nor did it happen in a moment of brilliant inspiration, or as the output of an intellectual algorithm. While Leibniz was surrounded by others in a common environment and fertile preconditions, he used pieces that existed among current discussions on ideas and integrated them in a unique way with intuition and *a priori* concepts and thoughts. He then invented a set of entities that helped define a new space and explained the implications of using them within algorithmic methods that transcended limited applicability on specific problems to provide *exploration* and *discovery*. Ernst Cassirer (1874-1945) explains how for Leibniz, the justification of studying the infinitesimal, incommensurable stems from his belief in the power of the *principle of sufficient reason* [Leibniz's principle of sufficient reason states that everything must have a reason or cause for why it exists or occurs., that is, nothing happens without an explanation or justification] for his description and classification of the different types of truths (Cassirer, 1943, p. 376).

This will be important in his making a bridge to equate the solutions provided by an infinitary series to the algorithmic result from his *calculus summatorious* method [later renamed to *integration* (Cajori, 1925, p. 414)]. The later resistance to accept calculus was in part due to the difficulty of the transition, and in part to the uncertainty of operating with non-tangible, ill-defined quanta: infinitesimals/indivisibles in this case. It was necessary to ideate the quanta that describe the change within the process/space and the implication of using them in a framework. Siegmund Probst explains that Leibniz had been working since 1673 on a general method to solve a well-known problem in the seventeenth century: the tangent problem. In the direct approach, the problem consists in constructing a tangent to a given curve, and while there were several methods known, Newton and Leibniz discovered that this amounts to *differentiation* (in modern terms). In the inverse tangent approach, what is given is the tangent (or subtangent, or its perpendicular normal) and the curve needs to be constructed (Probst, 2024, p. 2). In modern terms and in simple cases, this can be solved by finding the antiderivative of a differential equation. Although people like Pierre de Fermat (1607-1665), Gregory of Saint Vincent (1584-1667), Isaac Barrow (1630-1677), and Isaac Newton (1643-1727) knew a lot about the connection between these problems, Leibniz was the first to make the connection explicit. Even though Huygens thought that Descartes had found a method for inverse tangents of some optical curves, Leibniz did not believe it and continued to work on the inverse tangent problem, and he begun to draw insights for Blaise Pascal's (1623-1662) work[9]:

> In Paris, Leibniz obtained another work published from Pascal's legacy, the *Traité du triangle arithmétique* (1665), whose content was closest to his own earlier studies in

[9] In the following, references to the Academy Edition of Leibniz's *Sämtliche Schriften und Briefe*, 1923ff, are given with the sigle LSB, series (Roman numeral), volume (Arabic numeral) and number of the text in the volume. The texts of the more recent volumes of the edition are freely accessible via the page https://leibnizedition.de/ (Probst *et al.*, 2023a, p. 54)

combinatorics. Leibniz returned to these topics in 1672, and Pascal's triangle providing a summation symbolism to write binomial coefficients in a concise form also inspired him to construct the Harmonic Triangle, which he used for his successful summations of infinite series by a difference method (LSB VII, 3 N. 30, 53). Since the spring of 1673, another work by Pascal became important for Leibniz: The Lettres de Dettonville (1659) on infinitesimal mathematics, published in a small edition during Pascal's lifetime. He probably was not able to acquire the book, but borrowed a copy from Christiaan Huygens (1629 - 1695). To this circumstance we owe detailed excerpts from the book he studied intensively, especially the Traité des sinus du quart de cercle (LSB VII, 4 N. 10). As repeatedly reported by Leibniz, the study of this book led him to the introduction of the characteristic triangle consisting of the infinitesimal elements of abscissa, ordinate and tangent at a curve point. With it he could finally perform what we now call integral transformations and find the infinite circle series still in 1673 (LSB VII, 6 N. 1) (Probst *et al.*, 2023b, pp. 54–55).

Consistent with the idea of finding a general method, he also was trying to facilitate what he would later describe as *blind thought*: “thinking with signs or symbols in place of the ideas or concepts that they stand for” (Arthur, 2014, p. 48) which occurs beyond arithmetic and algebraic calculation.

To exemplify these ideas and what he was working on at that time, I am including here the following text which has a couple of excerpts from a manuscript called *Dialogus* [*A Dialogue (August 1677)* draft in Leibniz’s hand in LH 4, 5, 3 Bl. 1–2] (Leibniz, 1999a, pp. 20–25) (Strickland, 2026). This text was written as a dialogue between 2 characters named *A* and *B* regarding philosophical and mathematical work. In this short dialogue, character A, who speaks for Leibniz, leads character B to accept that truths and falsehoods pertain to propositions, and then to formulate objections to Thomas Hobbes’ claim that truth depends upon the definitions of terms, and therefore upon human discretion (in other words, that truth is arbitrary) (Strickland, 2026):

B. That’s right, but yet I note that if characters can be used for reasoning, then there is some complex structure in them, an order that agrees with things, if not in the individual words (although this would be even better) then at least in their combination and inflection. And indeed this order, with some variations, corresponds in some way in all languages. And this gives me hope of escaping the difficulty. For although the characters are arbitrary, their use and connection nevertheless have something which is not arbitrary, namely, some analogy between characters and things, and the relations which different characters expressing the same things have to each other. And this analogy or relation is the basis of truth. For it means that, whether we use these or other characters, the result is always the same or equivalent, that is, corresponding analogously. Although perhaps it is necessary that some characters always have to be used for thinking.

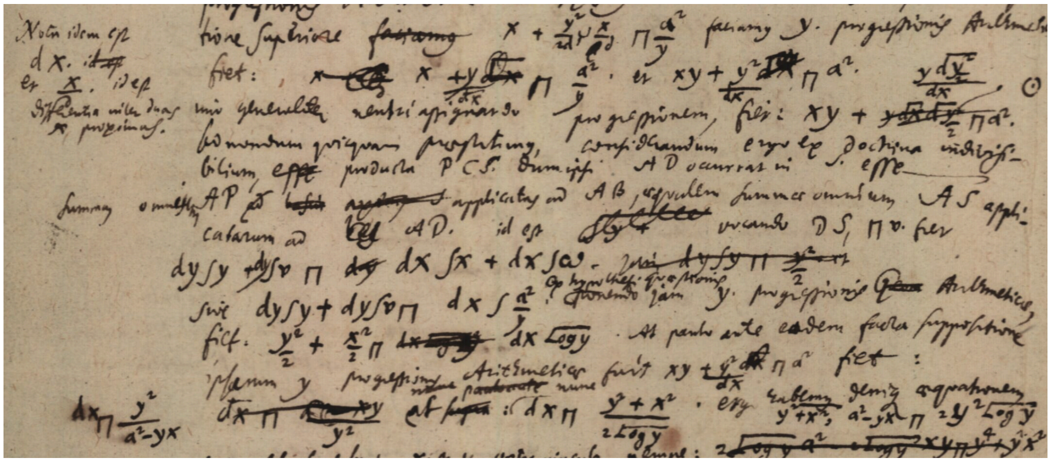
A. Well done! You have explained it very clearly. And the analytic or arithmetical calculus confirm this. For in numbers, things always work out in the same way, whether you use the decimal or, as some have done, the duodecimal progression, and what you have explained you may afterwards, in various ways, arrive at by calculations in grains or other countable matter, for the result will always be the same. In analysis as well, although different properties of things are more readily apparent from different characters. However, the basis of truth is always found in the very connection and placement of the characters, so that if you say a^2 is the square of a, by putting $b + c$ for a, you will have the square $+b^2 + c^2 + 2bc$, or by putting $d - e$ for a, you will have the square $+d^2 + e^2 - 2de$. In the former, the relation of the whole a to its parts b and c is expressed, in the latter, the relation of part a to the whole d, and the excess e over part a. Yet when substituting, it is apparent that the calculation always yields the same result. For in the formula $d^2 + e^2 - 2de$ (which is equivalent to a^2), in place of d let us substitute its value $a + e$, then for d^2 we will have $a^2 + e^2 + 2ae$, and for $- 2de$ we will have $- 2ae - 2e^2$… You see, whatever characters are taken at our discretion, so long as a definite order and rule is preserved in their use, everything always agrees. Therefore, although truths necessarily presuppose some characters, indeed, sometimes truths speak of the characters themselves (such as theorems dealing with casting out nines), yet they consist not in the arbitrary aspect of the characters but in what is permanent, namely, in their relation to things. And it is always true, without any discretion on our part, that when such-and-such characters are used, such-and-such a form of calculation will result, and likewise when others are used whose relation to the former ones is known, these being different but also preserving the relation to the former ones resulting from the relation of the characters, as is apparent by substituting or comparing. [Dr. Lloyd Strickland's translation from the original Leibniz's handwritten draft in Latin in LH 4, 5, 3 Bl. 1–2] (Leibniz, 1999a, pp. 24–25) (Strickland, 2026).

This dialogue from 1677 represents a crucial moment in Leibniz's intellectual development, particularly in his conception of the relationship between symbolic representation and truth.

The text illuminates Leibniz's emerging understanding of what we might now call the invariance of truth under different symbolic representations, a concept that would prove fundamental to both his mathematical and philosophical work. In the dialogue, Leibniz advances beyond the conventional wisdom of his time, particularly Hobbes's nominalist position that truth is merely a matter of definitions and therefore arbitrary. Instead, Leibniz articulates a sophisticated understanding of how different symbolic systems can capture the same underlying truths through what he terms "analogy or relation" between characters and things.

The significance of this text manifests in several key aspects: First, Leibniz uses mathematical examples, particularly from algebra and arithmetic, to demonstrate how different symbolic representations (such as decimal versus duodecimal systems) can express the same underlying truth. His example of algebraic substitution–$a^2 = (b+c)^2 = (d\text{-}e)^2$–serves as a concrete demonstration of how different symbolic expressions can preserve underlying relationships while using different characters. This understanding would prove crucial to his later development of differential calculus, where different notational systems could express the same fundamental relationships. Second, the text reveals Leibniz's emerging conception of what we might now call structural invariance. When he argues that "although characters are arbitrary, their use and connection nevertheless have something which is not arbitrary," he is articulating an early version of what would become a fundamental principle in modern mathematics and logic: that truth resides not in the symbols themselves but in the preservation of relationships between them. Third, this dialogue demonstrates Leibniz's broader philosophical project of developing a universal characteristic (*characteristica universalis*). His insight that different languages share an underlying "order" that "corresponds in some way in all languages" reflects his belief in the possibility of a universal symbolic system for expressing all rational thought. The text's historical significance extends well beyond its immediate context. Leibniz's understanding of the relationship between arbitrary symbols and invariant truth would influence the development of modern mathematical logic through figures like Frege and Russell. His insights about the preservation of structure under different representations anticipates aspects of modern abstract algebra and category theory. Furthermore, his recognition that different symbolic systems might make different properties "more readily apparent" presages important developments in mathematical notation and the role of representation in scientific understanding. This insight remains relevant to contemporary discussions in the philosophy of mathematics and computer science regarding the relationship between notation and understanding. This text thus represents a pivotal moment in the history of mathematical and logical thought, where Leibniz begins to articulate ideas that would prove foundational to modern mathematical logic, abstract algebra, and the philosophy of language. His synthesis of mathematical and philosophical insights in this dialogue exemplifies the interdisciplinary nature of his thought and its enduring influence on multiple fields of inquiry.

Let's see how the use of symbols and the concepts they represent played out in the development of an algorithm that could be used to represent a logical structure to be used in solving problems. On the 11 of November 1675, in the unpublished manuscript "Methodi tangentium inverse exempla" (Leibniz, 2008b, pp. 321–331) Leibniz was able to determine the cubic parabola as a solution in just a few lines that had been previously significantly challenging using existing methods, while at the same time he invented the differential and integral notation that we continue to use today. Over the next several months he continued to develop simple solutions for diverse problems and curves, which he saw as important evidence of the effectiveness of his methods (Probst and Mayer, 2008, pp. xxii–xxv). In the following figures you can see excerpts from both the 11 November 1675 manuscript (Leibniz, 1675, p. LH 35 V 9 Bl.1–2) and its published transcription by the Academy (Leibniz, 2008b, p. 325) where Leibniz uses both the differentiation symbol *d* representing the difference of two successive ordinates serving to find the slope of a tangent line and summation symbol ∫ representing the sum of ordinates that produces the area under a curve, and replaces Bonaventura Cavalieri's (1598-1647) term *omnis lineae*:



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Sed nondum quicquam praestitimus, considerandum ergo ex doctrina indivisibilium, producta PCS. dum ipsi AD occurrat in S. esse summam omnium AP applicatarum ad AB, aequalem summae omnium AS applicatarum ad AD. id est vocando DS, $\sqcap$ v. fiet $dy \int y + dy \int v \sqcap dx \int x +\, dx \int \omega$. sive $dy \int y + dy \int v \sqcap dx \displaystyle\int \frac{a^2}{y}$. ex hypothesi quaestionis. Ponendo jam y. progressionis Arithmeticae, fiet: $\dfrac{y^2}{2} + \dfrac{x^2}{2} \sqcap dx\,\overline{\operatorname{Log} y}$. At paulo ante eadem facta suppositione ipsarum y progressionis Arithmeticae fuit $xy + \dfrac{y^2}{dx}$ $\sqcap$ a^2. Fiet: $dx \sqcap \dfrac{y^2}{a^2 - yx}$ et nunc: $dx \sqcap \dfrac{\overline{y^2 + x^2}}{2\,\overline{\operatorname{Log} y}}$. Ergo habemus denique aequationem in qua solae supersunt x. et y. extra vincula, nempe: $\overline{y^2 + x^2}$, $a^2 - yx \sqcap 2y^2\,\overline{\operatorname{Log} y}$. Quae aequatio cum sit determinata locum dabit quaesitum. Et valde memorabilis est haec methodus, cum enim non sit hic in nostra potestate tot aequationes habere quot incognitas, poterimus tamen saepe plusculas obtinere aequationes, et earum ope quosdam terminos elidere, ut hoc loco $d\overline{x}$. Quae sola nobis obstabat. Singulae aequationes totam includebant quaestionis naturam, nec tamen ex iis erui poterat solutio, quod media facilia hactenus desint, conjunctio duarum aequationum rem compendio dedit.

Notice in the upper left corner of the manuscript that Leibniz changes the notation for differentials that he has been using to the one he will use going forward which he explains this way: “Note *dx* is the same as *x/d* that is, the difference between two adjacent *x*'s” [my italics][10] (Leibniz, 1675, p. LH 35 V 9 Bl.1v).

For Leibniz, the leap goes from the constraints of working with equal entities in ratios in specific cases to the description and use of infinitesimals and to the more general acceptance of numerical ratios that can then be implemented in various applications. Leibniz did not see the

[10] “Nota idem est dx. et x/d . id est differentia inter duas x proximas” (Leibniz, 1675, p. LH 35 V 9 Bl.1v)

examples he studied with Huygens as part of his education, but rather as the point of departure for his discoveries (Parmentier, 1989, pp. 12–13). With the publishing of the "*Nova Methodus pro Maximis et Minimis*…," (Leibniz, 2011, pp. 37–44) Leibniz leaves behind previous efforts by providing a method that works geometry differently, by applying calculation to the operations (functions in modern language) that define the curves. He was attempting to understand and mathematically describe continuous movement as seen in nature, whether in observable or unobservable entities. At issue were the adoption of different metaphysical ideas such as the corpuscular (atomic) nature of matter and the idea of continuity.

By analysing curves with traditional geometrical methods, approximations with geometrical figures (such as using a number of rectangles to fill the space under the curve), the introduction of errors was inevitable. Numbers were necessarily different unique entities that needed to be connected to a continuous line. Leibniz approached the problem by looking at quantities as representing segment lengths that were very, very small, and yet different from zero. Using these *vanishing*, intangible, incommensurable entities born from the mathematical description of the curves themselves, Leibniz could approximate the actual line so closely that the error disappeared for all practical purposes. Separating the infinitesimals from the physical and geometrical interpretation of the curve was a radical departure from scientists requiring experimentation and data to prove hypotheses, and from mathematicians providing rigorous proofs using Euclidean geometry, thus creating an intellectual space where it was necessary to address and use entities that were not representational of reality, and with no empirical status (Reyes, 2004, pp. 178–179). Let's see how he introduced his work in his first publication.

Leibniz's "*Nova Methodus pro Maximis et Minimis*…," (Leibniz, 2011, p. 37) opens with a figure showing several curves and their tangents, proceeding to explain that the tangents can be represented by arithmetic operations on small segments *d*x, *d*w, *d*y, *d*z (Leibniz, 2011, pp. 37–38). These small segments will come to be known as *infinitesimals*. Having posited the segments as represented by their numerical expressions, he proceeds to extend the operations to multiplication and division. Leibniz introduces notation that allows him to represent arbitrary segments with expressions that can be used in the same way in different curves and explains how the operations that represent the differences correspond to increases of the values associated with the curve if it is positive, or decreases if it is negative, with a maximum in a point at the apex of concavity or a minimum at a point at the bottom of a convexity (Leibniz, 1989, p. 107). Leibniz continues by calculating the *difference of the difference* (*differentiae differentiarum,* derivative of the derivative or second derivative in modern terminology) with a notation of *dd*x and uses it to associate the shape of the curve with the signs resulting from those operations. For example, Leibniz explains how when the *difference of the difference* transitions from negative to positive, the *concavity (concavitatem)* of the curve inverts and becomes *convexity (convexitatem)* where there is the greatest or least increase[11] (Leibniz, 2011, p. 39). Precisely here is where the novelty starts, as the geometrical interpretation of the curve follows the numerical operations on its symbolic representation (Diaz, 2023, p. 331).

Here is the departure from previous work, as the geometrical interpretation of the curve follows the numerical operations on its symbolic representation. The behaviour of the curve can be described by numerical analysis, and in doing so, Leibniz is creating an epistemic entity informed by mathematics in the study of the curve. The description of the method is semantic, teleological, and pedagogical. Leibniz develops not only a notation but a vocabulary that describes the behaviour of the curve, defining maximums and minimums by the arithmetical result that identifies them and by the use of descriptors such as inflection point (*punctum flexus contrarii*) (Leibniz, 1684a, p. 468). He anticipates the use of the method in a variety of applications by developing rules and examples in the calculations of examples from power and rational functions, and radical functions. Leibniz characterises his method as algorithmic (*algorithmo*) (Leibniz, 1684a, p. 469) and calls it *differential* [*differentiali*] (Leibniz, 1684a, p. 469). He elaborates on the usefulness of the approach by arguing that it can be extended to all curves without relying on hypotheses specific to particular curves or families of curves, an idea that was reflected in his *Dialogus* of 1677 (Leibniz, 1999a, pp. 20–25) (Strickland, 2026) and in *De Arte in Mathematicis* of 1679 (Leibniz, 2021) (Strickland, 2026). In doing so, Leibniz goes

[11] "(seu si positis v affirmativis etiam ddv *differentiae differentiarum* sunt affirmativae, vel negativis negativae) curva axi obvertit *concavitatem*; alias *convexitatem*: ubi vero est maximum vel minimum incrementum" (Leibniz, 2011, p. 39)

beyond the specificity of a solution of a problem to the universality of a method that can be explicitly used in a myriad of applications and fields. In his defence of the method, Leibniz explains that in progressively difficult problems, calculations by traditional methods might considerably increase in complexity, which his approach avoids, while using a notation that is not particular to a specific case (Leibniz, 1684a, pp. 471–472).

The approach above also relaxes the restrictions when operating within specific problems such as movement, which would ease the realism requirement to understand differentials in terms of variations in time such as was used by Newton, and in doing so it permits a true generalised analysis for the first time (Diaz, 2023, pp. 332–333). I will now describe how these ideas represented the beginning of a mathematical revolution à la Kuhn.

Alternate Views - Competing frameworks regarding the invention of calculus.

We now know and it has been accepted for quite some time that Newton and Leibniz came up with similar ideas independently (Boyer, 1988, p. 299) with Newton having thought of them first and Leibniz having published them first (Lodge and Strickland, 2020, p. 7).

> By the end of the second third of the seventeenth century all the rules needed to handle such problems [higher mathematics needed for dynamics and Keplers' astronomy among others] in areas and rates of change, in maxima and tangents, were available. The time was now ripe to build the infinitesimal analysis into the subject we know as *the calculus*. No specific new invention was needed; the techniques were at hand. What was wanting was a sense of the universality of the rules… This awareness was achieved first by Isaac Newton, in 1665-66, and again independently by Gottfried Wilhelm von Leibniz in 1673-76. (Boyer, 1970, p. 75)

In my view, and in the context of this thesis, it is less important to look at primacy of thought and more important to study the process of acceptance of the ideas and their incorporation into new fields of knowledge, and the original approach to those ideas is an important part of the analysis, as it plays a role in understanding the alternate views and competing frameworks that are always part of the process of acceptance. In the case of the invention of calculus, the original approaches are very different, and Ernst Cassirer (1874-1945) succinctly describes that difference:

> Whereas Newton started out with the study of certain natural *phenomena*-with an investigation of optical phenomena and with a theory of the motion of the moon, Leibniz, on the other hand, began with a *logical analysis of truth*… Leibniz fully acknowledges the value of empirical truth. But to his mind empirical truth is only a small part, a fragment, a single sector, of the universe of truth. Behind individual statements of empirical fact, it is the task of the philosopher to discover the necessary forms of thought. In physics we find factual truth; in logic, arithmetic, geometry, we have necessary or eternal truth (Cassirer, 1943, p. 374).

Leibniz's calculus faced bitter challenges, particularly from Newton and his followers. Reyes argues that the adoption of the calculus developed by Leibniz and Newton evolved from a network of discourses and brought with it a change in how humans perceive, think about, and discuss nature, and it became accepted knowledge through wide communication, discussion, and clarification between established thinkers (Reyes, 2004, p. 165).

From a philosophical perspective, the differences between Leibniz's and Newton's approaches to calculus reflect their broader philosophical views. Leibniz saw infinitesimals as a tool for reasoning about the behaviour of functions and believed that they could be used to develop a rigorous foundation for calculus. In contrast, Newton was an empiricist who believed that knowledge should be based on observation and experimentation. Leibniz believed in a relational theory of space and time, in which they are defined by the relationships between objects. He saw infinitesimals as a way to reason about these relationships and believed that they could be used to develop a unified theory of physics. Newton, on the other hand, believed space and time were absolute, and existed independently of objects. He saw his calculus as a tool for understanding the behaviour of physical systems in this absolute framework. Newton's views and conceptions as a physicist were constrained by a mathematical language specific to the problem at hand while Leibniz's approach was philosophical and concerned with developing explanations in a universal language not restricted to specific problems. This is where the leap to a more general method and understanding breaks with the existing paradigm, opening the path for its use in other applications.

Boyer explains the nature of the difference between Leibniz’s and Newton’s approaches to calculus (Boyer, 1988, pp. 187–223). We think of Leibniz as a mathematician and Newton as a scientist, no matter how brilliant a mathematician he was, Newton's mathematical methods were extremely difficult to replicate and generalise, and therefore were not adopted, while Leibniz’s were used extremely successfully by others and over time became the new methodological standard, the new paradigm. The approach could be applied to higher order differentials and sums, and while it was a logical next step in the application of the method, at first it was difficult to see phenomena explained by the mathematical operations. Instead, explanations began with mathematical descriptions that could later explain and predict phenomenological behaviour. This is crucial to understand the gestalt shift in mathematics in the 18th century. Replicability and mechanisation of approach made it possible for the new mathematics to be generally used. What made it difficult for Newton’s followers to replicate was a combination of the difficulty in the language used to describe his method and the specificity of its goal. Despite the later efforts to generalise its results and methods by Colin Maclaurin (1698-1746) (Maclaurin, 1747) and others, it was not possible for British mathematicians to emulate or expand its use and this stagnated British mathematics for more than a century.

There were sociological obstacles to the acceptance of Leibniz’s ideas. In Britain in general, and in the *Royal Society* in particular there was a distaste for the personality and style of Leibniz, who frequently made grandiose statements about the value and prospects of his projects and inventions, and this might have stemmed from the beginning when he was trying to become accepted into the *Society* in 1673. Niccolò Guicciardini explains how after the Stuarts were reinstated, many natural philosophers in the Royal Society wanted to assure others that they wouldn't assert absolute or dogmatic conclusions. The society did not welcome politically biassed philosophers or dogmatic theologians, instead fostering a harmless, moderate scepticism. Thus, any talk aimed at achieving certainty was viewed with suspicion, while scepticism and probabilism were endorsed in some of the Royal Society's most significant manifestos such as Hooke’s masterpiece on microscopy (Guicciardini, 2016, p. 396). The aversion to public conflicts and controversies regarding the validity of ideas can accommodate in part Newton’s aversion to publish his work and discuss it with others, thus affecting its acceptance and spread. The opposite is true to Leibniz in his successful campaign to spread his ideas and make them part of existing intellectual discourse.

In the *Cambridge Companion to Newton* (Iliffe and Smith, 2016a), and in an intriguing reassessment of Isaac Newton's mathematical legacy, Rob Iliffe makes a compelling case for Newton’s surprisingly limited impact on the historical development of mathematics, despite his undeniable genius. This perspective challenges the conventional narrative surrounding Newton's contributions, particularly in relation to the calculus. The argument centres on several key points. Firstly, Iliffe highlights Newton's reluctance to publish his mathematical work, especially on calculus, until the early 18th century. This delay allowed Leibniz and his school to establish a dominant position in the field, with a decade of frequent publications preceding Newton's. The lack of access to Newton's systematic methods, despite glimpses in the *Principia* (Newton, 1999), hindered the growth of a robust Newtonian mathematical tradition (Iliffe and Smith, 2016b, pp. 16–17).

Newton's approach is characterised as that of a skilled problem-solver rather than a revolutionary mathematician. This view is reinforced by his mathematical conservatism, which stands in stark contrast to the forward-thinking Leibnizian school. While Leibniz and his followers embraced symbolic manipulation as the future of mathematics, developing clear notations to this end, Newton remained rooted in classical geometry. His reluctance to devise perspicuous notations for calculus is seen not as a lack of ability but as a deliberate stance against a symbol-dominated mathematical revolution (Iliffe and Smith, 2016b, pp. 17–18). An exception is the dot notation for fluxions (time derivatives) which he did not invent or appear in the earlier versions of the *Principia* until mid-December 1691, almost four and a half years after the book first showed in the bookshops (Whiteside, 1970, p. 118).

The *Principia* itself serves as evidence of Newton's conservative approach, and in the early 1680s, after a vigorous review of classical mathematics, Newton appears to have concluded that the true roots of all mathematics lay in classical geometry (Iliffe and Smith, 2016b, p. 17). Whiteside's analysis of the *Principia* supports this assessment:

> The very form of the *Principia* is, it is needless to say, heavily "Euclidean", but Newton's "Propositions", "Theorems", "Problems", "Lemmas" and "Scholia" are mere expository frameworks inherited from his enforced study, as a subdued Trinity College undergraduate, of Isaac Barrow's 1655 Cambridge edition of the *Elements* and they are manifestly retained in his own subsequent mathematical writings purely as a literary convenience (Whiteside, 1970, p. 118).

Contrary to popular belief, there is no indication that Newton initially derived his celestial mechanics results using symbolic calculus before recasting them geometrically. Instead, he primarily employed his "method of first and last ratios," an extension of synthetic geometry incorporating limits. This approach, while elegant, diverged from the symbolic methods that would later prove superior in tackling complex problems in celestial mechanics. Ultimately, it fell to Leibniz's followers to reformulate Newton's work using symbolic calculus, revealing the power of this approach in physics. This shift marked a fundamental change in scientific problem-solving, moving from geometric representation to the formulation of differential equations in purely symbolic form (Iliffe and Smith, 2016b, pp. 18–19).

Richard Brown summarises the unique value of Leibniz's calculus:

> … no other mathematician, not even Newton, had constructed an algorithmic approach to infinitesimal analysis quite like Leibniz's. It is not simply the fact that Leibniz's notation of *d*x, *d*y, ∫ *y d*x is convenient. Its real advantage is that manipulating the notation by itself according to simple rules, without much attention to the underlying geometric structure, one is led almost without conscious thought to new approaches and solutions to what had been considered difficult problems… Leibniz could give in a few lines solutions to problems that, say, Barrow or Huygens could certainly have also solved, but in several pages of complex arguments (Brown, 2012, p. 174).

Épistémè-Socialisante - Socialising Episteme:

In a challenge to our understanding of scientific progress, highlighting the complex interplay between individual genius, publication practices, and broader mathematical philosophies in shaping the course of mathematical history takes us to the next phase in the process of additions to new fields of knowledge where social cognition develops knowledge beyond an individual’s capability to solve problems, as we will see in this section.

Gestalt shift

There is a gestalt shift in Leibniz’s leap in understanding mathematical methods and the language used in describing and solving problems. Severing the attachment to reality allowed an understanding that was disconnected from strictly geometric arguments and deductive logic to infer the applicability of the method beyond specific cases. The difficulty in transitioning from infinitary techniques in series to algebraic methods in integrals of ‘functions’ (in modern terms) arose from the reluctance to accept their equivalence, and thus, accepting a break with existing paradigms in 17th century mathematics. The transition uses the concept of infinitesimals as a bridge, and it was the philosophical approach that made possible the break with the existing paradigm by developing a general problem-solving ‘process’ and a general ‘language' that permitted seamless communication and diffusion. The ease of implementation of the new methods and their success in solving existing and new problems and applications is one of the critical factors in their eventual adoption.

By 1676, Leibniz had long recognised that the summation of the terms of a series is inverse to their difference, and yet, there were no systematic applications to quadratures and tangents. The link between these concepts is the essence of the usefulness and novelty of calculus. What Leibniz put together was a notation and algorithm to solve a classical problem in a manner that could become a universal method and could be applied to many problems beyond specific cases and the set of classical problems. In modern integral calculus we teach that a Riemann sum (in the limit with an infinite number of terms) is the same as the integration of a function that describes the curve. This is an example of the power of calculus:

$$\lim_{n \to \infty} \sum_{i=0}^{n} (\mathrm{y_i}\ (\mathrm{x_{i+1}} - \mathrm{x_j})) = \int_{x1}^{x2} ydx$$

The notation on the left side of the equation describes an infinitary sum whose origin is the quadrature (area under the curve) of a specific curve. The notation on the right side describes the area under the curve, found by adding infinite rectangles that approximate the full space under the curve (giving origin to the infinitary series on the left). The sigma notation (left) is a tool to write more efficiently the description of the solution of the problem and is solved with a formula as a method to efficiently carry out the summation of the series, such as the formula to add consecutive numbers in an arithmetic series. The integration formula in a definite integral (right) allows for an algorithmic solution of a family of functions that makes it easier to obtain the area under the curve (the quadrature) without recurring to infinite series techniques. If we have two different descriptions of a solution of a problem that produce indistinguishable solutions, then it does not matter which one we choose since they are both true representations of reality. The system is so powerful that to solve the problem it is only necessary to apply the algorithm, without necessarily understanding the details of the problem or being an advanced mathematician (Diaz, 2023, pp. 333–334).

The gestalt shift originated by Leibniz's invention and subsequent adoption, expanded with the work, pedagogical methods, and communication from key figures like L'Hôpital, the Bernoullis in the late 17th century, and later with Euler's text and work in the 18th century. The adoption of calculus stemmed from a Kuhnian revolution in mathematics, with fierce challenges and a lengthy process of adoption in which political, sociological, historical, and pedagogical factors played a role.

Support for new paradigm

The change in paradigm did not happen without bitter controversies and discussions in a revolutionary process à la Khun. In the debate about calculus and its implementation and uses, the different approaches of Newton and Leibniz had implications for the influence and acceptance of the new tools in science and mathematics. The editor of the eight volumes of the *Mathematical Papers of Isaac Newton* (1967 to 1981), professor D. T. Whiteside at Cambridge University describes the inaccessibility of Newton's Principia hindering the diffusion of his mathematical methods:

> Quite bluntly, the logical structure of Newton's book is slipshod, its level of verbal fluency none too high, its arguments unnecessarily diffuse and repetitive, and its very content on occasion markedly irrelevant to its professed theme: the theory of bodies in motion… The *Principia [my italics]* was and is accessible in its detail only to the mathematically sophisticated. In Newton's own lifetime only a handful of his contemporaries working without distraction at the frontiers of current research-maybe only the Dutch scientist Christiaan Huygens, the German *uomo universale* Leibniz, the able Swiss mathematician Johann Bernoulli, the French priest Pierre Varignon, the Huguenot expatriate Abraham De Moivre and Newton's most able editor Roger Cotes-had, each in his own way, achieved a working knowledge of the Principia's technical content (Whiteside, 1970, pp. 116–117).

The second phase of the epistemological process of additions to the Body of Knowledge implies the social cognition associated with discussions and support for the competing frameworks. Good examples of the exchanges that took place during this phase are some of the arguments from Maclaurin regarding Newton's methods and ideas and the success in acceptance of Leibniz's calculus vs Newton's: Maclaurin defends Newton's slowness to publish as his reluctance to engage in controversy out of modesty (Maclaurin, 1747, pp. 10–15). Maclaurin contrasts Newton's explanations of gravity with Leibniz's, where the former seemed to lean towards physics and the latter towards mathematics and philosophy. Maclaurin explains Leibniz's success by virtue of his theological argument that God (the Deity) had made a most perfect world, and that this argument resonated with others (Maclaurin, 1747, p. 80). Maclaurin explains Leibniz's introduction of monads (Maclaurin, 1747, p. 81) and afterwards proceeds to define space, body, etc., in an approach to the *continuum vs quanta* issue (Maclaurin, 1747, p.

100). Following the reasoning of science vs mathematics/philosophy in the two approaches, he leads the discussion and defence of Newton's views by explaining their root in the study of motion (Maclaurin, 1747, p. 104). In a separate text explaining Newton's theory of fluxions, he describes indivisibles (Maclaurin, 1742, p. 1), followed by a description of the continuum, metaphysical arguments, and the idea of infinitesimals (Maclaurin, 1742, pp. 39–45). Maclaurin wrote the Treatise of Fluxions to answer Berkeley's attack on Newton's methods for their lack of rigour (O'Connor and Robertson, 2017). These arguments are either apologetical or seek to clarify Newton's language, and contrast with expositions in the texts from L'Hôpital (L'Hôpital, 1696), Jean Bernoulli (Bernoulli, 1742) (Bernoulli, 1914)[12] (Bernoulli, 1924), Leonhard Euler (1707-1783) (Euler, 1748) and others that concentrate on didactic descriptions of the implementation of Leibniz's methods in a variety of problems.

Another aspect of social cognition that is essential in the acceptance of the new paradigm is the discussion of the entities chosen in the new framework and their notation. Leibniz experimented for decades with different symbols and communicated on the topic with mathematicians that he knew and he strived to accommodate to their preferences, this in letters to Jakob Bernoulli, Johann Bernoulli, John Wallis, Guido Grandi, Oldenburg, Hermann, Huygens, L'Hôpital, Tschirnhaus, Zendrini and Wolff, and in an example of these communications, Johann Bernoulli and Leibniz reached a compromise, adopting Bernoulli's name *Integral* calculus and Leibniz's symbol of integration (Cajori, 1925, pp. 413–414).

