# Formulating Opinion Dynamics from Belief Formation, Diffusion and Updating in Social Network Group Decision-Making: Towards Developing a Holistic Framework

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ABSTRACT

Interactions in social networks have become an integral part of people's daily lives. In various decision-making situations, individuals usually hold diverse prior beliefs and engage in communication with their social connections to make informed decisions. However, most existing research focuses on isolated steps of this process, overlooking the overall complexity of decision-making in social networks. To bridge this important research gap, our paper aims to explore the key steps involved in the process and develop a holistic framework for analyzing how individuals form, exchange and update beliefs, ultimately leading to opinion dynamics and group decision behaviors in a social network. Specifically, relevant literature that focuses on different steps will be reviewed and drawn together to characterize the decision-making process in a comprehensive and systematic manner: individuals form initial beliefs following the principle of multiple criteria decision-making intuitively, information propagates in the social network and affects individuals' beliefs differently in a form of social influence, beliefs evolve through dynamic interactions with others, and eventually individuals make their decisions, leading to group decision behaviors in the social network. Applications will be briefly discussed to illustrate the practical implications of this research. Finally, conclusions and future research outlook will be discussed in detail. It is expected that the holistic framework developed on the basis of the comprehensive literature review can provide in-depth insights into decision analysis in social networks and shed light on future research and applications toward effective integration of decision science, operational research, and social network analysis.

## 1. Introduction

With the advancement of Internet technology, more and more people have become Internet users. To meet the diverse needs of these users, various types of social networking applications have been developed, such as Twitter for microblogging, WhatsApp for instant messaging, and LinkedIn for professional networking. In social networks, the most important component is the interaction between individuals. Through these interactions, information can be disseminated, and individuals can learn from their social connections, update their opinions, and make decisions collectively. These topics are of particular interest to researchers in decision science and network science. To explore and characterize these complex interactions and

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decision-making processes under different scenarios, various social network-based models have been developed (Jusup et al., 2022). For example, the centrality measures (Lü et al., 2016), deep reinforcement learning (Fan et al., 2020), evolutionary algorithm (Liu et al., 2019), and trust propagation (Urena et al., 2019) have been applied to determine the importance of individuals and their interactions. The linear threshold models, independent cascade models, and epidemic models have been developed to characterize the information propagation process in the social network (Zhang et al., 2016). The DeGroot (Hunter and Zaman, 2022), Ising model (Ising, 1925), probabilistic inference model (Acemoglu and Ozdaglar, 2011), and Dempster-Shafer theory (Ni et al., 2021) have been employed to describe the opinion dynamics of agents in the network (Sîrbu et al., 2017). In addition, these models have been applied to a range of real-world problems to assist governments in making decisions, help regulatory agencies in responding to emergencies, support companies in adopting profitable strategies, etc. For example, multi-dimensional opinion dynamics models (Liu and Rong, 2022) have been developed to explore the intervention effects of varying official responses during emergency events, including removing comments compulsively and debunking misinformation in time. Agent-based models (Wang et al., 2020b) have been applied to show the counterproductive results of aggressive political campaigns and the reasons why political polarization emerges. Through the $k$-shell decomposition analysis (Kitsak et al., 2010), the most efficient spreaders are found to be located within the core of the network rather than the most highly connected or the most central people, thereby providing insights for the designing of efficient propagation strategies.

Generally speaking, researchers usually focus on only one type of process between interpersonal interactions, such as the establishment of trust, the dissemination of information, and the dynamics of opinions. However, these processes are interdependent in social networks. For example, the evolution of network structure will lead to changes in information propagation paths and the objects of exchange of opinions (Wang et al., 2020b). The trust established between individuals can provide a reliable reference for the opinions they exchange and thus promote opinion dynamics (Li et al., 2021). The dissemination of information in the network will bring in new opinions to further change the opinions of the agents (de Arruda et al., 2022). Nevertheless, most studies cannot formulate the whole process in its entirety.

To address this important research gap, in this paper we delve into this whole process and review comprehensively recent work that focuses on various steps of this process, underscoring the importance of belief formation, diffusion, updating, and opinion dynamics in characterizing social network group decision-making behaviors. A holistic and comprehensive framework of this whole process is developed in Figure 1, and the details are discussed in the sections that follow. Before elaborating on the framework, fundamental definitions and characteristics of social networks and users are introduced in Section 2 to provide readers with a foundational understanding. In this framework, each individual within the social network begins by forming their own belief about a course of action, based on various factors, often guided by a multiple criteria decision-making (MCDM) process (Part 1 in Figure 1). In this process, individuals usually consider multiple factors, such as personal knowledge, current cognition, social context, and other influences, to form their prior beliefs on a specific decision. These relevant factors, along with MCDM approaches are discussed in detail in Section 3. However, individuals do not operate in isolation. Before reaching a reasonable decision, they often engage in discussions and seek input from friends, colleagues, and experts on social media platforms, which leads to belief updates (García-Zamora et al., 2022; Zha et al., 2020). During these interactions, individuals' influence abilities vary depending on their knowledge and their ability to spread beliefs. Therefore, each individual implicitly evaluates the influence ability of others in their social network before interacting with them. Influence ability refers to the extent to which a person's opinions are affected by others, which can be quantified by the weight of edges, importance, or trustworthiness of individuals (Part 2). Two characteristics can determine influence ability in the study of social network group decision-making, which are discussed in Section 4 in detail. The first is network interaction, such as classic centrality metrics (Lü et al., 2016). For example, celebrities and official institutions with large followings (high in-degree) can easily affect their followers' opinions, resulting in high influence ability. The second is belief similarity (Li et al., 2020; Deffuant et al., 2001), as people tend to trust others, who hold similar opinions. Moreover, information propagation within the network affects belief updating, making information diffusion models (Zhang et al., 2016) a crucial component of this review (Part 3). The timing and frequency of new information reception influences the extent to which the individual is affected by this information. We explore several typical information propagation model and their variations in Section 5, including linear threshold models, independent cascade models, and epidemic models.

A key step in this framework is how individuals interact and exchange their beliefs within social network structures (Dong et al., 2018; Lorenz, 2007; Hassani et al., 2022), where relationships are abstracted from broader societal contexts. Models from various disciplines have been proposed to account for different sources and expressions of beliefs and the consequences of belief updating (Part 4). Hence, this framework is developed to focus on the evolution of individuals' beliefs in general scenarios, rather than solely on group outcomes. By updating beliefs and evolving network topology, groups of individuals may reach different collective states, such as consensus, polarization, or fragmentation, exhibiting various group behaviors (Sîrbu et al., 2017; Dombi and Jónás, 2024). Dynamical models from various fields are introduced in Section 6 to illustrate how individuals update their beliefs. Ultimately, individuals make decisions and change their behaviors that are beneficial to themselves by considering internal uncertainties, the costs and benefits of events, the decisions of external groups, and their updated beliefs in the opinion dynamics (Part 5), which also follows an MCDM process. Details about how individuals make decisions and change behaviors are illustrated in Section 7. Moreover, the decision-making process is cyclical: decisions affect individuals' prior beliefs and personal judgments in subsequent similar events. Therefore, this work divides the decision-making process into two broad phases: belief formation and decision-making based on multiple criteria through the MCDM process (Parts 1 & 5), and collective decision-making, shaped by interactions with others in social networks (Parts 2-4).

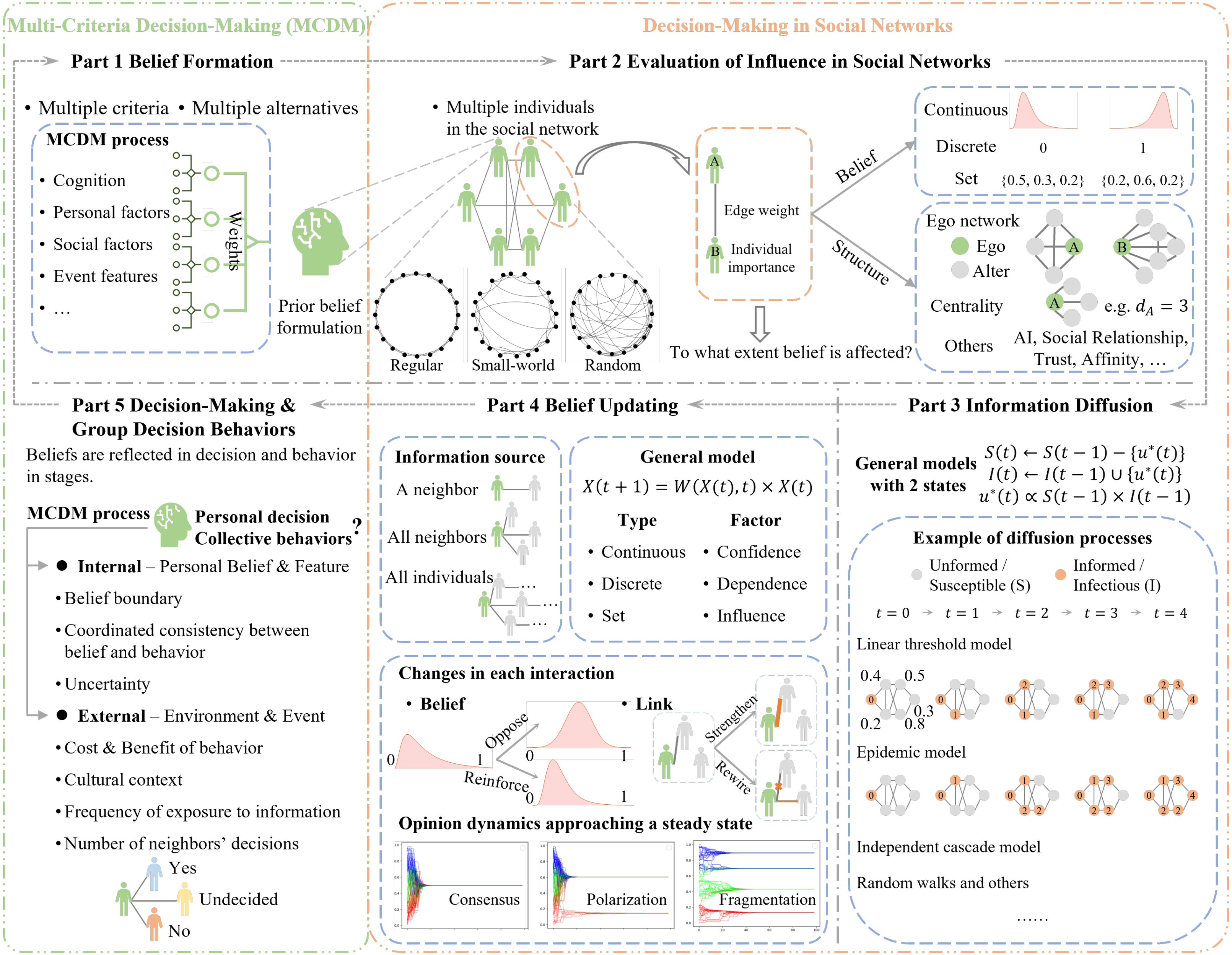


**Figure 1:** A holistic framework developed in this work, with each part introduced individually in sections ranging from Section 3 to Section 7.

The main contributions of this work are summarized as follows.

(1) A holistic framework is developed to characterize the complex process of group decision-making in social networks, based on a comprehensive review of relevant studies that focus on individual steps within the decision-making process.

(2) Two often separate phases in the literature on social network group decision-making are integrated coherently: (a) a multiple criteria decision-making process, where individuals consider multiple factors to form beliefs and make decisions, and (b) a group decision-making process where individual beliefs and opinion dynamics are shaped by social interactions. This integration leverages the strengths of both multiple criteria decision analysis and social network analysis, providing a more comprehensive understanding of decision-making in social networks.

