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Coupling analysis between wave resonance in the moonpool and heave motion of the twin hulls with mooring effect

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Abstract

Fluid resonance in the moonpool formed by two identical rectangular hulls in water waves is investigated by employing the Computational Fluid Dynamics (CFD) package OpenFOAM[®]. The influence of the vertical stiffness on the behavior of the moonpool resonance coupling with the heave motion response is presented. Numerical simulations show that the free surface oscillation in the moonpool exhibits a two-peak variation with the incident wave frequency, defined as the first and second peak frequencies. A local Keulegan-Carpenter (KC) number is introduced for describing the influence of the fluid viscosity and flow rotation on the fluid resonance and heave motion resonance. At the first peak frequency, the free surface oscillation and heave motion response show an in-phase relationship, where the increase of the vertical stiffness can increase the relative motion between them. This finally leads to an increase in the KC number, indicating the increased effect of the energy dissipation with the increase of the vertical stiffness. At the second peak frequency with an out-of-phase relationship between the free surface oscillation and heave motion response, the variation of the KC number is not sensitive to the vertical stiffness. Correspondingly, the influence of the energy dissipation is not much dependent on the vertical stiffness.

Keywords: Wave resonance, Moonpool, Heave motion, KC Number, Energy dissipation, Mooring effect

1 1. Introduction

Moonpool is a vertical opening through the deck and hull of ships or barges, which can be used to carry out marine operations such as pipe laying or diver recovery. The riser system also passes through a moonpool in offshore structures, such as drilling ships, Spar platforms, Floating Liquefied Natural Gas (FLNG) and Floating Production Storage and Offloading (FPSO) systems. In practical engineering, careful design is required in order to avoid excessive free surface resonance in the moonpool and the associated motion response of the floater. Therefore, an in-depth understanding of the fundamental physics of the fluid resonance in the moonpool is essential for marine operation and structure safety.

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Faltinsen et al. (2007) and Molin et al. (2018) derived a semi-analytical solution for the fluid resonance in 9 the moonpool, where the piston- and sloshing-mode resonances were formulated and solved via an approxi-10 mate eigenvalue problem. Sun et al. (2010) and Feng and Bai (2015) simulated the free surface resonance in 11 the moonpool between two barges in a side-by-side arrangement by using the second-order and fully nonlinear 12 potential flow models, respectively. However, the application of the potential flow model was reported to 13 over-predict the free surface amplitude around resonant frequencies, which is attributed to the absence of 14 fluid viscosity and flow rotation. This has been extensively confirmed by experimental measurements, such 15 as Saitoh et al. (2006), Iwata et al. (2007) and Zhao et al. (2017b), among others. To resolve this problem, 16 Newman (2004) and Chen (2004) adopted an improved potential flow model based on the introduction of 17 artificial damping to suppress the exaggerated resonant amplitudes. The keys of this method are the formula-18 tion of the damping term and the selection of the damping coefficient, where many efforts have been devoted 19 to (Lu et al., 2011a,b; Liu and Li, 2014; Feng et al., 2018; Tan et al., 2019, 2021). Another alternative method 20 is Computational Fluid Dynamics (CFD) simulations based on the Navier-Stokes solvers, which has become 21 more popular with the development of numerical and computing techniques (Lu et al., 2010; Fredriksen et al., 22 2014; Moradi et al., 2015; Jiang et al., 2018; Gao et al., 2019; Wang et al., 2019). The CFD methods are able 23 to capture the energy dissipation caused by the fluid viscosity and flow rotation, which are closely dependent 24 on the numerical scheme and the mesh density. 25