There are other social aspects that play a critical role in supporting the new ideas. Charlotte Wahl examines the crucial role played by expatriate mathematicians in the early dissemination of Leibniz's differential calculus, focusing on Rudolf Christian von Bodenhausen, Johann Bernoulli, and Nicolas Fatio de Duillier. These figures served as bridges between different mathematical communities, but their work was also influenced by national sentiments and rivalries. Bodenhausen, working in Florence, promoted Leibniz's calculus but faced resistance from Italian mathematicians who favoured classical geometric methods, highlighting tensions

[12] "The first integral calculus. A selection from Johann Bernoulli's mathematical lectures on the method of integrals and other material written for the use of the Marquis de l'Hospital in the years 1691 and 1692".

between different mathematical traditions. Johann Bernoulli successfully spread knowledge of the calculus in the Netherlands despite initial scepticism and came to prefer Dutch academic life over his native Basel[13]. Fatio de Duillier, working in England, became an admirer of Newton and played a controversial role in the priority dispute between Newton and Leibniz over the invention of calculus. Wahl argues that these expatriates' experiences illustrate both the potential for cross-cultural scientific exchange and the challenges posed by national rivalries and prejudices in the early modern scientific community. Their stories highlight the tension between cosmopolitanism and nationalism in the Republic of Letters during this period. While expatriate mathematicians had unique opportunities to bridge different mathematical traditions, they could also exacerbate rivalries or become marginalised figures. Wahl suggests that the limited personal contacts between mathematical communities around Leibniz and Newton, partly due to national divisions, contributed to the intensity of their priority dispute. Overall, the paper demonstrates how the dissemination of new mathematical methods was shaped by complex personal, institutional, and national factors beyond purely scientific considerations (Wahl, 2014).

Expansion and Pedagogy

The process of addition to knowledge stems from active and confrontational debates on the articulation of the new ideas. Understanding emerges from a dialectical approach within the scientific community before new knowledge gains the status of a correct conceptualisation. Concepts, methods, and their interpretation then spread through textbooks and teaching and by their use in new applications. An example of this phase is seen in the discussion of usefulness of the new methods as in the exchanges between Leibniz and Huygens in a letter from 17 September 1693 where Huygens talks about his work with Monsieur le Marquis de l'Hospital regarding quadrature results using Leibniz’s calculus and asks for clarification and confirmation of some results. He also respectfully expresses that he fails to understand the reason for its use since it seems that it is purely a procedural operation: “For I did not find it satisfactory that differential calculus should yield anything other than what is asked of it” (Huygens, 1693). He

[13] But when his Brother Jakob died in 1705, Johann returned to Basel to succeed him at the university and stayed there for the rest of his life.

then proceeds to ask Leibniz for the purpose of the second derivative [*dd*x] and enquiries if he has found any major problems that require their use so he (Huygens) can feel motivated to study them. In the original French text, Huygens' tone is formal and somewhat cautious, and it expresses curiosity and willingness to learn more about the subject.:

> I am now somewhat acquainted with it, although I still understand nothing about the second differentials (*dd*x), and I would very much like to know if you have encountered any important problems where they must be employed, so that it may give me the desire to study them (Huygens, 1693, p. 509).

Leibniz addresses the teleology of his framework and the use of double derivatives on diverse disciplines in an answer to Huygens dated 11 October 1693. Leibniz explains that he has often used the double derivatives in understanding the tendencies originated by gravity, questions regarding centrifugal speeds, and the movements of the stars. He also points out that Bernoulli has used them in understanding the shapes of sails:

> As for second differentials (*dd*x), I have often had need of them; they are to first differentials (*d*x) as the impetus of gravity or centrifugal urgings are to velocity. Mr. Bernoulli notes in the Acts of Leipzig of last year that he has employed them for the curves of sails. And I had already employed them for the motion of stars in the same journal (Leibniz, 1693, p. 539)

The successful adoption of calculus arose from the communications with others in the development of applications. Charlotte Wahl has suggested that there are a number of examples recently transcribed and published in the Leibniz Academy Edition *Third series: Mathematical, scientific and technical correspondence Volume 9* [A III 9] (Leibniz, 2022b): Correspondence with Jacob Bernoulli: differential equation $dy=(x^2+y^2)dx$, integration of functions involving polynomials and roots; correspondence with Johann Bernoulli: six problems, which are then further discussed, in particular partial fraction decomposition for the integration of rational functions; another problem concerning the rectification of algebraic curves, which is solved in a later response by Leibniz; correspondence with Varignon: Singularities of algebraic curves and tangents at singular points and determining central forces from their orbits. There are additional sociological factors influencing the incorporation of calculus to the mainstream of Science and the Body of Knowledge, as I will briefly present next.

Textbooks and New applications

Following in the tradition of presenting methods and examples and developing applications set by Leibniz, and adding to the development and propagation of the calculus in the eighteenth century, Leonhard Euler (1707-1783) published several papers in Volume VII of the the *Miscellanea Berolinensia* (*Berlin Miscellanea*) in 1743. The first of the papers was titled "Determinatio orbitae cometae qui mense Martio huius anni 1742 potissimum fuit observatus" [Determination of the motion of a comet which can be observed in March of this year, 1742] (Euler, 1743b) [the paper was written in 1742 and published in 1743]. The second of Euler's papers in the *Miscellanea Berolinensia* was called "Theoremata circa reductionem formularum integralium ad quadraturam circuli" [Theorems concerning the reduction of integral formulas to the quadrature of the circle] (Euler, 1743c), and another one of them being a short treatise on homogeneous linear differential equations with constant coefficients called "De integratione aequationum differentialium altiorum graduum" [On the integration of differential equations of higher degrees] (Euler, 1743a).

The first calculus textbook was written by the Marquis de L'Hôpital (L'Hôpital, 1696) and follows the pedagogical approach and structure of modern texts, with a presentation of several problems and their solutions. For example, section IX, proposition 1 presents the problem of a fraction where both numerator and denominator approach zero and where the ratio is the same as that of the differentials at the same point (L'Hôpital, 1696, p. 145). The description of the solution that follows the stated problem is known as the L'Hôpital Rule (also attributed to Bernoulli) and is an example of the tone and purpose of the text which contributed to the expansion of calculus. L'Hôpital's approach was successfully used in communications and publications of other scholars addressing different applications, thus contributing to the acceptance of Leibniz's calculus notation and methods.

Further diffusion and acceptance ensued from the exchange of ideas and preparation of textbooks with pedagogical objectives, for example between Leibniz and Rudolph von Bodenhausen, regarding the study of the catenary and an approximation to the value of e

(Raugh and Probst, 2019). Textbooks like those of L'Hôpital (published in French) in 1700 followed Carré's *Méthode pour la mesure des surfaces*, a textbook on integration. Raphson's *History of Fluxions* (1715) taught Newtonian calculus in English. Later those of Euler (Euler, 1748) (Euler, 1796), L. A. de Bougainville, and Maria Gaetana Agnesi (Agnesi, 1748) succeeded in spreading the use of calculus among different audiences (H. Bos, 1993). Agnesi's treatise was intentionally written in Italian instead of Latin with the aim to attract readers that were less welcomed in Latin Scholar circles, such as younger readers or women, and it became the first calculus book written in vernacular and the first mathematics book published by a woman, and its recognition earned her the appointment to the chair of mathematics and natural philosophy at Bologna in 1750 by Pope Benedict XIV (1675-1758) (J. Willard Marriott Library University of Utah, 2017).

Particularly valuable in the establishment of calculus and analysis in the eighteenth century was a textbook from Euler with a "pedagogical lucidity" that makes large portions of it survive to this day in college textbooks:

> … over these well-known textbooks there towers another, a work which appeared in the very middle of the great textbook age and to which virtually all later writers admitted indebtedness. This was the *Introductio in analysin infinitorum [Introduction to the Analysis of the Infinitee]* of Euler, published in two volumes in 1748. Here in effect Euler accomplished for analysis what Euclid and Al-Khowarizmi had done for synthetic geometry and elementary algebra, respectively. The function concept and infinite processes had arisen by the seventeenth century, yet it was Euler's Introductio which fashioned these into the third member of the mathematical triumvirate comprising geometry, algebra, and analysis. From the point of view of leading textbooks, then, one might refer (with, of course, some oversimplification) to geometry as ancient, algebra as medieval, and analysis as modern. (Boyer, 1951, pp. 223–224)

During the 18th century, problems in celestial mechanics, hydrodynamics, elasticity, and in general rational mechanics grew in importance and scope with interest expanding and expeditions sent to check results. Rational mechanics provided a language and the concepts for the new methods of analysis, and with these influences, it conferred prestige on the new methods of calculus (H. Bos, 1993, p. 119). In France, influential military and civil engineering educational institutions adopted textbooks on hydraulics. By the end of the century, the knowledge of pure mathematics required for admission increased along with what was taught in those schools, and the influential École Polytechnique made calculus part of the curriculum (H. Bos, 1993, pp. 121–122), thus establishing it and analysis within the mainstream of academia and the professions.

Connaissance Éclairante - Adoption to Body of Knowledge:

Becoming Knowledge

The invention of calculus necessitates examination of both methodological development and underlying motivations. Whereas Newton concentrated on descriptive approaches and specific motion-related problems, Leibniz pursued broader philosophical inquiries oriented toward developing a universal method for discovery and communication. This fundamental distinction illuminates the evolutionary trajectory of calculus from specific problem-solving techniques to a comprehensive theoretical framework applicable across scientific domains. The pedagogical progression of calculus illustrates this development: from particular methodological approaches—utilising limits for slope calculations and infinite series for area determinations—to the more generalised theoretical framework of derivatives and integrals. Derivatives elucidate rates of change, while integrals quantify cumulative effects, representing a transition from practitioner-level problem-solving to sophisticated framework application in novel scientific contexts. The eighteenth-century establishment of calculus and analysis as a new paradigm fundamentally derived from its reproducibility, implementability, and pedagogical accessibility.

Leibnizian conceptual frameworks transcend their mathematical origins, manifesting in diverse scientific contexts. The relationship between rate and accumulation, integration as unified summation, the continuum-quanta dialectic, and the significance of normative language in theoretical understanding resonate throughout scientific thought. Initial resistance to calculus emerged from both transitional conceptual difficulties and epistemological uncertainty regarding abstract quanta such as infinitesimals. This pattern of resistance recurs with subsequent conceptual innovations: quantum mechanics, information bits, evolutionary traits, computational processes, and logical formalism in number theory. These contributions to the Body of Knowledge necessitate the conceptualisation of appropriate quanta for describing change within specific theoretical frameworks. Unlike Kuhn's characterisation of discovery as either identification or interpretation, these developments constitute genuine inventions of novel ontological spaces that expanded human understanding, emerging not from isolated moments of insight but through sustained engagement with complex theoretical problems.

Summary

This chapter's analysis of Leibniz's invention of calculus illuminates fundamental patterns in how transformative knowledge emerges and becomes established. Through careful examination of this case study, we can identify several crucial dynamics in the *o-é-c process* of knowledge creation. The historical analysis demonstrates how Leibniz's breakthrough emerged from a unique confluence of intellectual and social forces in 17th century Europe. His broad philosophical quest for a universal method of discovery, combined with the period's mathematical challenges, created conditions for genuine innovation rather than mere incremental advancement. The chapter reveals three critical phases in the establishment of calculus as a new field. First, Leibniz's intellectual leap represented not merely technical innovation but a fundamental reconceptualisation of mathematical thinking, shifting from geometric to algebraic paradigms. Second, the social process of knowledge validation involved complex interactions between competing frameworks, notably with Newton's approach. Third, the institutional embedding of this knowledge through textbooks, scientific societies, and educational curricula demonstrated how theoretical innovations become established knowledge. This analysis provides empirical support for the *o-é-c model*'s key proposition: that transformative additions to knowledge require both revolutionary individual insight and systematic social processes of validation and integration. The case of calculus demonstrates how new theoretical frameworks can emerge from philosophical inquiry while requiring robust social and institutional mechanisms for their establishment and preservation.

This examination sets the stage for our subsequent analysis of Bohr's complementarity principle, suggesting patterns in how fundamentally new knowledge domains emerge and become established across different scientific contexts.

Chapter 4 - Bohr and Complementarity

This is the second case study of this thesis, and similarly to the examination of Leibniz, the invention of calculus and its evolution into the field of analysis, in this chapter I will study Niels Bohr, the invention of complementarity, and its evolution into the field of quantum mechanics. The framework for that study is the *o-é-c model* with each one of its phases and components describing the process towards the expansion of the body of knowledge with the new field.

Ouverture Ontologique - Ontological Opening

Historical Environment; the sociological atmosphere

The period from 1885 to 1965 witnessed profound changes in Europe's political, cultural, and scientific landscapes. Spanning two world wars and dramatic societal transformations, this era formed the backdrop against which Niels Bohr (1885-1962) developed his revolutionary ideas on complementarity and quantum mechanics. The *fin de siècle* was characterised by remarkable contrasts: unprecedented progress and optimism coexisted with deep anxieties about the future and modern society. As Eric Hobsbawm argues in The Age of Empire: 1875-1914, this period was “the most significant in the formation of modern thought then current” (Hobsbawm, 2009, p. 4). The era saw the rise of nationalism and imperialism alongside increasing challenges to traditional modes of thought and representation. Philosophically, there was a decisive move away from Enlightenment positivism. Thinkers like Friedrich Nietzsche (1844-1900) challenged the notion that science alone could fully explain reality, arguing that all knowledge is perspectival and influenced by individual experiences and cultural contexts. In a

> … famous passage[14] [Nietzsche] bluntly rejects the idea, dominant in philosophy at least since Plato, that knowledge essentially involves a form of objectivity that penetrates behind all subjective appearances (Anderson, 2024, p. section 6.2 Perspectivism).

William James, pioneering pragmatism and radical empiricism, similarly advocated for a more expansive view of experience beyond what could be captured by scientific methods alone. His

[14] *On the Genealogy of Morality*, Maudemarie Clark and Alan Swensen (trans.), Indianapolis: Hackett, 1998 (1887) page 12

concept of *pure experience* as the fundamental reality underlying both subjective and objective aspects of the world would later find resonance with quantum mechanical interpretations (Goodman, 2022, p. Essays in Radical Empiricism (1912)). This intellectual climate, which questioned the supremacy of rational, mechanistic explanations, later proved receptive to the counterintuitive ideas of quantum mechanics. Forman notes:

> The belief in a rational world order was shaken by the way the war ended and the peace dictated; consequently one seeks salvation in an irrational world order (Forman, 1971, p. 13).

Paradoxically, this era also saw tremendous optimism about scientific progress. Classical physics had achieved remarkable successes, leading physicists like Albert Michelson (1852-1931) to declare in 1894:

> The more important fundamental laws and facts of physical science have all been discovered, and these are so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote (Kragh, 1999a, p. 3).

However, anomalies like blackbody radiation and the photoelectric effect posed challenges that classical physics struggled to explain (Kragh, 1999a, pp. 5–9). These puzzles created a sense of unease among physicists. Bohr, born in 1885, came of age in this intellectual climate, shaped by the tension between classical certainty and emerging doubts—a recurring theme in his scientific career.

World War I and Its Aftermath (1914-1925)

The outbreak of World War I in 1914 shattered the relative stability of the previous decades. The war had a profound impact on the scientific community, disrupting international collaborations and redirecting research efforts towards military applications. Many scientists found themselves working on projects related to the war effort, from chemical weapons to submarine detection. The scale and brutality of the conflict shook the foundations of European civilisation and challenged the notion of scientific progress as an unalloyed good. This realisation would have lasting effects on the relationship between science and society, contributing to a more ambivalent public attitude towards scientific advances. The aftermath of the war brought further upheaval. The Treaty of Versailles (28 June 1919) redrew the map of Europe, creating new nation-states and sowing seeds of resentment that would contribute to future conflicts. In the scientific realm, the treaty had significant consequences.

> … the experience of general social crisis after the war affected scientists' mentality and inspired their talk about 'crisis in science'. The latter notion often implied not merely the economic difficulties of the profession, but also crises in the conceptual foundations of existing knowledge. Scientists became much more willing, in comparison with relative normal and stable times, to revise or entirely abandon fundamental principles and commitments of their respective disciplines… such culturally amplified criticisms were directed not only at basic concepts of classical physics, but even at some key assumptions of the quantum theory of the atom, which had only been around for a decade but was about to become labelled as, characteristically, the 'old' quantum theory (Kojevnikov, 2011, p. 343).

Economically, the post-war period was characterised by instability and hyperinflation, particularly in Germany:

> Inflation had gathered pace in Germany during the war, when the national debt rose almost thirtyfold, paper money in circulation over twentyfold. Prices were about five times higher in 1918 than they had been before the war, and the currency had lost about half its earlier value (Kershaw, 2015, p. 96).

This had severe consequences for scientific institutions and funding. Many scientists struggled to maintain their research programs. Bohr, working in Denmark, was somewhat insulated but acutely aware of his German colleagues' challenges. His efforts to maintain scientific connections across national boundaries would become increasingly important in this fractured landscape. Despite these difficulties, crucial developments occurred in physics. Einstein's theory of general relativity received dramatic confirmation with the 1919 solar eclipse observations, bolstering confidence in theoretical physics. Bohr's 1913 model of the atom opened new avenues of research, setting the stage for rapid progress in the early 1920s (Heilbron and Kuhn, 1969, p. 290).

Interwar Period (1925-1939)

In her description of the dialectic evolution of the Copenhagen Interpretation of Quantum Mechanics, Mara Beller (1945-2004) argues:

> This chapter further elaborates the stand, now commonplace in the historiography of science, that ideas can properly be understood only by an analysis of their local theoretical and sociopolitical emergence and use (Beller, 1999, p. 172)

The emphasis on the role of the observer and the limits of classical description in the Copenhagen interpretation can be seen as paralleling broader cultural shifts away from rigid determinism and absolute objectivity, as Forman explains:

> … the one inarguable "lesson" of quantum mechanics, and the statistics integral to it, is that individuality does not exist in the atomic world. This clear implication, so much at variance with the cultural values of Germany and the West, physicists either suppressed, or, flying unchallenged in the face of the facts, maintained the diametrically opposite proposition… The physicists allowed themselves, and were allowed by others, to make the theory out to be whatever they wanted it to be - better, whatever their cultural milieu obliged them to want it to be (Forman, 2011, pp. 213–214).

The interwar period saw the rise of totalitarian regimes in Europe, with fascism in Italy and Germany, and Stalinism in the Soviet Union. Ideological considerations began influencing research priorities and theoretical interpretations. Initially, the Weimar Republic provided fertile ground for scientific research despite economic challenges:

> Weimar scientists, especially atomic scientists, reacted to the worsening economic conditions by developing increasingly innovative strategies to obtain research support for themselves and their students (Cassidy, 2009, p. 77).

This period saw the emergence of quantum mechanics, with crucial contributions from German physicists like Werner Heisenberg (1901-1976), Max Born (1882-1970), and Pascual Jordan (1902-1980). However, the rise of Nazism in the 1930s had devastating consequences for German science. The expulsion of Jewish scientists and those deemed politically unreliable led to a brain drain that significantly weakened German physics.

> The Third Reich promulgated its first anti-Jewish on April 7 [1933] ... The new law abruptly stripped a quarter of the physicists of Germany, including eleven who had

> earned or would earn Nobel Prizes, of their positions and livelihood (Rhodes, 2012, p. 185).

The ideological attack on the so-called *Jewish physics*, which included relativity and aspects of quantum theory, further undermined the German scientific establishment (Cassidy, 2009, pp. 208–209). Culturally, existentialism and phenomenology emerged, emphasising individual experience and the limits of rational understanding. In the philosophy of science, logical positivism and the Vienna Circle[15] (1924-1936) sought to establish a rigorous, empirically grounded approach to scientific knowledge:

> Their radically anti-metaphysical stance was supported by an empiricist criterion of meaningfulness and a broadly logicist conception of mathematics. They denied that any principle or claim was synthetic *a priori* (Uebel, 2024).

These philosophical currents influenced the interpretation of quantum mechanics. The Copenhagen interpretation, developed by Bohr and his colleagues, resonated with these broader intellectual trends:

> Quantum mechanics is the fundamental theory of atomic and subatomic physics. It is the basic set of rules held by the theorist… These rules were found in 1925-26 and elaborated in the following years in a scientific milieu whose language was German and whose center lay in Germany. Of course, Niels Bohr's institute was in Copenhagen. And P.A.M Dirac [1902-1984], a young research student at Cambridge University, was among the earliest and most important contributors. Overwhelmingly, however, the creation of quantum mechanics was an enterprise of Germans and Austrians. Consequently, as a product of Germany it may be appropriately be considered in relation to German culture (Forman, 2011, p. 203) .

Despite the political tensions, this era also saw important developments in international scientific cooperation. The Solvay Conferences, which brought together leading physicists to discuss fundamental problems in their field, played a crucial role in the development of quantum theory. Bohr was a central figure at these conferences, using them as a platform to present and refine his ideas on complementarity.

[15] "The Vienna Circle was a group of early twentieth-century philosophers who sought to reconceptualise empiricism by means of their interpretation of then recent advances in the physical and formal sciences". Source: https://plato.stanford.edu/archives/sum2024/entries/vienna-circle/

World War II and the Dawn of the Atomic Age (1939-1950)

The outbreak of World War II disrupted the scientific world once again. The war effort saw unprecedented mobilisation of scientific resources, most notably in the Manhattan Project to develop the atomic bomb. This project raised profound ethical questions about the responsibility of scientists. Bohr, who escaped Nazi-occupied Denmark in 1943, advocated for international control of atomic weapons. In an open letter to the United Nations, he later wrote:

> The very fact that knowledge is in itself the basis for civilisation points directly to openness as the way to overcome the present crisis… full mutual openness, only, can effectively promote confidence and guarantee common security (Bohr, 1950, p. 13).

The post-war period saw a reorganisation of science on a grand scale. Government funding for research increased dramatically, giving rise to the era of Big Science. The onset of the Cold War further politicised science, with the nuclear arms race becoming central to international relations. This political context influenced scientific discourse, including debates about the interpretation of quantum mechanics (Kaiser, 2012, pp. 16–23).

Existing Paradigm; working within the accepted view

The development of quantum theory occurred within a specific European academic environment, exemplifying the tension between adhering to accepted theories and proposing revolutionary ideas. This tension aligns with Thomas Kuhn's model of scientific revolutions, where periods of "normal science" are interrupted by paradigm shifts. Niels Bohr navigated a complex landscape of scientific theories, philosophical perspectives, and institutional structures rooted in the successes of classical physics but increasingly challenged by new findings.

Quantum antecedents and controversies before the 20th century

The first European theory of the atomic theory of matter began in the fifth century BCE with the work of Leucippus (5th c. BCE), his student Democritus (born about 460 BCE), and its elaboration by Epicurus (341-270 BC) a century after (Guthrie, 1965a, pp. 382–383). We received the conceptualisation of atoms from Aristotle's account of Democritus ideas:

> Democritus, for his part, asserts that no element arises from another element. Nevertheless for him the common body is a principle of all things, differing from part to part in size and in shape (Aristotle, 1995b, pp. 346, 203a 34-203b 2).

For Democritus and the atomists, one of the two fundamental realities (the other one being the *void*) is that the primordial essence of everything are indivisible entities – *atoms*[16] – which are eternal, infinite in number, perfectly solid and unchanging, and that diverge from each other only in their geometrical attributes, their situation, and their organisation. Being indivisible by physical means and yet having structure was an idea shared by eminent scientists until the late nineteenth century. From the Aristotelian perspective and tradition, macroscopic objects in the world that we sense are clusters of these *atoms*, and their changes stem from additions or rearrangements of the *atoms* that compose them (Guthrie, 1965b, pp. 393–394) (Pais, 1995, pp. 43–44) (Sassi, 2020, p. 136). The atomists' views were opposed beginning with Anaxagoras (500-428 BCE) and Aristotle with the position that matter is continuous and infinitely divisible. The respect and influence commanded by Aristotle eclipsed the atomists' views until the seventeenth century when the opposing views were debated in several contexts. René Descartes argued that there cannot exist atoms that by nature were indivisible while Leibniz posited his doctrine of indivisible *monads*, and Newton and Leibniz were entangled in a debate of the nature of light with Huygens proposing a wave description and Newton arguing for a particle description. This understanding of the structure of matter lasted without much change since the original speculations of ancient Greek philosophy until he British chemist and physicist John Dalton (1766-1844) began to publish in 1808 his ideas on how *atoms of the same kind* that were simple or compound (the equivalent of what we now call molecules) and could be accounted for in a quantitative theory that was able to explain and predict a great variety of facts in what became the origin of modern chemistry (Pais, 1995, p. 44).

Arguments increased in the nineteenth century in both Chemistry and Physics. Max Planck argued in 1884 that the entropy law proposed by Boltzmann (Ludwick, 1844-1906) (where the process of entropy expansion is irreversible and described by mechanical statistics) was

[16] from άτομο singular, άτομα plural - absence of or privative to cut

incompatible with the assumption of the existence of finite *atoms* (Pais, 1995, p. 47). Part of the issue was the discussion of reducing all phenomena to mechanical processes, and the idea of cause and effect. Irreversibility of natural phenomena was very difficult to accept. Eventually, Boltzmann ideas began to take hold. Maxwell proposed a diameter for a hydrogen molecule in 1873 and although something inside the atom was emitting spectral lines of light, this did not mean that the *atom* could be taken apart, and yet, by 1999, J.J. Thomson had discovered the electrons and announced that the atom had been split, Marie Curie (1867-1934) explained in 1900 that the radioactive atoms were divisible by other means than chemical reactions, and Rutherford proposed the concept and calculation of the *half-life*[17] (Pais, 1995, pp. 48–50). The efforts to describe the *atom* were increasing. Early in the nineteenth century, Thomas Young (1773-1829) and Augustin Fresnel (1788-1827) had ended the controversy between Huygens and Newton by showing the wave description of light in favour of Huygens description. On Sunday 7 October 1900 Planck wrote down the first quantum law. His formula for spectral density fitted the entire experimental spectrum and he submitted a paper containing the new physical law on 14 December 1900 marking the birth of quantum physics. The theory was related to radiation energy and wave descriptions, and yet it implied that energy is quantised. Einstein might have been the first to realise that science had entered crisis with that publication, and he proposed in 1905 that under certain circumstances light behaved like particles that eventually were called photons. By 1925, quantum mechanics showed that both Huygens and Einstein were right (Pais, 1995, pp. 69–72).

The Intellectual Environment around the *fin de siècle*

The learned societies that saw their origin in the eighteenth century continued to grow in number and popularity, and by the *fin de siècle* you could find a local one in almost every city in Germany, France, Britain, Italy, Russia, the Netherlands, and the United States. Physicists, however, worked in a closed community, and as physical research moved away from phenomena removed from direct observation, their activities and communications of results

[17] Half-life is defined as the time required for half of any given quantity of a radioactive substance to decay. Rutherford established that decay rate is proportional to the number of radioactive atoms present, demonstrated that this process follows an exponential decay pattern, and proved decay rate is independent of external conditions (temperature, pressure, etc.).

appeared with progressively less frequency in such societies. The exception to this pattern was Great Britain, with the *Royal Institution of Great Britain* founded to pursue the application of science to public welfare and also had an important component of fundamental research in chemistry and physics (Pippard, 1995, pp. 3–5). The dissemination of scientific knowledge in this period occurred through both formal and informal channels. Scientific journals played a crucial role, with Germany leading the way through publications like *Annalen der Physik* and *Zeitschrift für Physik*. The *Physikalische Zeitschrift*, founded in 1899, pioneered the rapid communication of preliminary results through its inclusion of abstracts and research letters. While *Nature* gained influence in Britain under Sir Norman Lockyer's physics-oriented editorship, German publications remained the primary vehicles for announcing new discoveries. A significant development in scientific communication came with the emergence of international conferences, culminating in the 1911 Solvay Conference where Ernest Solvay brought together 24 leading physicists to address the emerging challenges in quantum theory. Prior to the establishment of regular conferences and summer schools, scientific consensus emerged through a combination of journal publications, learned society meetings, and extensive correspondence between colleagues. The German university system proved particularly influential in shaping physics education and research. Its flexible structure, which allowed students to progress from formal instruction to guided research with minimal bureaucratic obstacles, attracted international graduate students to work under eminent professors. This approach, emphasising the creation of new knowledge rather than the preservation of established wisdom, positioned German institutions at the forefront of physics education and research (Pippard, 1995, pp. 6–16). Academic status and recognition played important roles in shaping careers and research directions. The importance of priority in discoveries was heavily emphasised, as reflected in the numerous priority disputes in the history of physics. Awards and appointments, particularly the Nobel Prize, carried significant prestige and could influence the acceptance of new ideas. The European university system, particularly in Germany, played a central role in the advancement of physics. Research institutes, such as the Kaiser Wilhelm Institute in Germany, provided additional venues for scientific work. The importance of mentorship and academic lineages was significant, with many of the key figures in quantum physics having studied under established physicists. There is an aspect that will play an important role in the approach to the wicked problem of the crisis in science regarding the experimental data on atomic physics and its interpretation:

> The culture of Germanic academe upheld the strong ideal of scientific genius required to partially double as a philosopher. Culturally, a truly great scientist was expected not only to make discoveries in a specific field of research, but to go into it in such depth as to contribute to a general philosophical outlook, and to such conclusions that would be meaningful to all members of educated culture, transcending narrow professionalism and disciplinary boundaries… Besides its strong interest in philosophising, the academic culture that produced quantum mechanics was also extremely sensitive to hierarchy, with both of these concerns closely linked (Kojevnikov, 2011, pp. 344–345).

A related factor in situating the addressing of the science crisis at that time is that although junior researchers owned the results of their experiments, the senior professors demanded control of the interpretation of those results. This is one of the aspects that would result in the influence of the leaders of the 3 main centres of development of quantum theory: Arnold Sommerfeld (1868-1951) at Munich, Max Born at Göttingen, and Niels Bohr at Copenhagen. This would affect the later emphasis on claiming ownership of the theory as opposed to primacy of its development, with Bohr securing its role as the theory's leader with the final word on the developing interpretation, while Erwin Schrödinger, who was a senior professor (having succeeded Max Planck at the Friedrich Wilhelm University in Berlin in 1927) lost control of the field when he could not establish the acceptance of his philosophical interpretation and was relegated to being a contributor of a part of the mathematical formalism of the theory (Kojevnikov, 2011, pp. 345–346).

Classical Physics at the Turn of the 20th Century

Before 1890, professors working in the United States and many other countries were not expected to produce and maintain independent scientific research (Kragh, 1999b, p. 19). In 1900 classical physics was defined as the combination of mechanics, optics, acoustics, thermodynamics, and electromagnetism, with applied physics (concerned with technology) making the rest of the majority of topics of interest (Kragh, 1999b, p. 26).

The foundation of the existing paradigm was classical physics, a body of knowledge that had achieved remarkable success in explaining a wide range of natural phenomena. At its core were Newtonian mechanics and Maxwell's electromagnetism (James Clerk Maxwell - 1831-1879), supplemented by the laws of thermodynamics and statistical mechanics. Newtonian mechanics, with its concepts of absolute space and time, had provided a powerful framework for understanding motion and forces. Its crowning achievement was perhaps the explanation of planetary motions and the prediction of new planets based on gravitational perturbations:

> Newton's success in predicting quantitative astronomical observations was probably the single most important reason for the theory's triumph over its more reasonable but uniformly qualitative competitors (Kuhn, 2012, p. 153).

Maxwell's electromagnetism, developed in the latter half of the 19th century, unified the previously separate phenomena of electricity and magnetism. It also predicted the existence of electromagnetic waves, which were subsequently discovered by Heinrich Hertz. The success of Maxwell's theory in explaining light as an electromagnetic phenomenon led to a widespread belief in the existence of a luminiferous ether as the medium for electromagnetic wave propagation (Kuhn, 2012, p. 74). Thermodynamics and statistical mechanics, developed by scientists such as Rudolf Clausius (1822-1888), Ludwig Boltzmann (1844-1906), and Josiah Willard Gibbs (1839-1903), provided a framework for understanding heat, energy, and the behaviour of large collections of particles. The laws of thermodynamics, particularly the second law with its concept of entropy, had profound implications for understanding the direction of physical processes and the nature of time.

Emerging Challenges to Classical Physics

Despite its successes, classical physics began to encounter significant challenges in the late 19th and early 20th centuries. These challenges came from various quarters and would ultimately lead to the quantum revolution (Brown, Pais and Pippard, 1995, p. ix). Even though Planck had discovered the *quantum of action*[18] in 1900 and Einstein had formulated *relativity* theory in 1905, by 1910 there were still very few papers dealing with relativity, quantum theory or atomic structure and they were usually grouped under other categories. Nonetheless, around the turn of the 20th century there was a growing sense that physics, the way to understand it and teach it could no longer maintain the trajectory it had ten years earlier. One of the most pressing problems was the blackbody radiation problem. Classical physics, in the form of the Rayleigh-Jeans law, predicted that a blackbody would emit infinite energy at high frequencies - the so-called "ultraviolet catastrophe." This clear contradiction with experimental observations demanded a resolution. It was Max Planck (1858-1947) who, in 1900, introduced the radical idea of energy quantisation to resolve this issue, meaning it could only be emitted or absorbed in discrete amounts, or *quanta* (Pippard, 1995, pp. 23–24). The idea of discontinuity in physics did not come from Plank but from Ludwick Boltzmann's statistical treatment of irreversibility, which was fundamental in the road followed by Planck in his research (Kuhn, 1987, p. ix). It was during a lecture on 14 December 1900 in front of the German Physical Society when he presented the theoretical basis for his law using Boltzmann's relation between entropy and probability, and in doing so, he showed a relation of proportionality to the probability appropriate to equilibrium radiation. At that time and until after the publication of his *Lectures on the Theory of Thermal Radiation* in 1906, Planck did not think that he was introducing a radical change in physics by introducing an understanding of energy in quanta: "In the Lectures, as in the papers written six years before, Planck's conception of his theory remains *classical* [my italics]" (Kuhn, 1987, p. 120)

Early Quantum Theory

The response to these challenges led to the development of early quantum theory, a transitional phase between classical physics and modern quantum mechanics. At the heart of this development was Niels Bohr's atomic model, introduced in 1913 (Heilbron and Kuhn, 1969) (Rechenberg, 1995, p. 157). The paper that would make him a world-renowned scientist was

[18] The quantum of action discovered by Max Planck is known as Planck's constant – denoted by the symbol $\hbar \approx 6.62607015\times10^{-34}$ joule-seconds (J·s) – a fundamental constant that quantifies the smallest possible unit of energy transfer, establishing the discrete, or *quantised* nature of energy transfer at atomic and subatomic levels.

enclosed in a letter sent to Rutherford on 6 March 1913 announcing the “first chapter in the constitution of atoms” (Pais, 1995, p. 82). Bohr's model, which postulated quantised energy levels for electrons in atoms, was a radical departure from classical physics. It successfully explained the discrete spectra of hydrogen and provided a theoretical basis for Rydberg's empirical formula[19]. Bohr's model was further refined by Arnold Sommerfeld, who introduced elliptical orbits and explained the fine structure of spectral lines. These developments formed the core of the "old quantum theory," which, despite its successes, was recognised as an incomplete and somewhat *ad hoc* approach. In 1916, Einstein found a better way to understand Planck’s black-body radiation law and link it to the quantum jumps conceptualised by Bohr, giving his (Einstein’s) work the lasting property of introducing probabilities in quantum mechanics, which troubled Einstein immensely (Pais, 1995, pp. 93–94). This would play a role in the famous Bohr-Einstein debates to come later regarding the character of quantum mechanics.