Through this work, we aim to provide researchers with a holistic framework for formulating the entire process from belief formation to decision-making in a general social network context. Its applications across various fields are discussed in Section 8. Finally, the conclusions and the future outlook are discussed in Section 9.

## 2. Background of social network

A social network is a social structure composed of a group of social entities and their interactions, which encompass various activities, such as project collaboration among employees and opinion exchanges among experts. In the social network model $\mathcal{G}(\mathcal{N}, \mathcal{E})$ (Newman, 2018), individuals and connections between them can be denoted by nodes and edges in the set $\mathcal{N} = \{1, 2, \dots, |\mathcal{N}|\}$ and $\mathcal{E} = \{(i, j) : i, j \in \mathcal{N}\}$, respectively. The structure of social networks can be expressed mathematically as an adjacency matrix $\mathbf{A}_{|\mathcal{N}|\times|\mathcal{N}|}$, and its element $a_{ij}$ is defined as,

$$a_{ij} = \begin{cases} 1, & \text{if } (i, j) \in \mathcal{E} \\ 0, & \text{otherwise} \end{cases}. \tag{1}$$

$a_{ii} = 1$ means there is a self-loop for node $i$. It is an undirected network when the adjacency matrix $\mathbf{A}$ is symmetric, but becomes a directed network when $\mathbf{A}$ is asymmetric. A simple example of the directed edge is that individual $i$ follows $j$ but $j$ does not follow $i$ on Twitter. In this case, the weight or length of all edges is the same (its value is 1), so all edges are treated equally. However, relationships between people are generally different, such as the frequency of communication between friends. Therefore, the weighted network was developed to model the interaction between individuals with the weight matrix $\mathbf{W}_{|\mathcal{N}|\times|\mathcal{N}|}$.

There are several types of social networks (Newman, 2018). The simplest models refer to completely regular networks, such as ring networks and lattice networks. The other extreme is completely random networks, where the shortest distance of the path between nodes is small. In general, random graphs are initially composed of $|\mathcal{N}|$ isolated nodes, and edges are randomly added by some fixed rules. One typical random graph is constructed by the Erdős–Rényi (ER) model, where any two nodes are connected independently with probability $p$. Real-world networks are usually between the two extremes by introducing disordered information, such as rewiring the edges in regular networks. A typical one is the Watts–Strogatz (WS) small-world network, characterized by its high clustering coefficient and the small average shortest path length $L \propto \log |\mathcal{N}|$. This originated from the 'small-world' experiments and is the prototype of the theory of six degrees of separation. Another representative is the Barabási–Albert (BA) scale-free network, where the preferential attachment mechanism causes the power-law degree distribution $P(k) \sim k^{-\gamma}$, where $\gamma \in (2, 3)$. The three typical networks are shown in Figure 2.

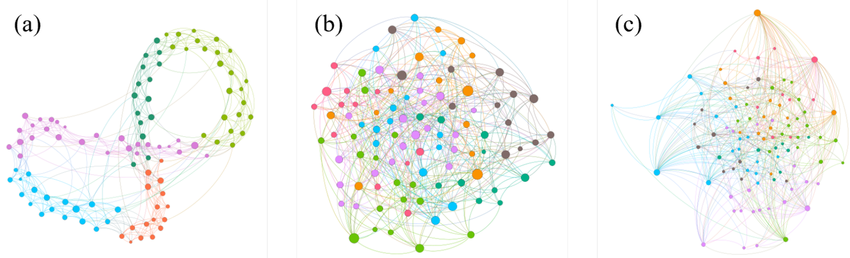


**Figure 2:** Examples of the three fundamental networks, including (a) Watts–Strogatz, (b) Erdős–Rényi, and (c) Barabási–Albert networks, where the size and color indicate the degree and community of nodes.

In contrast to the aforementioned single-layer network, multi-layer social network, also known as 'multiplex network', 'multirelational network' or 'network of network', has been developed to account for different types of social relations or actions (Wasserman and Faust, 1994; Kivelä et al., 2014). This concept is supported by both sociologists (Wasserman and Faust, 1994) and anthropologists (Whitaker Jr, 1970). In a multi-layer network $\mathcal{M}(\mathcal{G}, \mathcal{E}_{\mathcal{I}})$ with $m$ layers, there are both intra-layer and inter-layer connections. Here, $\mathcal{G} = \left\{\mathcal{G}_{(\alpha)}\left(\mathcal{N}_{(\alpha)}, \mathcal{E}_{(\alpha)}\right), \alpha \in \{1, 2, \dots, m\}\right\}$ represents a family of simple graphs, and $\mathcal{E}_{\mathcal{I}}$ describes the inter-layer connections, where endpoints belong to different layers. Here, inter-layer connections can occur between replica nodes across layers or distinct nodes representing different entities in different layers. More detailed descriptions can be found in (Kivelä et al., 2014; Boccaletti et al., 2014). In the networks mentioned above, interactions are typically pairwise, represented by tuples of nodes in $\mathcal{E}$. However, researchers have also identified multi-way interactions in networks called hypergraphs $\mathcal{G}_{\mathcal{H}}(\mathcal{N}, \mathcal{E})$, reflecting group activities in social networks (Zlatić et al., 2009; Çatalyürek et al., 2022). The set of hyperedges, $\mathcal{E} = \left\{\mathcal{E}^{(1)}, \mathcal{E}^{(2)}, \dots, \mathcal{E}^{(n)}, \dots, \mathcal{E}^{(K)}\right\}$, includes edges containing more than two nodes. Specifically, a hyperedge in $\mathcal{E}^{(n)}$ contains $n$ nodes and is represented as an $n$-tuple $(i, j, \dots)$, where $i, j, \dots \in \mathcal{N}$. Hyperedges $\mathcal{E}^{(1)}$ and $\mathcal{E}^{(2)}$ correspond to the set of self-loops and simple edges, respectively. Therefore, hypergraph structures can be described by a set of adjacency tensors $\left\{\mathbf{A}^{(n)}, n = 2, 3, \dots, K\right\}$, where an element of tenser $\mathbf{A}^{(n)}$, representing an $n$-edges, is defined as,

$$a_{ij}^{(n)} = \begin{cases} 1, & \text{if } (i, j, \dots) \in \mathcal{E}^{(n)} \\ 0, & \text{otherwise} \end{cases}. \tag{2}$$

There are still weighted and directed hypergraphs (Arcagni et al., 2024; Boccaletti et al., 2023), analogous to the simple networks discussed above. More details about higher-order networks and hypergraphs can be found in recent works (Ferraz de Arruda et al., 2024; Marques et al., 2025; Arcagni et al., 2017).

The characteristics around nodes can be described by several factors. For example, the degree of node $i$ describes the size of its neighborhood set in undirected networks, $k_i = \sum_{j\in\mathcal{N}} a_{ij} = \sum_{j\in\mathcal{N}} a_{ji}$. It is different in directed networks, which consists of out-degree and in-degree, $k_i = k_i^{out} + k_i^{in} = \sum_{j\in\mathcal{N}} a_{ij} + \sum_{j\in\mathcal{N}} a_{ji}$. Eigenvector centrality (Bonacich, 2007), an extension of degree centrality, takes into account the importance (or score) of neighbors, thereby treating neighbors differently. It is defined as $x_i = \frac{1}{\lambda}\sum_{j\in\mathcal{N}} a_{ij}x_j$, where $\lambda$ is the largest eigenvalue of $\mathbf{A}$. This means an individual can be influential in a social network either by knowing (1) many people or (2) a few highly important individuals. PageRank, an algorithm used to rank websites in the Google search engine (Brin and Page, 1998), is similar to eigenvector centrality but typically applies to directed networks. To account for link weights in weighted networks, the node strength is defined by $s_i = \sum_{j\in\mathcal{N}} w_{ij}$. In addition, the $H$-index considers the degree of neighbors to illustrate the impact of higher-order neighbors (Korn et al., 2009), where the largest value $h$ satisfies that node $i$ has at least $h$ neighbors with a degree larger than or equal to $h$. The core number from $k$-core decomposition has been further developed (Kitsak et al., 2010) to assess whether a node is located in the core part or periphery of the network. To characterize nodes, not only the information from neighbors but also the information about paths can be applied. The simplest centrality, eccentricity, is obtained by the maximum distance from this node to other nodes, $EC_i = \max_{j\in\mathcal{N}} d_{ij}$, where $d_{ij}$ is the length of the shortest path. In addition, the betweenness centrality that can control the information flow is defined by $B_i = \sum_{i\neq s, i\neq e, s\neq e} n_{se}^i / n_{se}$, where $n_{se}$ is the number of the shortest path between nodes $s$ and $e$ and $n_{se}^i$ is the number of the above paths passing through node $i$. Interested readers are referred to (Lü et al., 2016) for more detailed information on local characteristics.

As for the network characteristics, the density $\rho = 2|\mathcal{E}|/|\mathcal{N}|(|\mathcal{N}|-1)$ can indicate the number of existing edges compared to that of possible edges, thereby differentiating networks with different sizes. In social networks, not all individuals are connected, resulting in disconnected groups. Generally, the largest connected groups (i.e., the giant component) includes a significant proportion of individuals. In addition, community (also called cluster or module) is a common structure in the study of statistics, dynamics, and social influence, where nodes are tightly connected within communities but loosely connected between communities. It is usually caused by common locations, roles, and interests among individuals in social networks, resulting in different frequencies of communication between people (Fortunato and Newman, 2022). Assortativity, the tendency of individuals to connect with others who have similar degrees, is commonly observed in social networks, whereas technological and biological networks tend to exhibit disassortativity, where high-degree nodes are more likely to connect with low-degree ones. To measure this tendency, the assortativity coefficient (Newman, 2002, 2003) has been developed based on the Pearson correlation coefficient of degree between connected nodes,

$$r = \frac{\sum_{ij} ij\left(e_{ij} - q_i q_j\right)}{\sigma_i \sigma_j}, \tag{3}$$

where $\sigma$ is the standard deviation of degree distribution $q$, and $e_{ij}$ is the joint probability distribution. Other node attributes can replace degree centrality to assess connection tendencies. To determine if a network exhibits small-world or scale-free properties, the average shortest path length $\ell = \sum_{i \neq j} d_{ij}/|\mathcal{N}|(|\mathcal{N}| - 1)$ and the average clustering coefficient (transitivity) $C = 3\times$ number of triangles / number of all triplets (Wasserman and Faust, 1994) are compared with an equivalent random network. Therefore, small-worldness can be measured by $\sigma_{SW} = C\ell_r/C_r\ell$ or $\omega_{SW} = \ell_r/\ell - C/C_r$ (Telesford et al., 2011). Thus far, both local and global network characteristics that affect decision dynamics have been widely explored in this section.

## 3. Belief formation

In social network group decision-making, individuals typically form their own prior beliefs about events and behaviors before interacting with others. These beliefs are often shaped by personal attributes, social factors, and the characteristics of the event itself. This process is considered Part 1 of the overall framework in Figure 1. A belief is generally defined as the mental acceptance or conviction in the truth or reality of an idea (Das et al., 2019; Schwitzgebel and Zalta, 2011). It can be characterized as the propositional attitude, involving both a specific meaning expressed in sentence form and a mental stance on the validity of the proposition (Schwitzgebel and Zalta, 2011), while also encompassing subjective experiences. In literature, the formation of beliefs is widely discussed from social-psychological perspectives, often incorporating an understanding of uncertainty. Uncertainties encountered, including those related to the reliability of verbal information, are themselves manifestations of beliefs (Wyer and Albarracin, 2005). The majority of beliefs likely remain unconscious or outside of immediate awareness, yet their content pervades various aspects of life (Connors and Halligan, 2015).