The above work mainly considered the fluid resonance in the moonpool under wave diffraction or radiation 26 scenarios. That is, these studies only focused on the free surface oscillation in the moonpool between either 27 fixed hulls in water waves or hulls undergoing forced motions in the still water. However, it is an interaction 28 problem between the incident water wave and the floating bodies in practical engineering. Fredriksen et al. 29 (2015) and Ravinthrakumar et al. (2020) measured the fluid resonance in the moonpool coupling with the 30 motion response of the floater. Zhang et al. (2022) conducted laboratory tests of the coupling effect on the 31 free surface oscillation for a sway-box section adjacent to a fixed structure. In addition to experimental 32 measurements, numerical simulations are also effective tools for this topic. Li and Teng (2021) simulated the 33 free surface oscillation in the moonpool formed by twin-floating barges with the roll motion response under 34 wave actions. Gao et al. (2021, 2024) studied the influence of heave motion response on the fluid resonance in 35 the moonpool in regular and irregular waves. Jing et al. (2023, 2024) simulated the roll motion effect on the 36 wave resonance in the moonpool induced by higher-order harmonic components of incident waves. According 37 to the laboratory tests and numerical simulations, the significant effect of motion response on the behavior 38 of fluid resonance in the moonpool can be confirmed. 39

In addition, the mooring stiffness plays an important role in the behavior of floaters in practical engineering, which may further affect the coupling actions between the fluid resonance in the moonpool and the motion response of the floater. However, to the best of the authors' knowledge, very few studies have been paid on this topic, and the only relevant study is found in Lu et al. (2020) for the fluid resonance in the narrow gap formed by a box-wall system, where the heave and sway responses of the box section with different

mooring stiffnesses were investigated. Moreover, as mentioned above, the energy dissipation associated with 45 the fluid viscosity and flow rotation significantly affects the behavior of fluid resonance in the moonpool. A 46 further understanding of the influence of energy dissipation during the coupling actions between the fluid 47 resonance and the motion response of the moored floater is expected. In this paper, numerical simulations 48 are carried out for the fluid resonance in the moonpool formed by the free-heave twin hulls coupling with the 49 vertical stiffness effect. The influence of vertical stiffness on the behaviour of coupling actions is the major 50 contribution of this study to offshore engineering. In collaboration with the quantitative description of the 51 local Keulegan-Carpenter (KC) number and the examination of vortex shedding in the flow patterns, the 52 energy dissipation during the coupling process with various mooring stiffnesses is revealed, which highlights 53 the academic contribution of this study. 54

55 2. Mathematical formulation

Two numerical models, that is, the potential flow model and the viscous fluid flow model, are adopted to carry out the investigations in this study. The potential flow model is efficient, so it is used for the general description of the behaviour of the heave motion response and the free surface oscillation. However, it cannot capture the realistic physical phenomena around the peak frequencies due to the ignorance of the fluid viscosity and flow rotation. Therefore, the viscous fluid flow model is employed for the detailed analysis of the coupling actions between the heave motion and the fluid resonance. The model details are provided below.

63 2.1. Viscous fluid flow model

The Navier-Stokes equations for two-phase incompressible turbulent flows in the Arbitrary Lagrangian-Eulerian (ALE) reference system are adopted as the governing equations,

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0, \tag{1a}$$

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$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho (u_j - u_j^m) u_i}{\partial x_j} = \rho g_i - \frac{\partial p}{\partial x_i} + \mu_e \frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \tag{1b}$$

where x_i and u_i are the Cartesian coordinate component and the fluid velocity component in the *i*th direction, respectively; u_i^m is the velocity component due to mesh deformation in the ALE frame. g_i is the gravitational acceleration; p is the fluid pressure. ρ and μ_e are the fluid density and effective dynamic viscosity, respectively. $\mu_e = \mu_m + \mu_t$, where μ_m is the fluid molecule viscosity and μ_t is the turbulent viscosity.

The Re-Normalization Group (RNG) $k - \varepsilon$ two-equation formulations are selected for closing the governing equations,

$$\mu_t = C_\mu \frac{k^2}{\varepsilon},\tag{2}$$

⁷³ where $C_{\mu} = 0.09$ is a theoretical model constant, and the time-dependent advection-diffusion equations for ⁷⁴ the turbulent kinematic energy k and its dissipation rate ε can be written as,

$$\frac{\partial\rho k}{\partial t} + \frac{\partial\rho(u_j - u_j^m)k}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right) + \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} - \rho\varepsilon,\tag{3a}$$

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$$\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial \rho (u_j - u_j^m) \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right) + C_{1\varepsilon} \frac{\varepsilon}{k} \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} - \rho C_{2\varepsilon} \frac{\varepsilon^2}{k}, \tag{3b}$$

where the model constants $C_{1\varepsilon}$, $C_{2\varepsilon}$, σ_k and σ_{ε} are 1.42, 1.68, 0.71942 and 0.71942, respectively. They were derived theoretically in the RNG turbulent model (Yakhot and Orszag, 1986; Yakhot and Smith, 1992).