By the early 1920’s quantum theory required the use of increasingly sophisticated methods, and systems like the hydrogen atom with an electron whose motion was treated relativistically were subjected to external electric and magnetic fields, requiring the formulation of classical mechanics to explain their conditional periodicity.

> Physicists concerned with applying quantum theory to atomic structure were forced by 1920 to learn the classically mechanical methods that astronomers had invented and cultivated to study the conditionally periodic motions of planets. Pauli and Heisenberg learned celestial mechanics under Sommerfeld, Born, and Bohr and then applied it to atomic and molecular spectra (Serwer, 1977, p. 200).

The correspondence principle, formulated by Niels Bohr in 1918, represents a fundamental bridge between quantum mechanics and classical physics, serving as a crucial theoretical foundation in the development of quantum theory. This principle establishes that quantum mechanical predictions must converge with classical physics at macroscopic scales and high quantum numbers, where quantum effects become negligible. This convergence is particularly evident in the context of atomic spectra, where radiation frequencies during quantum

[19] Rydberg's empirical formula is a mathematical expression that relates the energy of an atom's electron to its principal quantum number (*n*). The formula is: $1/n^2$ where n is the principal quantum number, which is a measure of the energy level of an electron in an atom. The formula was developed by Swedish physicist Johannes Rydberg (1854-1919) in the late 19th century.

transitions correspond to classical electromagnetic predictions at high quantum numbers. The principle's significance manifested primarily as a heuristic guide during quantum mechanics' formative period, ensuring theoretical developments remained consistent with established classical laws while extending into the quantum domain. This methodological approach proved instrumental in understanding atomic spectra and electron behaviour, particularly in explaining spectral line formation and electron transitions between energy levels. Furthermore, it influenced the development of quantum selection rules and transition probabilities, providing a theoretical framework for understanding how quantum states evolve and manifest physically.

This reformulation of the correspondence principle emphasises its dual role as both a theoretical bridge between classical and quantum physics and a practical tool for understanding atomic phenomena. Its enduring significance lies in establishing quantum mechanics as an extension of classical physics rather than a separate framework, thereby maintaining theoretical continuity while accommodating new quantum phenomena. Regarding the "picture of atomic constitution," Bohr explained in 1921:

> … the so-called "principle of correspondence," by the establishment of which it has been possible – notwithstanding the fundamental difference between the ordinary theory of electromagnetic radiation and the ideas of the quantum theory – to complete certain deductions based on the quantum theory by other deductions based on the classical theory of radiation (Bohr, 1921, p. 104).

In the following quoted text, Bohr explains how the *correspondence principle* arises from the ideated model of the atom in comparison with classical cosmology models. It also suggests its need as it is not possible to directly observe the atomic structure:

> … it has been possible to construct mechanical pictures of the stationary states which rest on the concept of the nuclear atom and have been essential in interpreting the specific properties of the elements. In the simplest case of an atom with only one electron, such as the neutral hydrogen atom, the orbit of the electron would be in classical mechanics a closed ellipse, obeying Kepler's laws, according to which the major axis and frequency of revolution are connected in a simple way with the work necessary for a complete separation of the atomic particles…

> The *correspondence principle* [my italics] expresses the tendency to utilise in the systematic development of the quantum theory every feature of the classical theories in a rational transcription appropriate to the fundamental contrast between the postulates and the classical theories (Bohr, 1925, pp. 35–37).

Niels Bohr's *correspondence principle* was a pivotal concept that ensured quantum mechanics would replicate classical physics in the appropriate limits, providing a coherent and unified understanding of physical laws across different scales and becoming instrumental in subsequent developments in the field.

Social Cognition in Quantum theory

Expanding on Helen Longino's work, earlier in this thesis I argued the importance of social cognition in the development of a new field. This is particularly true in the development of quantum mechanics, which stemmed from intensely social interactions, discussions, and pressures of the academic community involved and invested in a new understanding of science. Helen Longino emphasises the importance of such scientific communities in shaping knowledge:

> … I develop an understanding of scientific inquiry as a set of necessarily social rather than individual practices. The result is a picture of scientific inquiry as a group endeavour in which models and theories are adopted/legitimated through critical processes involving the dynamic interplay of observational and experimental data and background assumptions. Since contextually located background assumptions play a role in confirmation as well as in discovery, scientific inquiry is, thus, at least in principle, permeable by values and interests superficially external to it (Longino, 1990, p. 13).

A particularly good example of this social cognition is the interaction between Heisenberg and Pauli during the 1924 - 1926 period and their influence on each other in the early development of quantum physics. A detailed recounting and convincing analysis of their relationship and work with each other is presented by Daniel Serwer in his paper "Unmechanischer Zwang [non-mechanical force]: Pauli, Heisenberg, and the Rejection of the Mechanical Atom. 1923-1925". They each had a personal and distinctive approach to physics and their professional and personal interactions are due in no small measure to their common intellectual formation. They were both doctoral students under Sommerfeld in Munich, where they met and worked together in 1920. Pauli worked under Max Born in Göttingen in the winter of 1921-1922. Heisenberg worked with Born during 1923-1924 while Pauli was at Copenhagen working as Bohr's assistant, and they both attended Bohr's lectures at Götingen in 1922. Heisenberg later visited Pauli at Copenhagen in 1924 and later spent fall and winter of 1924 as Bohr's assistant. Much of Heisenberg's and Pauli's preparation came from their exposure to the different goals and styles of Bohr and Sommerfeld:

> From before 1918 through at least 1922, Sommerfeld and Bohr were the poles around which the application of the quantum theory to atomic structure revolved. Their styles, aims, and methods were different. Sommerfeld's aim was primarily to solve problems. Bohr's was to create the postulates and procedures of a consistent theory. Sommerfeld's methods were the quantum conditions, applied mathematics, and empirical regularities. Bohr's all-purpose method in the early 1920's was the Correspondence Principle, which required agreement between the results of the quantum theory and classical mechanics in the limit of high quantum numbers… He [Bohr] used models and in general preferred their physical character to a more abstract and mathematical approach, but at the same time he was very careful to mark out their limits of applicability and to avoid unjustifiable implications from what he said about them. Hedged with reservations, Bohr's writing tended to understate both success and failure… Sommerfeld was the engineer of quantum; Bohr was its philosopher (Serwer, 1977, pp. 192–193).

Bohr's influence on both Pauli and Heisenberg was probably connected to two areas; they were both mostly concerned with the adequacy of quantum theoretical postulates, which was connected to Bohr's work, and in addition to coveted personal and legitimate manner, the *correspondence principle* had two sides. On the one hand, there was the radical distinction between the quantum theory and the classical theory. On the other hand, there was the promise of success if the right professional recognition, both of them were dependent on the financial support that Copenhagen and Bohr could provide during a critical period of financial constraints in Germany and restricted availability of academic positions (Serwer, 1977, p. 193). Working under Bohr, Pauli and Heisenberg were motivated by craving his attention and respect, and yet, they had two very different approaches to the theoretical problems they were facing. They had access to the same ideas, people and environment, and yet, being exposed to the same stimuli, they reacted differently. "Pauli no longer believed in any atomic models. The task of physics now [in 1924] was not to stamp out individual problems but to build them into a new theory” (Cassidy, 2009, p. 128) Pauli had been influenced by the logical positivism of Ernst Mach[20] (1838-1916) and in his mind he was dismissing a convoluted theory using unobservable entities and capricious changes to classical theory, therefore he advocated for legitimacy and consistency, with agreement with experimental results being secondary in priority. Heisenberg had followed the demand of a new quantum mechanics advocated by Max Born and therefore he had been driven primarily by adjusting ideas to obtain agreement with experimental outcomes. The result of their intense interactions and clashing styles and ideas both in person and via letters resulted in Pauli sending a paper and letter including the key concept of the *exclusion principle*[21] to Bohr in late 1924 (and published in February the following year) (Serwer, 1977, pp. 254–255) while Heisenberg’s revolutionary paper “Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen[22] [on Quantum-theoretical Reinterpretation of Kinematic and Mechanical Relationships]” was published on September 1925 and it provided the long awaited breakthrough to the new atomic physics working exclusively with experimental data instead of unobserved atoms:

> That advance precipitated the culmination of the quantum revolution of the first decades of the twentieth century, a revolution that reached its conclusion just two years later in Heisenberg’s uncertainty principle and the Copenhagen interpretation of Bohr, Heisenberg, and Born (Cassidy, 2009, p. 134).

A second example of social cognition is the combined effect of the work of four scientists at the end of the revolutionary time marking the transition between the old quantum theory that started in 1900 and the new quantum mechanics: Heisenberg, Schrödinger, Born, and Bohr. This end phase begins in 1925 with Heisenberg's paper in quantum mechanics, continues with Schrödinger’s wave mechanics and Born’s comments on probability and causality in 1926, and culminates with Heisenberg’s derivation of the uncertainty relations and Bohr’s introduction of *complementarity* at Como in 1927. Their work was revolutionary and their papers, along with Planck’s on the discovery of the quantum theory in 1900, Einstein’s introduction of the

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[20] Mach argued that all knowledge should be derived from direct sensory experiences. He believed that sensations are the fundamental components of knowledge, and scientific concepts should be linked to observable phenomena. This will play a role in Hesenberg’s developing a framework using only observable characteristics.
[21] It states that no two identical fermions—particles with half-integer spin like electrons, protons, and neutrons—can occupy the same quantum state simultaneously within a quantum system. This means that in an atom, each electron must have a unique set of quantum numbers, effectively preventing electrons from all collapsing into the lowest energy level.
[22] Heisenberg, W. (1925) *Zeitschr. F. Physic*, 30 , p 879-93

light-quanta in 1905, and Bohr's hydrogen atom model in 1913 all had in common that "they contained at least one theoretical step which (whether the respective authors knew it then or not) could not be justified at the time of their writing" (Pais, 2002, p. 251).

We can follow the tight weaving of ideas in a chronology presented by Abraham Pais: Louis De Broglie (1892-1987) expanded the light quanta from Einstein's 1905 paper to associate wave-like behaviour of matter in his PhD thesis of 1924, and Einstein linked waves with gas particles independently of De Broglie early in 1925. Pauli posited the *exclusion principle* followed closely by Heisenberg's seminal paper in which he concluded that the old quantum theory only partially agreed with experiment and this by accident, and therefore he developed a quantum mechanics in which only relations between observable quantities were taken into consideration using an obscure mathematical instrument called *matrix.* The unfamiliar mathematics of Heisenberg and inspiration from DeBroglie's work led Schrödinger to develop a solution from differential equations to derive wave mechanics which reproduced experimental data, and it became proven to be equivalent to matrix theory by Pauli and by Paul Dirac in Cambridge. Wave mechanics was easier to operate with by most physicists not proficient in complex mathematics, but Heisenberg's use of observable quantities seemed closer to the sought after physics of the atom. The clash between the high mathematical technology tools just developed went beyond its facility to use to include their assumed picture of nature. Schrödinger posited that waves are the only reality, while particles are only useful inventions and argued for a monistic view by describing waves holding together without expanding over space and time. A couple of months later in June 1926, Born declared that the wave function found by Schrödinger must be interpreted as probability, an idea that was already in the mind of Dirac and Wigner. Born's paper pointed laser-like to the problem of determinism and causality, demanding a revision of thermodynamic-statistical principles. Schrödinger resisted the interpretation and its implications, but he was in the minority, along with Einstein. Born might not have realised how profound his contribution was at the time that he was helping bring the quantum revolution to an end. In March 1927 Heisenberg published the *uncertainty relations*[23]

[23] They state that certain pairs of physical properties—like position and momentum—cannot both be precisely known simultaneously. The more accurately one property is measured, the less accurately the other can be determined. Mathematically this is expressed as, $\Delta p \Delta x \geq h/2\pi$ and

(Pais, 2002, pp. 252–262). The following passage from Pais clearly explains the implications of Heisenberg's paper:

> I have often felt that the expression 'uncertain relation' is unfortunate since it has all too often invoked imagery in popular writings utterly different from what Heisenberg very clearly had in mind, to wit, that the issue is not: what don't I know? But rather: what can't I know? In common language, 'I am uncertain' does not exclude 'I could be certain' . It might therefore have been better had the term 'unknowability relations' be used (Pais, 2002, p. 262).

De Broglie's hypothesis that particles could exhibit wave-like properties was soon confirmed experimentally by Clinton Davisson (1881-1958) and Lester Germer (1896-1971) in 1927. The final key piece will be Bohr's introduction of *complementarity* at Como and later at the Solvay Conference of 1927. Bohr provides a second example of social cognition with this excerpt from his paper "The Genesis of Quantum Mechanics - 1962", translated and published posthumously:

> In rendering some of my recollections from the old days, it has above all been on my mind to stress how the close collaboration among a whole generation of physicists from many countries succeeded step by step in creating order in a vast new domain of knowledge. In this period of development of physical science, in which it was a wonderful adventure to participate, Werner Heisenberg occupied an outstanding position (Bohr, 1963a, p. 78).

A third example of social cognition is the role of scientific conferences in the early development of quantum theory. The Solvay Conferences exemplified community-based knowledge production, providing a forum for intense debate and collaboration that was crucial to the development of quantum mechanics. As we will see later in this chapter, the famous debates between Bohr and Einstein at these conferences not only shaped the interpretation of quantum theory but also highlighted the philosophical implications of the new physics. In 1962, Bohr prepared an article reflecting on the importance of the Solvay meetings and the development of quantum physics. The following excerpt from that article highlights the importance of that example of social cognition, and Bohr's own recollection of the conferences' role for him:

$\Delta E \Delta t \geq h/2\pi$ where Δx is the uncertainty in position, Δp is the uncertainty in momentum, ΔE is the uncertainty in energy, and Δx is the uncertainty in time.

> The series of conferences originally convened, just fifty years ago, at the far-sighted initiative of Ernest Solvay and continued under the auspices of the International Institute of Physics founded by him, have been unique occasions for physicist to discuss the fundamental problems which were at the centre of interest at the different periods, and have thereby in many ways stimulated modern development of physical science… (Bohr, 1962, p. 79).

This recollection from Bohr serves as an introduction to the next section regarding Niels Bohr's *wicked problem* triggering the development of *complementarity*.

Wicked Problems

As I described in chapter 1, *wicked problems* could not accommodate typical assumptions of widespread consensus on goals, causal theory sufficiently developed to permit prediction, and effective instrumental knowledge are no longer applicable and therefore it is not possible to solve these problems with traditional approaches. Niels Bohr's lecture at Como was not the result of an attempt to address a well-defined problem but the answer to an awareness of the inherent difficulty of understanding a quandary without the background of interpretation possible through the lens of the definition of a *wicked problem.* The Classical Physics model described Nature as composed of a network of elementary particles subjected to mindless forces in an objective and flat Newtonian model of space and time. Einstein's General Relativity Theory rejected the Newtonian model of cosmic Space-Time and instead introduced the idea that local energy densities create local curvatures which replace the concept of Newtonian gravity. Werner Heisenberg's Quantum Mechanics showed that elementary particles matter, and that energy could only have definitive discrete values; that is, they were "quantised." Erwin Schrödinger showed that matter, energy, and momentum were also governed by wave-motions that interfered with each other in Space-Time that yielded in measurement only quantised matter, energy, and momentum, and in numbers only as directed by probability distribution (Heelan, 2016, p. 1).

> … European founders of quantum physics [Bohr, Sommerfeld, Heisenberg, Pauli, Dirac, Schrödinger, Einstein, Wigner and others] realized that the new physics seemed to overturn the traditional foundations of physics as natural philosophy founded on the intuition of Space and Time. Consequently, more than physics was challenged; also

> challenged were the ontologies of both classical physics and common sense. The problematic aspect seemed to focus on the differentiation of meaning attached to terms, such as "subject" and "object," "observer" and "observed," as these terms functioned in a variety of discursive frameworks used by the pioneers of QM [quantum mechanics] (Heelan, 2016, p. 2).

The response to these challenges led to the development of quantum theory, first in the form of the old quantum theory and then in the more comprehensive framework of quantum mechanics. This development was not a smooth, linear process, but rather a period of intense debate, competing theories, and philosophical soul-searching. It was in this context that Bohr developed his ideas on *complementarity*, which sought to provide a coherent philosophical framework for understanding the seemingly paradoxical nature of quantum phenomena. Bohr's work, while rooted in the specific problems of quantum physics, had broader implications for our understanding of the nature of scientific knowledge and the relationship between observer and observed.

The following excerpt from Bohr's 1962 article shows his reflection on the importance of the Solvay meetings and the development of quantum physics highlights the importance of that example of social cognition, and Bohr's own recollection of the conferences' role for him:

> … I was asked at the conference to give a report on the epistemological problems confronting us in quantum physics and took the opportunity to enter upon the question of an appropriate terminology and to stress the viewpoint of *complementarity*[24] [my italics]. The main argument was that unambiguous communication of physical evidence demands that the experimental arrangement as well as the recording of the observations be expressed in common language, suitably refined by the vocabulary of classical physics. In all actual experimentation this demand is fulfilled by using as measuring instruments bodies like diaphragms, lenses and photographic plates (Bohr, 1962, p. 91).

[24] Regarding the use of the word *complementarity:* During my work visit at the Niels Bohr Archive (NBA), Københavns Universitet (University of Copenhagen) in the summer of 2022, early in July, I contacted Dr. Jan Faye, Associate Professor Emeritus also at KU for some advice regarding Bohr's "principle of complementarity." After graciously offering to help via email, on 7 July 2022 he pointed out that "Although it is very common to talk about Bohr's *principle of complementarity*, I want to emphasise that Bohr never called complementarity 'a principle' himself." Since I was deeply surprised and puzzled by his comment, I asked Dr. Joas, the Director at NBA, about it and he too became surprised. After a search in his database, he confirmed that there was only one instance of use of the word *principle* by Bohr in reference to *complementarity*, in one communication sent in 1959 that was likely prepared by one of his assistants. Dr. Faye's statement is also published elsewhere (Folse, 1985, p. 18) (Faye, 2022, p. 3).

I also propose that the notoriously difficult definition of *complementarity* in its original enunciation, as well as afterwards needs to be analysed within the context of Bohr’s attempt to address an ill defined situation–*wicked problem*–in which different scientists viewed and described in conflicting terms, and where the objective and teleology of scientific efforts regarding the quantum world was not agreed upon. To do this, I will draw concepts from a couple of sources regarding the origin of *complementarity* (Holton, 1988, pp. 151–154) (Beller, 1992, pp. 147–151).

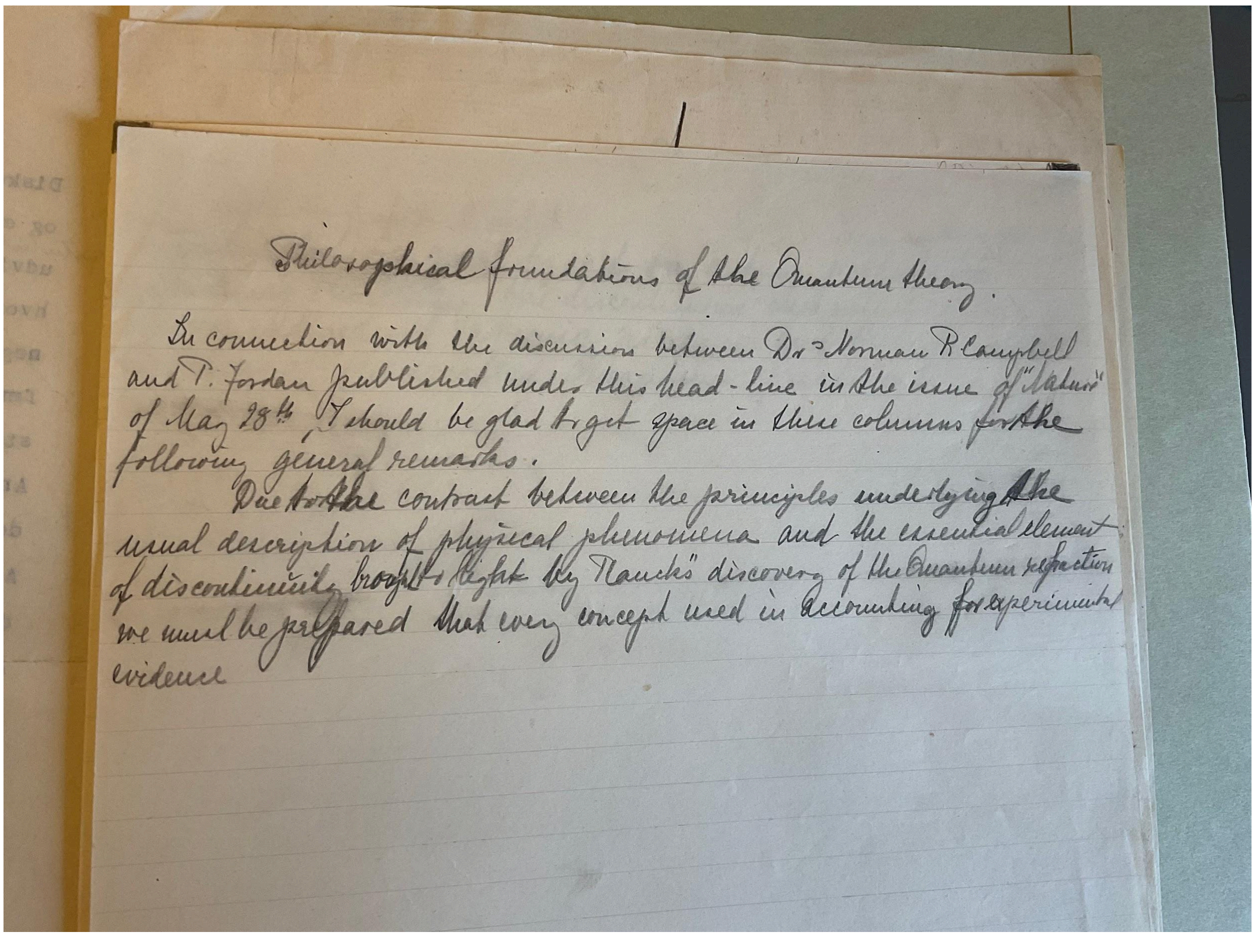
When Niels Bohr delivered his seminal lecture at Como in 1927, he confronted one of the most profound challenges in the philosophy of science—a *wicked problem* inherent in the quantum revolution. This problem was characterised by the resistance to consensus on the description and implications of quantum phenomena, and even the very definitions of observable results were under contention. Bohr's development of the concept of *complementarity* was not merely a theoretical advancement; it was a strategic response to this intractable dilemma that defied conventional scientific problem-solving approaches. The *wicked problem* Bohr faced stemmed from the fundamental dichotomies between classical and quantum physics. Classical physics operated under assumptions of continuity, determinism, and the ability to observe systems with negligible interference. In contrast, quantum physics revealed a world where discontinuity, probabilistic behaviour, and significant observer effect were intrinsic features. By adopting *complementarity*, Bohr addressed the *wicked problem*'s core characteristics: *complementarity* accepted that quantum phenomena could not be fully described within the confines of classical concepts; the idea of *complementarity* did not seek a final solution but provided a framework for ongoing exploration; *complementarity* reframed solutions as context-dependent rather than universally applicable; and it acknowledged that experimental results in quantum mechanics could not definitively confirm one model over another in all contexts. Furthermore, Bohr's insistence on the heuristic and theoretical adequacy of wave mechanics over an exclusively particulate ontology demonstrated his commitment to a more nuanced understanding. He argued that while waves might not be a literal depiction of reality, their superpositional nature offered a more fruitful framework for explaining interactions, especially in cases involving bound electrons and collisions—areas neglected by others who focused primarily on free particles. Niels Bohr's development of *complementarity* was a profound response to a *wicked problem* at the heart of the quantum revolution. By recognising the necessity of multiple, complementary descriptions and the limitations of classical concepts when applied to quantum phenomena, Bohr provided a way forward that embraced complexity rather than oversimplifying it. His approach allowed physics to move beyond the impasse created by conflicting theories, acknowledging that the pursuit of understanding in the quantum realm requires a willingness to accept and navigate inherent contradictions. This paradigm shift was instrumental in advancing quantum mechanics and has had lasting implications for the philosophy of science.

Intellectual Leap

I argue that while first posted by Bohr for his lecture at Como in 1927, *complementarity* is the product stemming from the social cognition process that characterised both the development of quantum mechanics and Bohr’s stylistic method of science that relied heavily on dialog and discussion. Earlier, in December 1925, Bohr and Einstein met in Leiden for the celebration of the golden anniversary of Lorentz’ doctorate. Max Jammer (1915-2010) argues that while not much is known about their dialog on that occasion, it seems indubitable that Bohr, who by then had accepted Einstein’s theory of light quanta, focused on the paradoxes stemming from the application of classical physics to quantum mechanics. As a follow up from their conversation in Leiden, Bohr sent a letter on April 13 1927 to Einstein along with the reprint of Heisenberg’s paper on the indeterminacy relations pointing out that our concepts’ limitations overlap with our observational capabilities, which is an indication that Bohr foresaw his complementarity interpretation in April 1927 (Jammer, 1974, p. 125), and in his translation of the letter to Einstein, Jammer quotes Bohr to show how the acceptance of Einstein’s conception of the *photon*[25] within the framework of the wave-particle duality was fundamental for Bohr’s ideation of *complementarity* (Jammer, 1974, pp. 125–126). There is no extant copy of the text given in his lecture in Como on “The Quantum Postulate and the Recent Development of Atomic

[25] The term was introduced by G. N. Lewis (1875-1946) in 1926

Theory." Prior to the lecture, on May 28 1927, the British physicist and philosopher of science Norman Robert Campbell (1880-1949) published a note titled "Philosophical Foundations of Quantum Theory"[26]. Inspired by this note, Bohr began to work on an answer with the same title. He abandoned the project after several drafts and the substance became the seed from which the Como lecture stemmed, helped by his assistant Oskar Benjamin Klein (1894-1977) (Bohr, 1985, p. 26). The picture reproduced here is of one of the unpublished manuscripts held at the Niels Bohr Archive[27] with Bohr's first attempts to draft the answer to Campbell's note:



Philosophical foundations of the Quantum theory.

In connection with the discussion between Dr. Norman R Campbell and P. Jordan published under this head-line in the issue of "Nature" of May 28th, I should be glad to get space in these columns for the following general remarks.

Due to the contrast between the principles underlying the usual description of physical phenomena and the essential element of discontinuity brought to light by Planck's discovery of the Quantum of action we must be prepared that every concept used in accounting for experimental evidence

Not much is known about the origin of the ideas in Bohr's mind in his work between July and September of 1927, however, Jammer argues that "one thing seems to be certain: Bohr's conception of *complementarity* [my italics] originated from his final acceptance of the

[26] *Nature* 119 (1927) 779

[27] Picture taken with the generous permission of the NBA during my working visit in July 2022

wave-particle duality" (Jammer, 1966, p. 345) which he had been opposing in relation to Einstein's light quanta.

Complementarity is mentioned for the first time in a manuscript in Bohr's handwriting[28]:

> … the theory exhibited a duality when one considered on the one hand the superposition principle and on the other hand the conservation of energy and momentum – *Complementarity* aspects of experience that cannot be united into a space-time picture based on the classical theories… (Bohr, 1985, p. 61).

Kalckar[29] believes that the the text of one of the manuscripts on hold at the Niels Bohr Archive (NBA) and dated September 13 1927 is probably the closest to the text Bohr used during the lecture even though it differs substantially from the version published in Nature (Bohr, 1928) the following year (Kalckar, 1985, pp. 28–29). Here is an excerpt of that text[30]:

> We see [thus] that a limitation of the group in extension in space and time is [depending?] on a *Complementarity* [my italics] conjugated to a limitation in accuracy with which energy and momentum can be defined. Indeed we may say that according to the quantum theory the possibility of a space time coordination is *complementary* [my italics] to the possibility of a causal description (Bohr, 1985, pp. 78–79).

A little later, he would present similar material in October 1927 at the fifth Solvay Conference in Brussels with all the founders of quantum theory in attendance: from Planck, Einstein, Marie Curie, and de Broglie, to Heisenberg, Pauli, Schrödinger, Dirac, and Born among others. Afterwards, he sent a version of the lecture material for publication in *Nature*.

The following is an excerpt from the version of the lecture containing the notion of *complementarity* that was published in *Nature* in 1928 and giving it access to a far wider audience. My preference for this version does not correspond to a historicity choice because the purpose of choosing it is not to identify the first formulation but to understand the

[28] Niels Bohr worked on the concept of *complementarity*, its significance, and its applications for the rest of his life, and the exposure to discussion, craft of its presentation in different media, context and times made it change its articulation. For the purpose of the thesis, I consider the version published in *Nature* in 1928 (dated 1927) the best fit to explain its early enunciation as it benefitted from being presented at both Como and later at the Solvay conference, however, what follows is a brief historical sketch of its earliest evolution from the handwritten drafts to the *Nature* publication.
[29] Editor of the Niels Bohr Collected works vol 6 covering 1927
[30] The notations on brackets are Kalckar's unless otherwise specified

evolution of the knowledge associated with it not as a solution but as the interpretation of the problem from the interaction with others in its clarification and evolution, and thus, Bohr published this version after many discussions with a number of thinkers after the unveiling of these ideas at Como:

> Now, the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. After all, the concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed. Ultimately, every observation can, of course, be reduced to our sense perceptions. The circumstance, however, that in interpreting observations use has always to be made of theoretical notions entails that for every particular case it is a question of convenience at which point the concept of observation involving the quantum postulate with its inherent "irrationality" is brought in.
>
> This situation has far-reaching consequences. On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case according to the quantum postulate, any observation will be impossible and, above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an ambiguous definition of the state of the system is naturally no longer possible, and there can be no questions of causality in the ordinary sense of the word. *The very nature of the quantum theory thus forces us to regard the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and definition respectively* [my italics]. Just as the relativity theory has taught us that the convenience of distinguishing sharply between space and time rest solely on the smallness of the velocities ordinarily met with compared to the velocity of light, we learned from the quantum theory that the appropriateness of our usual causal space-time description depends entirely upon the small value of the quantum of action as compared to the actions involved in ordinary sense perceptions. Indeed, in the description of atomic phenomena, the quantum postulate presents us with the task of developing a "complementarity" theory the consistency of which can be judged only by weighing the possibilities of definition and observation (Bohr, 1928, p. 580) (Bohr, 1961, pp. 54–55).

I agree with Jammer in that while the notion of *complementarity* is difficult to define, the notion of *complementarity interpretation* can be ascribed to the Copenhagen school and Jammer suggests the following:

> A given theory *T* admits a *complementarity interpretation* [my italics] if the following conditions are satisfied: (1) *T* contains (at least) two descriptions D_1 and D_2 of its substance-matter; (2) D_1 and D_2 refer to the same universe of discourse *U* (in Bohr's case, microphysics); (3) neither D_1 nor D_2 if taken alone, accounts exhaustively for all phenomena of *U*; (4) D_1 and D_2 are mutually exclusive in the sense that their combination into a single description would lead to logical contradictions… *complementarity* [my italics] is the answer to the following question: What are we to do when we are confronted with such a situation, in which we have to use two concepts that are mutually exclusive, and yet both of them necessary for a complete description of the phenomena? (Jammer, 1974, p. 104).

It is important to note that the Jammer's definition is presented with no details or explicit connection to nuclear physics. This is consistent with Bohr's own development of the notion of *complementarity* over time and its implications in a wide variety of contexts and fields. Niels Bohr's development of *complementarity* represented a profound intellectual leap in the history of quantum mechanics and the philosophy of science. The intellectual leap embodied in *complementarity* can be characterised by several key aspects: reconceptualising the nature of physical description, embracing apparent contradictions, redefining the relationship between observer and observed, and expanding the scope of scientific explanation. Let's examine each of these aspects in detail:

Reconceptualising the nature of physical description

Bohr's *complementarity* fundamentally challenged the classical ideal of a complete, unified description of physical reality. In classical physics, it was assumed that all properties of a system could, in principle, be measured simultaneously and with arbitrary precision. Quantum phenomena, however, seemed to defy this assumption. This is how Bohr described the reconceptualising in an address at the Congress of Copenhagen, October 1960, arranged by *La Fondation Européenne de la Culture*:

> From the beginning, the use of mathematics has been essential for the progress of the physical sciences. While Euclidean geometry sufficed for Archimedes' elucidation of

> fundamental problems of static equilibrium, the detailed description of the motion of material bodies demanded the development of the infinitesimal calculus on which the imposing edifice of Newtonian mechanics rests. Above all, the explanation of the orbital motion of the planets in our solar system, based on simple mechanical principles and the law of universal gravitation, deeply influenced the general philosophical attitude in the following centuries and strengthened the view that space and time as well as cause and effect had to be taken as a priori categories for the comprehension of all knowledge. The extension of physical experience in our days has, however, necessitated a radical revision of the foundation for the unambiguous use of our most elementary concepts, and has changed our attitude to the aim of physical science. Indeed, from our present standpoint, physics is to be regarded not so much as the study of something *a priori* given, but rather as the development of methods for ordering and surveying human experience. In this respect our task must be to account for such experience in a manner independent of individual subjective judgment and therefore objective in the sense that it can be unambiguously communicated in the common human language (Bohr, 1963b, pp. 9–10).

Embracing apparent contradictions

Perhaps the most striking aspect of Bohr's intellectual leap was his willingness to embrace apparent contradictions as essential features of quantum description. The wave-particle duality of light and matter, which had puzzled physicists for decades, became for Bohr not a problem to be resolved, but a key to deeper understanding. In his words:

> … an experimental arrangement aiming at ascertaining where an atomic particle, whose position at a given time has been controlled, will be located at a later moment implies a transfer, uncontrollable in principle, of momentum and energy to the fixed scales and regulated clocks necessary for the definition of the reference frame. Conversely, the use of any arrangement suited to study momentum and energy balance decisive for the account of essential properties of atomic systems implies a renunciation of detailed space-time coordination of their constituent particles.
>
> Under these circumstances it is not surprising that with one and the same experimental arrangement we may obtain different recordings corresponding to various individual quantum processes for the occurrence of which only statistical account can be given. Likewise, we must be prepared that evidence, obtained by different, mutually exclusive experimental arrangements, may exhibit unprecedented contrast and even at first sight appear contradictory. It is in this situation that the notion of *complementarity* [my italics] is called for to provide a frame wide enough to embrace the account of fundamental

> regularities of nature which cannot be comprehended within a single picture. Indeed, evidence obtained under well-defined experimental conditions and expressed by adequate use of elementary physical concepts exhausts in its entirety all information about the atomic objects which can be communicated in common language (Bohr, 1963b, pp. 11–12).