### 3.1. Relevant social-psychological aspects

An individual's perspective regarding a given event is intricately shaped by a multifaceted interplay of internal and external determinants, where some typical factors are shown below:

- *Internal determinants* consist of personal experiences and traits, cognitive processes, and cultural backgrounds (Wyer and Albarracin, 2005; Connors and Halligan, 2015).
- *External determinants* contain media outlets, societal influences, political and ideological affiliations, and figures of authority (Friedkin and Johnsen, 1990; Keren, 2014).

For example, in the domain of health beliefs, Rosenstock (1974) developed the health belief model and conducted a systematic review of the determinants of individuals' health beliefs. The factors influencing health beliefs can be categorized into perceived susceptibility, perceived seriousness, perceived benefits and barriers to taking action, and cues to action. Among these, Langlie (1977) identified the most impactful factor as the "perceived internal locus of control". This concept, originally proposed outside the health context, suggests that individuals who believe they can control what happens to them are more likely to take action. Despite beliefs arising from disruptions in direct experiential encounters, it has been found that beliefs may also originate from social interactions, exposure to media in the social environment, and secondary sources like books, newspapers, and television (Langdon, 2013; Enders et al., 2021; Druckman et al., 2021). Hence, it is vital to acknowledge the dynamic nature of the belief formation process, marked by the intricate interplay of these diverse elements. Moreover, it is essential to recognize the fluidity of beliefs, susceptible to change and evolution over time as individuals encounter new information and diverse experiences (Connors and Halligan, 2015; Ecker et al., 2022).

Numerous researchers underscore the pivotal role of social psychology in comprehending the process of belief formation because beliefs are not developed in isolation (Bar-Tal, 2000; Galesic et al., 2021). Analyzing the development of individual beliefs from the perspective of social psychology involves examining how social, cognitive, and emotional factors interact to shape an individual's belief. This perspective affords an avenue to investigate the mechanisms through which individuals are subject to social influence, thereby offering critical insights into phenomena including but not limited to conformity, self-fulfilling prophecy, groupthink, and persuasion (Baron, 2005). Normative and informational social influence are defined based on the psychological needs that lead humans to conform to the expectations of others, such as compliance and deindividuation (Cialdini and Goldstein, 2004). It has been found that people often seek to 'fit in' amongst friends and colleagues and to be liked and respected by other members of their social group. Moreover, individuals often value the opinions of others in their social groups and seek to maintain their standing within the group. As a result, they adjust their attitudes and behaviors to align with group norms. At the same time, when individuals feel uncertain about their own knowledge, they turn to others for information, hoping to receive accurate and reliable insights.

Following the seminal work on conformity by Asch and Guetzkow (1951), the study of social influence has gradually reached the culmination (Becker et al., 2017; Capuano et al., 2017). More work has been done in this period than any other, especially in the core areas of social influence, such as deindividuation, obedience, and reactance. From social influence theory, which was proposed by Kelman (1958), three broad categories of social influence were identified, including compliance, identification, and internalization. Specifically, compliance is defined as cooperation motivated by the desire for social acceptance rather than behavior according to a request, and people are influenced to comply because they wish to avoid negative social consequences or to gain social approval. Identification occurs when individuals adopt the induced behavior to create or maintain a desired and beneficial relationship with another person or a group. Internalization happens when individuals receive influence after perceiving the content of the induced behavior as valuable, where the content indicates the opinions and actions of others. Overall, the field of social influence saw a transformative moment with Cialdini (2009), marking a new era where research was systematically integrated across disciplines under the umbrella of social influence. Six fundamental principles of social influence are then identified by Cialdini (2009), including reciprocity, commitment and consistency, social proof, authority, attractiveness, and scarcity. The study of social influence gained further credibility in 2006 with the establishment of the journal *Social Influence*. An overview of research on social influence in the field of social psychology is demonstrated in Figure 3.

Beyond the factors that shape beliefs, the process of belief formation itself has become a key area of study, often drawing from disciplines such as psychology, sociology, economics, statistical physics, and applied mathematics (Acemoglu and Ozdaglar, 2011; Enders et al., 2021; Castellano et al., 2009). Extensive research has provided a wealth of empirical findings and theoretical models on the structure and formation

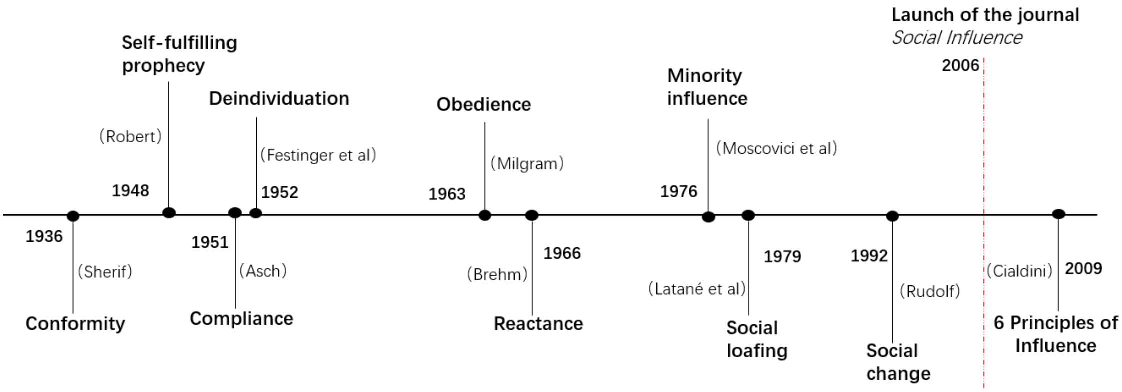


**Figure 3:** Seminal research of social influence in social psychology.

of beliefs. For instance, the trust-structural-cognitive model (Robbins, 2016) offers a theoretical framework that examines the origins and effects of trust in daily human interactions, which in turn influence belief formation. Similarly, preference modeling (Moretti et al., 2016) is an important approach for understanding and representing an individual's beliefs about multiple objects, including their preferential order and similarity. Moreover, a comprehensive synthesis of belief formation has been conceptualized as a five-stage, non-recursive progression, spanning from precursor events to the ultimate effects of beliefs (Connors and Halligan, 2015). This model integrates insights from both cognitive and neuropsychological studies. Ni et al. (2021) developed a criterion hierarchy to analyze personal beliefs toward vaccination by considering perceived disease risks (such as susceptibility and severity) alongside vaccine-specific issues (such as safety, effectiveness, and convenience). Beliefs in this model are established through an evidential reasoning approach. Across these methods, multiple internal and external factors need to be considered when shaping personal beliefs. This complexity makes MCDM approaches particularly useful due to its adaptability in accounting for multi-level and multi-attribute factors, leading to reasonable belief formation. Therefore, the following section will review typical multiple criteria decision-making approaches.

### 3.2. Multiple criteria decision-making

In line with the previous discussion, the formation of beliefs over a decision-making problem can be influenced by a multitude of internal and external factors (Enders et al., 2021; Keren, 2014; Druckman et al., 2021). Moreover, uncertainty serves as a driving force behind belief formation, prompting individuals to continuously acquire knowledge and re-evaluate their subjective judgments (Seitz and Angel, 2020). Hence, belief formation is regarded as an ongoing dynamic process. Scholars have observed that individuals' beliefs significantly influence their decision-making processes, wherein decision-making entails the cognitive process of selecting action plans among multiple alternatives and executing actions (Simon, 1959). However, understanding belief formation in a complex social environment requires consideration of multiple criteria, which is now a prominent feature of contemporary decision-making processes (Seitz and Angel, 2020; Porot and Mandelbaum, 2021; Ni et al., 2021).

The process of belief formation lays the foundation for how individuals assess, evaluate, and interpret information when making decisions involving multiple criteria or objectives. Usually, multiple criteria or attributes are involved in the process of assessing alternatives with diverse weights (Ding et al., 2020). In this process, individuals hold subjective beliefs or preferences about the importance of these criteria and the performance of alternatives (Seitz and Angel, 2020; Ni et al., 2021). Notably, subjective beliefs play a significant role as they guide the weighting of criteria and the evaluation of alternatives (Shafer, 1976; Sasaki, 2023). The scientists systematically investigated decision-making processes necessitating the consideration of multiple criteria, introducing the concept of multiple criteria decision-making (MCDM) (Greco et al., 2024; Sahoo and Goswami, 2023). Research on MCDM began in the 1960s on economics and became an active research field in the 1970s, where early research mainly focused on methods rather than the structure of MCDM. The methodologies and theories are gradually being valued due to a further comprehensive MCDM process. This process mainly relies on a variety of methods to solve different types of problems. Typically, there is no unique optimal solution for decision-making problems, thus, the incorporation of individuals' preferences becomes more important (Psomas et al., 2021). In addition, uncertainty associated with criteria weights and performance assessments is a vital factor for this process because it is believed that individuals cannot have completely certain attitudes and knowledge (Shafer, 1976). Therefore, due to MCDM's ability to incorporate uncertainty, human expertise, and subjective judgments, it has been considered to be an important tool in belief formation. Methods defined in this discipline are based on various principles and use different scoring, weighting, and aggregation procedures (Cinelli et al., 2022), thus, a synopsis of prevalent MCDM methodologies is offered in Table 1. More details about MCDM approaches can be found in the comprehensive reviews (Sahoo and Goswami, 2023; Alvarez et al., 2021).

MCDM can participate in the belief formation process in various ways (Wu and Barnes, 2010; Ni et al., 2021). However, it is essential to understand the manifestation of beliefs at the first stage. Researchers utilize various methods and techniques to represent beliefs in their research, depending on the nature of the research question, the research context, and the underlying theoretical framework. For instance, beliefs gathered through surveys from individuals or groups can be represented using commonly employed tools, including Likert scales, semantic differential scales, and visual analogue scales (Dean et al., 2021). These tools allow for the measurement of the intensity or strength of beliefs on specific topics. Moreover, rich descriptions of participants' beliefs collected by qualitative methods, such as interviews and online survey (Hickman et al., 2021), can be analyzed to identify patterns and themes related to beliefs, thus, researchers could categorize and quantify beliefs through a corresponding coding scheme. Additionally, quantitative methods such as evidence reasoning and probability theory can represent beliefs as probability distributions or belief structures (Xia and Liu, 2014), bringing out the uncertainty in beliefs as well. Therefore, belief formation can be demonstrated by analyzing the MCDM process. For example, Tam and Tummala (2001) utilized Likert survey data collected through research to elucidate the beliefs of 20 staff members regarding 23 distinct selection criteria encompassing quality, delivery, performance history, and other factors pertinent to the choice of suppliers in the telecommunications system domain. Furthermore, the Analytic Hierarchy

**Table 1**
A brief summary of MCDM methodologies.

| Approaches | Methods | Descriptions | Reference |
|---|---|---|---|
| Value measurement models | Weighted Sum | Solve single-dimensional problems through additive utility assumption. | (Fishburn, 1967) |
| | Weighted Product | Solve single-dimensional problems by multiplication utility assumption. | (Bridgman, 1922) |
| | Analytical Hierarchy Process (AHP) | Decompose problems into a hierarchical structure and use pairwise comparisons. | (Saaty, 1980) |
| | Simple Multi-Attribute Rating Technique | Rate alternatives based on a linear combination of criteria scores. | (Edwards, 1977) |
| | Multi-attribute Utility | Consider multiple attributes and individual preferences to maximize utility in ranking. | (Keeney and Raiffa, 1993) |
| | Evidence Theory | Combine mass functions from different sources with ignorance and uncertainty. | (Shafer, 1976) |
| | Evidential Reasoning | Combine evidence from multiple sources to make decisions under uncertainty. | (Yang and Singh, 1994) |
| Goal, aspiration or reference level models | The technique for order preference by similarity to ideal solutions (TOPSIS) | Evaluate the proximity of alternatives to the ideal and farthest-from-ideal solutions. | (Hwang et al., 1981) |
| Outranking relations | Preference ranking organization method for enrichment evaluation | Rank alternatives based on pairwise comparison with several criteria. | (Brans et al., 1986) |
| | The elimination and choice translating reality | Handle quantitative and qualitative criteria to provide outranking relations. | (Roy, 1968) |

Process is employed to incorporate the beliefs of multiple individuals with different conflicting goals, thereby achieving consensus decisions (Tam and Tummala, 2001).