The Volume of Fluid (VOF) method is adopted to track the fluid and capture the free surface motion. In the method, the fraction indicator α is defined as 0 for air and 1 for water, and any intermediate value between 0 and 1 represents a mixture of air and water. The distribution of α satisfies the following advection equation,

$$\frac{\partial \alpha}{\partial t} + (u_i - u_i^m) \frac{\partial \alpha}{\partial x_i} = 0.$$
(4)

The spatial variation of fluid density and dynamic viscosity can be obtained according to the fraction indicator α in the computational cell as follows,

$$\rho = \alpha \rho_W + (1 - \alpha) \rho_A, \tag{5a}$$

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$$\mu = \alpha \mu_W + (1 - \alpha) \mu_A, \tag{5b}$$

where the subscripts W and A represent the Water phase and Air phase, respectively. In numerical simulations, the contour with $\alpha = 0.50$ is used to represent the interface between the water and air phases.

The toolbox 'waves2Foam' proposed by Jacobsen et al. (2012) is utilized to generate the incident wave and avoid the internal wave reflection, where relaxation zones are defined at the inlet and outlet boundaries of the numerical wave flume. The exponential relaxation function,

$$\phi_R(\chi_R) = 1 - \frac{\exp(\chi_R^{3.5}) - 1}{\exp(1) - 1}, \qquad \chi_R \in [0, 1], \tag{6}$$

 $_{90}$ is applied within the relaxation zone,

$$\vartheta = \phi_R \vartheta_C + (1 - \phi_R) \vartheta_T,\tag{7}$$

⁹¹ where ϑ is either the velocity u_i or the fraction indicator of water α , and the subscripts C and T represent ⁹² the Computed value and Target value, respectively.

The simulations are conducted in the Supercomputer Center of Dalian University of Technology by using the OpenFOAM[®] package with version 3.0.0. The initial condition in the numerical computations is the still state, which is the zero velocity and hydrostatic pressure in the numerical wave flume. The no-slip boundary condition is imposed at the solid wall including the hull surface and seabed. At the upper boundary of ⁹⁷ the numerical wave flume, the Dirichlet and Neumann types of boundary conditions are prescribed for the ⁹⁸ pressure and velocity respectively. An incident fifth-order Stokes wave is adopted at the inlet boundary, ⁹⁹ where the gradient of pressure is set to zero. The governing equations (1a)-(1b) and the advection transport ¹⁰⁰ equation (4) are solved based on the Finite Volume Method (FVM). The velocity and pressure are decoupled ¹⁰¹ by the PISO (Pressure Implicit with Splitting of Operators) algorithm (Issa, 1986). The time increment ¹⁰² is determined automatically according to the Courant-Friedrichs-Lewy (CFL) condition, where the largest ¹⁰³ allowed Courant Number is set to $C_r = 0.20$.

104 2.2. Potential flow model

The classical linear potential flow model is also adopted in this study for the purpose of comparison. The boundary integral equation based on the Rankine source $G(\mathbf{x}, \mathbf{x_0}) = \frac{1}{2\pi} (\ln r + \ln r')$ is established, which has been described in the previous work in Jiang et al. (2017). The higher-order boundary element method (HOBEM) with quadratic isoparameteric elements (Teng and Eatock Taylor, 1995) is employed to discrete the boundary integral equation. The detailed theoretical formulation is omitted here, as it is well-known in many textbooks.