This approach represented a dramatic break from the traditional scientific goal of resolving contradictions. Instead, Bohr proposed that certain contradictions were fundamental and irreducible, reflecting the limitations of our classical concepts when applied to the quantum realm.

Redefining the relationship between observer and observed

Another crucial aspect of Bohr's intellectual leap was his reconceptualisation of the relationship between the observer and the observed in quantum experiments. In classical physics, it was assumed that the act of observation could, in principle, be made arbitrarily small, allowing for an objective description of reality independent of observation. Bohr, however, argued that in quantum physics, the interaction between the measuring apparatus and the system being measured could not be neglected or reduced below a certain limit set by Planck's constant. This led to a fundamental revision of the concept of objectivity in physics. Let’s see the argument in Bohr’s words:

> While within the scope of classical physics we are dealing with an idealization, according to which all phenomena can be arbitrarily subdivided, and the interaction between the measuring instruments and the object under observation neglected, or at any rate compensated for, it was stressed that such interaction represents in quantum physics an integral part of the phenomena, for which no separate account can be given if the instruments shall serve the purpose of defining the conditions under which the observations are obtained. In this connection it must also be remembered that recording of observations ultimately rests on the production of permanent marks on the measuring instruments, such as the spot produced on a photographic plate by impact of a photon or an electron. That such recording involves essentially irreversible physical and chemical processes does not introduce any special intricacy but rather stresses the element of irreversibility implied in the very concept of observation. The characteristic new feature in quantum physics is merely the restricted divisibility of the phenomena,

> which for unambiguous description demands a specification of all significant parts of the experimental arrangement (Bohr, 1963b, pp. 91–92).

This insight challenged the traditional subject-object distinction in science and philosophy, suggesting a more intertwined relationship between the knower and the known.

Expanding the scope of scientific explanation

Perhaps the most ambitious aspect of Bohr's intellectual leap was his attempt to expand the scope of complementarity beyond physics to other domains of knowledge. Bohr saw complementarity not just as a solution to specific problems in quantum mechanics, but as a general epistemological principle with broad applicability. In his later writings, Bohr explored the potential relevance of complementarity to fields as diverse as biology, psychology, and anthropology. In his address at the opening meeting of the International Congress of Light Therapy in Copenhagen, August 1932, he said:

> Just as the general concept of relativity expresses the essential dependence of any phenomenon on the frame of reference used for its coordination in space and time, the notion of *complementarity* [my italics] serves to symbolize the fundamental limitation, met with in atomic physics, of the objective existence of phenomena independent of the means of their observation. The revision of the foundations of mechanics, extending to the very idea of physical explanation, not only is essential for the full appreciation of the situation in atomic theory but also creates a new background for the discussion of the problem of life in their relation to physics (Bohr, 1987, p. 7).

It's important to note that Bohr's intellectual leap, while profoundly influential, was not universally accepted. Einstein, in particular, remained sceptical of Bohr's interpretation, leading to the famous Bohr-Einstein debates that spanned decades. These debates, far from diminishing the significance of Bohr's ideas, served to clarify and refine them. In this sense, even the resistance to Bohr's ideas played a crucial role in their development and in shaping the broader discourse on quantum foundations.

The intellectual leap represented by *complementarity* had far-reaching consequences, both within physics and beyond. Within physics, it provided a framework for understanding and applying quantum mechanics, even in the absence of a fully satisfactory ontological interpretation. It allowed physicists to make progress in areas like atomic and nuclear physics, solid state physics, and quantum chemistry, without becoming paralysed by foundational questions. Beyond physics, *complementarity* influenced fields ranging from biology to philosophy. It contributed to a broader questioning of absolute knowledge and recognition of the role of perspective in understanding, and so, I think it is appropriate to finish this section with the words Bohr chose to finish the paper published in *Nature*:

> Indeed, we find ourselves here on the very path taken by Einstein of adapting our modes of perception borrowed from the sensations to the gradually deepening knowledge of the laws of Nature. The hindrances met with on this path originate above all in the fact that, so to say, every word in the language refers to our ordinary perception. In the quantum theory we meet this difficulty at once in the question of the inevitability of the feature of irrationality characterising the quantum postulate. I hope, however, that the idea of *complementarity [my italics]* is suited to characterise the situation, which bears a deep-going analogy to the general difficulty in the formation of human ideas, inherent in the distinction between subject and object (Bohr, 1928, p. 590).

Alternate Views - Competing frameworks

As mentioned earlier in the recollections from Bohr, the presentation of the ideas regarding *complementarity* produced much dialog and disputation, beginning at the Solvay conference in 1927. The debates between Niels Bohr and Albert Einstein on the foundations of quantum mechanics, as recounted in Bohr's paper "Discussion with Einstein on Epistemological Problems in Atomic Physics" (Bohr, 1949, pp. 32–66) (Bohr, 1983, pp. 9–49) (Jammer, 1974, pp. 121–132) represent a pivotal moment in the development and acceptance of quantum theory. Analysing these discussions through the lens of Helen Longino's social epistemology provides valuable insights into how scientific knowledge is constructed and validated through critical discourse within scientific communities.

In agreement with Longino's social epistemological perspective, I have claimed that knowledge is social in both its generating process, and in its acceptance and utilisation within an epistemic community in controlling and predicting activities. N. R. Hanson stresses the importance of controversy for research science: researchers sometimes appreciate data differently:

> It is important to realise… that sorting out differences about data, evidence, observation… may require a comprehensive reappraisal of one's subject matter (Hanson, 1985, p. 19).

Those discursive interactions become constructive and justificatory, and in doing so they provide the justification needed to become a knowledge generating activity. As an example, for Niels Bohr, his dialogue with Einstein on Quantum Mechanics forced him to develop the tenets of *complementarity* (Bohr, 1983, pp. 17–18) in increasingly precise language (Pais, 1991, p. 363). It is worth it for its illustrative power on some of the arguments made in this chapter to quote here *in extenso* several pieces of Niels Bohr thoughts on his dialogues with Einstein:

> ... the many occasions through the years on which I had the privilege to discuss with Einstein epistemological problems raised by the modern development of atomic physics have come back vividly to my mind and I have felt that I could hardly attempt anything better than to give an account of these discussions which, even if no complete concord has so far been obtained, have been of greatest value and stimulus to me. I hope also that the account may convey to wider circles an impression of how essential the open-minded exchange of ideas has been for the progress in a field where new experience has time after time demanded a reconsideration of our views. (Bohr, 1983, p. 9)

He further reflects:

> The new progress in atomic physics was commented upon from various sides at the International Physical Congress held in September 1927, at Como in commemoration of Volta. In a lecture on that occasion, I advocated a point of view conveniently termed *complementarity* [my italics] suited to embrace the characteristic features of individuality of quantum phenomena, and at the same time to clarify the peculiar aspects of the observational problem in this field of experience. For this purpose, it is decisive to recognize that, *however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms* [Bohr's italics]. (Bohr, 1983, p. 17).

Bohr acknowledges the challenges posed by Einstein's critiques:

> During the discussions [at the Solvay Institute in Brussels in October 1927 after the Como presentation], where the whole subject was reviewed by contributions from many sides and where also the arguments mentioned in the preceding pages were again presented, Einstein expressed, however, a deep concern over the extent to which causal account in space and time was abandoned in quantum mechanics. (Bohr, 1983, p. 20).

Einstein's attitude gave rise to ardent discussions within a small circle (Bohr, 1948, p. 21).

> Our talks about the attitude to be taken in face of a novel situation as regards analysis and synthesis of experience touched naturally on many aspects of philosophical thinking, but, in spite of all divergencies [Bohr's spelling] of approach and opinion, a most humorous spirit animated the discussions. On his side, Einstein mockingly asked us whether we could really believe that the providential authorities took recourse to dice-playing (*' . . . ob der liebe Gott würfelt '*) ['whether the good Lord is rolling dice', Bohr's italics], to which I replied by pointing at the great caution, already called for by ancient thinkers, in ascribing attributes to Providence in every-day language. (Bohr, 1983, p. 26).

> It is, of course, true that in atomic physics we are confronted with a number of unsolved fundamental problems, especially as regards the intimate relationship between the elementary unit of electric charge and the universal quantum of action; but these problems are no more connected with the epistemological points here discussed than is the adequacy of relativistic argumentation with the issue of thus far unsolved problems of cosmology. Both in relativity and in quantum theory we are concerned with new aspects of scientific analysis and synthesis and, in this connection, it is interesting to note that, even in the great epoch of critical philosophy in the former century, there was only question to what extent a priori arguments could be given for the adequacy of space-time coordination and causal connection of experience, but never question of rational generalizations or inherent limitations of such categories of human thinking. (Bohr, 1983, p. 47).

Those discussions, as well as other interactions between many of the founding scientists in Quantum Physics produced the justification for the acceptance of the so-called *Copenhagen interpretation of Quantum Mechanics.* One of the central themes in Bohr and Einstein discussions was the nature of physical reality and the completeness of quantum mechanical descriptions. Einstein challenged the probabilistic interpretations of quantum theory. This objection reflected Einstein's belief in deterministic physical laws and complete descriptions of reality. Bohr, in contrast, argued for the necessity of probabilistic descriptions and complementarity in quantum phenomena. This fundamental disagreement highlights what Longino describes as the "underdetermination of theories by evidence" (Longino, 2002c, p. 126). She explains:

> The underdetermination problem, as I understand it, does not depend on wholistic views about meaning, or on a metaphysical or semantic view about observation and theory. It is an epistemological problem: the consequence of the gap between evidence and hypotheses produced by our postulation of different entities and processes in our explanations of phenomena than occur in our descriptions of those phenomena…
> The logical problem of underdetermination shows that empirical reasoning takes place against a background of assumptions that are neither self-evident nor logically true. The identification of good reasons is similarly context-dependent, whether this is accomplished by the scientist in action or by the philosopher in reflection (Longino, 2002c, pp. 127–128).

The debates between Bohr and Einstein demonstrate how even brilliant scientists can interpret the same evidence differently based on their background assumptions and philosophical commitments. The debates were not just about empirical facts, but about the conceptual frameworks used to interpret those facts. This historical perspective illuminates the evolving context within which the Bohr-Einstein debates took place and sets the stage for the analysis of the social dimension in the *Épistémè-Socialisante*.

Épistémè-Socialisante - Socialising Episteme:

Gestalt shift

The Copenhagen Interpretation represented a pivotal epistemological and ontological framework in quantum mechanics, crystallising during the watershed 1927 Solvay Conference through the intellectual synthesis of primarily Bohr and Heisenberg. This interpretative paradigm fundamentally reconceptualised the relationship between measurement, observation, and physical reality at the quantum level, introducing core principles such as complementarity, wave-particle duality, and quantum indeterminacy. The term *gestalt shift* aptly characterises how this interpretation marked a radical departure from classical mechanical determinism, establishing a new philosophical foundation that acknowledged inherent probabilistic elements in nature and the inseparability of the observer from the observed phenomenon. This framework proved instrumental in both theoretical advancement and experimental practice, providing physicists with a coherent (though philosophically contentious) conceptual apparatus for navigating the counterintuitive aspects of quantum phenomena. To understand the creative process behind the *gestalt shift* that led to the emergence of new physics during the remarkably productive period of 1925–1927, we can consider the methodological approach described by Heelan as the *movement of thought*. Heelan observes:

> The movement of the thought in the context of creative discovery is very mysterious. The creation of a new hypothesis is not the product merely of logical rules. The creative process uses epistemological heuristic stratagems to generate conjectures with the hope of understanding the nature of the problem and in the process eventually to create the new revolutionary science (Heelan, 2016, p. 32).

The initial debates between Werner Heisenberg's particle mechanics framework and Erwin Schrödinger's wave theory laid the groundwork for the subsequent dialogues between Niels Bohr and Albert Einstein. As Pais notes: "Through most of his life Bohr learned new science from discussions within others; reading was secondary" (Pais, 1991, p. 109). Einstein's probing questions and arguments compelled Bohr to refine and nuance his ideas. While both scientists initially had their supporters, over time the force and logic of Bohr's arguments gained broader acceptance. Not necessarily through definitive proof, but by effectively addressing and exhausting criticisms with increasingly nuanced explanations, Bohr's position prevailed. The gradual acceptance of Bohr's Copenhagen interpretation within the physics community exemplifies what Longino refers to as the *socialization of cognition* (Longino, 2002b, p. 97). She argues that scientific knowledge is produced through social processes of critical interaction, not merely individual reasoning:

> … discursive interactions are integral to both observation and reasoning in the sciences. The results of both reasoning and observation, then, are socially processed before incorporation into the body of ideas ratified for circulation and use, or are treated as having been so processed. The critical dimension of cognition is a social dimension, requiring the participation of multiple points of view to insure that the hypotheses

> accepted by a community do not represent someone's idiosyncratic interpretation of observational or experimental data (Longino, 2002b, p. 106)

Bohr's account of his dialogues with Einstein illustrates how the quantum mechanical worldview emerged through sustained debate and scrutiny within the physics community, rather than through a single decisive experiment or argument. Importantly, these debates were not mere academic exercises but had profound implications for the practice of physics. Heisenberg further elaborates on this point:

> When we speak of a picture of nature provided by contemporary exact science, we do not actually mean any longer a picture of nature, but rather a picture of our relation to nature. The old compartmentalization of the world into an objective process in space and time, on the one hand, and the soul in which this process is mirrored, on the other–that is, the Cartesian differentiation of *res cogitans* and *res extensa*–is no longer suitable as the starting point for the understanding of modem science. In the field of view of this science there appears above all the network of relations between man and nature, of the connections through which we as physical beings are dependent parts of nature and at the same time, as human beings, make them the object of our thought and actions. Science no longer is in the position of observer of nature, but rather recognizes itself as part of the interplay between man and nature. The scientific method of separating, explaining, and arranging becomes conscious of its limits, set by the fact that the employment of this procedure changes and transforms its object; the procedure can no longer keep its distance from the object. The world view of natural science thus ceases to be a view of "natural" science in its proper sense (Heisenberg, 1958, p. 107).

The acceptance of the Copenhagen interpretation represented a gestalt shift to a new Kuhnian paradigm, wherein objectives, procedures, and emphases changed, introducing a new ontology of entities. The significance lies in how this interpretation not only offered a pragmatic methodology for conducting atomic physics but also precipitated profound implications for scientific epistemology, challenging traditional notions of causality, measurement, and the nature of physical reality itself.

Support for new paradigm

With the first ideas and papers in 1925-1926 came much confusion and heated debate. Heisenberg and the first readers of his first paper had an inadequate grasp of what was happening since the mathematics was alien and the physics was muddied. Later in the year Pauli used the matrix mechanics to derive the same Balmer formula that Bohr had derived[31] for the discrete spectrum of hydrogen, converting Bohr into a supporter (Pais, 2002, p. 255). Schrödinger produced the wave forms to suggest physical principles in the context of quantum mechanics and proposed that reality could only exist in the form of waves and that particles were only a useful ideation. Born recognised the usefulness of Schrödinger's formalism but not his interpretation and he had the insight that Ψ (the wave equation), as opposed to the electromagnetic field, does not represent physical reality. Born introduced probability without using the wave equation and instead deriving it as a consequence of conservation consistent with the wave description which was seen as obvious by many, particularly in Copenhagen. Einstein did not accept the converging ideas, and Planck, Max von Laue (1879-1960), and Schrödinger, who had been at the forefront of the once leading Berlin school, gravitated between scepticism and opposition (Pais, 2002, pp. 260–261). In reference to the probability interpretation of Schrödinger's wave theory, Pais recalls:

> … in fact, when I worked in Copenhagen in 1928 it was already called "Copenhagen Interpretation"– I do not think I ever realized that Born was the first to put it forward. In response to a query, Casimir [Hendrik Casimir (1909-2000)], who started his university studies in 1926, wrote to me: 'I learned the Schrödinger equation simultaneously with the interpretation. It is curious that I do not recall that Born was specially referred to. He was of course mentioned as co-creator of matrix mechanics' (Pais, 2002, p. 261).

It is at this point that the physics world, particularly the younger generation began to accept the new ideas, while the reticent "old guard" began to fade into a vocal but nonetheless secondary and less influential role. In March 1927 Heisenberg published the uncertainty relations, and Bohr presented *complementarity* in September of 1927. By then Born, Heisenberg, and Jordan had indicated the close connection between quantum fields and quantum statistics. Work and ideas began to grow beyond the German niche and stemmed from the melding of the views. Even though, led by Einstein and Schrödinger, there was still opposition to Bohr's views among

[31] The Balmer formula empirically described the wavelengths of the visible spectral lines of hydrogen. Niels Bohr sought to explain the discrete spectral lines observed in hydrogen using a new atomic model in 1913 and he proposed that electrons orbit the nucleus in specific, quantised orbits without emitting radiation. By calculating the energy differences between quantised orbits, Bohr was able to derive an expression for the wavelengths of emitted light. His derivation matched Balmer's empirical formula, thereby providing a theoretical foundation for the observed spectral lines of hydrogen.

some scientists, the vast majority of the physicists in general accepted the *complementarity* interpretation without reservations during the first two decades after its inception and "*they became primarily interested in its applications to practical problems and its extension to unexplored regions* [my italics]" (Jammer, 1974, p. 247), impressed by quantum mechanics spectacular successes in all fields of microphysics.

Beyond that point, research in different areas began to produce a flurry of results that began to expand quantum physics with emphasis on theoretical research and experimentation of nuclear particles. Working at Cambridge, Dirac began to develop quantum mechanics from a generalisation of classical Hamiltonian dynamics[32] with a new method called *transformation theory* that circumscribed both matrix and wave mechanics as special limiting cases (Jammer, 1966, pp. 228–234) (Brown, Pais and Pippard, 1995, p. 360). Using that method Dirac derived an equation for the electron that was relativistic, published in 2 papers in 1928 (Dirac, 1928a) (Dirac, 1928b), that was a groundbreaking achievement in quantum mechanics and theoretical physics. Before Dirac's work, physicists struggled to reconcile quantum mechanics with Einstein's special relativity; existing equations (like Schrödinger's) did not properly account for relativistic effects, and the electron's observed spin had no theoretical explanation. The equation's significance is manifold: Dirac's equation was the first successful attempt to reconcile quantum mechanics with special relativity for elementary particles like electrons; this unification was crucial for the development of quantum field theory. It was the first theory to naturally explain why in experiments, electrons exhibit spin–a form of intrinsic angular momentum–($\frac{1}{2}\hbar$)[33] without artificially adding it. The electron's spin is associated with a magnetic moment, meaning the electron behaves like a tiny magnet. Dirac's equation provided a theoretical foundation for the electron's magnetic moment, predicting its value accurately and explaining phenomena like the fine structure in atomic spectra. In its most striking feature, an unexpected but profound implication of Dirac's equation was the existence of particles identical in mass but opposite in charge to electrons—the positron, which was discovered in 1932 by Carl Anderson (1905-1991). At the same time, Dirac invented notations that became part of our

[32] the Hamiltonian formalism can be used to express the equations of motion in both special and general relativity
[33] Where $\hbar$ is the reduced Planck constant or $h/2\pi$

language like *q-numbers* where *q* stands for *quantum* or *queer; c-numbers* where *c* stands for *classical* or *commuting*. Like finding an "extra" solution to a mathematical equation, Dirac's work suggested that for every particle, there must exist an antiparticle with opposite charge - a concept that revolutionised our understanding of matter and the universe. This discovery is considered one of the greatest triumphs of mathematical physics and twentieth century science, where pure theoretical work predicted a fundamental aspect of nature before it was observed. Dirac's work paved the way for understanding the behaviour of all fermions[34], not just electrons. It has been instrumental in the development of the Standard Model of particle physics (Jammer, 1966, pp. 152–153; Brown, Pais and Pippard, 1995, pp. 217, 231; Brown, 1995, pp. 360–362; Pais, 2002, pp. 286–292). In another example of the establishing paradigm, in the early nineteen-thirties, Enrico Fermi (1901-1954) and others begun to use neutrons in nuclear studies[35], finding that in collisions with an atom nucleus neutrons were frequently captured with similar behaviour shown in radiative capture, which could not be explained at the time until the solution[36] was presented by Bohr early in 1936 in a lecture to the Danish Academy on 24 January 1936 and later published in *Nature* (Bohr, 1936a, pp. 344–348) (Peierls, 1986, pp. 151–156). Bohr continued to work on other aspects of the interactions at the subatomic level, and “By the summer of 1936, Bohr’s approach to nuclear reactions had become the generally accepted treatment.” (Peierls, 1986, p. 39).

The new paradigm was a combination of ideas, and a framework that coalesced in 1927 into what became known as the *Copenhagen Interpretation*. Hanson summarises it thus:

> Almost all practicing quantum theoreticians of the 1928-1938 period accepted some form of the Copenhagen Interpretation. They would have claimed, as they now claim in retrospect, that this interpretation affected even their manipulations with the formalism. They were orientated in terms of the Copenhagen Interpretation, and evaluated proposed lines of research in this manner too… As a matter of historical fact,

[34] Fermions are one of the two fundamental classes of particles in physics (the other being bosons). Fermions are particles that have half-integer spin (½, 3/2, 5/2, etc.) like electrons, protons, and neutrons. They follow the Pauli Exclusion Principle: two identical fermions cannot occupy the same quantum state simultaneously which is why electrons in atoms occupy different energy levels, leading to the periodic table's structure and they obey Fermi-Dirac statistics, which describes their statistical behaviour. The fermionic nature of electrons explains why matter is solid and does not collapse, different elements have different chemical properties , and conductors and insulators behave differently.

[35] This work will later produce the first sustained nuclear chain reaction, a preliminary step to the development of nuclear weapons at the Manhattan Project.

[36] “...the point was that the constituents of the nucleus are interacting very strongly with each other, and with the incident particle, so that its kinetic energy is shared between many particles, with the result that none of them can escape until enough energy happens to be concentrated on one of them” *(rudolph peirl pages14-15)*

> it was a triumphant invention of an orderly interpretative pattern for a cluster of facts which before had been simply chaotic (Hanson, 1959, pp. 325–326).

Expansion and Pedagogy

There is a critical component to the socialisation of the framework and ideas and that is the expansion through pedagogical means such as lectures, textbooks, teaching, and new applications by the members of the community formed around the gestalt shift, all of which helps in the acceptance of the new paradigm and the training of the new generation of scientists. In the case of quantum mechanics this is especially true, and I will describe several examples.

Lectures and imagery

Although Niels Bohr was not particularly known for electrifying lectures, he nonetheless was charismatic and spent much time preparing his lectures and the language to be used in his communications. Heelan elaborates on this point:

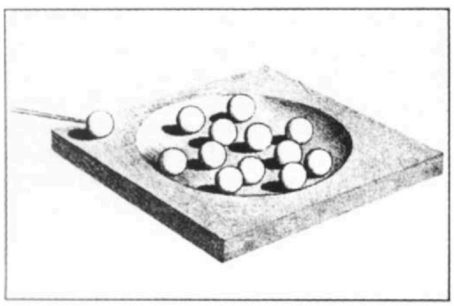
> Bohr's philosophy was intuitively bound up with his reflections on language. *We are immersed in language* [my italics], he would say. Language was not only the vehicle of human communication, its norms of usage constituted a set of objective meanings which limited what could be meant or stated in words. Language was a priori to every description. Either a descriptive concept was already in the language, or it could be paraphrased or used figuratively in the language. If in a particular case none of these was true, the particular case was incommunicable, and if incommunicable, it did not belong to the most public of all realms, that of what we call 'reality' (Heelan, 2016, p. 65).

Let us see this point in Bohr's chosen words to start his address in front of the audience at the auditorium of the *Istituto Carducci* on September 16 1927 in Como:

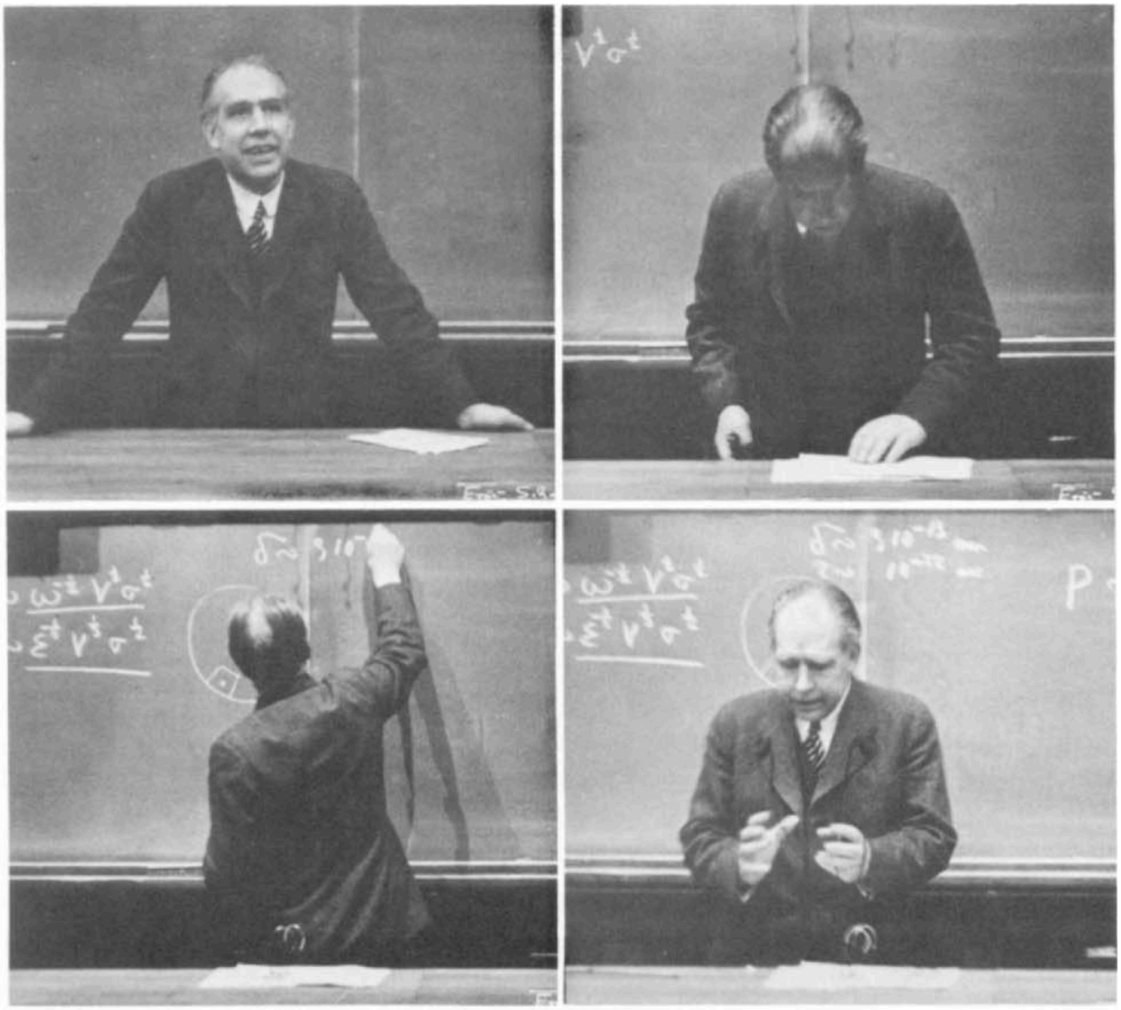
> I shall try, by making use only of simple considerations and without going into details of technical mathematical character, to describe to you a certain general point of view which I believe is suited to give an impression of the general trend of the development of the theory from its very beginning which I hope will be helpful in order to harmonize

> the apparently conflicting views taken by different scientists (Bohr, 1927, p. 565) (Jammer, 1974, p. 86).

In 1936 he had gained general acceptance of his ideas on nuclear reactions and the absorption of neutrons during collisions, and he was frequently invited to lecture on those themes. He would try to convey pictures rooted in the language of experience that would facilitate the understanding of the ideas and, in a representative case, at a lecture to the Chemical and Physical Society of University College, London, on February 11 he introduced a brilliant figure (reproduced here) to represent how a slow neutron would get trapped in the nucleus of an atom and it was thus explained:

> Imagine a shallow basin with a number of billiard balls in it as shown in the accompanying figure. If the basin were empty, then upon striking a ball from the
>
> 
>
> outside, it would go down one slope and pass out on the opposite side with its original velocity. But with other balls in the basin, there would not be a free passage of this kind. The struck ball would divide its energy first with one of the balls in the basin, these two would similarly share their energies with others, and so on until the original kinetic energy was divided among all the balls. If the basin and the balls are regarded as perfectly smooth and elastic, the collisions would continue until the kinetic energy happens again to be concentrated upon a ball close to the edge. This ball would then escape from the basin and the remainder of the balls would be left with insufficient total energy for any of them to climb the slope. The picture illustrates, therefore, "that the excess energy of the incident neutron will be rapidly divided among all the nuclear particles with the result that for some time afterwards no single particle will possess sufficient kinetic energy to leave the nucleus" (Bohr, 1936b, p. 351).

One year later he would expand to give lectures in Paris, and at several Universities in the US and Canada, including the University of California (Berkeley and Los Angeles), Duke, Harvard, Johns Hopkins, Michigan, Princeton, Rochester, and Toronto. From the US, Bohr continued to Japan and Moscow. Here you can see Bohr lecturing in Princeton in 1937 (Peierls, 1986, p. 40):



Along with the in-person lectures, there was a progressive growth in the number of written versions of lectures that were requested for special themed issues of journals, such as the one published for the *Annalen der Physik* honouring Planck's 80th birthday, where he used the opportunity to recount a detailed historical account of the development of nuclear physics, including some of his own ideas (Peierls, 1986, pp. 39–42). His care on the language and the intelligibility of the ideas, as well as his method of doing science through discussion with others can be seen in the following extract that exemplifies how Bohr was aware of the intuitive nature of the conjectures in a model contained in the note submitted to *Nature* regarding nuclear photo-effects (Bohr, 1938). Immediately afterwards he sent a copy of the note to Felix Bloch (1905-1983) and Pauli (both became Nobel prize winners in 1954 and 1945 respectively for their work in QM) requesting their opinion:

> In connection with the discussions with Bothe in Bologna it may interest you and Pauli to see the little note on nuclear photo-effects which I have just sent to 'Nature' and of which I enclose a copy. It seems to me that the argumentation is not only very natural from a theoretical standpoint, but also a very plausible description of the experimental facts, and I shall be very happy to hear the criticism of the more learned gentlemen… (Bohr, 1986, p. 44).

Lectures, papers and conferences continued over the years. The proof of nuclear fission in 1939 brought a flurry of work with people and laboratories in the US, and with many occasions to lecture on the subject (Peierls, 1986, p. 75). The lectures and their influence did not stop during the war. Bohr visited the Manhattan project in secret brought to it by the Allies. After the war, his influence and reputation continued to grow. In 1955 the Massachusetts Institute of Technology Committee on General education recommended the establishment of a new lectureship with the name Karl Taylor Compton. For its Inaugural Compton Lecture, MIT chose Niels Bohr[37] and to this date, it has been the best-attended Compton Lecture ever given, with 1500 people in attendance coming from a variety of locations and Universities around MIT. Bohr gave six Compton lectures, three seminars, and five question-and-answer sessions with students on the theme, "The Philosophical Lesson of Atomic Physics" (MIT, no date).

The Niels Bohr Archive at the University of Copenhagen holds a series of unpublished manuscripts from Bohr with his notes in preparing for the Karl Compton Lectures[38] he gave at MIT at the end of 1957. The papers present a picture of the organisation of Physics in Bohr's mind and the description of its evolution from the end of the nineteenth century to the current views at the time of the Lectures. In an outline dated 23.5.57, Bohr prepared material for 5 sections, the first of which is on Classical Physics and Atomic theory which reflects the view one would have had of the *themes* in physics before the development of the quantum theory:

> I. Classical physics and atomic theory
> Foundation of mechanics and electromagnetism
> Causal description

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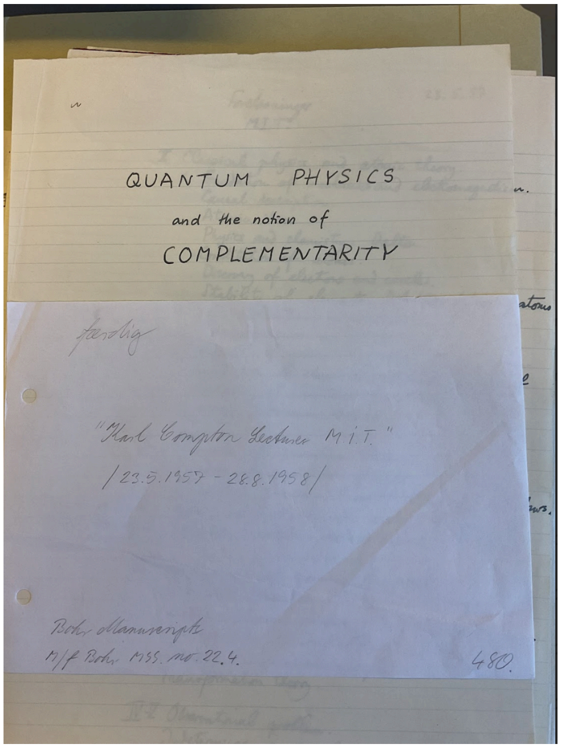
[37] Source: MIT's Niels Bohr | MIT Compton Lectures

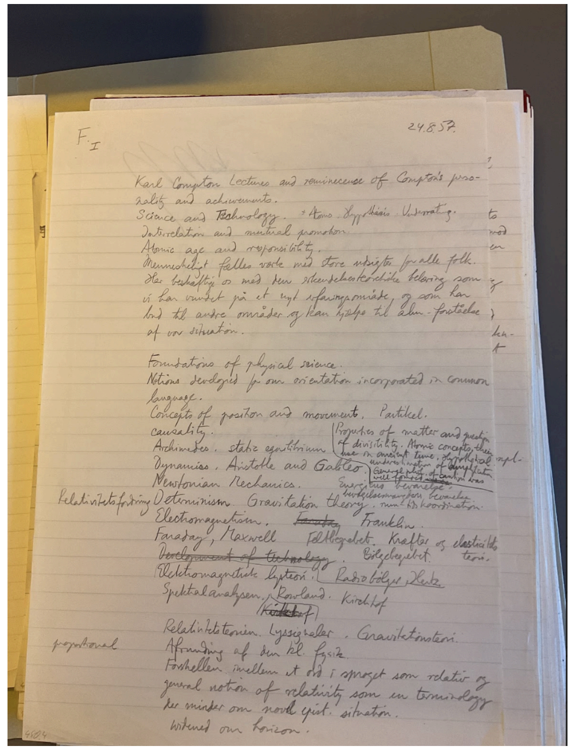
[38] I was granted permission to examine and copy some of the manuscripts while I was doing primary research on Bohr's papers as a visiting faculty at the NBA in July 2022. I am indebted to its Director, Dr. Christian Joas, and the staff at the NBA for their generous welcome, help, and support while I was there.