## 4. Evaluation of influence in social networks

In social networks, each individual, including bots (des Mesnards et al., 2022), typically has multiple neighbors. The validity of information received from different users must be assessed as it is received, as this information can influence an individual's opinion to varying degrees, depending on personal characteristics and other factors. This evaluation is discussed in Part 2 of the framework. While this has been explored across different fields, we focus on its implementation in social networks by quantifying the degree of influence of connected individuals based on the presence of edges in various types of networks. During the belief updating process, different terms have been used to describe the degree of influence between individuals, such as weight, trust, confidence, and reputation (Friedkin and Johnsen, 1990; Fan et al., 2020; Sherchan et al., 2013). Although there are slight differences between these concepts – people may completely trust family members in daily matters due to family bonds, but the degree of influence may vary in professional contexts due to differences in expertise and domain knowledge – all these terms ultimately describe how much an individual accepts the ideas of others. The degree of influence is primarily based on two fundamental types of personal profiles. The first is the structural characteristics of the individual within social networks, such as their positions, connectivity, and the nature of their relationships with others. The second type is the individual's belief profiles, which encompass their personal experiences and existing beliefs. These profiles are utilized in various scenarios and can be effectively integrated to evaluate an individual's influence in the social network.

By considering the structural characteristics and belief profiles, a comprehensive understanding of influence evaluation in social networks can be achieved.

### 4.1. Structure-based approaches

Social networks primarily consist of users and their connections, so statistical structural properties are often used to describe the validity of the information. In most early models, all of an individual's neighbors were treated equally, meaning they all had the same level of influence. However, the concept of confidence was later introduced to represent how firmly an individual adheres to their beliefs (Friedkin and Johnsen, 1990), allowing for distinctions in how different users influence one another. In order to distinguish the influence from others, the centrality measure is widely used to describe the structure characteristics of individuals. For example, the opinion of an individual with many friends is a relatively more important source for others, reflecting the degree centrality of individuals; and the quality of friends also matters (Jia et al., 2015) – the opinion is of high importance for others if an individual has few but knowledgeable friends – reflecting eigenvector centrality and the PageRank centrality. This has been reflected in the social power ranking (Jia et al., 2015). The topological information from both direct and second-order neighborhood agents can be also considered to determine the weight, including the self-persistence degree and degree centrality measures of agents. Some centrality measures that consider different types of information have been reviewed in Section 2.

Recently, various models have been developed to identify the importance and influential ability of individuals based on the topological structure of networks. Below, we briefly introduce some typical methods:

- *Artificial intelligence algorithms*: A deep reinforcement learning framework, FINDER (Fan et al., 2020), can be trained on small synthetic networks and applied to identify key players in different real-world scenarios. The training process operates as a Markov decision process, involving interactions between agents' states, actions, and rewards within the environment. Another deep reinforcement learning algorithm (Ma et al., 2022) has been designed to evolve the deep $Q$ network to identify vital nodes.

- *Evolutionary optimization and operation approaches*: A branch-and-cut algorithm with Benders reformulation (Güney et al., 2021) has been developed to identify the set of individuals with the maximum influential ability, significantly outperforming typical methods in solution runtime. In this work, the problem is defined as a maximal covering location problem with the objective $\max \sum_{\omega \in \Omega} p_\omega \mu_\omega$, where $p_\omega$ and $\mu_\omega$ indicate the probability and contribution of scenario $\omega \in \Omega$. In addition, a discrete moth-flame optimization method (Wang et al., 2021) addresses the unreliability of communication channels to identify influential spreaders by enhancing the processes of population initialization, selection, updating, and mutation. A game-theoretic approach (Liu et al., 2024) considers non-additive fuzzy measures provided by individuals, determining their importance based on their connections via a gravity model.

- *Mathematical physics methods*: Research has shown that individuals at the core of the network, rather than those who are the most highly connected, are the most influential spreaders (Kitsak et al., 2010). In addition, this issue has been addressed from the perspective of optimal percolation (Morone and Makse, 2015), where the energy of a many-body system is minimized. Fractal-based algorithms (Wen and Cheong, 2021; Wen and Deng, 2020) have also been applied to describe the local structure around each node, thereby identifying its influential ability.

Different types of networks have garnered attention in this field (Zhou et al., 2023; Klages-Mundt and Minca, 2022; Wen et al., 2024a), including weighted, directed, and temporal networks. While machine learning models demonstrate promising performance, they often lack explainability regarding why a particular group of users exerts the strongest influence. On the other hand, mathematical physics models offer explanatory mechanisms but may struggle to yield reasonable results across networks of diverse sizes and types. Therefore, integrating these approaches to analyze topological characteristics in different scenarios is necessary for comprehensively quantifying users' influence in social networks.

Trust, a concept extensively explored in sociology and psychology, serves as a metric for quantifying individuals' reliability within interactions. Although definitions vary across disciplines, trust is generally understood as the confidence one entity believes another will behave in the expected way (Sherchan et al., 2013). In the context of social networks, this concept extends to social trust, reflecting the social capital inherent in the richness of the connections between individuals. Social trust exhibits several key properties: it is subjective and self-reinforcing due to individual cognition, propagates yet remains non-transitive among a group of individuals, and is dynamic, influenced by new information and experience. More details about its properties can be found in (Sherchan et al., 2013). In group decision-making, trust between individuals plays a pivotal role, facilitating information sharing and opinion exchange (Urena et al., 2019; Wang et al., 2024b, 2020a), which in turn, enhances collective decision-making and consensus-reaching among individuals.

To determine the trust between individuals, two primary approaches can be employed. The first, rooted in network structure analysis, suggests that individuals linked to highly connected peers typically command greater trust. This is often modeled using frameworks like the Web of Trust or Friend-Of-A-Friend, wherein trust networks are constructed for each individual (Wu et al., 2019; Gong et al., 2020). This method considers both social relationships and feedback from social connections to evaluate the trust level between them. However, it overlooks direct interactions between individuals in the group, including their nature, frequency, and intensity. To bridge this gap, the second approach focuses on interactions within a group. For instance, in the STrust model, trust is evaluated based on the positive interactions in a group (Nepal et al., 2011). Specifically, it considers the popularity trust, indicating the trustworthiness of an individual from others in the group, and the engagement trust, reflecting the trust this individual has towards the group. While insightful, this approach tends to neglect network topological structure, leading to incomplete information consideration. Hence, a hybrid model that integrates both perspectives could offer a more comprehensive evaluation of social trust (Trifunovic et al., 2010).

Furthermore, estimating unknown trust can leverage known and available trust values from others. For example, if individual $i$ trusts $j$, and $j$ trusts $k$, it is likely that individual $i$ will also trust $k$, a principle known as direct trust propagation. This concept extends through mechanisms such as transpose trust, co-citation, and trust coupling (Guha et al., 2004). These propagation methods can be integrated into a matrix $C_B = w_1 B + w_2 B^T B + w_3 B^T + w_4 B B^T$, where $B$ is the belief matrix and $W = (w_1, w_2, w_3, w_4)$ represents the weight coefficients. This framework is also capable of estimating distrust among individuals. As trust propagates through multiple pathways in the network, the propagated trust needs to be aggregated to estimate the missing trust $t_{ij}$ between individuals $i$ and $j$. The ordered weighted averaging approach (Li et al., 2021) is a notable technique in this field. Other approaches, such as those rooted in quantum theory (Wang et al., 2024b), can also be employed to aggregate trust. Nonetheless, some algorithms think that trust propagation prefers the shortest path in the social network, and it can balance the number and cost of trust propagation (Wu et al., 2019; Wang et al., 2024a). In addition, on the propagation path, trust stability and discounting should be considered over longer social distance (Wang et al., 2024b). Interested readers can refer to (Urena et al., 2019; Sherchan et al., 2013; Wang et al., 2020a) for more details about estimating trust using different kinds of methods, such as machine learning, diffusion models, and structural features.

### 4.2. Belief-based approaches

The belief profile can be also considered to determine the weight of individuals. The trivial way to consider the difference of belief profiles between individuals in belief updating is through the bounded confidence model (Bernardo et al., 2024), including the Deffuant–Weisbuch model (Deffuant et al., 2001), and the Hegselmann–Krause model (Hegselmann et al., 2002). Here, individuals only trust and communicate with others whose beliefs are within the range of confidence, that is, the difference in beliefs $|x_i(t) - x_j(t)|$ is below a given bounded confidence $\varepsilon$. Individuals who are outside the confidence set will not be trusted, and opinions cannot be exchanged between them. More details of the bounded confidence model and its variants will be introduced in Section 6.2.

Individuals may exhibit cognitive dissonance if they experience conflicting beliefs (Festinger, 1957), which can increase their psychological stress (Li et al., 2020). To reduce cognitive dissonance, individuals usually choose to (1) accept information that is more consistent with their existing beliefs or (2) reject or ignore conflicting information. Therefore, a cognitive dissonance-based opinion model (Li et al., 2020) has been developed that (1) assigns weights to others who are in the confidence set and (2) breaks ties with individuals who have conflicting beliefs and connects with individuals who support its opinion. In addition, it has been found that alternative response behaviors are effective in reducing cognitive dissonance in a group (Whitaker et al., 2021). The impact of reconciling cognitive friction is investigated on different networks to examine the sensitivity of behavior to network structures in coping with alternative dissonance. Many modified models have been developed to consider different factors and characteristics in social relationships. For example, the local world opinion from agents' common neighbors is introduced to measure the difference in opinions and network structure (Dong et al., 2022), and more than one type of communication mechanism is considered to assign weights to neighbors based on a mixed opinion dynamics model (Wu et al., 2023). A framework for estimating the extent to which personal beliefs are influenced by others can be found in Figure 4. It details the key components and processes involved in quantifying the influence of social connections on individual beliefs, which are essential for understanding the degree of influence within social networks.

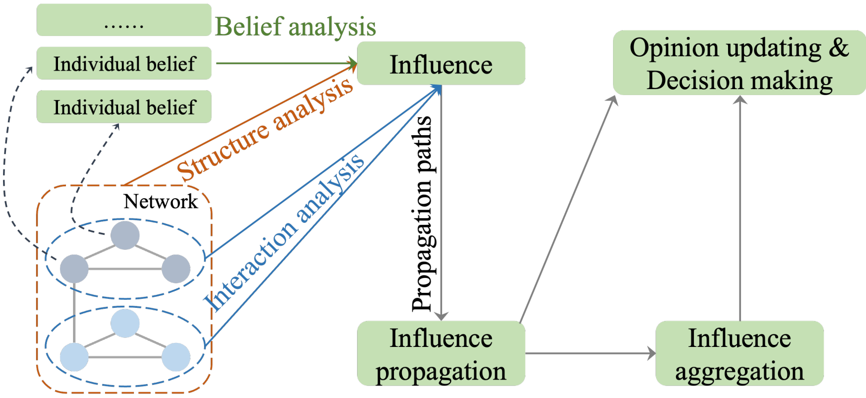


**Figure 4:** Illustration of evaluating the extent to which personal beliefs are influenced by others through interactions, facilitating opinion updating and decision-making.