3. Mesh convergence and numerical validation

The wave interaction with two identical free-heave hulls with a moonpool in the centre is investigated in 112 this study, as shown in Fig. 1. The large-amplitude piston-type free surface oscillation in the moonpool is 113 focused, which is one of the most important issues in the coupling analysis. The hulls of breadth B_L and 114 height B_H with the moonpool of draft D and breadth B_{mp} are situated in a numerical wave flume with the 115 water depth h. In this study, the twin hulls are rigidly connected and only the heave motion is free. Two 116 relaxation zones are situated on the left and right sides of the numerical wave flume, respectively, where the 117 lengths of the zones are generally about 1.5 - 2.0 times the wavelength λ in the numerical simulations. A 118 total of four wave gauges are equipped to record the free surface elevations. G2 and G3 with a distance of 119 0.25λ between them are situated 1.5 λ from the left side of the hulls for separating the incident and reflected 120 waves; while G4 located 1.5 λ from the right side of the hulls is adopted for measuring the transmission wave. 121 The wave gauge G1 is situated in the middle of the moonpool for recording the free surface oscillation in 122 the moonpool. Numerical simulations have confirmed that there is no sloshing-type free surface oscillation 123 in this study. Finally, it should be noted that all of the free surface amplitudes in this work are calculated in 124 the space-fixed coordinate system. 125

Numerical simulations begin from the mesh convergent test in this study, where the geometries of the twin hulls with a moonpool are $B_L = 0.360$ m, $B_H = 0.360$ m, $B_{mp} = 0.180$ m and D = 0.180 m. The water depth of the numerical wave flume is h = 1.030 m and the incident wave height is $H_i = 0.03$ m, where the relationship between the wave height and wave amplitude is $H_i = 2A_i$. The non-uniform meshes are adopted to minimize the number of meshes and save computational resources. In general, the square fine



Figure 1: Sketch of the numerical wave flume with two identical hulls.

meshes with a high resolution are adopted in the moonpool to account for the boundary layer effects and to 131 accurately capture the large-amplitude free surface oscillation. In the outlet relaxation zone, the rectangular 132 coarse meshes with a large aspect ratio up to 1/20 (height/length) are adopted, which are helpful to dissipate 133 the reflection wave. The rectangular meshes with a medium resolution are utilized in the regions of the inlet 134 relaxation zone and incident wave path. Tab. 1 represents the detailed schemes of convergence tests for four 135 types of meshes, where the superscripts 'w' and 'mp' are defined as the meshes in the wave propagation path 136 and inside the moonpool, respectively. A typical mesh scheme around the hulls is illustrated in Fig. 2, which 137 is Mesh 3 in Tab. 1. 138

Table 1: Detailed information of four different mesh schemes for the mesh convergence study.

Parameters	Physical meaning	Mesh 1	${\rm Mesh}\ 2$	Mesh 3	${\rm Mesh}\ 4$
N_x^w	No. of cells per wave length λ	87	109	130	151
N_y^w	No. of cells along box height B_H	71	82	102	112
δy^w_{min}	Minimal cell height on free surface (mm)	5.33	4.21	3.47	3.07
N_x^{mp}	No. of cells along moon pool breadth ${\cal B}_{mp}$	45	58	68	78
N_y^{mp}	No. of cells along box height B_H	120	150	180	210
δy^{mp}_{min}	Minimal cell height on free surface (mm)	1.25	1.00	0.83	0.71
	Total number of cells for the case	159985	244358	363712	458362



Figure 2: Typical grid system in this study around the hulls.



Figure 3: Mesh convergent tests for the evolutions of heave motion response of twin hulls and free surface oscillation in the moonpool. (a) Heave motion response, (b) Free surface oscillation.

Fig. 3 shows the evolutions of the heave motion response $\zeta_2(t)$ of the hulls and the free surface oscillation $\eta(t)$ in the moonpool, where the incident wave frequency $\omega = 6.00$ rad/s is selected. After a short transient period, the steady-state evolutions can be observed after t = 25 s, implying that the inlet and outlet zones have a satisfactory performance in generating and eliminating the incident and transmission waves, respectively. Numerical simulations also indicate that there is very little discrepancy between the steady-state results of Mesh 3 and Mesh 4, implying that the convergent solutions can be produced by Mesh 3.