Atomic ideas
Physics and chemistry. Dalton
Constitution of atoms
Discovery of electrons and nuclei
Stability of elements. Distinction between atoms and molecules.
Thermodynamics
Properties of gases.
Limitations of classical mechanics
Brownian motion
Discovery of quantum of action. Energy (Bohr, 1957, p. 2)

In a later outline dated 24.8.57, Bohr had a substantial change in the approach to the lecture, choosing to present a picture of physics stemming from its historical development and the philosophical and epistemological questions associated with it:

> Introduction with reminiscence of Compton's personality and achievements. Science and Technology. Interrelation and mutual promotion, Atomic age and responsibility.
>
> Foundations of Physical Science
> Notions developed for our orientation incorporated in common language
> Concepts of position and movement. Particles.
> Causality
> Archimedes. Static equilibrium
> Dynamics. Aristotle and Galileo.
> Newtonian mechanics. Energy. Properties of matter and questions of divisibility. Atomic concepts and their use in ancient times.
> Relativism forming [?] Determinism. Gravitation theory
> Electromagnetism. Franklin.
> Faraday, Maxwell, Radiology, Hertz
> Spectral Analysis, Rowland, Kirchhoff
> Relativity theory, Gravitational theory (Bohr, 1957, p. FI).





These two versions illustrate the perspectives that affect the introduction of a problem associated with an unobservable entity and the historical interest in it, as Bohr had done much earlier when he introduced *complementarity* in 1927. In the former outline, the topics follow a "theme" base covering different areas in nuclear physics, while in the later the emphasis is on the historical evolution of our understanding of Nature, matter, and science, topics very dear to Bohr. The expansion is not uniquely due to Bohr. Many of the founders followed similar patterns, before and after the war. The first series of lectures in the US born from *complementarity* were given by Heisenberg in Chicago in 1929 with his stated aim to make known the *Kopenhagener Geist der Quantentheorie*–Copenhagen spirit of quantum theory–with the central concept being "the complete equivalence of the corpuscular and wave concepts" (Heelan, 2016, p. 69).

Textbooks

In his *Revolutions*, Kuhn highlighted in his analysis of scientific change the critical role of textbooks in the training of every generation of scientist with their aim being both persuasive and pedagogic, becoming a crucial source of the epistemological process in which future scientists determine and learn how research will be conducted, “what is good question, what is a satisfactory answer… even what is viewed as a possible object of research” (Badino and Navarro, 2013, pp. 8–9). As Kuhn described:

> These textbooks expound the body of accepted theory, illustrate many or all of its successful applications, and compare these applications with exemplary observations and experiments (Kuhn, 2012, p. 10).
>
> Scientists, it should already be clear, never learn concepts, laws, and theories in the abstract and by themselves. Instead, these intellectual tools are from the start encountered in a historically and pedagogically prior unit that displays them with and through their applications. A new theory is always announced together with applications to some concrete range of natural phenomena; without them it would not even be a candidate for acceptance. After it has been accepted, those same applications or others accompany the theory into the textbooks from which the future practitioner will learn his trade (Kuhn, 2012, p. 47).

As in Kuhn’s description of the establishment of a new paradigm after a scientific revolution, the acceptance of the Copenhagen interpretation of quantum mechanics was followed by research in mycrophysics turning its emphasis to the resolutions of practical problems and new applications. While all textbooks written between 1930 and 1950 embraced *complementarity*, the majority concentrated on the new mathematical techniques needed to solve problems, and left little emphasis in conceptual meaning, usually including the logical relationship between the concepts of position and momentum defined as *complementarity* with a reference to Bohr’s writing as a footnote (Jammer, 1974, pp. 247–248).

A good example of the expansion of the Copenhagen interpretation beyond Germany through textbooks is how it was introduced and taught at Cambridge in the first decade of its establishment as the new paradigm. In 1914, Charles Galton Darwin (1887-1962, the grandson of Charles Darwin), who maintained contact with Bohr in Copenhagen, was about the only one trying to disseminate concepts on quanta. By 1927, both the *Mathematical Tripos* and the *Natural Science Tripos* included courses in quantum topics where students were expected to answer questions that only months earlier had escaped the grasp of some of the best scientific minds, with an example in the following question in the final exam of 1928:

> Show how the Heisenberg matrix of a *q*-number is determined from the normalised Schrödinger characteristic functions (Eigenfunktionen) of the problem concerned. Illustrate it by the problem of the rigid rotator (molecule) (Navarro, 2013, pp. 231–232).

In the 1920s, quantum theory could only be taught by young people, like Ralph Fowler (1889-1944) and Darwin, converted to the novel theory since people like Thomson strongly opposed it and Rutherford was basically experimental in his program. Darwin left Cambridge in 1922 and Fowler began to teach quantum physics, translated key papers from German and invited Heisenberg and others to lecture at Cambridge, and became a mentor to students like P.A.M. Dirac (Navarro, 2013, pp. 231–248). From that environment came the very important textbook of Paul Dirac. While still a PhD student at Cambridge in 1925, he developed *q*-number algebra, his version of quantum mechanics, and was given the task to teach in the Easter term of 1926 the first course in a British University on the new developments of quantum mechanics, with several notable students that went on to make significant contributions. Among those students was Julius Robert Oppenheimer (1904-1967). "Quantum Mechanics (Recent Developments)" evolved into a regular course in quantum mechanics containing what was known at the time, and that he continued to teach until the 1960s[39]. The materials of the lectures evolved into his monumental *Principles of Quantum Mechanics* (Dirac, 1930). It was meant to be faithful to how Dirac thought the material should be presented, that is, in a concise and coherent comparison between calculated quantities and those found experimentally, and do this using a coherent symbolic calculus:

> To him, the new physics was basically a formal scheme that allowed the calculation of experimental results, while it had nothing to say about ontological questions. The proper way to presenting quantum mechanics must necessarily be abstract, he wrote , for the new theory is "built up from physical concepts which cannot be explained in terms of things previously known to the student, which cannot be explained adequately in words at all [Dirac 1930]" (Kragh, 2013, p. 250).

[39] You can still see Dirac lecturing on the introduction to quantum mechanics. As an example, here is a link to his lecture 1 of 4, delivered in Christchurch, New Zealand, in 1975: https://youtu.be/2GwctBldBvU?si=yS5DhlKYzRqMkBLw

The book became an immediate success and the standard work on quantum mechanics, and it is still in print and used in advanced courses (Dirac, 2010). With a number of positive reviews from the likes of Pauli, Heisenberg, Einstein (who did not agree with Dirac's view but appreciated the clarity and depth of his exposition), Oppenheimer, Felix Bloch (1905-1983), and others, *Principles* was translated into German (1930), French (1931), Russian (1932), and Japanese (1936) and was sold and used extensively and broadly (Kragh, 2013, pp. 253–255).

> Dirac, never much of a philosopher, was in general agreement with the Bohr-Heisenberg view of quantum theory, including the interpretation of the measurement process and the nature of the principle of indeterminacy. Although his book did not refer explicitly to philosophical issues, it did much to disseminate certain views of the Copenhagen school to a generation of young physicists. In accordance with Bohr, in the preface of the first edition, Dirac called attention to "the increasing recognition of the part played by the observer himself in introducing the regularities that appear in his observations" This he considered "very satisfactory from a philosophical point of view" (I,v). Also with regards to determinism and causality, he shared the view of Bohr and his circle of physicists (Kragh, 2013, pp. 259–260).

The influence of the *Principles* did much to expand the new paradigm in general beyond a generation of Cambridge physicists to the point that it was characterised with the epithet "bible" of quantum physics textbooks (Freire, 2022, p. 721).

A third example is a book that does not conform to traditional formats of textbooks, and yet, it qualifies as one of the cardinal texts in understanding the foundations of quantum mechanics: *The Conceptual Development of Quantum Mechanics* by Max Jammer (Jammer, 1966). Josep Simon describes it:

> Max Jammer worked on a comprehensive account for the conceptual foundations of quantum mechanics. His investigation proceeded implicitly through not only classical journal papers but also an international set of influential textbooks, as main sources to elucidate the conceptual, logical, epistemological and mathematical structure of quantum theory. Although often buried in footnotes, it also showed the importance of textbook writing for contemporary physicists to clarify or solve complex scientific and philosophical implications in the making of quantum mechanics as an evolving research field. In the process of writing his book, Jammer discusses parts of it with physicists

> involved in the facts he dealt with (e.g. de Broglie, Born, Heisenberg, Dirac, Hund, Heitler, van der Waerden, Fierz, Jost, Andrade, Slater, Fues, and Tank) and with philosophers of science (Holton, Feyeraband, and Kuhn). In its preface, he indicated that his book was 'neither a textbook nor a collection of biographical notes nor even the study of priority questions' but in fact it has become a classic textbook for any student of quantum physics history (Simon, 2022, pp. 713–714).

He followed his *Conceptual Development* with a second book originated from his lectures in a graduate course on the history and philosophy of modern physics given at Columbia University in 1968, titled *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective* and having the stated objective to serve as an analysis of the interpretations of quantum mechanics as well as to be used as a guide to the literature of the subject (Jammer, 1974). Both texts have been fundamental in my understanding of the subject while researching and writing this thesis.

To close this section and as an introduction to the next, which deals with teaching, I will add as examples a few important books that, like Dirac's *Principles*, arose from their authors' experience as lecturers and were instrumental in research and pedagogy: Max Born's *Vorlesungen über Atommechanik* (Lectures on Atomic Mechanics) (Springer, 1925), and its second volume together with Pascual Jordan *Elementare Quantenmechanik* (Elementary Quantum Mechanics) (Springer, 1930); Born's *Probleme de Atomdynamik (Dreissig Vorlesungen gehalten in Wintersemester 1925/1926 am Massachusetts Institute of Technology)* (Springer, 1926 translated in English as Problems of Atomic Dynamics, MIT Press, 1926) which stemmed from a 10 lecture series given during an American tour (Simon, 2022, p. 720).

Teaching

The rapid expansion of ideas was fueled, among other factors, by the hold gained in teaching positions beyond the original triumvirate of Sommerfeld, Born, and Bohr. I mentioned earlier the growth of the teaching of quantum mechanics at Cambridge. In 1927-1928 there was a massive revision of the chairs of theoretical physics at different Universities. Heisenberg became professor at the University of Leipzig, and Gregor Wentzel (1898-1978) left Leipzig to succeed Schrödinger at the University of Zurich, and Schrödinger succeeded Planck at the University of

Berlin. Pauli went to Zurich ETH[40] and Peter Debye (1884-1966) left Zurich ETH to accept an experimental professorship at Leipzig. Leipzig and Zurich began to collaborate closely with Munich, Göttingen, Copenhagen, and with each other (Rechenberg, 1995, p. 219). Similarly, their students began to cross pollinate with examples like Block going from Leipzig to Zurich, getting his PhD with Heisenberg and becoming assistant and lecturer with work in between at Utrecht with Hendrik Kramers (1894-1952) in 1929-1930 and with Bohr in 1931-1932. Rudolf Peierls (1907-1995) left Munich in 1928 to be at Leipzig, however, he completed his PhD with Pauli at Zurich and then continued to Rome and Cambridge in 1932-1933. Sommerfeld and his teaching of the wave-mechanical methods made Munich an attractive place with Edward Condon (1902-1974) and Linus Pauling (1901-1994) arriving in 1926, and William Houston (1900-1968) and Carl Eckart (1902-1973) following in 1927. The Americans brought quantum mechanics back to the US in the early 1930s and made their departments and Universities blossom in a trend that continued for the rest of the century. Sommerfeld also fostered applied quantum mechanics in the field of astrophysics with one of his American students, Norris Russell (1877-1957) helping to institute the physics of stellar atmospheres (Rechenberg, 1995, p. 219). In 1933 another massive shift occurred in the teaching and student development in quantum mechanics and physics in general, stemming from the Nazi Party getting control of the Government in Germany, and the Jews becoming labelled as unworthy of public service. Bloch left Heisenberg, and Hans Albrecht Bethe (1906-2005) left Sommerfeld. Max Born and James Franck left Göttingen along with a significant fraction of their best students and collaborators. With enormous effort from friends and colleagues, particularly from Copenhagen, Cambridge, the US, and Russia, the many emigrants from Germany went on to establish other centres in Western Europe and America, with examples like Bethe and Herbert Fröhlich (1905-1991) going to Bristol Laboratory to do metal research, and Von Neumann and Wigner going to Princeton. Pauling started a school of quantum chemistry at Caltech; Slater along with several visitors from Sommerfeld started the solid-state group at MIT, and Isidor Rabi (1898-1988) brought the molecular-ray method to Columbia University. With the German theoreticians of nuclear physics

[40] (German) *Eidgenössische Technische Hochschule Zürich;* (English) *Federal Institute of Technology Zurich*

emigrating en masse, this brought the consequence that quantum mechanics and its applications expanded into a truly international science (Rechenberg, 1995, p. 220).

New Applications

Once the new ideas began to spread, they quickly began to influence interdisciplinary work in the period of 1925-1932. An example is how Heisenberg's work led to breakthroughs in the epistemology of chemical binding forces, beginning with Walyter Heitler (1904-1982) and Fritz London (1900-1954) deriving the *covalent* binding force, and their modified work by Friederich Hund (1896-1997), Erik Hückel (1896-1980), Robert Mulliken (1896-1986), and Linus Pauling becoming *quantum chemistry*[41]. Further work by London and others provided a quantum theory of the Van Der Waals forces[42] in chemistry (Rechenberg, 1995, p. 213). In another example, the theory of solid state began to get developed with Pauli solving the first problem in metal physics, that of temperature-independent paramagnetism of alkaline earths. Léon Brillouin (1889-1969) provided an easily visualisable model of electron propagation in crystal lattices, and together with the work of others made possible an explanation of the metals and insulators behaviour, as well as the properties of semiconductors (Rechenberg, 1995, p. 216).

To finish this section and as preamble to the *Adoption to Body of Knowledge - Connaissance Éclairante Phase* I want to add a couple of additional examples combining the points made in the teaching and applications sections here. The first one is Enrico Fermi (1901-1954). Fermi escaped the Italian racial laws targeting jews the same year he was awarded the Nobel prize in physics in 1938, and he went to the University of Chicago where he led the development of the first self-sustaining nuclear chain reaction in 1942 (Fermi, 1946). He later worked at Los Alamos, and after the war helped establish the Institute for Nuclear Studies in Chicago and made important contributions to the study of *pions* and *muons* in particle physics, along with a number of patents for use of nuclear energy (Bretscher and Cockcroft, 1955).

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[41] The mathematical treatment of molecules in both inorganic and organic chemistry

[42] Van der Waals forces are relatively weak intermolecular forces that occur between all molecules. They arise from temporary fluctuations in the electron distribution around molecules, creating brief dipoles that attract each other. While they're weaker than covalent or ionic bonds, they're important for many phenomena like why gases can be liquefied, how geckos can walk on walls, or why waxes and oils stick together.

The second example is J. Robert Oppenheimer. He obtained his PhD at Göttingen with Max Born in 1927 and developed the Born-Oppenheimer approximation for molecular wave functions and worked on the theory of positrons, quantum electrodynamics, quantum field theory, and the Oppenheimer–Phillips process in nuclear fusion in which a deuteron, entering a heavy nucleus, is split into proton and neutron, one of these particles being retained by the nucleus while the other is re-emitted. He brought back quantum mechanics to Berkeley where he became full professor in 1936 and he created his great School of Theoretical Physics, and he began to develop the field of astrophysics with his students, with significant contributions like the theory of cosmic ray showers, neutron stars and black holes before going to lead the Manhattan project (Bethe, 1968):

> In addition to this massive scientific work, Oppenheimer created the greatest school of theoretical physics that the United States has ever known. Before him, theoretical physics in America was a fairly modest enterprise, although there were a few outstanding representatives. Probably the most important ingredient he brought to his teaching was his exquisite taste. He always knew what the important problems were as shown by his choice of subjects… In his classroom teaching he always applied the highest standards. He was much influenced by Pauli's article in the *Handbuch der Physik* [Handbook of Physics] which provided the deepest understanding of quantum mechanics then and even now (Bethe, 1968, p. 396).

In the next section I will explicate the final epistemological evolution to become part of the Body of Knowledge.

Connaissance Éclairante - Adoption to Body of Knowledge

Becoming Knowledge

> I do believe that one can discern general themes in the history of discovery in science
>
> Abraham Pais

The first inceptions that caused the crisis in classical physics as well as the transition to modern atomic physics and quantum mechanics while fitting the definitions of *invention* and *discovery* explored earlier in this thesis, correspond to the Kuhnian discovery-*that* and a discovery-*what*, and yet, the interpretation and correct conceptualisation of those inventions and discoveries was not immediate, and frequently, did not originate in the discoverers themselves. As mentioned earlier in this chapter, after his discovery of the quantum theory, Planck did not understand or accept its consequences for years. In March 1905, Rutherford gave the Silliman Lectures at Yale in New Haven. The text of his lectures does not include a critical description of radioactive materials that he had discovered: the characteristic lifetime for each radioactive substance[43]. He also did not mention Planck's quantum of action discovered in 1900 or its implications (Pais, 2002, p. 129). In another example of this, Pais quotes an autobiographical note from Einstein:

> 'All my [Einstein's own] attempts ... to adapt to the theoretical foundation of physics to this [new type of, Pais' insertion] knowledge failed completely. It was as if the ground had been pulled out from under one, with no firm foundation to be seen anywhere, upon which one could have built'[44] In writing these lines he referred quite specifically to his early recognition of the mysteries revealed by Planck's discovery of the quantum theory (Pais, 2002, p. 130).

In the same theme, Einstein wrote of light as a quanta, however, it took him 11 more years to describe light as momentum quanta giving it the definition of a particle having energy and momentum, and it would take another 7 years for someone else, Compton, to do the experimental discovery of the *photon*, "an individual object, capable of exchanging energy and momentum with another particle (the electron)." (Pais, 2002, pp. 135–136).

The development of knowledge is not due to the interpretation of one individual, nor is it the result of a quick change of understanding and acceptance of the new entities, frameworks and language, Pais provides an interesting and illustrative anecdote. In the summer of 1931, Robert Oppenheimer was giving a lecture on Dirac's equation that was attended, among others, by Pauli and Kramers, when suddenly Pauli stood up and purposely approached the blackboard and grabbed a piece of chalk muttering that "everything is wrong anyway" and kept repeating it

[43] Rutherford discovered that each radioactive substance has a characteristic lifetime known as its *half-life*. This is the time it takes for half of a given amount of the radioactive material to decay into another element or isotope. His work showed that radioactive decay is a spontaneous and constant process unique to each substance, unaffected by external conditions like temperature or pressure. This discovery was fundamental in later understanding nuclear physics and the behaviour of radioactive elements.
[44] A. Einstein, "Autobiographical notes', in *Albert Einstein: philosopher scientist* , p.45, Ed. P.A. Schilpp, Tudor, New York 1949

in front of the chalkboard until Kramer asked him to give a chance to the speaker and Pauli sat back down. This lecture happened shortly after Dirac had proposed the anti-electron (later called *positron*), and although this might not have been in his mind during the lecture, he (Pauli) did not seriously consider the notion true before the discovery of the *positron* by Anderson (Pais, 2002, p. 130). “If, in the early years of this century, some physicists could not accept the quantum theory, that was because to them it seemed (as indeed it was) to be too complicated a hodgepodge of *ad hoc* rules” (Pais, 2002, p. 136) . The Copenhagen Interpretation marked a decisive break from classical deterministic physics, introducing probabilistic and non-deterministic principles as essential features of quantum mechanics. By synthesising concepts such as wave-particle duality, the uncertainty principle, and complementarity, it established a comprehensive theoretical framework that transformed both the theoretical foundations and experimental methodologies of atomic physics.

The interpretation's contemporary significance manifests across multiple domains. In current scientific practice, it serves as the primary pedagogical framework for quantum mechanics and guides experimental protocols, particularly in measurement theory. Its conceptual vocabulary—including terms such as wave function collapse, superposition, and complementarity—has become fundamental to both technical discourse and popular science communication. The interpretation's influence extends to emerging fields, informing developments in quantum biology, quantum computing, and quantum information theory, where it provides essential conceptual foundations for understanding phenomena such as entanglement and quantum communication.

Chapter 5 - Turing and Artificial Intelligence

> "No man adds very much to the body of Knowledge. Why should we expect more of a machine?" [Turing, Lecture to L.M.S. Feb. 20 1947 on ACE, p. 31] (A. Turing, 2004a, p. 394)

My third case study begins with Turing's work which constitutes the preliminary stage to the development of artificial intelligence and its acceptance as part of the body of knowledge. Rather than approaching AI through pure mathematical formalism, Turing developed his ideas through an innovative blend of mathematical logic and mechanical analogies. This distinctive approach emerged from both his institutional context at Cambridge and his personal intellectual style.

Ouverture Ontologique - Ontological Opening

Historical Environment; the sociological atmosphere

The 1930s in Britain represented a complex intersection of social upheaval, intellectual ferment, and looming international crisis that would profoundly shape both Alan Turing's work and the broader development of computing. This period, marked by economic uncertainty and growing political tensions, created a unique environment for scientific and mathematical innovation. Cambridge University, where Turing would spend his formative academic years, exemplified the British scientific establishment of the 1930s. The institution maintained its traditional collegiate structure while grappling with modernisation and new scientific developments. King's College, Turing's academic home, represented a more progressive element within Cambridge, known for its relatively liberal atmosphere and openness to new ideas in both science and society. The mathematical tradition at Cambridge during this period was particularly strong, building on the legacy of figures like G.H. Hardy (1877-1947) and J.E. Littlewood (1885-1977). The *Mathematical Tripos*, though somewhat reformed from its Victorian incarnation, remained a rigorous and prestigious programme that emphasised pure mathematics over applications. This environment would significantly influence Turing's approach to mathematical problems, encouraging both abstraction and innovation (Newman, 2013, pp. 3–12) (Hodges, 2014, pp. 103–117). The 1930s represented a crucial period in mathematical logic and foundations. The aftershocks of Gödel's incompleteness theorems, published in 1931, were still reverberating through the mathematical community. The Vienna Circle's logical positivism influenced British thought, while Ludwig Wittgenstein's (1889-1951) presence in Cambridge contributed to intense discussions about the nature of mathematical truth and meaning. The foundational crisis in mathematics had evolved from its early twentieth-century form but remained influential. Questions about the relationship between logic, computation, and mathematical truth – central to Turing's later work – were actively debated. The work of Bertrand Russell (1872-1970) on mathematical logic remained influential, though increasingly challenged by new developments in mathematical logic (Davis, 2018, pp. 121–126). While Britain lagged behind Germany and the United States in some areas of industrial technology, it maintained strengths in specific sectors. Telecommunications and radio research, particularly at institutions like the National Physical Laboratory, continued to advance. These developments would later prove crucial to computing and code-breaking efforts. The computing landscape of 1930s Britain still centred on human computers – individuals, often women, who performed calculations by hand or with mechanical calculators. Large-scale computation, when needed, was organised through these human computing bureaus. Mechanical calculating devices existed but were primarily used for business rather than scientific applications. The educational system that shaped Turing and his contemporaries remained highly selective. Public schools like Sherborne, which Turing attended, emphasised classical education while beginning to accommodate more modern scientific subjects. The path from public school to Oxbridge was well-established for academically gifted students, though access remained largely restricted by class and gender. Career opportunities for mathematicians typically led to academia, teaching, or actuarial work. Pure research positions were limited, and the practical applications of advanced mathematics were not yet fully appreciated. This would change dramatically with the approach of war, but in the 1930s, theoretical work often seemed removed from practical concerns (A. M. Turing, 2004, pp. 205–209). Turing began attending University at Cambridge in 1931. The academic environment at King’s where Turing did his undergraduate studies had some privileges, not the least due to John Maynard Keynes (1883-1946) and the fortune he had brought, but in particular it was characterised by acceptance and moral autonomy that extended to politics and sexuality (Hodges, 2014, pp. 90–92) “... although Alan’s self-contained nature placed him on the edge of King’s society, he was protected from the harshness of the outside world” (Hodges, 2014, p. 100).

Existing Paradigm; working within the accepted view

The intellectual landscape that Alan Turing encountered in the 1930s was dominated by fundamental questions about the nature of mathematics, mechanical computation, and the mind. These intersecting domains would prove crucial to the development of his revolutionary ideas about computability and intelligence, and thus I will describe the environment associated with each one of them.

Mathematics

Mathematics was influenced by David Hilbert's (1862-1943) ambitious programme to establish the complete formalisation of mathematics that stood as the dominant paradigm in mathematical foundations when Turing began his academic career. In his landmark 1900 address to the International Congress of Mathematicians, Hilbert declared that there were no unsolvable problems (Reid, 1970, pp. 69–73). Hilbert posted a list of what he labelled the most important problems of mathematics, instituting an ambitious programme that was a quest for absolute certainty in mathematics, and its key objectives included the demonstration that every mathematical statement could be either proved or disproved; and the development of a decision procedure (*Entscheidungsproblem,* German for "decision problem") for determining the truth or falsity of any mathematical statement (Hilbert, 1930) (Copeland, 2004, pp. 46–48). By the early 1930s, Hilbert's programme faced significant challenges. Central to this period was Kurt Gödel's (1906-1978) incompleteness theorems, which had a profound impact on the mathematical community and directly influenced Alan Turing's work. Kurt Gödel's incompleteness theorems in 1931 delivered what many viewed as a fatal blow. In the first

Incompleteness Theorem Gödel proved that in any consistent formal system capable of expressing basic arithmetic, there exist true statements that are unprovable within that system. In the second Incompleteness Theorem he also showed that such a system cannot demonstrate its own consistency. Gödel's theorems challenged the previously held belief that mathematics could be both complete and consistent, and they led to a foundational crisis in the mathematical community and originating intense debates about the nature of mathematical truth and proof. Philosophers and mathematicians like Ludwig Wittgenstein and members of the Vienna Circle were deeply engaged with these ideas (Davis, 2018, pp. 99–101) (Raatikainen, 2022, p. 5. The History and Early Reception of the Incompleteness Theorems).

The Fall of 1931 saw Turing at the start of his study in one of the world's most important centres for Mathematics, Cambridge, and it was there that he worked on two books that would be very influential on him; in 1932 with John von Newmann's (1903-1957) text on the foundational mathematics of Quantum Mechanics, *Mathematische Grundlagen der Quantenmechanik* (Neumann, 1996), and in 1933 with Bertrand Russell's text on the *Introduction to Mathematical Philosophy* (Russell, 1993), in addition to books on quantum mechanics by Heisenberg and Schrödinger (Hodges, 2014, p. 101). His graduation from Cambridge University in 1934 with an outstanding degree in mathematics was followed by a successful dissertation in probability theory which won him a Fellowship of King's College, Cambridge, in 1935 (Hodges, 2014, pp. 119–121). King's College provided Turing with a nurturing environment for homosexualism and unconventional opinions that proved to be essential later in the development of his ideas. He likely met von Newman in 1935 which started a series of important interactions that happened in a number of ways over the following years. Later, following von Newman's advice and support he was elected as a Visiting Fellow at Princeton University which was becoming the new Göttingen and found himself working at the side of some of the best mathematicians in the world, including von Newman, the British mathematician G. H. Hardy, and under the direction of the most respected of American logicians, Alonzo Church (1903-1995), who later made important contributions to theoretical Computer Science (Hodges, 2014, pp. 121–123).

In the Spring of 1935, Turing took a course on the foundations of mathematics with the topologist M. H. A. (Max) Newman (1897–1984), during which he was introduced to Gödel's work and to the problem of how the truth of a statement concerning infinitely many instances can be deduced using a finite number of deductive logic rules. An example of this problem would be to find how we can believe the truth of a statement such as that for all *a, b, c* and *c* not equal to *0*, if *a=b* , then *(a/c)=(b/c)*. In the example, the word all refers to infinite instances, however the conditional statement implies that it can be deduced with a countable number of rules of deductive logic. It was a Newman's lecture that prompted Turing into the research that resulted in the invention of the universal *Turing Machine*[45] (A. M. Turing, 2004, p. 206). Earlier during 1928 David Hilbert had specified three questions fundamental for Mathematics: Is Mathematics consistent? (is it free from contradictions?), Is Mathematics complete? (can every statement be proved or disproved?), and is Mathematics decidable? (is there a process that can decide if a statement is true or false?) (A. M. Turing, 2004, pp. 46–47). Up to that point there had been a great effort to find the foundations of Mathematics on Logic, with fundamental work on that area from Georg Cantor (1845-1918), Gottlob Frege (1848-1925), Bertrand Russell, Alfred North Whitehead (1861-1947), and others. Gödel's Incompleteness theorem from 1931 had shown that it was not possible to obtain both consistency and completeness. This means that there will always be statements about numbers that will not be possible to prove either as true or false using a finite number of statements previously accepted as true and a finite number of rules (Hodges, 2014, pp. 117–119). This left open the question of decidability, that is to say, is it possible to find a method or procedure that when applied to any given proposition it will decide whether the proposition is either provable or not? The proof of Gödel's theorem was the end point to Newman's lectures. It is at this stage where Turing reached the outer edge of knowledge with Newman discussing the possibility to find "a *mechanical process* which could be applied to a mathematical statement, and which could come up with the answer as to whether it was provable" (Hodges, 2014, p. 120). At that time, Turing decided to attack the third part of Hilbert's question, the problem of decidability (the *Entscheidungsproblem*), that is, if any given proposition is provable. Solving this problem was not that of traditional mathematical precise

[45] A *Turing Machine* is an abstract device that can compute anything that's algorithmically computable.

definition of, in this case, a method, but of a philosophical approach to a definition that was universal and invulnerable to attacks in its generality. He began working on the question of decidability on his own for a year until April 1936 when he finished “On Computable Numbers” (A. Turing, 2004b). Unbeknownst to Turing while he was working on the *Entscheidungsproblem*, there were two other approaches to the problem (one from Gödel and one from Church) and both trying to produce a definite, mathematically expressible definition of an “effectively calculable function” (Hodges, 2014, pp. 117–122). Turing’s cardinal work for computer science and later AI begins with his 1936 paper "On Computable Numbers" in which he introduced what is now called a *Turing Machine*: “It is possible to invent a single machine which can be used to compute any computable sequence” (A. Turing, 2004b, p. 68). He used it to prove certain numbers are not computable and ultimately to show the *Entscheidungsproblem* was unsolvable:

> The results of §8 [section 8: Application of the diagonal process in page 246] have some important implications. In particular, they can be used to show that Hilbert Entscheidungsproblem can have no solution (Turing, 1936, p. 259).

Computability

There are attempts to build machines that could compute before the twentieth century with examples of devices built by Blaise Pascal (1623-1662) and Gottfried Leibniz in the eighteenth century with the possibility of doing arithmetic calculations, but they were not programmable. In the nineteenth century Charles Babbage (1791–1871) designed a machine called the “Analytical Engine” that could do most computation functions and Ada Lovelace (1815–1852) allegedly devised programs for the Analytical Engine but it was never built (Nilsson, 2010, pp. 54–55). The concept of computation in the 1930s remained primarily tied to human calculation (Copeland, 2004, p. 40). There were attempts to mechanise scientific investigation such as the MIT machine created in the 1930s to analyse differential equations associated with physics problems connected to continuous functions, which implied working with rates of change in specific engineering or physics situations (Gleick, 2011, p. 172). The US in the 1930s began to receive an influx of physicists and mathematicians leaving Germany or visiting for work, and the locus of many of them was Princeton University and the Institute for Advanced Study to the point of surpassing the earlier talent pool at Göttingen, with examples like Hermann Weyl

(1885-1955), Albert Einstein, John von Newmann, and Kurt Gödel; UK mathematicians like G. H. Hardy and local figures like Alonzo Church, Stephen Kleene (1909-1994) and J. Barkley Rosser Sr. (1907-1989) who were brilliant students of Alonzo Church that became influential mathematicians later. Some of the problems they were working on were associated with algorithms[46] for calculations. Church introduced a novel notation associated with mathematical functions called *λ notation*[47], also known as *lambda calculu*s, which is a formal system for expressing computation based on function abstraction and application, however, he was not aware at that time of its connection with what was possible by using algorithms:

> It is clear that, in the case of any *λ*-definable function of positive integers, the process of reduction of formulas to normal form provides an algorithm for the effective calculation of particular values of the function (Church, 1936, p. 349).

Gödel introduced and lectured on a class of functions called *recursive*[48] which had the critical property of having values that could be computed with an algorithm (Davis, 2018, pp. 103–106). It is at this point where Turing begins to work on computable numbers, and Hilbert's *Entscheidungsproblem*. This is how Turing introduced his paper:

> The "computable" numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means. Although the subject of this paper is ostensibly the computable *numbers*, it is almost equally easy to define and investigate computable functions of an integral variable or a real or computable variable, computable predicates, and so forth… According to my definition, a number is computable if its decimal can be written down by a machine (Turing, 1936, p. 230) (A. Turing, 2004b, p. 58).

The problem asked for the possibility to find an algorithm that can take any mathematical statement as input and definitively answer whether it is possible to prove that statement, this is

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[46] "Intuitively, an algorithm is a set of instructions allowing the fulfilment of a given task. Despite this ancient tradition in mathematics, only modern logical and philosophical reflection put forward the task of providing a definition of what an algorithm is, in connection with the foundational crisis of mathematics of the early twentieth century… The notion of effective calculability arose from logical research, providing some formal counterpart to the intuitive notion of algorithm and giving birth to the theory of computation." (Angius, Primiero and Turner, 2024, p. 3. Algorithms)

[47] Lambda notation is a formal system that became one of the foundations of computer science and functional programming, and its basic idea is that a *λ* expression represents a function without needing to give it a name. It shows a) what goes into the function (the input parameter), and b) what the function does with that input. This is an example: λx.x+1 This represents a function that takes a number x and adds 1 to it. In modern programming, this would be like writing def f(x): return x + 1 # in Python or x => x + 1 # in JavaScript.

[48] "Such functions take their name from the process of *recursion* by which the value of a function is defined by the application of the same function applied to smaller arguments" (Dean and Naibo, 2024, p. 1)

to say, imagine a theoretical computer that could take any mathematical statement (like "there are infinitely many prime numbers") an always give you a yes/no answer about whether that statement can be proven true or false, and do this in a finite amount of time. In 1936, both Alan Turing and Alonzo Church independently proved that such an algorithm cannot exist. Church's recursive functions and Turing's functions that can be computed with a machine are thought to be equivalent in what is called the "Church–Turing Thesis" in an unproven claim for which no counterexample has ever been found (Nilsson, 2010, p. 56).

Another important and seminal contemporary contribution to the development of practical computation was Claude Shannon's Master's thesis at MIT "A Symbolic Analysis of Relay and Switching Circuits" (Shannon, 1938, pp. 713–723). In his paper, Shannon demonstrates how any circuit can be represented by a set of equations corresponding to ordinary algebraic algorithms and producing results calculated by the same propositions used in the symbolic study of logic (Shannon, 1993a, p. 471). This was a fundamental paradigm shift in the design of electrical and electronic circuits that moved closer the possibility of a physical construction of a computer. In the following decade, Shannon would cross paths with Turing and discuss ideas regarding mechanical intelligence while they were both working at Bell Laboratories in New Jersey. From that experience, Shannon later proposed to him to collaborate on the field, as we will see later in this chapter. Considered to be the event originating artificial intelligence as a field of study, John McCarthy (1927-2011), Marvin Minsky, Nathaniel Rochester (1909-2001), and Claude Shannon organised a conference at Dartmouth[49] in the summer of 1956 that brought together researchers exploring the possibility of developing machines that could simulate human intelligence. Sadly, by that time they organised the conference, Alan Turing had died.