## 5. Information diffusion models

In social networks, belief updates are primarily influenced by two factors. First, new information spreads through the network, reaching individuals at different times and affecting their cognitive processes in various ways (de Arruda et al., 2022; Keppo et al., 2022; Ferraz de Arruda et al., 2024). In this passive process, both the timing and frequency of the new information reaching individuals play a crucial role in shaping their beliefs, which corresponds to Part 3 of the framework. Hence, some fundamental models that explore information propagation within social networks will be reviewed in this section.

Second, interactions and exchanges of opinions with friends in the network involve proactive behavior, which can also alter individuals' cognitive states. Through active discussions and sharing of perspectives, individuals' beliefs evolve, forming Part 4 of the framework. A more detailed examination of opinion dynamics models will be presented in the subsequent section.

Hence, this section will focus primarily on reviewing several fundamental and noteworthy models that explain the dynamics of information propagation and diffusion within social networks. In the next section, we will delve deeper into opinion dynamics models.

### 5.1. Linear threshold models

Numerous mathematical models have been developed to characterize the information diffusion process, thereby analyzing the diffusion patterns and controlling the spread of misinformation and viruses. One of the most fundamental models is the linear threshold model (LTM) (Granovetter, 1978), which was developed to characterize collective behavior. In this model, there are two states for each agent, including active and inactive states. The model assumes that individuals are likely to make decisions based on the actions already taken by their neighbors, exhibiting herd-like behavior. An individual can imitate its neighbors' behavior if it surpasses a threshold $\phi$, chosen from a distribution $f(\phi)$ based on memory and exposure history. Initially, only a small part of individuals are randomly designated as active while the rest remain inactive. In each step, the inactive individual $i$ becomes active if the fraction of its active neighbors exceeds $\phi_i$, while active individuals retain their states. This process continues until no more individuals can be activated, achieving system stability. An example of this process is shown in Figure 5 (a).

This model has been combined with numerous models, such as competitive diffusion models (Yang et al., 2020) and non-Markovian processes (Wang et al., 2016), to describe information diffusion and rumor propagation, investigate information cascades, and explore the impact of modular structure in the information propagation process. To incorporate the memory of past exposures to the information, generalized LTMs (Dodds and Watts, 2004) have been developed to consider the impact of inactivated individuals in the past $t'$ steps. Specifically, a new state, removed, is introduced in this model, where an activated individual will recover if the impact this individual received from neighbors is less than the threshold. Under specific settings, this model can degenerate into the independent interaction model, stochastic threshold model, and deterministic threshold model (Dodds and Watts, 2004).

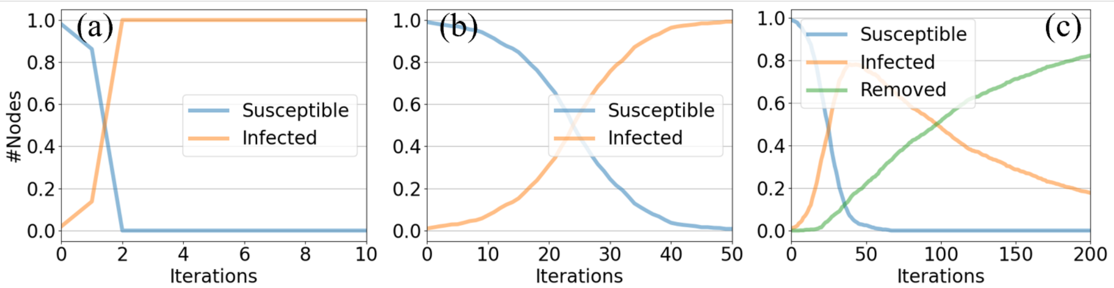


**Figure 5:** Examples of information propagation models on an Erdős–Rényi (ER) network with $|\mathcal{N}| = 1000$ nodes and connection probability $p = 0.2$, including (a) LT model with $\phi = 0.03$, (b) SI model with $\beta = 0.001$, and (c) SIR model with $\beta = 0.001$ and $\gamma = 0.01$. Results were averaged over 200 realizations.

## 5.2. Independent cascade models

The independent cascade model (ICM) is another fundamental model, initially developed from interacting particle systems to study marketing dynamics (Goldenberg et al., 2001). Similar to the linear threshold model, individuals in ICM are categorized into two states: active and inactive. At the onset of information diffusion ($t = 0$), all individuals are inactive except for the sources. Each inactive individual $i$ can be activated by its active neighbor $j$ with probability $p_{ji}$, independently of the influence of other active neighbors. In addition, each active individual $i$ attempts to activate its inactive neighbor $j$ only once, with no further influence regardless of success or not. The diffusion process continues until no more individuals can be activated, reaching time $t = t_{end}$. If $\mathcal{A}_t$ indicates the set of active nodes at time $t$, the process follows

$$\mathcal{A}_0 \subseteq \mathcal{A}_1 \subseteq \cdots \subseteq \mathcal{A}_t \subseteq \mathcal{A}_{t+1} \subseteq \cdots \subseteq \mathcal{A}_{t_{end}} \subseteq \mathcal{N}, \tag{4}$$

implying that active individuals cannot revert to being inactive during the process. The diffusion probability of each edge $p_{ij}$ can be estimated using the expectation-maximization algorithm based on past propagation (Saito et al., 2008), making this model applicable to real-world networks without known diffusion probability.

This model has been extended to various scenarios, including time-delay and negative information propagation (Gruhl et al., 2004). Given that these models traditionally consider discrete time, a continuous-time ICM (Saito et al., 2009) has been developed, where the time-delay $\delta$ on edge $(i, j)$ follows an exponential distribution with parameter $r_{ij}$. In marketing scenarios, where individuals can be influenced multiple times with time restrictions by their neighbors, a continuously activated and time-restricted ICM (Kim et al., 2014) has been developed to better describe the diffusion progress.

After characterizing the information diffusion process, it is crucial to know how to control the information coverage size when the system reaches stability. Positive information is typically expected to spread widely, while rumors (negative information) should be minimized to reduce their impact, leading to the influence maximization problem and contamination minimization problem, respectively. For example, the influence maximization problem can be defined as,

$$\arg \max_{S \subseteq \mathcal{N}, |S|=k} \sigma(S), \tag{5}$$

where $\sigma(S)$ quantifies the influence of a set $S$ of $k$ individuals. This NP-Hard problem in networks is challenging due to the vast number of candidate sets $S$ and the complexity of quantifying the actual influence of individuals (Kim et al., 2014). Several review papers (Li et al., 2018, 2023) have comprehensively reviewed how to incorporate IC-based and LT-based approaches to address these issues.

### 5.3. Epidemic models

Epidemic models are extensively used to mathematically describe the propagation of information and infectious diseases (Chowell et al., 2016). These models date back to Daniel Bernoulli's study of smallpox spread in 1760 and were later solidified by Kermack and McKendrick in 1927 (Kermack and McKendrick, 1927). The simplest example is the Susceptible-Infectious (SI) model, where the susceptible and infectious individuals correspond to inactive and active states, respectively. In this model, a susceptible individual becomes infected through contact with its infected neighbors with a constant probability $\beta$. This process can be described using a system of ordinary differential equations,

$$\begin{cases} \frac{dS(t)}{dt} = -\beta S(t) I(t), \\ \frac{dI(t)}{dt} = \beta S(t) I(t). \end{cases} \tag{6}$$

Moreover, the model can be extended to include recovery, with infected individuals recovering at a probability $\gamma$. If individuals have transient immunity post-recovery, the Susceptible-Infectious-Susceptible (SIS) model is used, where recovered individuals can become infected again. However, if individuals gain

**Table 2**
A brief summary of information diffusion models.

| Objective | Level | Model | Example |
|---|---|---|---|
| Information Diffusion Models | Macro level | Epidemic model | SAIDE model (Cheong et al., 2020) |
| | | | SIDARTHE model (Giordano et al., 2020) |
| | | Bass model | Bass model (BM) (Bass, 1969) |
| | | | BM with free sampling (Han and Zhang, 2018) |
| | Micro level | Threshold model (TM) | LTM (Granovetter, 1978) |
| | | | Non-Markovian LTM (Wang et al., 2016) |
| | | | Generalized LTM (Dodds and Watts, 2004) |
| | | | Competitive LTM (Yang et al., 2020) |
| | | Cascade model | ICM (Goldenberg et al., 2001) |
| | | | ICM with time delay (Gruhl et al., 2004) |
| | | | Continuous-time ICM (Saito et al., 2009) |
| | | | Continuously activated and time-restricted ICM (Kim et al., 2014) |
| | Others | | Linear influence model (Yang and Leskovec, 2010) |
| | | | External influence model (Myers et al., 2012) |

permanent immunity after recovery, the Susceptible-Infectious-Recovered (SIR) model is applicable,

$$\begin{cases} \frac{dS(t)}{dt} = -\beta S(t)I(t), \\ \frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t), \\ \frac{dR(t)}{dt} = \gamma I(t), \end{cases} \tag{7}$$

where $\gamma$ indicates the recovery probability. The basic reproduction number $R_0 = \beta/\gamma$ determines the dynamics of the infection. Examples of the SI and SIR process are shown in Figure 5 (b) and (c).

Gradually, additional compartments have been incorporated to better describe the features of different diseases. For example, the SEIR model includes individuals who have been exposed but are not yet infectious, and the SIRV model incorporates vaccination during the process. During the outbreak of COVID-19, several SIR-based models have been developed to consider new states (Cheong et al., 2020), such as the SIDARTHE model (Giordano et al., 2020) which considers susceptible, infected, diagnosed, ailing, recognized, threatened, healed, and extinct individuals. Regardless of the number of compartments, the sum of individuals in each compartment must equal the total number of nodes in the network. These epidemic models have been further explored using various mathematical and physical methods, including homogeneous and heterogeneous mean-field methods, pair-based methods, and generating function methods. A brief summary of information diffusion models, including other typical approaches like the Bass model (Bass, 1969) and linear influence model (Yang and Leskovec, 2010), can be found in Table 2. More comprehensive details on these information propagation models, including random walk proportion and time-varying network diffusion processes, can be found in Zhang et al. (2016).

## 6. Belief updating and opinion dynamics

Variations in belief are influenced not only by an individual's social psychological traits but also by their social milieu. In social networks, interactions and communication with others can easily shape individuals' feelings and attitudes. Empirical evidence suggests that individuals update their opinions and beliefs as a mix of their own and others' opinions with weights, a concept present in early works (DeGroot, 1974). The mixing mechanism, known as the convex combination, is considered fundamental in synthesizing diverse information in the information integration theory. Therefore, various opinion dynamics and belief updating mechanisms have been formulated to explore this issue within the social cognitive structures (Sîrbu et al., 2017; Choi et al., 2023; Jia et al., 2015).

In this part (i.e., Part 4) of the holistic framework, we focus on how individuals interact with their peers to update their beliefs in a general scenario, without being constrained to specific events. This differs from large-scale group decision-making (LSGDM) (Li et al., 2022; Hassani et al., 2022; Urena et al., 2019), which emphasizes how a group of experts reaches consensus on a particular decision event. Generally, for social issues of common concern, it is challenging for all individuals to achieve consensus. For instance, in political elections and climate change discussions, polarization and fragmentation often occur due to the echo chamber effect in social media (de Arruda et al., 2022; Wang et al., 2020b; Druckman et al., 2021), resulting in diverse beliefs, decisions, and behaviors. Therefore, consensus-reaching, a crucial process in LSGDM, emerges as a result of belief updating through interactions within social networks, particularly when experts aim to achieve consensus on a given event (DeGroot, 1974; Korbel et al., 2023; Ni et al., 2021). Our comprehensive framework is designed to guide individuals through the entire process from belief formation to decision-making, rather than focusing on any specific step. The details of how individuals interact within social networks and update their beliefs, encompassing both general scenarios and specific applications such as LSGDM, will be introduced below.