Numerical results for the amplitudes of the heave motion response $\zeta_2(t)$ of twin hulls and the free surface oscillation $\eta(t)$ in the moonpool, that is, ξ_2 and A_{mp} , at four different wave frequencies are shown in Fig. 4. In the figure, the amplitudes ξ_2 and A_{mp} are calculated according to the averaged values of $\zeta_2(t)$ and $\eta(t)$ between 80 - 120 s, respectively. The comparisons show that the variation of mesh density has little effect on the amplitudes of heave motion and free surface when the number of cells exceeds 3×10^5 . Again, Mesh 3 is able to produce convergent solutions, and hence it is adopted as the baseline for the following numerical analysis.

The numerical accuracy of the present turbulence model is validated against the experimental data in 152 Faltinsen et al. (2007) for the twin hulls with sharp edge profiles, firstly. For the purpose of comparison, 153 the potential flow solutions are also included in the figure. The twin hulls undergoing synchronous heave 154 oscillations are adopted in Faltinsen et al. (2007), where the geometries of the hulls and moonpool are the 155 same as the present setup in the convergent tests. Correspondingly, the left inlet relaxation zone is replaced 156 with an outlet relaxation zone, by which the process in the laboratory tests in Faltinsen et al. (2007) can be 157 simulated, correctly. Two forced heave amplitudes, $A_b = 2.5$ mm and 5.0 mm, are selected, and the values at 158 the wave gauges G1 and G4 (700 mm from the right side of Hull B) are measured. Fig. 5 shows the variations 159



Figure 4: Mesh convergent tests for the amplitudes of heave motion response of twin hulls and free surface oscillation in the moonpool. (a) Heave motion response, (b) Free surface oscillation.

¹⁶⁰ of wave amplitudes and phase shifts at G1 and G4 with the heave frequency $\omega^2 B_{mp}/g$. The potential ¹⁶¹ flow model over-predicts the free surface amplitudes around the resonant frequency, which is attributed to ¹⁶² neglecting the fluid viscosity and flow rotation. A good agreement between the present RNG results and the ¹⁶³ experimental data can be observed, for both the wave amplitudes and the phase shifts.



Figure 5: Numerical validation of the wave amplitudes and phase shifts at G1 and G4 versus the heave frequency for twin hulls under the heave amplitudes $A_b = 2.5$ mm and 5.0 mm. (a) Amplitude, $A_b = 2.5$ mm, (b) Phase shift, $A_b = 2.5$ mm, (c) Phase shift, $A_b = 2.5$ mm, (d) Phase shift, $A_b = 5.0$ mm.

To further validate the heave motion response of twin hulls and the free surface oscillation in the moonpool, 164 another physical experiment in a wave flume with the water depth h = 1.0 m in Fredriksen et al. (2015) is 165 simulated, where the twin hulls have the breadth $B_L = 0.201$ m and height $B_H = 0.201$ m and the moonpool 166 has the draft D = 0.097 m and breadth $B_{mp} = 0.100$ m. The incident wave is given in the form of a wave 167 steepness in their work, where two sets of wave steepness, $A_i/\lambda = 1/120$ and 1/90, are selected in the present 168 study. In the experiments, only the heave motion of the twin hulls is free, which means there is coupling 169 actions between the heave motion of the hulls and the fluid resonance in the moonpool. Again, the potential 170 flow solutions are included in the figure for the purpose of comparison. As shown in Fig. 6, the over-prediction 171 can be observed in the potential flow solutions in the regions of T = 7.5 - 0.80 s and 0.9 - 1.1 s, respectively, 172 for both the free surface and the heave motion amplitudes. The present viscous fluid flow model can work 173 well in predicting the heave motion response and the free surface oscillation, which are in good agreement 174 with the experimental measurement. According to the comparisons, it can be confirmed that the present 175 viscous numerical wave flume is reliable for simulating the coupling actions between the heave motion and 176 the fluid oscillation in the moonpool. 177



Figure 6: Numerical validation of the amplitudes of heave motion response of twin hulls and free surface oscillation in the moonpool. (a) $A_i/\lambda = 1 / 120$, A_{mp}/A_i , (b) $A_i/\lambda = 1 / 120$, ξ/A_i , (c) $A_i/\lambda = 1 / 90$, A_{mp}/A_i , (d) $A_i/\lambda = 1 / 90$, ξ/A_i .