Turing's conceptual framework for computation provided the theoretical foundation upon which modern computer architecture would be built. His 1936 paper introducing the Turing Machine concept (A. Turing, 2004b) established the fundamental principles of computation, though the path from theoretical insight to practical implementation would require additional innovations. Von Neumann's seminal "First Draft of a Report on the EDVAC" (1945) represents a

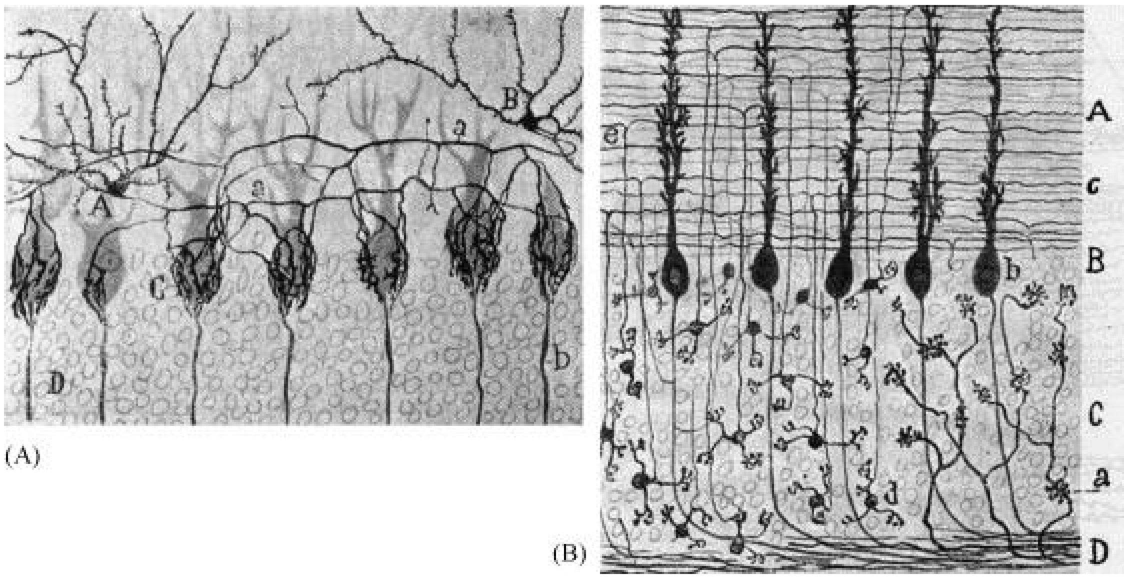
[49] Source: https://home.dartmouth.edu/about/artificial-intelligence-ai-coined-dartmouth

crucial bridge between Turing's abstract theory and practical computing architecture. His key contribution was synthesising existing concepts into a coherent architectural framework that unified programme storage, memory organisation, and computational control. The resulting "von Neumann architecture" transformed Turing's theoretical insights into implementable engineering principles through several key innovations: the stored programme concept, sequential memory organisation, centralised processing components, binary logic implementation, and systematic programme flow control. The significance of von Neumann's synthesis extends beyond its technical details; it created a conceptual architecture that separated software development from hardware implementation, establishing the foundation for modern computing paradigms. While these ideas built upon Turing's theoretical work and concurrent engineering developments at the Moore School, von Neumann's systematic exposition created the blueprint that would shape computer science's development as a discipline. This transformation from theoretical computation to practical architecture exemplifies how abstract mathematical concepts can evolve into engineering frameworks that fundamentally reshape technological possibilities.

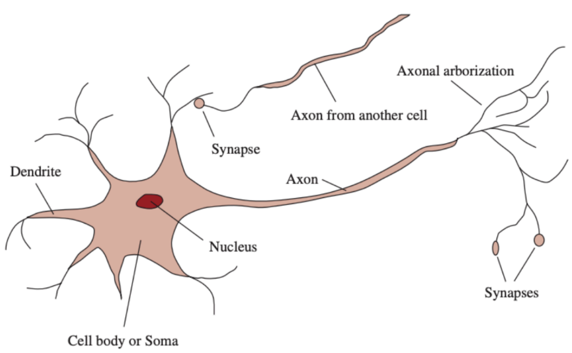
Mind

For millennia we have attempted to analyse and codify intelligence and the process of cognition. We can find a beginning with Aristotle’s logic and his syllogisms as replicable templates for patterns of reasoning, and continue later with Leibniz’s attempts to develop a language that could formulate all human knowledge, to Thomas Hobbes (1588–1679) as “a sort of empiricist, in that he thinks all of our ideas are derived, directly or indirectly, from sensation” (Duncan, 2024, p. 2.1 Sense and Imagination), and then with George Boole (1815–1864) and his logical principles of human reasoning represented in mathematical form, continuing to Friedrich Ludwig Gottlob Frege (1848–1925) and his system in which propositions and their components could be represented in graphical form (Nilsson, 2010, p. 33), and after that we arrive at Turing's work. In the early twentieth century, the Spanish neuroanatomist Santiago Ramón y Cajal (1852–1934) won the 1906 Nobel Prize in Physiology or Medicine for his work on the description and structure of the nervous system and the neurons as non-continuous system and establishing the information flow within the neurons (Y Cajal, 1888). Reproduced by

Lopez-Muñoz *et al*, the original drawings of neurons in Ramon y Cajal cardinal publication can be seen here below (López-Muñoz, Boya and Alamo, 2006, p. 397):



"The *neuron doctrine*[50] [my emphasis] constitutes the cornerstone on which, throughout the 20th century, all the neuroscientific disciplines were constructed" (López-Muñoz, Boya and Alamo, 2006, p. 391). The figure on the left represents the structure of a neuron (Russell and Norvig, 2022, p. 30). This intellectual environment provided both the tools and the problems that would guide Turing's revolutionary work and its evolution beyond its original conception. His synthesis of these various strands of thought – mathematical logic, mechanical computation, and questions about the nature of intelligence – would transform understanding of computation and lay the groundwork for the modern computer age and the expansion of the field of Artificial Intelligence.



[50] Ramón y Cajal's was the seminal work that originated the so-called *neuron doctrine* claiming that "… the various activities of the brain, including perception and thinking, are the result of all of [the] neural activity…" (Nilsson, 2010, p. 34)

Wicked Problems

Alan Turing's groundbreaking work in the 1930s and 1940s is situated in an intellectual landscape that presented multiple interconnected challenges that defied isolated solutions and it exemplified the characteristics of a quintessential *wicked problem* - one that defies complete definition, involves multiple stakeholders with competing perspectives, and resists definitive solutions, has no stopping rule, and solutions are not true-or-false but rather better-or-worse. This in short, is the *wicked problem* that Turing addressed most of his life:

> It is often supposed that computers began with heavy arithmetic, and that with this successfully achieved, computer scientists wandered to more ambitious fields. This may be so of others, but is quite untrue of Turing, who had *always* been *concerned with modelling the human mind* [my emphasis] (Hodges, 2021, p. 49).

The *Entscheidungsproblem*, or "decision problem," served as a focal point for deeper questions about the relationship between mechanical calculation and human thought, revealing fundamental uncertainties about the nature of computation and intelligence. The mathematical context was dominated by Hilbert's program, which sought absolute certainty in mathematics through complete formalisation. This apparently straightforward goal became increasingly complex as it intersected with questions of mechanical computation and the nature of proof. Gödel's incompleteness theorems had already demonstrated that mathematics could not be both complete and consistent within any formal system capable of basic arithmetic. This created the first layer of the *wicked problem*: how could one develop a rigorous foundation for mathematics when the very system used to establish that foundation was inherently incomplete?

Turing's approach to this aspect was revolutionary precisely because he reconceptualised the problem itself. Rather than attempting to directly solve Hilbert's *Entscheidungsproblem*, he reframed the question in terms of mechanical computation. His 1936 paper "On Computable Numbers" introduced the concept of the Turing Machine not merely as a mathematical construct but as a bridge between abstract mathematics and physical computation. This exemplifies a key characteristic of *wicked problems*: the solution approach fundamentally alters how we understand the problem itself. The practical computing challenges of the era formed another layer of complexity. While mechanical calculators existed, they were not programmable in any meaningful sense. The concept of computation itself was primarily associated with human calculation. The physical implementation of computational devices faced both theoretical and engineering challenges, as evidenced by the unfinished work of Babbage and Lovelace in the previous century. Turing's theoretical work provided a framework for understanding computation that would later prove crucial for practical implementation, but it also raised new questions about the relationship between abstract mathematical models and physical machines. The philosophical questions about mind and intelligence added yet another dimension to this *wicked problem*. The long tradition of attempting to formalise human reasoning, from Aristotle's syllogisms to Boole's logical algebra, suggested that mental processes might be reducible to formal operations. The emerging understanding of neurons as biological computing elements, particularly through the work of Ramón y Cajal and later McCulloch and Pitts, raised new questions about the relationship between biological and mechanical computation. The convergence of these ideas addressed a fundamental *wicked problem* that had puzzled philosophers and scientists for centuries: Understanding and replicating intelligence itself. This is best summarised by Turing's introduction to his paper on Computer Machinery and Intelligence: "I propose to consider the question, 'Can machines think? This should begin with definitions of the meaning of the terms 'machine' and 'think'." (Turing, 1950, p. 433).

Turing's groundbreaking contributions to artificial intelligence exemplify the interconnected nature of wicked problems in scientific advancement. His work demonstrates how fundamental challenges in mathematical logic, practical computing, and theories of mind converge to demand holistic solutions that transcend traditional disciplinary boundaries. Through his response to the Entscheidungsproblem, which led to the conceptualisation of the universal Turing Machine, he established a theoretical framework that simultaneously advanced mathematical logic and laid the foundation for modern computing. This theoretical work, when applied at Bletchley Park and the National Physical Laboratory, illustrated the practical implications of his abstract insights.

The recursive nature of these challenges manifests in how solutions in one domain inevitably generate novel questions in others, exemplified by how developments in practical computing raised fundamental questions about the relationship between human and machine intelligence. Shannon's implementation of Boolean logic in electronic circuits further demonstrates this interconnectedness, advancing both practical computing capabilities and theoretical models of neural networks. Turing's universal Machine concept serves as a paradigmatic example of how solutions to wicked problems must operate across multiple levels simultaneously, functioning as a mathematical proof, a blueprint for computing devices, and a model for understanding information processing in both mechanical and biological systems. The persistent relevance of his foundational queries in contemporary artificial intelligence and cognitive science underscores how these *wicked problems* resist definitive resolution, instead generating new questions as our understanding evolves. His work ultimately necessitated what the *o-é-c model* terms an "intellectual leap," establishing a new conceptual framework that would enable subsequent revolutionary developments in computing and artificial intelligence.

Intellectual Leap

The arc from Turing's computable numbers to mechanical intelligence and beyond is a stream of interconnected intellectual leaps that are indivisible from the ontological spaces created by his work and ideas. In his bibliographic memoir of the Fellows of the Royal Society, Max Newman attributes Alan Turing with the abilities of "a mathematical analyst" and of "a natural philosopher full of bold ideas" (Newman, 1955, p. 253). He had those and more. Turing's work stemmed from two uniquely personal characteristics, the first one being his character as an outsider trying to function in the mainstream, and the second one being his intellectual interests and approaches to problem solving. Alan Turing achieved everything he did, and we owe him so much to him and his work because he was both a mathematician *and* a natural philosopher. Prior to University, Turing attended preparatory schools and later Sherborne where he showed the first signs of those two characteristics; he found homosexual love while immersed in a social system that was opposed to it and prosecuted it (Hodges, 2021, p. 6), and an academic environment bent on providing him with a well-rounded education in which he selectively focused on science and mathematics while developing a problem-solving strategy that started from first principles instead of building from others' work and ideas, and that drew pleasure from the combination of theories and practical experimentation that he could perform on his own (Turing, 2013, p. 5).

First Leap

Cambridge's mathematics Tripos equally emphasised both pure mathematics grounded in absolute logical deduction, and applied mathematics focused on developing an interface between mathematics and fundamental and theoretical science, which meant physics in particular. With this approach and with the output of P.A.M Dirac and others, Cambridge's work was second to that of Göttingen where much of quantum mechanics theory had developed, Turing, however, gravitated towards pure mathematics as a way to abstract himself from the greater difficulties in the world. At Kings, he began to develop relationships, both sexual and intellectual with others (Hodges, 2014, pp. 74–81). Since his early high school years, Turing's intellectual approach showed his indifference to the mainstream and was connected to his own alienation due to his homosexuality, and a fascination for the contrast between the abstract and the physical. He approached problems from a pure mathematician's perspective and free reign of thought and then would try to find there was reflection of the answers in nature. By 1936, he was deeply interested in machines and their possibilities, which was highly unusual for a mathematician at Cambridge, as Turing was very interested in both philosophy and practical engineering to the point of writing his mother about it (Hodges, 2021, p. 37).

Working on the *decidability* question within the mathematical paradigm of the time (like Hilbert's programme represented or Cambridge approach to mathematics emphasised) implied to look for a method that could be applied to a stated proposition and decide if such statement was provable, however, Turing was working in isolation, disconnected from the mainstream mathematics and he was unaware of Gödel's or Church's approach that relied on established mathematical analysis. Instead, Turing approached the problem from an additional perspective, a philosophical one, and he reframed the question in terms of computable numbers instead of pure proofs. Labouring by himself until April 1936 he made his first major intellectual leap: the *entscheidungsproblem* was really an implementation of a new idea, that of *computability*: “The ‘computable’ numbers maybe described briefly as the real numbers whose expressions as a decimal are calculable by finite means” (Turing, 2013, p. 16). In doing so he formulated the details of a new framework, the process of a computer:

> We may compare a man in the process of computing a real number to a machine which is only capable of a finite number of conditions $q_1, q_{2,\ldots} q_R$, which will be called “*m*-configurations” (Turing, 1936, p. 231) (Turing, 2013, p. 17).
>
> Computing is normally done by writing certain symbols on paper. We may suppose this paper is divided into squares like a child's arithmetic book. In elementary arithmetic the two-dimensional character of the paper is sometimes used. But such a use is always avoidable, and I think that it will be agreed that the two-dimensional character of paper is no essential of computation. I assume then that the computation is carried out on one-dimensional paper, *i.e.* on a tape divided into squares…The behaviour of the computer [a human being performing the operations] at any moment is determined by the symbols which he is observing, and his "state of mind" at that moment (Turing, 1936, pp. 249–250).

This is the cardinal significance of Turing’s *movement of thought*[51]: In an analogue to the *gedankenexperiments* (thought experiments) used by Albert Einstein in discussing conceptual rather than actual experiments in the development of relativity and the discussions with Niels Bohr on quantum mechanics, Turing developed a *gedankenexperiment* to describe the concept of computation and to model an intelligent action performed by a human being. There were no machines capable of those functions at that time, Turing is not simply solving Hilbert’s problem within the constraints of the existing mathematical paradigm, nor is he writing a mainstream mathematical paper full of formalism, he is ideating a different space in which the notion of cognition and problem solving are the subject to be addressed, and he does so by modelling a human mind and initiating a thread that will continue and find further expression later in his ideas on mechanical intelligence.

[51] “The creation of a new hypothesis is not the product merely of logical rules. The creative process uses epistemological heuristic stratagems to generate conjectures with the hope of understanding the nature of the problem and in the process eventually to create the new revolutionary science” (Heelan, 2016, p. 32)

Turing introduced new entities by asking the questions of what does it mean to specify the infinite in terms of the finite, or to calculate an infinite number by defining the concept of a Turing machine, a theoretical model that defines a simple and powerful abstraction of computation designed to formalise the concept of algorithmic processes and to determine the limits of what can be computed. Turing defined the machine components thus (Turing, 1936, pp. 231–233, 243–246):

Infinite Tape: The machine has an infinitely long tape divided into discrete cells. Each cell can hold a symbol from a finite alphabet. The tape serves as both the input and unbounded memory storage.

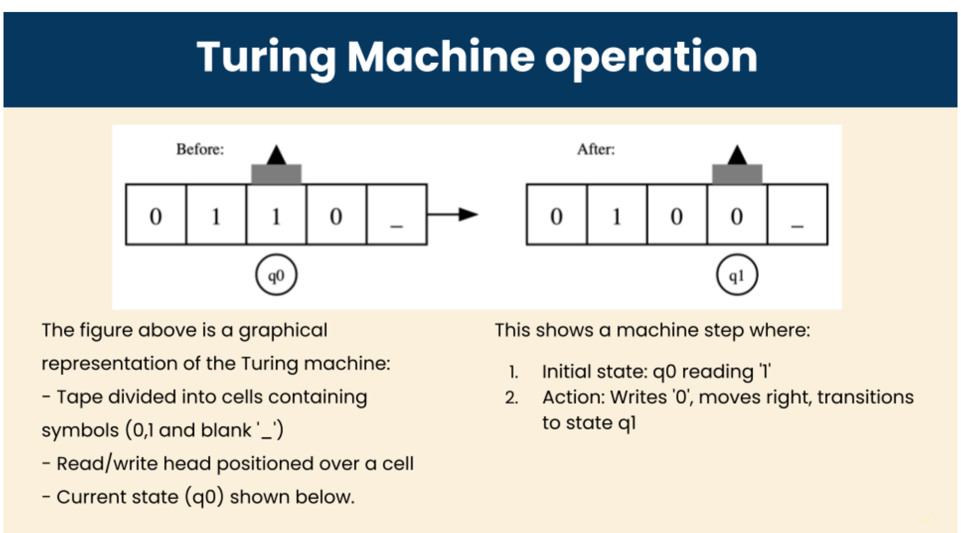
Tape Head: A read/write head moves along the tape, one cell at a time. It can read the symbol in the current cell, write a new symbol (possibly overwriting the existing one), and move left or right.

Finite Set of States: The machine has a finite number of states, including a start state and one or more halting (accepting or rejecting) states.

Transition Function: This function dictates the machine's actions. Given the current state and the symbol under the tape head, it specifies:

- What symbol to write in the current cell.
- Whether to move the tape head left or right.
- Which state to transition into next.

The figure below shows how the Turing machine would work:



Turing made two crucial discoveries about his machines: first, he showed that for any computable function (anything that can be calculated by following steps), you could build a specialised Turing machine to compute it, a concept that serves as a universal model for defining computability and complexity. More revolutionary was his second insight: instead of needing different machines for different problems, you could have one "universal" machine that could simulate any other Turing machine. It would work by reading a description (program) of any specialised machine from its tape, and by using this description to mimic exactly what that specialised machine would do. This second discovery is essentially the blueprint for modern computers - a single general-purpose machine that can run different programs to solve different problems, rather than needing a separate machine for each task, thus providing the groundwork for modern computing theory, and influencing the development of actual computers and programming languages. The 1936 intellectual leap in Turing is connected to the mechanisation and quantisation of the continuum formed by the mathematical space of Hilbert's problem, as every step of the Turing machine is an indivisible element in the composition of all possible mathematical operations. It is also a product of his tendency of finding connection between the abstract and the physical, and of his work associated with mainstream but not part of it in an analogue way to how his homosexual nature found refuge at Kings but not part of Bloomsbury and Keynes social circles. He knew of both but was more comfortable following his own search. He did benefit from the exposure to some of the best in the world and the frontier of knowledge like in Newman's lectures or later work with von Neumann and church at Princeton. This abstraction from mainstream continued in his work at Bletchley on Enigma. He was not working completely within the paradigm, he was working on his own created ontological space. There are no extant records, accounts, or surviving drafts of his thought process while working on the *entscheidungsproblem* (the work developed in isolation, and Newman only saw the final paper when it was completed) however, we can have a glimpse on his method from his text:

> No attempt has yet been made to show that the "computable" numbers include all numbers which would naturally be regarded as computable. All arguments which can be given are bound to be fundamentally, appeals to intuition, and for this reason rather unsatisfactory mathematically. The real question at issue is "What are the possible processes which can be carried out in computing a number?"

> The arguments which I shall use are of three kinds.
> (a) A direct intuition.
> (b) A proof of the equivalent of two definitions (in case the new definition has a greater intuitive appeal).
> (c) Giving examples of large classes of numbers which are computable.
> Once it is granted the computable numbers are all "computable" several other propositions of the same character follow. In particular, it follows that, if there is a general process of determining whether a formula of the Hilbert function calculus is provable, then the determination can be carried out by a machine (Turing, 1936, p. 249).

Turing begins with *a priori* conceptions -a direct intuition- and systematises them to find objectivisation of the ideas in a physical space. We can see there the elements described before: working from basic principles and, the reframing of the problem, the provision of a path to practical experimentation to support intuition.

Second Leap

The second step in the thread towards the analysis of the mind and mechanical intelligence is Turing's 1938 PhD thesis at Princeton: "Systems of Logic Based on Ordinals" (Turing, 1939). Gödel posited that it was not possible to intuit the truth of an unprovable proposition and at the same time to follow a set of given rules. Turing decided to study the possibility of formalisation of the actions of the mind that are not those of an algorithmic nature, that is, those arising from creativity and originality. "Turing focused on the action of seeing the truth of one of Gödel's unprovable assertions" (Hodges, 2021, p. 29). He asked if the theory of *ordinal numbers*[52] can contain a rule that could, knowing that an uncomputable number cannot be transformed into a computable one, use ordinal logic to bring an increased level of order to it. This is how Turing presented the goal in his paper:

> The purpose of introducing ordinal logics was to avoid as far as possible the effects of Gödel's theorem. It is a consequence of this theorem, suitably modified, that it is impossible to obtain a complete logic formula, or (roughly speaking now) a complete system of logic. We were able, however, from a given system to obtain a more complete one by the adjunction as axioms of formulae, seen intuitively to be correct, but which the Gödel theorem shows are unprovable in the original system; from this we obtained a

[52] The theory of *ordinal numbers* is the study of the rules and ways in which an infinite sequence can be expressed and organised

> yet more complete system by a repetition of the process, and so on. We found that the repetition of the process gave us a new system for each C-K ordinal formula. We should like to know whether this process suffices, or whether the system should be extended in other ways as well (Turing, 1939, p. 198).

Gödel's Incompleteness Theorem showed that in any sufficiently complex formal system of mathematics, there will always be true statements that cannot be proven within that system. What Turing is describing is a clever way to work around this limitation, he suggests that when we find one of these unprovable-but-true statements in our system, we can simply add it as a new axiom (a basic assumption we take as true). This gives us a stronger system that can prove more things. But then, by Gödel's theorem, this new system will also have its own unprovable-but-true statements. So we can repeat the process - add those new unprovable statements as axioms, creating an even stronger system, and so on. Turing discovered that this process of repeatedly strengthening the system could be organised using what are called Church-Kleene ordinal numbers (C-K ordinal formulas in his terminology). These are special numbers that extend beyond the regular counting numbers, allowing us to keep track of this infinite process of strengthening our logical systems. Each time we strengthen the system we move to the next ordinal number. The key question Turing posed at the end of the quoted text is whether this ordinal-based approach of repeatedly adding unprovable statements as new axioms is sufficient to capture all mathematical truth, or whether we need other methods as well. In other words, are there mathematical truths that would remain beyond our reach even after applying this process of strengthening our system through all possible ordinal numbers? This reflects Turing's deep interest in understanding the fundamental limits and possibilities of mathematical reasoning.

Turing reached quite a sobering conclusion in his paper. He demonstrated that ordinal logics, while interesting and powerful, did not fully solve the incompleteness problem as hoped. He showed that there were still fundamental limitations to what ordinal logics could achieve. Specifically, Turing proved that there could be no mechanical (what we would now call algorithmic) way to determine whether statements in an ordinal logic system were true. In modern terms, he showed that the problem of determining whether a formula is provable in an ordinal logic system remains undecidable. He also demonstrated that for any ordinal logic system, there would always be true arithmetic statements that remained unprovable in that system, even after extending it through ordinal progressions. In his paper, he says:

> Among the questions that we should now like to ask are
> (a) Are there any complete ordinal logics?
> (b) Are there any complete invariant ordinal logics?
> To these we might have added "are all ordinal logics complete? "; but this is trivial; in fact, there are ordinal logics which do not suffice to prove any number-theoretic theorems whatever. We shall now show that (a) must be answered affirmatively (Turing, 1939, p. 201).
>
> The question (b) must be answered negatively. Much more can be proved, but we shall first prove an even weaker result which can be established very quickly, in order to illustrate the method. I shall prove that an ordinal logic Λ cannot be invariant and have the property that the extent of $\Lambda(\Omega)$ is a strictly increasing function of the ordinal represented by Ω (Turing, 1939, p. 203).
>
> …it is sufficiently general to show that, with almost any reasonable notation for ordinals, completeness is incompatible with invariance (Turing, 1939, p. 209).

This was a somewhat disappointing result, as it showed that even this sophisticated approach of using ordinals to build increasingly powerful logical systems couldn't completely circumvent Gödel's incompleteness results. The fundamental barriers to complete mathematical knowledge that Gödel had identified proved to be remarkably robust, resisting even this creative attempt to work around them. This work by Turing helped establish that the limitations identified by Gödel were not just artifacts of particular formal systems, but reflected deeper, unavoidable constraints on formal mathematical reasoning itself. The paper demonstrated Turing's remarkable ability to combine deep mathematical insight with precise formal analysis, characteristics that would later prove crucial in his work on computability and artificial intelligence. We can have a glimpse of Turing's thoughts in 1939 before he returned to England and the start of the war with Germany in September which became the catalyst for him to move into the Government Code and Cypher School at Bletchley Park:

> Mathematical reasoning may be regarded rather schematically as the exercise of a combination of two faculties, which we may call *intuition* and *ingenuity*. The activity of the intuition consists in making spontaneous judgments which are not the result of conscious trains of reasoning. These judgments are often but by no means invariably correct (leaving aside the question what is meant by "correct"). Often it is possible to find some other way of verifying the correctness of an intuitive judgment…
> The exercise of ingenuity in mathematics consists in aiding the intuition through suitable arrangements of propositions, and perhaps geometrical figures or drawings. It is intended that when these are really well arranged the validity of the intuitive steps which are required cannot seriously be doubted (Turing, 1939, pp. 214–215).

Third leap

The progression led him to the interest behind another intellectual leap which happened in the 1940s; and the ideas on mechanical intelligence that would be exemplified by his later conversations with Shannon, his lectures on it and the paper in *MIND* as we will see later in this section. The war years marked a crucial transition in Turing's thinking about machine intelligence. His experience at Bletchley Park, while classified, proved transformative for his conceptualisation of mechanical computation. By 1941, his work on cryptanalysis had demonstrated how machines could produce sophisticated results through relatively straightforward mechanical processes. This practical experience shifted his theoretical perspective on machine capabilities, leading him to recognise that computational systems could exhibit complex behaviours without explicit programming for each outcome. Rather than viewing this as diminishing the significance of human intelligence, Turing began to see it as revealing new possibilities for mechanical intelligence. By the war's end, he had moved beyond purely theoretical constructs to actively pursuing the practical implementation of "building a brain," marking a crucial shift from abstract mathematical theory to applied artificial intelligence (Hodges, 2021, pp. 42–44). Beginning in 1941, he was particularly interested in how machines might solve problems by systematically searching through possible solutions; using "heuristics" (rules of thumb or practical guidelines) to guide this search; how to make machines play chess; and on the concept of machine learning. While working at Bletchley Park, he shared these ideas with his colleagues. He even wrote and distributed what was almost certainly the first-ever paper about artificial intelligence - though unfortunately this document has been lost to history. By 1948 he posited that "intellectual activity consists mainly of various kinds of search", eight years earlier than the independent same ideation by Herbert Simon and Allen Newell which became one of the central tenets of AI going forward (Copeland, 2006, pp. 353–355). At the end of 1945, Turing joined the Mathematics Division of the National Physical Laboratory (NPL) where he wrote and submitted in 1946 a report to its Executive Committee titled "Proposals for the Development of an Automatic Computing Engine (ACE)" which was the first thorough design for an electronic stored-programme digital computer[53] (A. M. Turing, 2004, pp. 362–366). Turing equated his design and work on the ACE to building a brain, and the report contained the earliest extant comments on artificial intelligence posted by Turing (A. M. Turing, 2004, p. 374). In the section describing the scope of the machine (section 8), Turing remarked:

> 'Can the machine play chess?' It could fairly easily be made to play a rather bad game. It would be bad because chess requires intelligence. We stated at the beginning of this section that the machine should be treated as entirely without intelligence. There are indications however that it is possible to make a machine display intelligence at the risk of making occasional serious mistakes. By following up this aspect the machine could probably be made to play very good chess (Turing, 1946, p. 16) (A. Turing, 1992, p. 41).

That text gives us a glimpse of Turing's next intellectual leap. Although not explicit in Turing's quoted text above, the wording he chose suggests that when the level of expectation of failure is similar to that of a human, that is, when the possibility of "occasional serious mistakes" is allowed, then it is possible to show a behaviour that requires intelligence (playing chess). The thought of adjudicating the nature of intelligence to a performance that is indistinguishable from that of a human is an idea that will be explored later in his 1950 paper "Computing Machinery and Intelligence" (Turing, 1950).

Fourth Leap

In 1947 Turing applied for a one-year Sabbatical leave at Cambridge where he planned to follow his thoughts in the direction of AI. Copeland quotes a 1947 letter from Sir Charles Galton Darwin (1887-1962), Turing's Supervisor at the NPL, containing Darwin's description of Turing's purpose during his leave:

[53] Von Neumann's earlier report for the EDVAC in May 1945 did not contain much engineering detail, in particular regarding electronic circuitry (A. M. Turing, 2004, p. 365)

> To extend his work on the machine [the ACE, Copeland's note] still further towards the biological side. I can best describe it by saying that hitherto the machine has been planned for work equivalent to that of the lower parts of the brain, and he [Turing, Copeland's note] wants to see how much a machine can do for the higher ones; for example, could a machine be made that could learn by experience? This will be theoretical work, and better done away from here (Copeland, 2004, p. 400).

By summer of 1948, Turing had written a far-sighted report with the first Turing's programme on Artificial Intelligence. The title was "Intelligent Machinery", and it included the first introduction of the *Turing Test*, the imitation game made famous in his 1950 paper. He approached the work as he had done before, from first principles and not from the work of others. As a curious note, he only has 3 references at the end of the paper: 1 from Alonzo Church from 1936 titled "An Unsolvable Problem of Elementary Number Theory" (Church, 1936), 1 from Gödel from 1931, the first one on incompleteness titled "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I" (On Formally Undecidable Propositions of Principia Mathematica and Related Systems I), and his own paper "On Computable Numbers, with an Application to the Entscheidungsproblem" (Turing, 1936). This is Turing's abstract for "Intelligent Machinery":

> Abstract
> The possible ways in which machinery might be made to show intelligent behaviour are discussed. The analogy with the human brain is used as a guiding principle. It is pointed out that the potentialities of the human intelligence can only be realised if suitable education is provided. The investigation mainly centres round an analogous teaching process applied to machines. The idea of an unorganized machine is defined, and it is suggested that the infant human cortex is of this nature. Simple examples of such machines are given, and their education by means of rewards and punishments is discussed. In one case the education process is carried through until the organization is similar to that of an ACE.
>
> I propose to investigate the question as to whether it is possible for machinery to show intelligent behaviour. It is usually assumed without argument that it is not possible. Common catch phrases such as 'acting like a machine', 'purely mechanical behaviour' reveal this common attitude… [he then presents a number of objections] (A. M. Turing, 1992, p. 107).

> … In this section I propose to outline reasons why we do not need to be influenced by the above-described objections… [he proceeds to refute the earlier objections, followed by the ideas on AI implementation in the rest of the paper] (A. M. Turing, 1992, p. 108).

The paper contained a number of ideas that proved foundational for the field of AI:

The states of machines and a systematic framework

Turing was concerned about the systematisation of the framework describing *Intelligent Machinery*. This is how Turing introduces his ideas in the paper:

> It will not be possible to discuss possible means of producing intelligent machinery without introducing a number of technical terms to describe different kinds of existent machinery.
>
> '*Discrete*' and *'continuous' machinery*. We may call a machine 'discrete' when it is natural to describe its possible states as a discrete set, the motion of the machine occurring by jumping from one state to another. The states of 'continuous' machinery on the other hand form a continuous manifold, and the behaviour of the machine is described by a curve on this manifold.
>
> All machinery can be regarded as continuous, but when it is possible to regard it as discrete it is usually best to do so. The states of discrete machinery will be described as 'configurations'.
>
> *'Controlling'* and *'active' machinery*. Machinery may be described as 'controlling' if it only deals with information.
>
> In practice this condition is much the same as saying that the magnitude of the machine's effects may be as small as we please, so long as we do not introduce confusion through
>
> Brownian movement, etc. 'Active' machinery is intended to produce some definite physical effect.

Examples	A Bulldozer	Continuous Active
	A Telephone	Continuous Controlling
	A Brunsviga	Discrete Controlling
	A Brain (probably)	Continuous Controlling, but is very similar to much discrete machinery
	The ENIAC, ACE, etc.	Discrete Controlling
	A Differential Analyser	Continuous Controlling.

> We shall mainly be concerned with discrete controlling machinery (A. M. Turing, 1992, pp. 109–110).

This passage is remarkably significant as it lays several foundational conceptual distinctions that would prove crucial for computer science and AI. First, Turing establishes the fundamental distinction between *discrete* and *continuous* systems. This dichotomy became essential for understanding different approaches to computation and intelligence. By defining discrete systems as those that jump between distinct states, while continuous systems move smoothly through a continuous manifold, Turing provided a framework for classifying different types of computational processes. This distinction remains relevant today in debates about whether human cognition and artificial neural networks are better understood as discrete or continuous systems. Second, Turing makes a profound observation that while all machinery could theoretically be viewed as continuous, it's often more useful to treat systems as discrete when possible. This insight foreshadowed the dominant paradigm of digital computing, where we abstract away from the underlying continuous physical processes to work with discrete states. This principle of useful abstraction became central to computer science. Third, his distinction between "controlling" (information-processing) and "active" (physical effect-producing) machinery is particularly prescient. This separates the abstract notion of computation from its physical implementation - a separation that would become crucial for theoretical computer science and AI. When Turing defines controlling machinery as dealing purely with information, he's essentially describing what we now call information processing systems, laying groundwork for understanding computation as manipulation of abstract symbols. His example table is fascinating because it places the brain alongside other information-processing systems, suggesting it might be understood as a computational device. His notation that the brain is "probably" continuous but similar to discrete machinery is particularly insightful - it anticipates modern debates about whether neural computation is fundamentally discrete or continuous. Finally, Turing's focus on "discrete controlling machinery" effectively predicted the direction of computer science and AI development. Modern digital computers and most AI systems are exactly what he described - discrete systems that process information. This framework helped establish the theoretical foundations for understanding both artificial and natural intelligence in computational terms.