### 6.1. Basic introduction

In the opinion dynamics process, the belief profile of $|\mathcal{N}|$ individuals at the $t$th step can be represented by $\mathbf{X}(t) = \left(x_1(t), x_2(t), \ldots, x_i(t), \ldots, x_{|\mathcal{N}|}(t)\right)^T$, where $x_i(t)$ denotes the belief of individual $i$ at time $t$. This belief can take continuous values, discrete values, or sets. Starting with an initial belief profile $\mathbf{X}(0)$, determined using the methods described in Section 3, individuals interact with others to either update their opinions or modify their connections (Santos et al., 2021; Wang et al., 2020b). Specifically, when individuals engage with others who share similar opinions, their beliefs may be reinforced, or the connection between them strengthened. Conversely, interaction with opposing opinions may challenge beliefs or lead individuals to rewire their connection toward someone with more aligned opinions (Grabisch et al., 2023). Through repeated interactions and updates under various models, three possible outcomes can emerge:

- Consensus: $\lim_{t\to\infty} x_i(t) = c$, for $\forall i \in \mathcal{N}$ and $\forall \mathbf{X}(0) \in \mathbb{R}^{|\mathcal{N}|}$.
- Polarization: $\lim_{t\to\infty} x_i(t) = c_1$ or $c_2$, for $\forall i \in \mathcal{N}$ and $\forall \mathbf{X}(0) \in \mathbb{R}^{|\mathcal{N}|}$.
- Fragmentation: $\lim_{t\to\infty} x_i(t) = c_1, c_2, c_3, \ldots$, for $\forall i \in \mathcal{N}$ and $\forall \mathbf{X}(0) \in \mathbb{R}^{|\mathcal{N}|}$.

Here, $c_i$ represents a constant. Refer to Part 4 of Figure 1 for a diagram illustrating these states.

Belief updating models are typically classified into three types based on the nature of the belief variable, including continuous models with real-valued variables (e.g., $x_i(t) \in [0, 1]$), discrete models with limited candidates (e.g., $x_i(t) \in \{0, 1\}$), and probabilistic inference models that account for uncertainty (Dong et al., 2018). These models are generally represented by the following framework,

$$\mathbf{X}(t+1) = \mathcal{W}(\mathbf{X}(t), t) \times \mathbf{X}(t), t = 0, 1, 2, \ldots, \tag{8}$$

where $\mathcal{W}(\mathbf{X}(t), t)$ represents the general form of the weight matrix $\mathbf{W}_{|\mathcal{N}|\times|\mathcal{N}|}$, with elements that are either constants (DeGroot, 1974; Deffuant et al., 2001) or functions of $\mathbf{X}(t)$ or $t$ (Hegselmann et al., 2002). Each element $w_{ij}$ indicates the trust and weight of individual $i$ places on individual $j$'s belief, constrained by $0 \leqslant w_{ij} \leqslant 1$ and $\sum_{j\in\mathcal{N}} w_{ij} = 1$. These weights are influenced by several factors, including confidence, influence ability, and dependence. Furthermore, through the weight matrix $\mathbf{W}$, individuals can interact with varying numbers of others to update their beliefs, such as engaging with one neighbor, all neighbors, or a subset of individuals with specific characteristics.

### 6.2. Update rules for continuous opinions

Continuous opinions, represented by real-valued variables $\mathbb{R}^{|\mathcal{N}|}$, typically fall within intervals like $x_i(t) \in [0, 1]$ and $x_i(t) \in [-1, 1]$. Periodic boundary conditions can also be applied to signify the same meaning of extremes within the interval (Baumann et al., 2021). The DeGroot model (DeGroot, 1974) is a classical linear combination model where individuals update their beliefs $x_i(t)$ by taking a weighted average of the opinions of their connected neighbors, $x_i(t+1) = \sum_{j\in\mathcal{N}, a_{ij}\neq 0} w_{ij} x_j(t), t = 0, 1, \ldots$. In this model, individuals constantly and unconditionally trust their neighbors, leading to the fixed weights $w_{ij}$ between individuals. The sufficient and necessary condition for reaching a consensus has been explored by DeGroot (1974). The Friedkin-Johnsen (FJ) model (Friedkin and Johnsen, 1990) extends the DeGroot model by introducing individual self-confidence $\alpha_i$

$$x_i(t+1) = \alpha_i x_i(0) + \left(1 - \alpha_i\right) \sum_{j\in\mathcal{N}, j\neq i} w_{ij} x_j(t). \tag{9}$$

where individuals can adhere to their initial belief with $\alpha_i \in [0, 1]$ and accept opinions from others with $1 - \alpha_i$. An extended version with varying weights (Hegselmann et al., 2002) has been developed to reflect evolving influence. The system's equilibrium equation is expressed as $\mathbf{X}(\infty) = \Gamma\mathbf{X}(0) + (\mathbf{I} - \Gamma)\mathbf{W}\mathbf{X}(\infty)$, where $I$ is the identity matrix and $\Gamma = diag(\alpha_i)$ is the diagonal matrix of self-confidence. Stability and convergence of the FJ model are discussed in (Parsegov et al., 2017). Through observing 1288 individuals' behavior, (Friedkin and Bullo, 2017) investigates how truth prevails in a group of independent individuals when the influence of each statement is based on its truthfulness.

Since individuals on social media encounter several relevant statements simultaneously, researchers investigate if these statements follow a shared logic constraint structure. For example, *Statements B* and *C* become true if *Statement A* is true, forming a belief system. To describe how $|\mathcal{N}|$ individuals update their beliefs on $m \geq 2$ inter-dependent statements within the same logic constraint structure, a method (Friedkin et al., 2016) was developed,

$$\mathbf{X}(t+1) = \Gamma\mathbf{X}(0) + (\mathbf{I} - \Gamma)\mathbf{W}\mathbf{X}(t)\mathbf{C}^T, \tag{10}$$

where $\mathbf{W}_{|\mathcal{N}|\times|\mathcal{N}|}$ is the weight matrix, $\Gamma = diag(w_{ii})$ indicates self-confidence, and $(\mathbf{I}-\Gamma)$ reflects openness to interpersonal influences. $\mathbf{X}_{|\mathcal{N}|\times m}$ represents belief certainty on $m$ statements from $|\mathcal{N}|$ individuals, with $x_{ij}$ ranging in $[0,1]$. $\mathbf{C}_{m\times m}$ describes inter-dependencies between $m$ statements. Analyzing three relevant statements involved in a political decision (Friedkin et al., 2016) revealed the critical role of statement inter-dependency in complex interpersonal influence networks. Moreover, a multidimensional FJ model (Parsegov et al., 2017) was developed to generate belief systems from interpersonal influences networks, with detailed mathematical discussion and analysis of the matrix of multi-issues dependence structure $\mathbf{C} \in \mathbb{R}^{m\times m}$. The logic matrix was also introduced to multidimensional DeGroot models (Ye et al., 2019) to explore its impact on consensus reaching.

A special case of the DeGroot model is the bounded confidence (BC) model, where individuals only trust neighbors within a confidence set $\mathcal{I}\left(i,\mathbf{X}(t)\right) = \left\{ j \mid \left| x_i(t) - x_j(t) \right| \leq \varepsilon, a_{ij} \neq 0 \right\}$ (Li et al., 2022). Here, the given bounded confidence $\varepsilon$ that considers the psychological factor determines communication and information exchange. The BC model is homogeneous with uniform $\varepsilon$ values and heterogeneous otherwise. The typical BC models include the Deffuant–Weisbuch (DW) model (Deffuant et al., 2001) and the Hegselmann–Krause (HK) model (Hegselmann et al., 2002). In the DW model, opinions are updated by $x_i(t+1) = x_i(t) + \mu\left(x_j(t) - x_i(t)\right)$ if $j \in \mathcal{I}\left(i,\mathbf{X}(t)\right)$, where $\mu \in [0, 0.5]$ controls convergence towards another one (Sîrbu et al., 2017). In the HK model, opinions are updated by

$$x_i(t+1) = \sum_{j\in\mathcal{I}(i,\mathbf{X}(t))} w_{ij} x_j(t), t = 0, 1, \ldots, \tag{11}$$

and the weight is determined by

$$w_{ij}(t) = \begin{cases} 1/\left|\mathcal{I}\left(i,\mathbf{X}(t)\right)\right|, & j \in \mathcal{I}\left(i,\mathbf{X}(t)\right) \\ 0, & j \notin \mathcal{I}\left(i,\mathbf{X}(t)\right) \end{cases}. \tag{12}$$

Notably, the DW model involves asynchronous communication with randomly selected pairs, while the HK model features synchronous communication among all individuals in the confidence set $\mathcal{I}\left(i,\mathbf{X}(t)\right)$. Hence, they suitably model pairwise interaction and group meetings (Castellano et al., 2009), respectively. The bounded confidence $\varepsilon$ plays a crucial role in the final stage state for both models (Castellano et al., 2009). With a sufficiently large $\varepsilon > \varepsilon_c$, individuals tend to reach consensus, while smaller values may lead to polarization or fragmentation. The number of opinion clusters $n_c$ at the final stage is approximately $1/(2\varepsilon)$, as determined by Monte Carlo simulations. Further insights into parameter impacts, extensions, and applications in diverse contexts can be found in (Lorenz, 2007; Hickok et al., 2022).

Theoretical physicists have developed various models to describe the updating of individuals' opinions and group behaviors. For example, the Vicsek model (Vicsek et al., 1995), akin to the DeGroot model, was proposed in the context of flocking where individuals update their beliefs based on their neighbors' average state, which reveals collective motion without centralized control. Moreover, inspired by the Kuramoto model, the opinion changing rate model (Pluchino et al., 2005), was developed to capture individuals' inclination to change opinions,

$$dx_i(t)/dt = \omega_i + \frac{K}{|\mathcal{N}|} \sum_{j\in\mathcal{N}} a_{ij} \sin\left(x_j - x_i\right) e^{-\gamma |x_j - x_i|}, \tag{13}$$

where $x \in (-\infty, +\infty)$, $\omega_i$ indicates the natural opinion changing rate (intrinsic inclination), $K \geq 0$ indicates the global coupling strength, similar to weight, and the exponential factor makes individuals can only influence each other when the difference is within a certain threshold, akin to the BC model. The impacts of parameters, such as $K$, on the consensus have been also explored in real social systems (Pluchino et al., 2005). A similar model was developed (Baumann et al., 2021) to consider multidimensional topics, $dx_i/dt = -x_i + \sum_{j\in\mathcal{N}} a_{ij} \tanh(\beta x_j)$, where $\beta$ indicates controversy of opinion and the sensibility to the opinions of acquaintances. The usage of trigonometric functions enables the opinions to saturate to $\pm 1$.

The game theoretic approach is also useful to update individuals' beliefs in online social networks (Meng et al., 2023). For example, the asynchronous HK model is analyzed with the game-theoretic approach (Etesami and Başar, 2015), thereby providing a necessary condition for the finite termination time of the evolution to advance the analysis of the HK model. In the evolutionary game, beliefs can be updated by comparing payoffs with neighbors when connected individuals benefit when they have the same opinion, and pay a cost otherwise (Yang, 2016). An optimal ratio of cost to benefit has been found to cause the shortest consensus time. An incomplete information estimation method based on interaction indicators in cooperative evolutionary games has also been proposed (Liu et al., 2021a), which models the interaction between negative synergy, positive synergy, and independence. There are still several commonly used approaches/information combined with classical models (Jia et al., 2015), such as social power and the information accumulation system model.