178 4. Behavior of free surface oscillation and heave motion response

The motivation of this study is the coupling actions between the free surface oscillation in the moonpool 179 and the heave motion response of twin hulls with the vertical mooring stiffness. The geometries adopted in 180 the mesh convergent test, that is, the twin hulls with the breadth $B_L = 0.360$ m and height $B_H = 0.360$ m, 181 and the moonpool with the breadth $B_{mp} = 0.180$ m and draft D = 0.180 m, in the water depth h = 1.030182 m, are selected. The twin hulls are rigidly connected and only the heave motion is free. Six types of vertical 183 stiffness, k = 0, 1000, 4000, 7000, 14000 and ∞ , are selected, where the unit of stiffness is N/m. Note that 184 the cases of k = 0 and $k = \infty$ correspond to the no-stiffness hulls and fixed hulls, respectively. The twin hulls 185 are restricted to be free only in the z-axis direction, that is, only the heave direction is free. The potential 186 flow model and viscous fluid flow model are adopted in the numerical simulations, where a wide range of 187 incident wave frequencies from $\omega = 3.50$ - 9.50 rad/s are considered. The incident wave height is $H_i = 0.03$ 188 m in the viscous fluid flow simulations. 189

Fig. 7 shows the variation of the free surface oscillation A_{mp}/A_i in the moonpool and the heave motion 190 response ξ_2/A_i of the hulls against the incident wave frequency, including the influence of the vertical mooring 191 stiffness k. The potential flow model can firstly manifest a general impression of the behavior of the heave 192 motion response and the free surface oscillation. As illustrated in Fig. 7a, the free surface amplitudes in 193 the moonpool show the two-peak variation with the incident wave frequency for the cases of non-fixed twin 194 hulls. One of the peaks is smooth and bounded at the low frequency range, which is defined as the first peak 195 frequency $\omega_p^{(1)}$; while the other is a sharp spike at the high frequency range, which is defined as the second 196 peak frequency $\omega_p^{(2)}$. The values of these two peak frequencies $\omega_p^{(1)}$ and $\omega_p^{(2)}$ increase with the increase of 197 the vertical stiffness k, where the cases with k = 0 and $k = \infty$ are the two limitation cases. In the present 198 geometry, the range of the first peak frequency $\omega_p^{(1)}$ is from 4.55 rad/s to 5.30 rad/s; while the range of the 199 second peak frequency $\omega_p^{(2)}$ is from 6.15 rad/s to ∞ . This implies that the frequency range between $\omega = 5.30$ 200 rad/s and 6.15 rad/s has no resonant frequency for any vertical stiffness. It should be noted that there is 201 also no resonant frequency during the same range in the viscous fluid flow results in Fig. 7a, indicating that 202 it may be a real physics for the moored twin hulls with the moonpool system. 203

The variation of the heave motion response with the incident wave frequency is illustrated in Fig. 7b, 204 where only the single-peak value can be observed around the second peak frequency $\omega_p^{(2)}$. The comparison 205 between Figs. 7a and 7b shows that the values of the second peak frequencies $\omega_p^{(2)}$ for the fluid resonance 206 and the heave motion response are the same. At the first peak frequency $\omega_p^{(1)}$, the heave motion has no 207 peak values, implying that the large-amplitude free surface oscillation in the moonpool has little effect on 208 the heave motion response of the hulls. Another interesting phenomenon in Fig. 7b is the zero-amplitude 209 heave motion response around the incident frequency $\omega = 5.76$ rad/s. This is due to the fact that the vertical 210 wave exciting force approaches zero, where the ratio of wavelength and twin-hull breadth is $\lambda/(2B_L + B_{mp})$ 211 = 2.08. It can be understood that the vertical wave forces generated by the wave crest and wave trough are 212 generally cancelled with each other at this frequency. 213



Figure 7: Variation of heave motion response and free surface oscillation against the incident wave frequency with different vertical mooring stiffnesses. (a) Free surface oscillation, (b) Heave motion response.