The concept of machine learning through education

Turing proposed that rather than trying to programme a fully-formed adult intelligence, we should create machines that could learn and develop like children do - through experience and education. He suggested machines could be "trained" rather than completely pre-programmed, which was a radical departure from conventional computing at the time and anticipated modern machine learning approaches:

> If we are trying to produce an intelligent machine, and are following the human model as closely as we can, we should begin with a machine with very little capacity to carry out elaborate operations or to react in a disciplined manner to orders (taking the form of interference). Then by applying appropriate interference, mimicking education, we should hope to modify the machine until it could be relied on to produce definite reactions to certain commands (A. M. Turing, 1992, p. 118).

Turing described how machines could be modified through external interference (training) to achieve desired behaviours (Turing, 1950, p. 118), similar to modern concepts of supervised learning and reinforcement learning.

Neural networks, connectionism, and the importance of randomness and initiative

Though he called them "unorganised machines," Turing described what we'd now recognise as artificial neural networks - systems of connected nodes that could be trained through a process similar to learning. He proposed both fully connected networks[54] ("A-type machines") and networks with modifiable connections ("B-type machines") (A. M. Turing, 1992, pp. 113–115), and networks responding to pain and pleasure stimulus ("P-type machines") (A. M. Turing, 1992, pp. 122–125) laying groundwork for neural network architectures. Turing argued that truly intelligent machines would need some element of randomness or initiative rather than being purely deterministic (A. M. Turing, 1992, p. 113). This insight anticipated later developments in exploration vs exploitation trade-offs in reinforcement learning and the role of stochastic processes in AI systems.

[54] A-Type and B-Type are Boolean networks, and the P-Type was modified Turing machine with *pleasure* and *pain* inputs

Digital computers as universal machines and environmentalism
He emphasised that digital computers could, in principle, simulate any other machine or process that could be precisely defined - including potentially the human brain (A. M. Turing, 1992, pp. 110–112). This notion of computational universality became central to the field of AI. Turing stressed the importance of machines being able to interact with their environment and having "organs" (sensors and actuators) to do so (Turing, 1950, pp. 116–117). This previewed later work on robotics and embodied AI, suggesting that intelligence requires more than just abstract computation.

Turing turned in the paper on September 18 1948. Unfortunately, Darwin rejected the paper and denigrated it as youthful and inappropriate for publication and Turing never published in his lifetime (A. M. Turing, 2004, p. 401). Darwin presented a contemptuous evaluation of Turing's report to the Executive Committee at the NPL and the paper vanished inside of the NPL files and would have to wait until 1968 to be published long after Turing's death. With no small irony, in the other side of the Atlantic and on 20 September 1948 there was a lecture that turned out to be the first publication on the "theory of automata", that is, on the concept of discrete controlling machines, and it contained a highlight of the cardinal significance of the Universal Turing Machine. The lecture was delivered by von Newmann (Hodges, 2014, p. 489). Turing decided to leave NPL and accepted a post at Manchester University where Max Newman had ensured he would have his first full academic post. It is there where he prepared and published his famous paper in the influential philosophical journal *Mind*, titled "Computing Machinery and Intelligence" (Turing, 1950) which, among notable characteristics and in line with Turing's style of work, did not have substantial citations grounded in the psychological or philosophical literature (Hodges, 2021, pp. 51–52). He was once again consistent with his system of work and he added only 6 more references to the 3 that he had in "Intelligent Machinery", with the oldest one being from the Countess of Lovelace from 1842 on Babagge's Analytical Engine, and the newest one being *The History of Western Philosophy* by Bertrand Russell from 1940 giving us a hint of where his thoughts were when he was working on the paper. He brought to it many of the ideas explored in "Intelligent Machinery".

The Fifth Leap

In "Computing Machinery and Intelligence", Turing's next intellectual leap laid several foundational conceptual frameworks that would profoundly influence cognitive science, philosophy of mind, and artificial intelligence. Turing's most immediate contribution was methodological - replacing the philosophically problematic question "Can machines think?" with the empirically tractable "imitation game." As he argues: "Instead of attempting such a definition I shall replace the question by another, which is closely related to it and is expressed in relatively unambiguous words." (Turing, 1950, p. 433). This operational reformulation would later become a template for cognitive science's behaviouristic methodological commitments. Perhaps most significantly, Turing articulated what would later be formalised as the Physical Symbol System Hypothesis (PSSH) which posits that intelligent action arises from a physical system that manipulates symbols according to formal rules. He describes digital computers as consisting of:

> (i) Store.
> (ii) Executive unit.
> (iii) Control.
> The store is a store of information… In so far as the human computer does calculations in his head a part of the store will correspond to his memory.
> The executive unit is the part which carries out the various individual operations involved in a calculation. What these individual operations are will vary from machine to machine…
> We have mentioned that the 'book of rules' supplied to the computer is replaced in the machine by a part of the store. It is then called the 'table of instructions'. It is the duty of the control to see that these instructions are obeyed correctly and in the right order (Turing, 1950, p. 437).

To illustrate this concretely, Turing describes how symbolic information is physically instantiated: "The information in the store is usually broken up into packets of moderately small size. In one machine, for instance, a packet might consist of ten decimal digits." (Turing, 1950, p. 437) This physical implementation of symbolic processing would become central to the PSSH's claim that intelligence requires both symbols and the physical machinery to manipulate them. The key insight, which Turing articulates but does not fully formalise, is that intelligence emerges from the physical manipulation of symbolic representations according to systematic rules - an idea that would become foundational to cognitive science's computational theory of mind. This architecture, with its emphasis on symbolic storage and rule-governed manipulation, prefigured the computational theory of mind. Particularly notable is his assertion that "The control will normally take the instructions to be obeyed in the order of the positions in which they are stored," (Turing, 1950, p. 437) establishing the sequential nature of symbolic processing. Turing's treatment of discrete state machines was revolutionary in its linking of abstract computation to physical implementation. He acknowledges that "Strictly speaking there are no such machines. Everything really moves continuously." Yet he argues that we can meaningfully treat physical systems as implementing discrete states when "These states are sufficiently different for the possibility of confusion between them to be ignored." (Turing, 1950, p. 439). This insight - that physical continuity could implement logical discreteness - would become central to cognitive science's treatment of neural implementation. It's particularly notable in his discussion on page 451 of the "nervous system" which, though continuous, could be functionally analysed in discrete terms. Perhaps most prescient was Turing's treatment of machine learning. He introduced the "sub-critical" vs "super-critical" analogy comparing learning to nuclear chain reactions, suggesting that below a certain threshold of complexity/organisation, learning would fizzle out, while above it, it could become self-sustaining (Turing, 1950, p. 454). This anticipated later ideas about scaling laws and emergence in AI systems. He also proposed that instead of programming adult-level intelligence directly, we should "produce one which simulates the child's mind" and then subject it to an "appropriate course of education." (Turing, 1950, p. 456). This developmental approach, with its emphasis on learning rather than direct programming, anticipated modern machine learning. He specifically outlines the analogy between machine learning and evolution, identifying:

> Structure of the child machine = Hereditary material
> Changes = Mutations
> Natural selection = Judgment of the experimenter
> One may hope, however, that this process will be more expeditious than evolution. The survival of the fittest is a slow method for measuring advantages (Turing, 1950, p. 456).

This evolutionary framework for understanding learning and development would become influential in both cognitive science and AI.

The paper also tackles core philosophical issues that would define cognitive science. His treatment of the "argument from consciousness" (Turing, 1950, p. 445) and the "argument from various disabilities" (Turing, 1950, p. 447) anticipated and shaped decades of philosophical debate about machine intelligence. The arguments represent two major philosophical objections to machine intelligence that Turing preemptively addressed. The consciousness argument, articulated through Professor Jefferson's position on page 445, claims that true intelligence requires not just behaviour but subjective experience: "Not until a machine can write a sonnet or compose a concerto because of thoughts and emotions felt, and not by the chance fall of symbols, could we agree that machine equals brain." (Turing, 1950, p. 445). Turing dismantles this by showing it leads to solipsism - if we demand proof of inner experience, we cannot verify consciousness in other humans either. The "various disabilities" argument claims machines cannot possess certain human qualities like kindness, humour, learning, or creativity. Turing reveals this as arising from inductive bias based on existing simple machines. His rebuttal shows these limitations stem from current engineering constraints rather than any inherent impossibility (Turing, 1950, pp. 447–448). For example, regarding the claim that machines can't make mistakes, he distinguishes between "errors of functioning" and "errors of conclusion" (Turing, 1950, p. 449), demonstrating how apparent limitations often result from imprecise thinking about machine capabilities. Particularly notable is his response to Lady Lovelace's objection that machines can only do what we programme them to do. He argues (Turing, 1950, p. 450) that learning machines might develop capabilities beyond our explicit programming - a view that would become central to modern machine learning approaches. These arguments anticipated major themes in philosophy of mind: the relationship between behaviour and consciousness, the nature of intelligence versus its simulation, and the question of whether certain human qualities are uniquely biological or potentially implementable in machines. By systematically addressing them, Turing provided a framework for evaluating claims about machine intelligence that remains relevant to contemporary debates in AI ethics and philosophy. Turing's paper represents a watershed moment in the conceptual foundations of cognitive science and AI. Its linking of abstract computation, physical implementation, and learning/development provided a framework that continues to influence the field. The paper's methodological sophistication - moving from philosophical questions to empirical tests, from continuous physical systems to discrete computational ones, and from direct programming to learning - established patterns of thinking that would define these fields. The paper's influence extends beyond its specific claims to its general approach of treating mind and intelligence as amenable to mechanical/computational analysis while remaining sensitive to the philosophical complexities involved. This balanced approach helped establish cognitive science's characteristic blend of computational and philosophical thinking.

Alternate Views - Competing frameworks

The period between 1936 and 1956 saw the emergence of multiple competing frameworks for understanding computation and machine intelligence. These different approaches reflected not only technical differences but fundamental philosophical divergences about the nature of mind, computation, and intelligence. The complexity of the wicked problem that characterised the trigger for Turing's work led to alternate views of different aspects of his intellectual output, corresponding to the different fields in which he had a seminal influence: Mathematics and Logic, Computation, and Mind and Cognitive Science.

Mathematics and Logic: The Church-Lambda Framework

Shortly before Turing was able to publish his paper on computability, a second answer to Hilbert's problem emerged from Princeton, developed by Alonzo Church. Church's *lambda calculus* provided the first formal solution to the Entscheidungsproblem, appearing with an approach that was deeply rooted in mathematical logic (Church, "An Unsolvable Problem of Elementary Number Theory," 1936, p. 356). The lambda calculus offered a purely functional approach to computation, an abstraction in formal mathematical formalism developed by Church and without analogy to a physical process, and this contrasted sharply with Turing's more intuitive and mechanistic approach. R. I. Soare posits the question of why Turing and not

Church gets much of the credit for solving Hilbert's problem, and the answer is that in reframing the problem his solution went beyond and defined a new ontological space. This is how Soare presents the concluding answer in his paper:

> By any purely quantifiable evaluation Church's contribution was at least as important as Turing's. Gödel's Incompleteness Theorem (1931) and his proof (1940) of the consistency of CH and AC[55] were purely mathematical problems not requiring one to make mathematically precise an informal concept like calculability. However, characterising human computability was *not* a purely quantifiable process. Gödel (1946, p 84) wrote, 'one [Turing] has for the first time succeeded in giving an absolute definition of an interesting epistemological notion, i.e. one not depending on the formalism chosen' (Soare, 2013, p. 67).

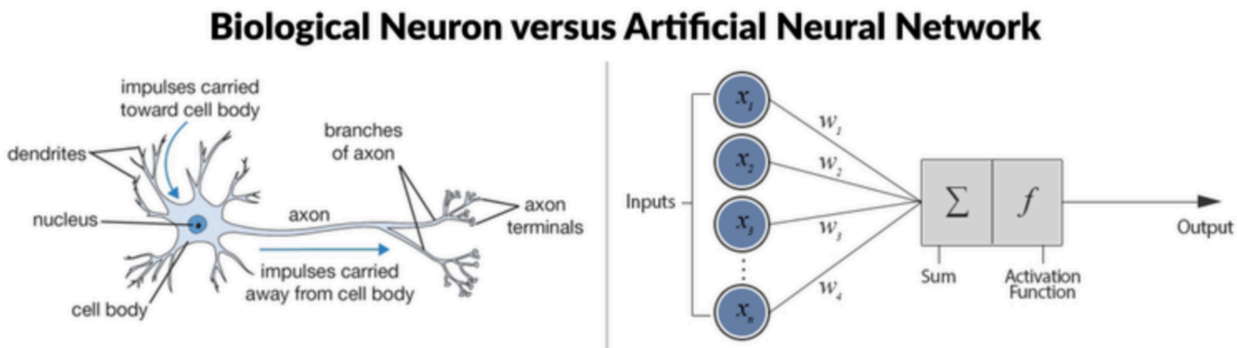
This fundamental difference in approach - Church's abstract formalism versus Turing's mechanistic analysis - would have lasting implications for how computation would be conceptualised. While both approaches were ultimately shown to be equivalent in power (the Church-Turing thesis), they suggested different ways of thinking about computation that would influence later developments in computer science and artificial intelligence.

Biological views

A distinctly different framework emerged from the work of Norbert Wiener and his colleagues, who approached the problem of machine intelligence from a biological and control-theoretical perspective. Wiener's conception of cybernetics offered an alternative to both Church's pure formalism and Turing's mechanical approach. The cybernetic approach emphasised (Russell and Norvig, 2022, p. 34): feedback mechanisms rather than sequential computation, continuous rather than discrete processes, biological analogies rather than mechanical ones, and systems thinking rather than algorithmic processing. In 1943, the American neurophysiologist Warren McCulloch (1899–1969) and logician Walter Pitts (1923–1969) claimed that the neuron was, in essence, a "logic unit." (Nilsson, 2010, p. 34). In their paper they proposed simple models of neurons and showed that networks of these models could perform all possible computational

[55] CH stands for the Continuum Hypothesis, which states there is no set whose size is strictly between that of the integers and the real numbers. AC refers to the Axiom of Choice, which states that for any collection of non-empty sets, there exists a function that selects one element from each set. The quote indicates that unlike computability theory (which required formalising intuitive concepts), proving these mathematical statements did not require defining informal notions rigorously.

operations: “Because of the “all-or-none” character of nervous activity, neural events and the relations among them can be treated by means of propositional logic” (McCullogh and Pitts, 1943, p. 99). McCullogh and Pitts’ idea can be represented by the following figure[56] showing the so-called *perceptron*[57]. Introduced by Rosenblatt (Rosenblatt, 1957) (Rosenblatt, 1961), a *perceptron* is an entity replicating the functionality of a network of biological neurons.



This framework suggested that; first, the brain could be understood as implementing logical operations, and second, neural networks could serve as a bridge between physical and logical descriptions. This perspective would later influence both artificial neural networks and cognitive science, offering a distinctly different approach to machine intelligence than either Turing's abstract machines or Wiener's cybernetic systems. After that there was a further confluence of fields:

> In the 1960s, Turing computation became central to the emerging interdisciplinary initiative cognitive science, which studies the mind by drawing upon psychology, computer science (especially AI), linguistics, philosophy, economics (especially game theory and behavioural economics), anthropology, and neuroscience (Rescorla, 2020, p. 3. The classical computational theory of mind).

Von Neumann's Pragmatic Synthesis

John von Neumann, who had interactions with all these competing schools of thought, developed a more pragmatic framework that focused on the practical implementation of

[56] Source: https://www.datacamp.com/tutorial/deep-learning-python

[57] A perceptron can be represented as a single-layer neural network that computes a weighted sum of its inputs, applies an activation function (usually a step function), and outputs a binary result (e.g., 0 or 1) (Rosenblatt, 1961, pp. 83–86)

computing machines (von Neumann, 1993). Von Neumann's approach emphasised: the physical implementation of logical operations (von Neumann, 2012, pp. 8–12); the importance of memory organisation (von Neumann, 2012, p. 14); the relationship between programme and data (von Neumann, 2012, pp. 20–24); and the necessity of error tolerance (von Neumann, 2012, pp. 25–28). This perspective would prove highly influential in the development of actual computing machines, offering a bridge between theoretical frameworks and practical implementation.

Shannon's Information Theory Framework

Claude Shannon's information theory provided yet another perspective on computation and intelligence, focusing on the fundamental properties of communication and information processing: "The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point." (Shannon, 1948, p. 379). Shannon's framework introduced concepts like: information as uncertainty reduction, channel capacity and noise, coding and redundancy, and the separation of syntax from semantics
This approach would influence both practical computing development and theoretical understandings of intelligence, offering a distinctly different perspective from Turing's original conception. The coexistence and interaction of these competing frameworks demonstrates the richness of the intellectual environment from which modern computing and artificial intelligence emerged. Each perspective captured important aspects of the problems at hand, and their synthesis would eventually contribute to our modern understanding of computation and intelligence. This multiplicity of approaches exemplifies how transformative knowledge often emerges not from a single framework but from the productive tension between competing perspectives.

Understanding phase - Épistémè-Socialisante:

Gestalt shift

The transformation in how computation and intelligence were conceptualised between 1936 and 1956 represents a profound gestalt shift in scientific understanding. This shift was not merely an accumulation of knowledge but a fundamental reconceptualisation of what computers could be and do, moving from seeing them as sophisticated calculators to viewing them as potential thinking machines. The initial transformation emerged through Turing's universal machine concept, which radically reframed the nature of computation. This reconceptualisation was particularly evident in how the field began to understand the relationship between human and machine computation. The gestalt shift manifested in three distinct but interrelated areas: First, in the understanding of computation itself. What began as a mathematical investigation into the limits of formal systems transformed into a new way of thinking about information processing. Claude Shannon captured this transformation in 1950:

> In information theory, one of the basic notions is that of the amount of information associated with a given situation… In everyday usage, information usually implies something about the semantic content of a message. For the purposes of communication theory, the "meaning" of a message is generally irrelevant; what is significant is the difficulty in transmitting the message from one point to the other (Shannon, 1993b).

Second, in the conceptualisation of intelligence. The field moved from viewing intelligence as an ineffable human quality to seeing it as potentially implementable in mechanical systems. Marvin Minsky would articulate this shift in 1961:

> But we should not let our inability to discern a locus of intelligence lead us to conclude that programmed computers therefore cannot think. For it may be so with *man*, as with *machine*, that, when we understand finally the structure and program, the feeling of mystery
> (and self-approbation) will weaken (Minsky, 1961, p. 27).

Third, in the relationship between human and machine capabilities. Rather than seeing machines as tools that merely amplified human computational abilities, they began to be viewed as potentially autonomous cognitive systems. Minsky observed in 1966:

> Once we have devised programs with a genuine capacity for self-improvement a rapid evolutionary process will begin. As the machine improves both itself and its model of itself, we shall begin to see all the phenomena associated with the terms "consciousness," "intuition" and "intelligence" itself. It is hard to say how close we are to this threshold, but once it is crossed the world will not be the same (Minsky, 1966, p. 260).

This shift was particularly evident in how researchers began to approach questions of machine intelligence. The change was not simply about technical capabilities but about the very framework within which such questions could be asked. The gestalt shift also manifested in how the field approached problem-solving. Early work focused on replicating human mathematical procedures, but the new perspective suggested that machines might solve problems in fundamentally different ways. The transformation was particularly evident in how the field began to understand learning and adaptation. Where early approaches focused on programming explicit rules, the new perspective suggested that machines might develop their own problem-solving strategies. This gestalt shift was not merely theoretical but had profound practical implications. It influenced how computers were designed, how programming languages were developed, and how researchers approached the challenge of artificial intelligence. The shift from seeing computers as fast calculating machines to viewing them as potentially intelligent systems opened new avenues for research and development that continue to influence the field today.

Stuart Russell would later explain the fundamental concept of an *agent* in AI, which he defines as any entity capable of perceiving its environment and taking actions in response. This definition is remarkably versatile, encompassing everything from humans and robots to corporations and even simple devices like thermostats. The core focus of artificial intelligence is developing the internal mechanisms that allow these agents to convert sensory inputs into appropriate actions. This involves creating systems for processing and interpreting incoming data, then determining suitable responses based on that information. All cognitive capabilities—whether that's reasoning, planning, or learning—ultimately serve this primary purpose of enabling effective action. What makes AI system design particularly complex is that it must account for three critical factors: the specific environment the agent will operate in, how the agent interfaces with that environment through its sensors and actuators, and what the agent needs to accomplish. These considerations significantly influence the architecture and capabilities required for the AI system to function effectively. This explanation from Russell effectively captures how AI development is fundamentally about creating systems that can intelligently process information and act upon it, rather than just building systems that can think or reason in isolation (Russell, 2021, p. 510)). Crucially, this transformation aligned with the *Épistémè Socialisante* phase of the *o-é-c process*, as it represented not just an individual insight but a collective reconceptualisation that fundamentally changed how the scientific community understood computation and intelligence. This new understanding would prove essential for the subsequent development and expansion of artificial intelligence as a field.

Support for new paradigm

The period following the initial conceptual breakthroughs saw the gradual development of institutional and intellectual support for the new paradigm of computation and machine intelligence. This support emerged through multiple channels and demonstrated the social processes essential to the *Épistémè Socialisante* phase of knowledge development. The Dartmouth Conference of 1956 marked a crucial moment in this process. As John McCarthy (then Assistant Professor of Mathematics at Dartmouth College) and his colleagues articulated in their proposal:

> We propose that a 2 month, 10 man study of artificial intelligence be carried out during the summer of 1956 at Dartmouth College in Hanover, New Hampshire. The study is to proceed on the basis of the conjecture that every aspect of learning or any other feature of intelligence can in principle be so precisely described that a machine can be made to simulate it. An attempt will be made to find how to make machines use language, form abstractions and concepts, solve kinds of problems now reserved for humans, and improve themselves (McCarthy *et al.*, 1955, p. 2).

This statement represented more than just a research agenda; it articulated a new paradigm for understanding intelligence and computation. The conference brought together key figures including Marvin Minsky, Claude Shannon, and Allen Newell, establishing artificial intelligence as a distinct field of inquiry. Each one of them made a proposal for the areas they were interested in researching, which will be a preamble of the different emphasis of future departments and laboratories where they will continue their work. For example, Shannon (then at Bell Telephone Laboratories) proposed to study application of information theory concepts to computing machines and brain models (McCarthy *et al.*, 1955, p. 5). Minsky (then at Harvard University) proposed to work on machine learning by developing and providing machines with a way for abstracting sensory material (McCarthy *et al.*, 1955, p. 6). McCarthy proposed to study the relation of language to intelligence (McCarthy *et al.*, 1955, p. 10). N. Rochester (then at I.B.M Corporation) proposed:

> <u>Process of Invention or Discovery</u>
> Living in the environment of our culture provides us with procedures for solving many problems. Just how these procedures work is not yet clear but I shall discuss this aspect of the problem in terms of a model suggested by Craik.[58]. He suggests that mental action consists basically of constructing little engines inside the brain that can simulate and thus predict abstraction related to the environment (McCarthy *et al.*, 1955, p. 7).

The development of early AI programs provided crucial empirical support for the new paradigm. Institutional support emerged through the establishment of dedicated research centres. At MIT, the Artificial Intelligence Laboratory, founded by Marvin Minsky and John McCarthy, provided a crucial organisational framework in which they concentrated on the problem of “how minds do common sense reasoning” (Minsky, 1988, p. 323). US President Dwight Eisenhower authorises funding for the Advanced Research Projects Agency and begins to flow into academic institutions. Industrial support began to emerge as companies recognised the potential of the new paradigm with research projects supported later by Schlumberger Corporation, Exxon, Xerox, Atari, Apple, and others (Minsky, 1988, p. 324). Academic support grew as universities began establishing computer science departments that incorporated artificial intelligence into their curricula. This institutional embedding helped normalise the new paradigm within the

[58] KJW Craik *The Nature of Explanation,* Cambridge University Press (reprinted 1952) p. 92

broader scientific community. This multifaceted support system - institutional, technical, theoretical, and practical - demonstrated how the new paradigm moved from individual insight to collective understanding, exemplifying the social processes essential to the *Épistémè Socialisante* phase of the *o-é-c model*. The success of this support system would prove crucial for the field's subsequent development and its eventual integration into the broader body of scientific knowledge.

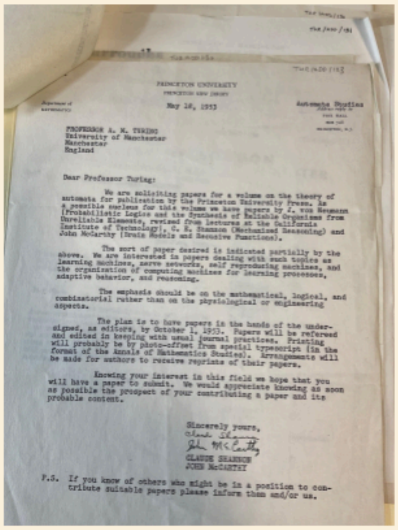
Expansion and Pedagogy

The transformation of artificial intelligence from a speculative venture into an established field of study required the development of systematic pedagogical approaches and institutional frameworks. This expansion phase proved crucial for translating initial insights into transmissible knowledge, demonstrating a key aspect of the *Épistémè Socialisante* phase of knowledge development.

Shannon and McCarthy were working on a text on the theory of automata in 1953, with a paper from von Neumann at its core and they tried to recruit Turing as a critical source for its development. As you can see below in the unpublished letter to Turing and his response[59], it was not to be and Turing died before they organised the Dartmouth Conference.

[59]The pictures reproduced here are of unpublished manuscripts held in the Alan Turing Archive showing the 1953 request from Shannon and McCarthy for contribution towards a book on the theory of automata and Turing's answer. Picture taken with the generous permission of the University of Manchester during my working visit in June-July 2022.

Engaging Turing in the next steps on AI



PRINCETON UNIVERSITY

May 18, 1953

Automata Studies

PROFESSOR A. M. TURING
University of Manchester
Manchester
England

Dear Professor Turing:

We are soliciting papers for a volume on the theory of automata for publication by the Princeton University Press. As a possible nucleus for this volume we have papers by J. von Neumann (Probabilistic Logics and the Synthesis of Reliable Organisms from Unreliable Elements, revised from lectures at the California Institute of Technology), C. E. Shannon (Mechanized Reasoning) and John McCarthy (Brain Models and Recursive Functions).

The sort of paper desired is indicated partially by the above. We are interested in papers dealing with such topics as learning machines, nerve networks, self reproducing machines, and the organization of computing machines for learning processes, adaptive behavior, and reasoning.

The emphasis should be on the mathematical, logical, and combinatorial rather than on the physiological or engineering aspects.

The plan is to have papers in the hands of the undersigned, as editors, by October 1, 1953. Papers will be refereed and edited in keeping with usual journal practices. Printing will probably be by photo-offset from special typescript (in the format of the Annals of Mathematics Studies). Arrangements will be made for authors to receive reprints of their papers.

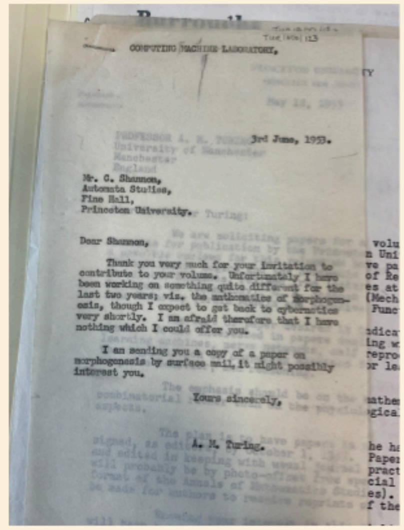
Knowing your interest in this field we hope that you will have a paper to submit. We would appreciate knowing as soon as possible the prospect of your contributing a paper and its probable content.

Sincerely yours,

CLAUDE SHANNON
JOHN McCARTHY

P.S. If you know of others who might be in a position to contribute suitable papers please inform them and/or us.

The pictures reproduced here are of unpublished manuscripts held in the Alan Turing Archive showing the 1953 request from Shannon and McCarthy for contribution towards a book on the theory of automata and Turing's answer. Picture taken with the generous permission of the University of Manchester during my working visit in June-July 2022



COMPUTING MACHINE LABORATORY,

3rd June, 1953.

Mr. C. Shannon,
Automata Studies,
Fine Hall,
Princeton University.

Dear Shannon,

Thank you very much for your invitation to contribute to your volume. Unfortunately I have been working on something quite different for the last two years; viz. the mathematics of morphogenesis, though I expect to get back to cybernetics very shortly. I am afraid therefore that I have nothing which I could offer you.

I am sending you a copy of a paper on morphogenesis by surface mail, it might possibly interest you.

Yours sincerely,

A. M. Turing.

Having developed the first time-shared system in the 1960s Dartmouth College took a leap and required all undergraduates to be "computer literate". The process began with the establishment of dedicated research laboratories that served as centres for both research and education. MIT's Artificial Intelligence Laboratory, under Minsky's direction, developed a distinctive approach to teaching artificial intelligence: Minsky was interested in studying human reasoning by focusing on pattern recognition and analysis (Minsky, 1988, p. 323). This institutional support was paralleled by developments at other centres: Stanford's Artificial Intelligence Project, established by John McCarthy in 1963, emphasised the logical and mathematical foundations of artificial intelligence. Carnegie Mellon's approach, led by Allen Newell and Herbert Simon, focused on cognitive simulation. The development of specialised programming languages provided crucial technical support for the new paradigm. McCarthy's LISP language, in particular, offered tools specifically designed for artificial intelligence research.

The development of foundational textbooks marked a crucial step in this pedagogical evolution. Among the earliest influential texts is von Newmann's *The Computer and the Brain* (von Neumann, 2012). It contained the collection of the lectures he developed after he was invited to deliver the Silliman Foundation Lectures at Yale University in 1956. They were destined to be his last contribution but he was never able to deliver them as he was hospital bound with cancer where he continued to work on them until his death in February 1957. The lectures were edited in the book that was posthumously published in 1957 after Yale University admitted the incomplete and fragmented manuscript for the record of the Silliman lectures. In his foreword to the third edition of *The Computer and the Brain* in 2012, Ray Kurzweil reflected on the achievements of the book thus:

> There are very few discussions in this book that I find to be at significant odds with what we now understand. We are not in a position today to describe the brain perfectly, so we would not expect a book from 1956 on reverse engineering the brain to do so. That being said, von Neumann's descriptions are remarkably up to date, and the details on which he bases his conclusions remain valid (von Neumann, 2012, p. xxviii).
>
> Even though it was remarkably early in the history of the computer when this manuscript was written, von Neumann nonetheless had confidence that both the hardware and the software of human intelligence would ultimately be available. That was the reason he prepared these lectures (von Neumann, 2012, p. xxx).

Marvin Minsky's "Semantic Information Processing" (1968) established early standards for teaching artificial intelligence and it arrived at a time where the field was still in its nascent phase:

> How can one make machines understand things? This body is a collection of studies in *artificial intelligence*, the science of making machines do things that would require intelligence if done by men.
>
> Most of the chapters are slightly edited Ph.D. theses and the book is to serve two purposes: to make the results of these dissertations more available to scientists, and to exhibit the work to students searching for new problems in this area (Minsky, 1968, p. v).

Below is a selection of some of the most influential Artificial Intelligence (AI) textbooks published over the last 50 years, organised by decade. While this list is not exhaustive, it highlights works that have shaped the field's development and have been widely cited and used in academic courses and research. I will refer to the latest edition that is available:

1970s: Patrick Winston's *Artificial Intelligence* introduced a more systematic approach to teaching the field and targeted computer scientists and engineers as well as psychologists, biologists, linguists, and philosophers (Winston, 1993, p. xxiii).
1980s: N. J. Nilsson's *The Quest for Artificial Intelligence: A History of Ideas and Achievements*. This text was specially meant for "AI researchers, students, and teachers who would benefit from knowing more about the things AI has tried, what has and has not worked, and good sources for historical and other information" (Nilsson, 2010, p. 14). Another influential book was by E. Charniak, and D. McDermott's *Introduction to Artificial Intelligence* (Charniak and McDermott, 1985).
1990s: E. Rich, and K. Knight's *Artificial Intelligence*, which brought an emphasis on neural networks and robotics (Rich and Knight, 1996).
2000s: C. M. Bishop's *Pattern Recognition and Machine Learning* (Bishop, 2016) and M.T. Jones' *Artificial Intelligence: A Systems Approach* (Jones, 2009)
2010s: K.P Murphy's *Machine Learning: A Probabilistic Perspective* (Murphy, 2012) and I. Goodfellow, Y. Bengio, and A. Courville's *Deep Learning* (Goodfellow, Bengio and Courville, 2016).

The 2020s have seen an explosion in specialised AI texts, especially in subfields like deep learning and reinforcement learning. The above selection focuses on a canonical, updated edition of an already influential work. Textbooks are an essential element in the patterns found and conforming to the *Épistémè Socialisante* phase and perhaps one of the most influential and broadly used textbooks in the field of AI, is S. Russell (UC Berkeley) and P. Norvig (Google) *Artificial Intelligence: A Modern Approach* (Russell and Norvig, 2022). Russell and Norvig's *Artificial Intelligence: A Modern Approach* (*AIMA*) represented a watershed moment in the systematisation of artificial intelligence knowledge, as it demonstrated how a field transitioned from disparate research threads to coherent academic discipline. Now in its fourth edition (2022), this seminal text accomplished what Thomas Kuhn identified as a crucial marker of mature science: the emergence of standardised pedagogical materials that define a field's theoretical foundations and methodological approaches. What sets *AIMA* apart is its unified approach to artificial intelligence. Rather than treating AI as a collection of disparate techniques, it presents a coherent view of the field organised around the concept of intelligent agents. The book's distinctive agent-based framework has profoundly influenced how current AI researchers conceptualise and approach artificial intelligence systems. By presenting AI through the lens of rational agents that perceive and act within their environment, *AIMA* has helped establish a coherent mental model that continues to influence system design and implementation across the field. This framework has proven particularly valuable as AI systems have grown more sophisticated, offering researchers a structured way to think about increasingly complex architectures. The text's distinctive contribution lies in its unifying theoretical framework centred on intelligent agents, a conceptual architecture that bridges the philosophical questions of intelligence with practical implementation concerns. This framework has proven remarkably robust, accommodating both classical symbolic approaches and modern neural network methodologies while maintaining philosophical coherence. The text's evolution through four editions mirrors the field's development, demonstrating how new technical achievements can be integrated into existing theoretical frameworks without losing conceptual clarity. AIMA's influence extends beyond mere pedagogy; it has helped establish what Michel Foucault would term an "episteme" - a structural framework that determines what constitutes valid knowledge within a field. Through its comprehensive treatment of both theoretical foundations and practical implementations, the text has created a shared conceptual vocabulary and methodological framework that enables coherent discourse across academic and industrial contexts. This standardisation of knowledge representation and transmission exemplifies the final phase of the *o-é-c process*, where individual insights become systematised into transmissible knowledge.