### 6.3. Update rules for discrete opinions

In the simplest scenario, there is a limited number of candidates of individuals' beliefs. For example, people usually need to choose between two options in real life, which can be described by the binary state $x_i(t) = 0, 1, \forall i, t$. The Sznajd model, a variant of the spin model, was first applied to model belief updating in a one-dimensional case (Sznajd-Weron and Sznajd, 2000). This model assumes that a group of individuals with the same belief has a larger impact on neighbors than a single individual – conformity, which is based on a simple concept "*United we Stand, Divided we Fall*". Notably, conformity increases with the influential ability of the group, but the more significant factor is unanimity. Other types of social influence (Frey and Van de Rijt, 2021) related to belief updating, such as anti-conformity and independence, can be also explained by social pressure. It has been found that the steady state of convergence depends on the initial distribution of beliefs. Recently, this model has been extended in different ways, and more information on the Sznajd model can be found in the review (Sznajd-Weron et al., 2021).

The voter model describes the binary choices of individuals distributed on the regular lattice (Holley and Liggett, 1975). In this linear model, individuals randomly select a neighbor and blindly imitate their views $x_i(t+1) = x_j(t)$. Therefore, the imitation of each individual is only related to one neighbor, where the group does not have a direct influence. In any $d$-dimensional hyper-cubic lattice system, there are only two types of possible consensus states, and the probability of reaching each consensus is determined by the initial distribution of opinions. Its extensions have been developed to consider different cases. For example, a nonlinear voter model (Yang et al., 2012) was proposed where individuals adopt a neighbor's belief (+1) by a power function with adjustable parameter $\epsilon$,

$$p_+ = \frac{n_+^\epsilon}{n_+^\epsilon + n_-^\epsilon}, \tag{14}$$

where $n_+$ ($n_-$) is the number of individuals who hold opinion $+1$ ($-1$) among the selected individual and its neighbors. They determined the optimal value of $\epsilon$ to obtain the fastest consensus in networks with different types of topology. More information about the voter model can be found in the review (Redner, 2019). In society, some individuals tend to follow the majority opinion, which can be modeled by the majority rule model. The final state in this model depends on the size of the selected group in each step. More information can be found in the review (Galam, 2008).

The social impact theory (Latané, 1981) was developed to model the impact of a group of individuals on the belief of a single individual, which depends on three factors: group size, personal strength, and interaction distance. Specifically, the personal strength is determined by its persuasiveness $I_p$ and supportiveness $I_s$,

$$I_i(t) = I_p\left(\sum_j \frac{f(p_i)}{g(d_{ij})}\left(1 - x_i(t)x_j(t)\right)\right) - I_s\left(\sum_j \frac{f(s_i)}{g(d_{ij})}\left(1 + x_i(t)x_j(t)\right)\right), \tag{15}$$

where $d_{ij}$ is the shortest distance between any pair of individuals, and $f(\cdot)$ and $g(\cdot)$ are the strength scaling function and a decreasing function, respectively. The belief can be updated by, $x_i(t+1) = -\text{sign}\left(x_i(t)I_i(t) + h\right)$, where $h$ indicates the noise. It has been found that spatially localized clusters can be caused by the social learning theory in the application of several types of networks.

Another typical model is the Ising model (Ising, 1925), which has been widely applied to update beliefs (Korbel et al., 2023). The total energy of interactions between individuals is described by

$$E = -J \sum_{i,j\in\mathcal{N}} x_i x_j - H \sum_{i\in\mathcal{N}} x_i, \tag{16}$$

where $J$ and $H$ represent the global interaction weight and external information, respectively. The first term indicates the degree of conflict of opinion between any two individuals and the second term indicates the relationship between each individual's opinion and the external environment. When individuals have the same beliefs as each other and the external field, the energy will be minimized.

Based on the kinetic exchange, individuals can update their beliefs $x_i(t+1) = \alpha_i x_i(t) + w_{ij} x_j(t)$ based on the degree of conviction $\alpha_i$ and the interaction with others $w_{ij}$ (Biswas et al., 2012), which is similar to the FJ model in the continuous form. In this model, any pair of individuals can exchange beliefs due to the limitation of the mean field, resulting in the nonequilibrium continuous phase transition between phases with and without consensus.

Furthermore, when a finite set of experts is asked to evaluate a finite set of alternatives, the preferences of individuals can be aggregated into a group belief that expresses collective preferences through several rounds of discussion, resulting in an appropriate number of solutions for a specific problem (García-Zamora et al., 2022; Meng et al., 2024; Tang and Liao, 2021; Shen et al., 2024). However, this group decision-making process differs from the previously discussed scenario, where the focus was on individual opinion interactions and updating, leading to individual decision-making within a social network. Here, the goal of group decision-making is for experts to reach a consensus through iterative group discussion. In the consensus-reaching process (CRP), experts iteratively exchange opinions about a specific event, adjusting their initial views to increase group consensus (Liang et al., 2024; Zhang and Li, 2023). The CRP usually involves four steps (Palomares et al., 2014), including opinion collection, consensus measurement, consensus control, and feedback generation. However, when a large number of experts with diverse opinions are involved in large-scale group decision-making (LSGDM), the process becomes more complex, especially in aggregating experts' opinions (García-Zamora et al., 2022). This necessitates additional steps, such as dimension reduction, behavior and cost management, and social network analysis. Various technologies have been incorporated into LSGDM to address various scenarios. For example, Wang et al. (2024b) applied the quantum theory to aggregate trust among individuals and developed a trust screening rule and a leadership incubation mechanism to enhance CRP. Similarly, Liu et al. (2021a) used the Shapley function and interaction indicator from cooperative game theory to estimate incomplete information about interaction features. Using the social influence network generated from this estimated information, experts exchange their opinions and eventually reach a consensus, leading to more reliable decision-making outcomes. Furthermore, Li et al. (2022) analyzed CRP based on the Manhattan distance and developed a feedback mechanism to adjust experts' beliefs slightly when consensus is elusive. Their study explored the impacts of self-confidence and bounded confidence on CRP, demonstrating effectiveness through numerical simulations. Despite these advancements, several challenges remain, such as managing conflicts, reducing the high costs of consensus-reaching, and selecting appropriate LSGDM models. Further details about LSGDM can be found in (García-Zamora et al., 2022; Tang and Liao, 2021).

### 6.4. Probabilistic inference

Individuals' initial opinions can be updated by communicating and receiving information/beliefs from others, thereby updating their opinions. This is similar to Bayesian models that use Bayes rules to estimate unknowns through priors and new information (Acemoglu and Ozdaglar, 2011; McCoy and Prelec, 2024). In detail, individuals have prior information $P(\theta)$ about state $\theta \in \Theta$. After receiving information $s \in S$ from their socially connected neighbors, they can combine new information $s$ to update their prior beliefs based on the Bayesian model,

$$P(\theta|s) = \frac{P(s|\theta)P(\theta)}{P(s)}. \tag{17}$$

However, it leads to two requirements for individuals:

- Have a complete set of prior: It is difficult to achieve both in practice and mathematics for large $\Theta$ due to the lack of prior knowledge and zero probability, respectively.

- Know the conditional probability $P(s|\theta)$ well: It strictly requires individuals to possess sufficient and reliable knowledge.

It has been applied to explore the relationship between Bayesian approaches and human rationality and study the effects of payments for ecosystem services on land-use decision-making (Sun and Müller, 2013). Different types of information are learned by Bayesian models, including the observation of others' actions – Bayesian observational learning, or communication – Bayesian communication learning.

In Bayesian observational learning, individuals make sequential decisions in the social network. Their decisions and actions are made based on historical behaviors and private signals (Lee, 1993), assuming all historical behaviors are observable. Here, the payoff $u_i$ of agent $i$ is defined as

$$u_i(x_i, \theta) = \begin{cases} 1, & \text{if } x_i = \theta \\ 0, & \text{if } x_i \neq \theta \end{cases}, \tag{18}$$

where decision $x_i$ and the underlying state $\theta$ of individual $i$ are binary $\{0, 1\}$. In this process, individuals can receive signals and observe past neighbor behaviors to determine their behaviors. Several models have been developed to update individuals' opinions based on the observation. For example, Fang et al. (2020) proposed a Bayesian social learning model, showing faster learning than individual Bayesian learning with theoretical support.

Bayesian communication learning allows individuals to learn from communication with others, yet selfish interests often hinder information sharing due to (1) lack of common interests and (2) time-consuming communication (Acemoglu and Ozdaglar, 2011), which has been widely applied in markets (Acemoglu and Ozdaglar, 2011). Payoff is defined by,

$$u_i(x_i, \theta) = \begin{cases} \delta^\tau \pi, & \text{if } x_i(\tau) = \theta \text{ and } x_i(t) = \text{'Wait' for } t < \tau \\ 0, & \text{otherwise} \end{cases}, \tag{19}$$

where $\theta$ is binary and $x_i(t) \in \{0, 1, \text{'Wait'}\}$. $\pi$ is the payoff from the correct decision and $\delta$ indicates the discount factor. Due to the complex signal conversion process, wrong signals lead to cognitive bias, such as confirmation bias, where individuals misinterpret new information as supporting their existing hypothesis. In addition, senders need to correctly express the belief, and receivers need to correctly receive the signal. A Bayesian decision-making model that captures this process (Rabin and Schrag, 1999) shows individuals may believe false opinions even though they can receive an infinite amount of new information.

Dempster–Shafer theory, a generalization of the Bayesian theory (Shafer, 1976; Liu et al., 2023), has been applied to update individuals' opinions through the perspective of decision-making (Wen et al., 2024b). In this approach, individuals' decision is binary, represented by the frame of discernment $\Theta = \{0, 1\}$, and individuals' belief belongs to the power set $2^\Theta = \{\emptyset, \{0\}, \{1\}, \Theta\}$. The basic probability assignment (BPA) $m(\cdot)$ indicates the belief assigned to elements in the power set, that is, the probability to support $m(1)$, refuse

$m(0)$, or not yet made the decision $m(\Theta)$, satisfying $m : 2^{\Theta} \to [0,1], m(\emptyset) = 0, \sum_{A \in 2^{\Theta}} m(A) = 1$, The belief can be updated by combining $m_i$ and other individuals' belief $m_o$,

$$m_i'(A) = \frac{\sum_{B \cap C = A} m_i(B) \cdot m_o(C)}{1 - \sum_{B \cap C = \emptyset} m_i(B) \cdot m_o(C)}, \text{ for } A, B, C \in 2^{\Theta}, \tag{20}$$

where the numerator represents the agreement of BPA, and the denominator is a normalization factor. It has been applied to investigate whether people have decided to vaccinate or not, or have not yet made a decision (Xia and Liu, 2014). The socially influenced vaccination decision-making process (Ni et al., 2021) has been further explored in the vaccination context by the recursive evidential reasoning approach, where the social influence is incorporated into the information aggregation process. Moreover, the BPA is exploited to define the organizational influence, thereby affecting the aggregation process of information in social networks (Liu et al., 2021b).

## 7. Decision-making and group decision behaviors

In the decision-making process, the final step (Part 5 of the framework) entails the actual implementation of a decision, which can be viewed as the outcome of the belief updating process. Decision-making informs which behavior should be adopted, and behavior change is the practical manifestation of the decision. This process involves consideration of several aspects. Specifically, when faced with an event, individuals assess whether to engage in certain behaviors based on the perceived costs and benefits (Rosenstock, 1966). Furthermore, individuals adjust their behaviors according to whether their current beliefs align with their psychological expectations and cognition thresholds (Martins, 2008; Baker et al., 2022), which are unique for each individual. In addition, these changes are influenced by societal factors, including the behaviors of peers, which can significantly affect individual decisions (Bandura and Walters, 1977). For instance, when deciding whether to vaccinate during a pandemic, individuals must weigh the benefits – such as reduced infectivity – against the risks – such as vaccine safety. They also consider their own professional knowledge, psychological expectations, and the vaccination decisions (decided to take vaccination or not or event have not decided) of colleagues and friends. This multifaceted decision-making process has been the subject of extensive research across various disciplines (Bossaerts and Murawski, 2015; Frederiks et al., 2015; Tang et al., 2021), including psychology, neuroscience, behavioral economics, and sociology. A brief overview of key studies in these fields will be provided below.