The potential flow model can only manifest a general understanding of the behavior of the free surface 214 oscillation in the moonpool and the heave motion response of the hulls, while the real physical phenomena 215 should be simulated by the viscous fluid flow model, especially around the two-peak frequencies $\omega_p^{(1)}$ and 216 $\omega_p^{(2)}$. As shown in Fig. 7a, the potential flow model over-predicts the free surface amplitudes in the moonpool 217 at the first and second peak frequencies $\omega_p^{(1)}$ and $\omega_p^{(2)}$. The over-prediction can also be found in the results 218 of heave motion amplitudes around the second peak frequency $\omega_p^{(2)}$, as illustrated in Fig. 7b. All of these 219 phenomena can be explained by the ignorance of the energy dissipation associated with the fluid viscosity 220 and flow rotation. Oppositely, a little discrepancy of the heave motion amplitudes between two numerical 221 models can be identified around the first peak frequency $\omega_p^{(1)}$ in Fig. 7b, implying that the energy dissipation 222 effect is not very significant on the occasion. 223

Furthermore, the influence of the vertical stiffness on the potential flow and viscous fluid flow results are identified in Fig. 7. Around the first peak frequency $\omega_p^{(1)}$, the increased discrepancy of the free surface amplitudes between two numerical results can be observed with the increase of vertical stiffness. It indicates that the increase of vertical stiffness is able to generate more significant energy dissipation around the first peak frequency $\omega_p^{(1)}$. The influence of the vertical stiffness on the peak frequencies between two numerical models can also be reflected in the figure. That is, the increased vertical stiffness is able to increase the difference of the $\omega_p^{(1)}$ and $\omega_p^{(2)}$ values between the potential flow and viscous fluid flow results.

Fig. 8 shows the influence of incident wave amplitude at different vertical stiffnesses, where the normalized 231 amplitudes of free surface oscillation A_{mp}/A_i in the moonpool and heave motion response of the twin hulls 232 ξ_2/A_i are presented. A general description of the decreased normalized amplitudes A_{mp}/A_i and ξ_2/A_i with 233 the increase of incident wave amplitude can be observed, which also deviates more from the potential flow 234 solutions. It implies the increased influence of the energy dissipation associated with the fluid viscosity and 235 flow rotation. Furthermore, the influence of the vertical stiffness on the variation of normalized values of 236 A_{mp}/A_i and ξ_2/A_i is quantified in Tabs. 2 and 3, which are the ratio of the normalized amplitudes and 237 the incident wave heights $H_i = 0.03$ m and 0.06 m at the first and second peak frequencies, respectively. 238 The decreased ratio can be observed at the first peak frequency $\omega_n^{(1)}$ with the increase of vertical stiffness in 239 Tab. 2, for both the normalized free surface amplitudes and the heave motion amplitudes. It demonstrates 240 the increased influence of the energy dissipation at the first peak frequency $\omega_p^{(1)}$ with the increase of vertical 241 stiffness. However, there is no simple regular pattern in the ratios at the second peak frequency $\omega_p^{(2)}$. 242 Numerical results show the ratio values at the second peak frequency are always around about 60% - 70% in 243 Tab. 3. It, in fact, implies that the energy dissipation plays a different role in the behavior of the free surface 244 oscillation and heave motion response under different vertical stiffnesses, which will be further discussed in 245 Sec. 5. 246



Figure 8: Variation of heave motion response and free surface oscillations against the incident wave amplitude with different vertical mooring stiffnesses. (a) Free surface oscillation, (b) Heave motion response.

Parameters		K = 0	K = 1000	K = 4000	K = 7000	K = 14000	$K = \infty$
	$H_i=0.01~{\rm m}$	1.530	1.862	2.680	3.166	3.606	4.134
A_{mp}/A_i	$H_i=0.03~{\rm m}$	1.459	1.701	2.405	2.835	3.096	3.107
	Ratio*	95.36%	91.35%	89.76%	89.56%	85.84%	75.15%
ξ_2/A_i	$H_i=0.01~{\rm m}$	1.040	0.998	0.729	0.570	0.345	0.000
	$H_i=0.03~{\rm m}$	1.020	0.914	0.647	0.496	0.296	0.000
	Ratio	98.06%	91.57%	88.72%	86.99%	85.89%	

Table 2: Normalized amplitudes of free surface oscillations A_{mp}/A_i and heave motion responses ξ/A_i under different incident wave heights at the first peak frequency $\omega_p^{(1)}$.