Adoption to Body of Knowledge - Connaissance Éclairante Phase

Becoming Knowledge

The transformation of artificial intelligence from a speculative field into established knowledge exemplifies the *Connaissance Éclairante* phase of the *o-é-c process*. This transformation involved not just the acceptance of specific techniques or theories, but a fundamental shift in how we understand intelligence, computation, and cognition itself. This cross-disciplinary influence extended into multiple domains. In cognitive psychology, the computer metaphor became more than an analogy - it provided a fundamental framework for understanding mental processes. The integration of artificial intelligence concepts into scientific methodology represented another crucial aspect of this transformation. The emergence of cognitive science as a distinct field demonstrated how artificial intelligence had become foundational knowledge. In neuroscience, artificial intelligence concepts became essential tools for understanding brain function. The incorporation of artificial intelligence into commercial and industrial applications marked another aspect of its transformation into established knowledge. The development of formal frameworks for machine learning, knowledge representation, and reasoning demonstrated how the field had developed robust intellectual foundations. The influence of artificial intelligence on philosophy of mind represented a particularly significant aspect of its transformation into established knowledge. The integration of artificial intelligence concepts into public discourse and policy discussions demonstrated another dimension of its establishment as knowledge. Perhaps most significantly, artificial intelligence concepts began shaping how we understand human capabilities and limitations. This transformation into established knowledge continues to evolve, with new developments in machine learning and neural networks generating fresh insights into cognition and computation. Stuart Russell captures this ongoing process and its linkage to the crucial aspect of ethics:

> Assuming that the theoretical and algorithmic foundations of the new model for AI can be completed and then instantiated in the form of useful systems such as personal digital assistants or household robots, it will be necessary to create a technical consensus around a set of design templates for provably beneficial AI, so that policymakers have some concrete guidance on what sorts of regulations might make sense. The economic incentives would tend to support the installation of rigorous standards at the early stages of AI development, because failures would be damaging to entire industries, not just to the perpetrator and victim. The question of enforcing policies for beneficial AI is more problematic, given our lack of success in containing malware. In Samuel Butler's Erewhon and in Frank Herbert's Dune, the solution is to ban all intelligent machines, as a matter of both law and cultural imperative. Perhaps if we

> find institutional solutions to the malware problem, we will be able to devise some less drastic approach for AI (Russell, 2021, p. 518).

The establishment of artificial intelligence as knowledge thus exemplifies the culminating phase of the *o-é-c process*, where new understanding becomes not just accepted but transformative, changing how we conceptualise fundamental aspects of mind, computation, and intelligence itself. This process continues today, as artificial intelligence generates new insights that reshape our understanding across multiple domains of knowledge.

Summary

The Turing case study demonstrates with remarkable clarity how transformative knowledge emerges through the interplay of individual insight, social development, and systematic integration into broader understanding. Through the lens of the *o-é-c process*, we can trace how Turing's original insights about computation and machine intelligence evolved into a field that has fundamentally transformed our understanding of mind, intelligence, and computation itself. This case exemplifies several key patterns identified in our analysis of transformative knowledge creation. First, like Leibniz with calculus and Bohr with quantum mechanics, Turing's breakthrough emerged during a period of profound social and intellectual upheaval - the aftermath of one world war and the onset of another - when traditional frameworks and authorities faced unprecedented challenges. This environmental context proved crucial for enabling radical reconceptualisation of established paradigms. Second, the case illuminates how truly transformative ideas often arise from addressing *wicked problems* that resist formulation within existing frameworks. Turing's approach to computability and machine intelligence required not just solving well-defined problems but creating new conceptual spaces within which such problems could be meaningfully formulated. His reconceptualisation of computation in terms of mechanical processes rather than mathematical formalism parallels Bohr's radical reframing of quantum phenomena through complementarity. The social dimension of knowledge creation manifests particularly clearly in this case. While Turing's initial insights were profound, their development into established knowledge required the collective efforts of multiple communities - mathematicians, engineers, psychologists, and philosophers. The transformation from individual insight to collective understanding proceeded through identifiable stages of social validation, refinement, and integration, demonstrating the essential role of the Épistémè Socialisante phase in knowledge creation. Most significantly, this case exemplifies how knowledge, once established, can illuminate previously unapproachable questions across multiple domains. Artificial intelligence has become not merely a technological achievement but a fundamental framework for understanding cognition, computation, and intelligence itself. This transformative power - the ability to generate new insights beyond its original domain - characterises the *Connaissance Éclairante* phase and distinguishes truly significant additions to human knowledge. The continuing evolution of artificial intelligence, from early computing theory to today's deep learning systems, demonstrates how transformative knowledge creates cascading insights that extend far beyond initial conceptualisation. Just as Leibniz's calculus provided tools for understanding continuous change across multiple domains, and Bohr's complementarity offered frameworks for comprehending seemingly contradictory phenomena, Turing's insights into computation and machine intelligence continue generating new understanding across an expanding range of fields. In synthesising these patterns, the Turing case provides compelling evidence for the *o-é-c model*'s utility in understanding how transformative knowledge emerges and becomes established. It demonstrates that significant additions to human knowledge require not just individual brilliance but systematic processes of social validation and integration. As artificial intelligence continues evolving, it offers an ongoing demonstration of how established knowledge can illuminate new paths for understanding, embodying the essence of *Connaissance Éclairante* and setting the stage for future transformative developments.

Chapter 6 - Analytical Conclusion

Lessons learned

The *o-é-c process* model proposed in this thesis illuminates fundamental patterns in how transformative knowledge emerges and becomes established within scientific communities. Through detailed analysis of three pivotal cases—Leibniz's development of calculus, Bohr's formulation of complementarity, and Turing's conceptualisation of computation and artificial intelligence—we can now see how this model provides a robust framework for understanding scientific advancement that bridges previous theoretical approaches while offering new insights into knowledge creation. The model's three phases—*Ouverture Ontologique, Épistémè Socialisante,* and *Connaissance Éclairante*—correspond to distinct but interrelated aspects of knowledge development that were previously treated separately in philosophical accounts. Where earlier frameworks emphasised either individual discovery (like Hanson) or social construction (like Longino), the *o-é-c model* demonstrates how these aspects work together in the creation of new knowledge domains. This synthesis helps resolve longstanding tensions in the philosophy of science between revolutionary and evolutionary accounts of scientific change. The following analysis examines specific patterns identified across the three cases, demonstrating how the *o-é-c model* enables us to understand both the particular features of each case and the general principles governing how revolutionary scientific insights become established knowledge.

The Systematisation-Socialisation-Expansion Cycle

While my thesis identifies multiple patterns across the cases, the most significant pattern is what I would call the "systematisation-socialisation-expansion" cycle. This pattern proves particularly important because it helps explain not just how new ideas emerge, but how they become established knowledge. Let me explain why this pattern is most significant by examining how it manifests across the cases. In Leibniz's development of calculus, we see him creating not just mathematical insights but a systematic framework with clear notation and operational rules. What makes this particularly significant is that it was not just about solving specific problems - it was about creating a new language for mathematical thought. The systematisation through notation and rules made it possible for others to learn and apply these ideas. This systematic foundation then enabled social processes of validation and refinement, particularly through pedagogical tools like L'Hôpital's textbook. Finally, this led to expansion beyond the original mathematical context into physics and other fields. I found this same pattern, but with different manifestations, in Bohr's case. His complementarity concept was not just a theoretical insight - it was systematised into a comprehensive framework for understanding quantum phenomena. This systematic foundation enabled the social processes of the Copenhagen interpretation's development, particularly through the intense discussions between physicists at various conferences and institutes. The framework then expanded beyond its original atomic physics context to influence fields from biology to philosophy. Turing's case provides perhaps the most dramatic example. His systematisation of computation through the abstract machine concept provided a foundation that others could build upon. This systematic framework enabled social processes of validation and refinement through both theoretical development and practical implementation. The expansion phase has been particularly dramatic, with computational thinking now influencing fields from cognitive science to biology.

This pattern is most significant for several reasons: First, it helps resolve a longstanding tension in philosophy of science between individual genius and social construction accounts of scientific progress. It shows how both elements are necessary - individual insights must be systematised in a way that enables social processes of validation and refinement. Second, it has practical implications for fostering scientific advancement. It suggests that supporting breakthrough ideas isn't enough - we need to ensure conditions that enable systematisation, social validation, and expansion beyond original contexts. Third, it helps explain why some brilliant insights fail to become established knowledge. Without systematic foundation and social processes of validation, even profound insights might remain isolated achievements rather than transformative knowledge. Fourth, it demonstrates the crucial role of pedagogy and institutional networks in knowledge creation - elements often overlooked in philosophical accounts of scientific progress. Lastly, this pattern's significance lies not just in its explanatory power but in its practical implications. Understanding how systematisation enables social processes which in turn enable expansion can help us better foster conditions for transformative scientific advancement today.

Historical Context and Institutional Transformation

Perhaps the most striking pattern emerging in the historical environments that catalysed these developments is that each case arose in periods of profound social and intellectual upheaval—post-war environments that lowered barriers to challenging established paradigms. This pattern suggests that transformative ideas often emerge when social disruption creates openness to fundamental reconceptualisation of established frameworks. This is a critical observation in my thesis that requires careful substantiation. The evidence comes from analysing both the historical contexts and the nature of intellectual resistance to new ideas during these periods. For Leibniz, the aftermath of the Thirty Years' War (1618-1648) created unique conditions for intellectual innovation. The Peace of Westphalia in 1648 fundamentally reshaped European society, weakening traditional authorities and creating openness to new ways of thinking. The evidence for this lies in several areas. First, the documents from the period show increasing challenges to established intellectual frameworks across multiple fields. Second, the growth of scientific societies and journals in this period demonstrates new channels for sharing controversial ideas. Most importantly, the religious upheavals of the time had already destabilised many traditional certainties, making it easier to challenge established mathematical paradigms. For Bohr's case, the evidence is particularly strong through Forman's work on Weimar culture. The post-WWI environment in Germany created what Forman calls a 'crisis of causality' that made physicists more receptive to acausal explanations. There is extensive documentation of how the cultural environment influenced scientific thinking during this period. The Copenhagen interpretation emerged in this context where traditional mechanical explanations were already being questioned. The evidence includes not just Bohr's own writings but extensive correspondence between physicists of the period showing how the post-war environment influenced their receptivity to radical new ideas. For Turing, the post-WWII period saw both institutional and intellectual transformations that facilitated the acceptance of computational thinking. First, there is clear documentation of how wartime work at Bletchley Park created new approaches to mechanical computation. Second, the migration of scientists during and after the war created new intellectual networks that helped spread these ideas. Third, the post-war funding environment, particularly in the United States, provided institutional support for developing these new frameworks. What's particularly compelling is how in each case, we see similar mechanisms at work. Post-war environments tend to weaken institutional resistance to new ideas, create new channels for scientific communication, and often provide both the practical problems and resources that drive innovation. We can trace these patterns through institutional records, scientific correspondence, and the rapid growth of new scientific communities in these periods. While one might argue that scientific advancement occurs in all periods, the evidence suggests that post-war environments are particularly conducive to fundamental paradigm shifts. This does not mean that all scientific revolutions require such conditions, but rather that these environments create conditions where radical new ideas face reduced resistance and find greater institutional support. We can see an example in mathematics and logic; Wolfram suggests that the intellectual leap to see implications requires a certain willingness to challenge authority, and he conjectures that von Neumann did not see the possibility of Kurt Gödel's 1931 theorem because he lacked the boldness of attempting to find a counterexample of Hilbert's ideas and challenge his programme for mathematics (Wolfram, 2016, p. 35). Environments characterised by instability create an atmosphere in which challenging the status quo might be generalised and more easily accepted.

Wicked Problems

A third crucial pattern appears in how each case featured *wicked problems* that defied description within existing frameworks. The challenges faced were not merely difficult problems awaiting solutions, but fundamental inadequacies in the conceptual tools available for their description. Notably, understanding these problems was inseparable from developing their solutions. Leibniz's infinitesimal calculus, Bohr's complementarity framework, and Turing's computational theory each emerged through the very process of attempting to articulate their respective problems.

Conceptual Tensions

A fourth striking commonality across cases is the presence of fundamental tensions between continuity and discreteness. Leibniz grappled with the relationship between geometric continuity and arithmetic discreteness, and he addressed the description of curves through infinitesimals. Bohr confronted wave-particle duality and the conceptualisation of *quanta* in atomic and nuclear physics. Turing addressed the interface between discrete computational steps and continuous human thought processes. These tensions proved fertile ground for innovative thinking. This pattern reveals how transformative advances often stem from addressing fundamental tensions that existing frameworks cannot resolve. The intellectual leaps in each case share a common pattern of creating new ontological spaces complete with novel conceptual entities and operational rules. Leibniz's infinitesimals and characteristic triangles using algebraic methods for differential and integral methodological rules; Bohr's complementarity, acausality, uncertainty and definition of observer, and Turing's abstract machines, computability, discrete state machines and machine learning represent more than solutions to specific problems—they constitute entirely new frameworks for understanding. This pattern suggests that truly transformative advances require not just solving bounded, well-defined problems but creating new ways of thinking about them.

The Nature of Intellectual Breakthroughs

The revolutionary frameworks that emerged shared several key characteristics. Each involved the creation of novel representational systems that could express previously inarticulable concepts: Leibniz's differential and integral notation and algebraic methods, Bohr's complementarity concept and the Copenhagen interpretation of quantum mechanics, and Turing's computation machine concept and later ideas on a framework for mechanical intelligence. These systems bridged previously separate domains of understanding, creating powerful new syntheses. Crucially, each framework introduced new conceptual entities that transcended existing paradigms. Leibniz presented infinitesimals and characteristic triangles as mathematical objects, Bohr established complementarity as a fundamental description and incorporated the ideas on quantisation, indeterminacy and probabilistic behaviour, and Turing defined the abstract concept of computation through his machine model and the novel ideas of discrete state machines and machine learning. These innovations were not merely theoretical constructs but systematically developed frameworks with their own operational rules and entities.

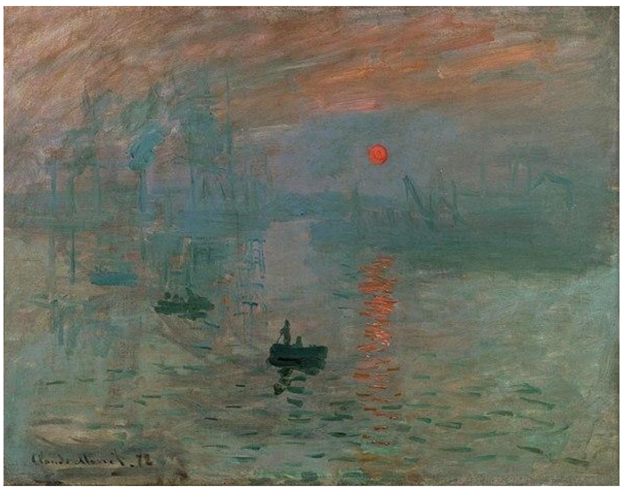
Competition and Criteria for Success

The history of scientific advancement repeatedly demonstrates that transformative frameworks often emerge alongside competing approaches that ultimately fail to gain lasting influence. This pattern of concurrent development and competition provides crucial insights into the criteria for successful scientific innovations. The Newton-Huygens debate on the nature of light, the Darwin-Wallace parallel development of evolutionary theory, the Einstein-Hilbert development of the equations on relativity, the Watson-Crick-Pauling race to uncover DNA's structure, Feynman-Schwinger methods for working in QED (Quantum Electrodynamics), or the Shannon-Wiener approaches to entropy and information, all exemplify this phenomenon. Successful frameworks consistently demonstrated three key characteristics: they offered superior explanatory power, provided more elegant mathematical or logical formalisation, and proved more amenable to practical application and extension. For instance, while Newton and Leibniz both developed calculus, Leibniz's notation and systematic approach proved more fertile for further mathematical development. Similarly, though Turing and Church both addressed Hilbert's decidability problem, Turing's machine-based approach provided a more intuitive and practically applicable framework that would later prove fundamental to computer science. This pattern continues in contemporary debates, such as the Chomsky-Norvig disagreement over approaches to artificial intelligence, where competing frameworks vie to provide the most productive path forward. The persistence of this pattern suggests that successful scientific frameworks don't merely solve immediate problems but provide fertile ground for future development and application across multiple domains.

Expansion beyond the original conceptions

Significantly, each framework evolved far beyond its original domain through processes of expansion and integration. Calculus transformed from a tool for analysing continuous change into a fundamental method and language of science. Quantum mechanics expanded from atomic physics to influence fields from computing to cosmology. This pattern suggests that truly transformative ideas contain within themselves the seeds of broader application. Pais posits that humans do not always understand the implications of the truly groundbreaking frameworks that sometimes they create. In particular, physicists are not revolutionary per se, requiring someone else to interpret the significance of inventions and discovery and so, Einstein understood Planck's quantum of action leading to atomic theory. Bohr understood Einstein's quantisation of light, Heisenberg's particle mechanics and uncertainty principle, and Schrödinger's wave mechanics, leading him to ideate complementarity in quantum mechanics and other fields (Pais, 2002, pp. 129–134). This is not unique to physics, an example in humanities is the evolution of Impressionism. Claude Monet (1840-1926) is widely considered a principal creator of Impressionism - in fact, the movement got its name from his 1872 painting "Impression, Soleil Levant" (Impression, Sunrise). Monet developed the core techniques that would define Impressionism: capturing the fleeting effects of light, using visible brushstrokes, painting outdoors (*en plein air*), and focusing on everyday scenes rather than historical or mythological subjects. Someone else took these ideas to greater heights: Vincent van Gogh (1853-1890) stands out as an artist who absorbed Impressionist techniques but transformed them into something far more expressive and emotionally charged. While Monet was interested in capturing objective visual effects of light and atmosphere, van Gogh used similar techniques - visible brushstrokes, bright colours, outdoor painting - to express intense inner feelings and spiritual meanings. His famous work "The Starry Night" shows how he took Impressionist methods but pushed them toward what would become Post-Impressionism: the stars and cypress trees pulse with an almost supernatural energy that goes well beyond mere visual impression. Van Gogh also expanded the emotional range of what painting could express. Where Impressionists typically painted pleasant scenes of middle-class leisure–think about

Édouard Manet (1832-1883)– van Gogh depicted subjects like peasant labourers, mental asylums, and night cafés, using modified Impressionist techniques to convey psychological depth and human struggle. His influence extended far beyond Impressionism, helping to pave the way for Expressionism and other modern art movements of the 20th century. Below you can see the 2 paintings side by side, *Impression* on the left and *Starry* on the right (images source: Wikipedia):



Social Validation and Knowledge Integration

The social processes through which these ideas gained acceptance show remarkable similarities. Each case demonstrates the crucial role of systematic presentation and pedagogical development. The success of these frameworks depended heavily on social processes of validation and transmission. Each case featured intense methodological competition with alternative approaches: Leibniz's success compared to Newton's parallel work highlights how systematic notation and clear exposition with clear communication and discussion are essential for enabling others to work within new conceptual frameworks. Similarly, Bohr's careful development of the Copenhagen interpretation through sustained dialogue within the physics community shows how social processes shape and refine revolutionary ideas. The clarity of the physical representation of computability made Turing’s approach spread widely and become part of computation discussions, foundations, and early implementations. In the socialisation process a framework with a philosophical perspective is also a common feature in the success of a framework as shown in the cases of Leibniz versus Newton, Bohr versus Einstein, and Turing versus Church. These competitions reflected deeper philosophical differences and ultimately contributed to the refinement and clarification of the successful frameworks. The transition from individual insight to established knowledge required systematic presentation and pedagogical development. Clear exposition proved crucial, as evidenced by L'Hôpital's calculus textbook, the Copenhagen interpretation of quantum mechanics nurtured in all the students and scientists passing through Bohr's Institute, and Turing's methodologically sophisticated presentation of computational theory and mechanical intelligence. Institutional networks played a vital role in knowledge transmission, with early adoption of texts and younger generations of researchers often providing crucial support for the new paradigms.

Conclusion

The patterns identified across our three case studies - Leibniz's calculus, Bohr's complementarity, and Turing's computation - reveal fundamental aspects of how transformative knowledge emerges and becomes established. The *o-é-c model* not only illuminates these historical developments but provides a novel framework for understanding how human knowledge advances through the interplay of individual insight, social processes, and systematic integration. This research builds upon and extends key theoretical frameworks in the philosophy of science, making several original contributions to our understanding of scientific progress: first, by demonstrating how individual insights become collective knowledge through identifiable phases, the *o-é-c model* bridges a crucial gap in existing theoretical frameworks. I build on Hanson's emphasis on the conceptual analysis of discovery, but my model goes beyond his focus on individual moments of insight. Hanson also argues that the idea of discovery should be analysed conceptually as it is fundamental in our understanding of science. I go further by proposing that the philosophy of science should also concern itself with ideation in discovery and the critical role of systematisation for the success and acceptance of inventions of theoretical frameworks. While Hanson was primarily concerned with how scientists come to see new patterns or relationships, the *o-é-c model* shows how these initial insights must be developed and validated through social processes before becoming established knowledge. Where Kuhn sees sharp breaks between paradigms, the *o-é-c model* shows how new knowledge can emerge through a more structured process. I extend Kuhn's concept of gestalt switches in an innovative way - rather than seeing them as sudden, community-wide shifts, I position them within the *Épistémè Socialisante* phase where they can be gradually adopted and refined through social processes. This allows for a more fluid understanding of how scientific communities adapt to new ideas. With the historical analysis done in the thesis, I extend Forman's insights about historical context by showing how cultural factors interact with both individual creativity and social validation processes. I extend Forman's framework by demonstrating how cultural contexts influence not just the initial emergence of ideas but also their subsequent development and acceptance. This investigation supports Longino's emphasis on the social dimensions of knowledge creation while extending beyond it. My model demonstrates how social processes interact with individual insights and institutional structures throughout knowledge creation and evolution. The patterns reveal how social processes do more than validate individual insights—they actively shape the development and evolution of new frameworks through critical discourse, pedagogical refinement, and practical application. My emphasis on pedagogy and textbooks as crucial elements in knowledge dissemination extends Longino's framework into more practical domains. The research not only aligns with but goes beyond Longino's concept of critical discursive interactions by showing how such interactions contribute to the systematic development of new fields. This creates a more complete picture of how knowledge becomes established within scientific communities. Second, by emphasising the interplay between individual creativity and social validation processes, this work complements and deepens the contributions of the Sociology of Scientific Knowledge[60] (SSK). I go beyond the relativistic tendencies of SSK by showing how systemic patterns like the *o-é-c process* operate across diverse historical and cultural contexts, balancing the subjective and objective dimensions of knowledge creation. Third, the analysis of *wicked problems* as catalysts for knowledge creation offers new insight into what drives scientific

[60] SSK treats scientific knowledge as fundamentally shaped by the social conditions in which it is created, rather than viewing it as purely objective truth discovered through neutral observation. It examines how scientists' social networks, institutional affiliations, funding sources, and broader cultural contexts influence what questions get studied, how research is conducted, and which findings become accepted as scientific fact.

advancement. The inability to formulate certain problems within existing frameworks - whether in Leibniz's approach to continuous change, Bohr's treatment of quantum phenomena, or Turing's investigation of computation - proves crucial for precipitating conceptual breakthroughs. This suggests that recognising and engaging with such problems, rather than avoiding them, may be essential for scientific progress. Fourth, the three-phase structure of the *o-é-c model* resolves tensions between revolutionary and evolutionary accounts of scientific change. By showing how radical insights are gradually refined and integrated through social processes, the model explains how discontinuous conceptual advances can lead to continuous knowledge development. This offers a more nuanced understanding than either purely revolutionary or purely evolutionary models. Fifth, my emphasis on pedagogy and textbooks as decisive elements in knowledge dissemination adds a crucial practical dimension often overlooked in theoretical accounts. The transformation of revolutionary insights into teachable frameworks proves essential for their broader acceptance and application. This highlights how the social processes of knowledge creation extend beyond validation to include systematic transmission and integration.

In returning to my opening questions about how we solve unprecedented problems and whether problem-solving is individual or social, this research demonstrates that transformative knowledge creation requires both individual insight and social processes. The analysis also validates the adoption of Minsky's concept of knowledge as "whichever mental models, processes, or agencies can be used to answer questions." Each of the case studies demonstrates how new conceptual frameworks become knowledge precisely by providing new ways to understand and address previously intractable problems. Leibniz's calculus offered new tools for understanding continuous change, Bohr's complementarity provided new ways to comprehend quantum phenomena, and Turing's frameworks enabled new approaches to understanding computation and intelligence. The remarkable consistency of these patterns across different historical contexts and fields - from mathematics to physics to computer science - suggests I have identified fundamental aspects of how human knowledge advances. In each case, we can see how individual insights (*Ouverture Ontologique*) develop through social processes (*Épistémè*

Socialisante) into transformative knowledge that illuminates previously unapproachable questions (*Connaissance Éclairante*). These findings have significant implications for contemporary challenges. As we confront increasingly complex global problems that resist traditional disciplinary approaches - from climate change to artificial intelligence ethics - understanding how transformative knowledge emerges becomes crucial. The *o-é-c model* suggests that addressing such challenges requires not just technical solutions but new conceptual frameworks and systematic processes for their social validation and integration. Moreover, my analysis demonstrates that truly significant advances in knowledge do more than solve specific problems - they create new ways of thinking that generate cascading insights across multiple domains. This pattern, evident in how calculus transformed our understanding of change, complementarity reshaped our view of physical reality, and computational thinking revolutionised our approach to intelligence, suggests criteria for recognising potentially transformative developments in contemporary fields. The research also reveals how environmental factors - particularly periods of social and intellectual upheaval - can facilitate the emergence of revolutionary ideas by weakening attachment to established frameworks. This insight has implications for how we might foster conditions conducive to transformative thinking in contemporary contexts. They suggest the importance of recognising when problems resist formulation within existing frameworks; supporting the creation of new conceptual frameworks and entities; facilitating social processes of validation and refinement; developing systematic approaches to teaching and transmitting new ideas; enabling the expansion of frameworks beyond their original domains. Understanding how transformative knowledge emerges and becomes established isn't just historically interesting - it's crucial for addressing today's complex challenges. This can be illustrated with the case of Artificial Intelligence Ethics: The rapid development of AI presents a perfect example of why we need frameworks like the *o-é-c model* for handling emerging *wicked problems*. We're currently witnessing all three phases of the model simultaneously: researchers are creating new conceptual frameworks for understanding AI capabilities (*Ouverture Ontologique*), the tech community is debating standards and best practices (*Épistémè Socialisante*), while some basic principles are already becoming established knowledge (*Connaissance Éclairante*). My model helps explain why progress in AI ethics seems uneven - different aspects are at different phases of the process. As we look toward future challenges, the *o-é-c model* offers both theoretical insight and practical guidance. It suggests that addressing complex problems requires not just individual brilliance or collective effort, but structured processes that enable the development, validation, and integration of new conceptual frameworks. Understanding these processes can help us better recognise and support potentially transformative developments across multiple fields. In an era of accelerating technological change and mounting global challenges, the ability to foster and integrate transformative knowledge becomes increasingly crucial. By illuminating the fundamental patterns through which such knowledge emerges and becomes established, this research contributes to our capacity to address the complex problems that will define our future. As we move forward, the insights provided by the *o-é-c model* may prove essential for cultivating the intellectual and institutional environments needed to meet these challenges. This research thus makes a distinctive contribution to our understanding of scientific progress while opening new avenues for investigation. As we continue to explore how transformative knowledge emerges and becomes established, the patterns and principles identified here provide a foundation for further theoretical development and practical application.

Limitations and questions for further research

This thesis has explored how transformative scientific knowledge emerges and becomes established through the o-é-c process, examining three seminal cases of knowledge creation across different historical periods. While the research has yielded valuable insights into the patterns of scientific advancement, it is essential to acknowledge the limitations of this investigation and identify promising directions for future research. The next section critically examines the boundaries and constraints of the current study while suggesting ways to extend and deepen my understanding of knowledge creation and establishment.

Conceptual and Methodological Limitations

The decision to focus on three specific cases—Leibniz's calculus, Bohr's complementarity, and Turing's computational theory—while illuminating, necessarily constrains the generalisability of my findings. These cases were chosen for their transformative impact and clear demonstration of the *o-é-c process*, but they represent only a fraction of significant scientific advances. Moreover, all three cases emerge from Western scientific traditions, potentially limiting my understanding of how knowledge creation occurs in different cultural contexts. The historical span of my cases (17th to 20th centuries) also presents limitations. While this timeframe captures important developments in modern science, it may not fully account for how contemporary factors like digital communication, global research networks, and rapid technological change affect knowledge creation and establishment. The accelerating pace of scientific discovery and the increasingly interconnected nature of research communities may introduce dynamics not captured by my historical cases.

Methodological Constraints

While the integrated methodological approach combining historical analysis, philosophical investigation, and cross-case pattern recognition has provided valuable insights, it also presents certain limitations that warrant acknowledgment. The strength of using these three complementary approaches helps mitigate some individual methodological weaknesses but also introduces complexities and potential blind spots that need to be considered. In terms of historical analysis, although I pursued deep contextual examination following Forman's approach to understanding scientific developments within their social and cultural environments, the available historical record inherently limits my investigation. While I examined correspondence, institutional documents, and broader cultural contexts, many informal interactions and undocumented influences that shaped these developments remain inaccessible. The cross-case pattern recognition methodology, while powerful for identifying commonalities across my three cases, presents challenges regarding generalizability. The patterns I identify may be particularly visible in these cases precisely because they represent successful, transformative developments in science. This raises questions about whether these patterns hold true for less dramatic or less successful attempts at knowledge creation, or whether they apply equally across different scientific domains and historical periods. Furthermore, the integration of these three methodological approaches, while providing multiple perspectives on my subject matter, introduces its own complexities. The challenge of maintaining consistent analytical depth across all three dimensions while ensuring their effective integration means that some aspects may receive more thorough treatment than others. The interplay between historical context, philosophical framework, and pattern recognition creates a rich but complex methodological landscape where some insights may be emphasised at the expense of others. Additionally, my methodology's emphasis on examining successful cases of knowledge creation may introduce an unintended selection bias. While I can trace how successful frameworks emerged and became established, I have less insight into contemporary competing frameworks that failed to gain acceptance. This limitation affects my ability to fully understand the factors that distinguish successful from unsuccessful attempts at knowledge creation.

Theoretical Framework Limitations

The *o-é-c process* model, while useful for understanding knowledge creation, has its own theoretical limitations. The distinct phases I identify—*Ouverture Ontologique, Épistémè Socialisante,* and *Connaissance Éclairante*—may suggest a more linear progression than actually occurs. In reality, these phases likely overlap and interact in complex ways that my model simplifies, as presented in the example on AI Ethics mentioned in chapter 6. Additionally, my focus on *wicked problems* as catalysts for knowledge creation may not fully capture other important drivers of scientific advancement. Incremental improvements, technological innovations, and serendipitous discoveries may follow different patterns that my framework does not adequately address.

Critical Reflection on Theoretical Boundaries

My research reveals several philosophical tensions that warrant further examination. First, the relationship between individual insight and social processes in knowledge creation remains complex. While my model incorporates both elements, the precise nature of their interaction and relative importance in different contexts requires deeper investigation. Second, the notion of *wicked problems* itself presents philosophical challenges. The distinction between truly *wicked problems* and those that are merely complex but tractable within existing frameworks is not always clear. This ambiguity raises questions about how we identify and categorise problems that require new conceptual frameworks.

Unresolved Questions

Several important questions remain unresolved by my current investigation: How do we account for failed attempts at knowledge creation? What distinguishes promising frameworks that ultimately fail from those that succeed? To what extent is the *o-é-c process* culturally dependent? How might different epistemic traditions approach the creation and validation of new knowledge? What role do institutional structures play in facilitating or hindering the progression through the three phases? How does the increasing specialisation of scientific knowledge affect the social validation process?

Implications and Future Research Directions

Future research could extend my framework in several promising directions with examples like examining knowledge creation processes in non-Western scientific traditions could reveal alternative patterns and enrich my understanding of how new frameworks emerge and gain acceptance; investigating how the *o-é-c process* manifests in current scientific challenges, such as climate change research or artificial intelligence development, could help validate and refine the model; and as an additional example, studying cases where promising theoretical frameworks failed to gain acceptance could provide valuable insights into the necessary conditions for successful knowledge creation.

Methodological Extensions and Refinements

The historical analysis component could be enriched through the incorporation of digital humanities approaches. While my investigation followed Forman's contextual method, future researchers could employ computational text analysis and data mining techniques to analyse larger corpuses of scientific writings, correspondence, and institutional documents. This could reveal patterns and connections that might not be apparent through traditional historical analysis. For instance, researchers could trace the evolution of key concepts and terminology across multiple scientific communities simultaneously, providing insight into how new frameworks disseminate and transform. The philosophical investigation component could be enhanced through greater integration with contemporary approaches in social epistemology and science studies. Future research could incorporate Actor-Network Theory[61] to better understand the role of non-human actors (such as experimental apparatus, computational tools, and institutional structures) in knowledge creation. Additionally, researchers could draw more extensively on feminist epistemology to examine how power relations and diverse perspectives influence the development and acceptance of new frameworks.

The cross-case pattern recognition methodology could be significantly expanded through the application of formal comparative methods. Future studies could develop more structured analytical frameworks for comparing cases across different temporal and cultural contexts. This might include developing specific metrics for measuring the progression through each phase of the *o-é-c process*, allowing for more systematic comparison across cases. Researchers could also employ qualitative comparative analysis[62] (QCA) techniques to identify necessary and sufficient conditions for successful framework establishment.

My methodology, focused on human actors and social processes, may need expansion to account for human-AI collaboration in knowledge creation. For instance, when an AI system identifies patterns that lead to new scientific insights, how does this affect the *Ouverture Ontologique* phase of my model? The validation process for AI-generated discoveries may follow different patterns than those we identified in my historical cases. The *o-é-c model* offers a valuable foundation for understanding knowledge creation, but it must evolve to address contemporary challenges. As we face increasingly complex global problems requiring new ways of thinking and understanding, the importance of comprehending how transformative knowledge emerges and becomes established only grows. This research provides not just a

[61] Actor-Network Theory (ANT) proposes that both human and non-human elements (like technologies, objects, concepts, and organisations) are equally important actors in creating and maintaining social networks

[62] Qualitative Comparative Analysis (QCA) is based on set theory and Boolean algebra, allowing researchers to systematically analyse complex causation patterns across multiple cases. We can think of it as a sophisticated way to identify combinations of conditions that lead to specific outcomes, rather than looking at single causes in isolation.

starting point for that understanding, but also a structured framework for its continued development.

And for this thesis,

“πρέπει να υπάρχει στάση σε κάποιο σημείο”... there must be a stop at some point[63]

[63] Aristotle. “Metaphysics book XII 1070a 4” in *The Complete Works of Aristotle - The Revised Oxford Translation Volume 2 (Bollingen Series LXXI-2)*, edited by Jonathan Barnes. 1st ed. Princeton, NJ, USA: Princeton University Press (1995): 1690

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