It has been found that beliefs and behaviors exhibit coordinated consistency. Cognitive science research indicates when individuals' behavior is inconsistent with their beliefs, they will experience psychological discomfort, known as cognitive dissonance (Gawronski, 2012). In order to alleviate this discomfort, individuals may adjust their beliefs or behaviors to achieve consistency. This point was further emphasized in behavioral economics, with concepts such as 'nudges' and 'choice architecture' highlighting how small changes in the environment can lead to significant shifts in behavior, often occurring unconsciously (Johnson et al., 2012). However, whether the changes are intentional or unintentional, they all suggest a tendency for beliefs and behaviors to align.

Moreover, traditional utility theory assumes utility maximization for rational decision-making (Stigler, 1950). However, behavioral economics acknowledges the existence of cognitive biases, leading to deviations from strict rationality. Therefore, individuals frequently make decisions that are deemed adequate rather than optimizing, a concept attributed to bounded rationality, initially introduced by Simon (1997). Bounded rationality acknowledges the constraints imposed by limited time and cognitive resources, prompting the utilization of heuristics and satisfying strategies in the decision-making process. This aligns with principles of prospect theory (Kahneman and Tversky, 1979), which asserts that individuals assess potential decision outcomes by considering perceived gains and losses relative to a reference point. In addition, the conceptual framework of belief boundaries that indicate the thresholds or limits present within an individual's belief system has been developed by psychologists to analyze human decision-making behavior within belief systems (Baker et al., 2022). Multiple quantitative models have been developed to depict this transformation. For example, the continuous opinion and discrete action model is a commonly applied model to analyze the action change in the decision-making process (Zhan et al., 2021; Martins, 2008), where individuals' perspectives on specific issues or topics are quantified as continuous variables but actions change when opinions cross a threshold. It has been a valuable framework for comprehending the interplay between continuous opinion dynamics in social networks and discrete actions. In the context of decision-making, behavioral changes are influenced not only by intrinsic individual factors but also by social factors such as social norms, peer influence, and cultural context (Wolske et al., 2020). Understanding how these social factors impact individuals' decision-making behaviors and even behavioral changes attracted researchers from different fields (Wen and Cheong, 2024; Tang et al., 2021, 2020), and two competing hypotheses have been posited (Centola and Macy, 2007).

- The spread of behavior can be treated as a contagion, like diseases, rendering small-world networks to be more effective in promoting the diffusion of behaviors due to the close connection.
- Social behavior requires reinforcement, not similar to simple contagion. Hence, multiple exposures are needed for individuals to adopt a behavior, resulting in the effectiveness of clustered networks with redundant ties.

Furthermore, behavior scientists suggested that there are two types of variables to determine the change the health behavior in the health belief model (Rosenstock, 1966): (1) the psychological state of readiness that the individual takes a specific behavior and (2) the extent to which this behavior is believed to be conducive to reducing the threat. Neuroscientists have demonstrated the extraordinary capacity of the human brain – neural plasticity, which implies that alterations in behavior, cognition, and experiences can lead to structural and functional changes within the brain. Furthermore, the mechanisms by which the brain acquires, stores, and retrieves information, encompassing processes like learning and memory, provide valuable insights into understanding behavioral changes (Foerde and Shohamy, 2011). For instance, it allows us to comprehend how behaviors are learned, reinforced, and transformed over time. Additionally, neuroscientists have posited the brain's reward system, involving the release of neurotransmitters such as dopamine, as playing a paramount role in incentivizing and reinforcing behaviors (Sanfey, 2007).

Behavior can be also learned through observing and imitating others, a process known as "modeling" (Bandura and Walters, 1977; Brady et al., 2021). When individuals observe others achieving positive outcomes from certain behaviors, they may be more inclined to adopt those behaviors themselves. This can influence the formation and updating of beliefs, leading to similar decision-making. As discussed earlier, the influence on information diffusion and belief updating can potentially result in behavioral consistency (Young, 2009). However, this research primarily focuses on the iterative process of individual belief formation and decision-making under social influence at a population level. The absorption of knowledge from the social environment is also considered a spontaneous action by individuals. Whether this spontaneity arises from personal awareness or societal influence, it serves as a driving force behind individual behavioral changes and actions (Heimlich and Ardoin, 2008). Overall, understanding action change is a pivotal step within the overarching framework, which requires an in-depth exploration of the impact of cognitive, emotional, and social factors.

## 8. Applications in various fields

As individuals constantly interact with others within complex social systems, their opinions and behaviors evolve. These interactions give rise to collective dynamics, which help explain various social phenomena. Such collective behaviors have been studied and applied in a range of contexts to gain a better understanding of social opinion dynamics and group decision behaviors (Thuy and Benoit, 2024; Wen et al., 2025). For example:

- *Online social media*: The dissemination of fake news is considered one of the most pressing and threatening issues (Friedkin and Bullo, 2017), thus, its application in fake news is reviewed. Additionally, its applications in sentiment analysis and community structure detection are reviewed.
- *Political election*: Users are always interested in political discourse and political elections, especially the echo chambers and polarization during the U.S. presidential campaign and Brexit (de Arruda et al., 2022; Del Vicario et al., 2017). Therefore, its applications in political elections are reviewed.
- *Epidemic and vaccination*: Individuals' attitudes toward vaccination during the epidemic can be easily affected by other individuals (Ni et al., 2021; Xia and Liu, 2014). Hence, some works to explore the change in the attitude to vaccination are reviewed.
- *Business decision-making*: It has been applied to finance and business decision-making to study its impact on corporate interests and consumer services (Sun and Müller, 2013; Kwan et al., 2024; Chao et al., 2021).

More details about applications are summarized in Table 3.

## 9. Concluding remarks

With the rapid advancement of the Internet and communication technologies, individuals' activities on social platforms have become more frequent and influential, affecting how beliefs are updated and decisions

**Table 3**
A brief summary of real-world applications of social network group decision-making models.

| Field | Application | Ref. |
|---|---|---|
| Online social media | Misinformation & Fake news propagation | (Liu and Rong, 2022; Friedkin and Bullo, 2017) |
| | Network & Community generation | (Korbel et al., 2023; Peng et al., 2023) |
| | Dynamic sentiments analysis | (Del Vicario et al., 2017; El-Diraby et al., 2019) |
| Political election | Echo chambers emergence & Political polarization | (Wang et al., 2020b; de Arruda et al., 2022) |
| | Impact of topic and sentiment | (Del Vicario et al., 2017; Zhu et al., 2020) |
| | Victory of the minority candidate | (Biswas and Sen, 2017; Biswas et al., 2021) |
| Epidemic | Vaccination hesitancy | (Ni et al., 2021; Xia and Liu, 2014) |
| | Impact of opinion from social media | (Du et al., 2021; Teslya et al., 2022) |
| Decision-making | Group decision-making | (Wang et al., 2024b; Li et al., 2022) |
| | Market & Business assessment | (Chao et al., 2021; Tong and Zhu, 2023) |

are made. Decision-making is no longer a solitary process but rather a complex, multi-step process shaped by continuous interactions with others. As individuals engage in these interactions, human beliefs are influenced by a wide range of factors, including personal characteristics such as knowledge and cognitive biases, social dynamics involving expert opinions and familial advice, environmental influences like social norms and peer pressure, and situational elements such as perceived risks and benefits. These multidimensional factors continuously affect decision-making, not only at an individual level, but also at the organizational and social community levels. As a result, understanding how humans make decisions within the context of social networks has become increasingly important.

In this paper, we examine the entire process from the formation of initial beliefs to final decision-making in social networks. By reviewing and synthesizing relevant studies, this work provides valuable insights for researchers studying these dynamic decision-making processes. Section 2 lays the foundation by outlining the basic concepts of network systems, highlighting both classic network types and their local and global characteristics. As described in Section 3, when individuals encounter specific problems, various factors influence the formation of initial beliefs. This section also discusses the development of MCDM models, which allow for the comprehensive consideration of these various influences. Since individuals are part of networks, their beliefs are shaped by interactions with others, as discussed in Section 4, which reviews methods for determining weights based on the local topology of networks and the divergence of individual beliefs. Section 5 and Section 6 provide detailed reviews of information diffusion models and opinion dynamics models, respectively, illustrating how beliefs evolve over time. After that, individuals will make decisions and take actions for specific events based on their updated beliefs (Section 7). These choices not only affect the current situation but also feed into their initial beliefs for future similar events. Finally, real-world applications in several fields are summarized in Section 8 to demonstrate the practical implications of the holistic framework developed in the paper.

This paper contributes to knowledge beyond a comprehensive literature review by developing a holistic framework that provides valuable, in-depth insights into social network group decision-making. The framework covers key processes such as belief formation, diffusion, updating, opinion dynamics and decision-making. It shifts the focus from viewing individuals as isolated decision-makers to seeing them within the broader context of social networks, where beliefs are continuously shaped by group interactions. This research paves the way for future studies to move beyond examining decision-making as a series of isolated steps. By adopting the perspective of decision-making within social networks, scholars can explore how individual beliefs are influenced by group dynamics and the overall network characteristics. Furthermore, this integrated approach holds great potential for advancing interdisciplinary research. By bridging decision science, operational research, and social network analysis, future studies can develop more robust models that reflect the complexity of real-world decision-making environments. Practical applications in domains such as public policy making, organizational behavior, and even technological innovation can be greatly enhanced through this holistic framework, enabling more informed decisions in increasingly interconnected and information-rich societies. Therefore, this study lays the foundation for further exploration and development of models that consider both interactions between individuals and the broader social structures in which they operate.

A spectrum of interesting and challenging problems are worth further research:

1. *Overall framework*: Most of the existing works mainly focus on specific problems but ignore the overall framework, which is significant for analyzing collective behaviors in social networks. Therefore, establishing effective and reasonable frameworks to fully consider the process from initial belief establishment to final decision-making is inevitable for future research in this field.

2. *Large-scale group decision-making and opinion dynamics*: With the popularity of the Internet and mobile devices, the scale of network systems is growing explosively in the real world. Individuals are affected by more and more factors and people on the Internet, making decision-making very complicated. Therefore, it has become urgent to propose realistic and novel methods to model large-scale group decision-making and opinion dynamics efficiently.

3. *Expression of beliefs under uncertainty*: The way individuals express their opinions or beliefs about events is usually expressed as discrete or continuous values, which often ignores the uncertainty they face in unfamiliar events. However, how to include uncertainty while expressing cognitive tendencies is very important to collective behavior, which is conducive to determining the weight and trust of information sources during complicated interactions.

4. *Information diffusion*: The current research in information diffusion systems primarily focuses on the enhancement of propagation models. However, within the framework based on belief formation and updating, the influence of individual attributes within the network can be seamlessly integrated into the information diffusion process, such as personal psychology and emotions. This integration will enhance the alignment of research with real-world scenarios, thereby providing a more effective foundation for predictive or intervention measures during implementation.

5. *Wider applications*: Understanding the entire process of belief formation, decision-making, and its integrated development can assist various stakeholders in gaining a comprehensive understanding of the characteristics and decision-making habits of their target audience. Furthermore, the entire framework can be traced through various stages, enabling continuous applications and capturing its dynamic evolution, rather than focusing solely on static outcomes.

## Declaration of competing interest

The authors declare that they have no conflict of interest.

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