* Ratio means the ratio of the result at $H_i = 0.03$ m and the result at $H_i = 0.01$ m.

Table 3: Normalized amplitudes of free surface oscillations A_{mp}/A_i and heave motion responses ξ/A_i under different incident wave heights at the second peak frequency $\omega_p^{(2)}$.

Parameters		K = 0	K = 1000	K = 4000	K = 7000	K = 14000	$K = \infty$
A_{mp}/A_i	$H_i=0.01~{\rm m}$	2.660	2.520	2.087	1.826	1.182	
	$H_i=0.03~{\rm m}$	1.718	1.777	1.533	1.207	0.805	
	Ratio	64.58%	70.53%	73.43%	66.09%	68.11%	
ξ_2/A_i	$H_i=0.01~{\rm m}$	0.712	0.714	0.892	1.022	0.811	0.000
	$H_i=0.03~{\rm m}$	0.529	0.551	0.688	0.690	0.561	0.000
	Ratio	74.29%	77.20%	77.22%	67.51%	69.11%	

²⁴⁷ 5. Keulegan-Carpenter Number

Numerical simulations in Sec. 4 described the general behavior of the fluid resonance and heave motion 248 response with different vertical stiffnesses, where the two-peak variation is the most important behavior for the 249 twin-box with the moonpool system. To better understand the essential mechanism behind the phenomena, 250 the detailed characteristics of the free surface oscillation and heave motion response are further discussed in 251 this section. Figs. 9 and 10 show the time histories of the free surface elevation in the moonpool and the 252 heave motion response of the hulls, where the first and second peak frequencies $\omega_p^{(1)}$ and $\omega_p^{(2)}$ with the vertical 253 stiffnesses k = 0 and k = 7000 N/m are selected. As shown in Fig. 9, the free surface oscillation and heave 254 motion response have the in-phase relationship at the first peak frequency $\omega_p^{(1)}$. Further comparison indicates 255 that the amplitudes of the free surface elevation and heave motion response at the first peak frequency $\omega_p^{(1)}$ 256 are very close when the vertical stiffness is k = 0; while the heave motion amplitude is evidently smaller than 257 the free surface amplitude when the vertical stiffness is k = 7000 N/m. As for the second peak frequency $\omega_p^{(2)}$ 258

in Fig. 10, the out-of-phase relationship can be observed between the free surface oscillation and the heave motion response for both k = 0 and 7000 N/m. The above in-phase and out-of-phase relationships at the first and second peak frequencies are also applicable to other vertical stiffnesses, which has been confirmed according to the careful examinations of the CFD simulations.



Figure 9: Time histories of heave motion responses and free surface oscillations with k = 0 and k = 7000 N/m at the first peak frequency $\omega_p^{(1)}$. (a) k = 0, $\omega_p^{(1)} = 4.50$ rad/s, (b) k = 7000 N/m, $\omega_p^{(1)} = 5.00$ rad/s.



Figure 10: Time histories of heave motion responses and free surface oscillations with k = 0 and k = 7000 N/m at the second peak frequency $\omega_p^{(2)}$. (a) k = 0, $\omega_p^{(2)} = 6.00$ rad/s, (b) k = 7000 N/m, $\omega_p^{(2)} = 7.20$ rad/s.

Moreover, a local Keulegan-Carpenter (KC) Number is defined as,

$$KC = V_A T / D, \tag{8}$$

where V_A is the amplitude of the averaged relative vertical fluid velocity along the moonpool bottom. T is the period of free surface oscillation in moonpool and D is the draft of the hulls. It is the relative velocity between the fluid motion and the heave response. The relative time-dependent space-averaged vertical velocity $\tilde{v}(t)$ can be computed as follows,

$$\tilde{v}(t) = \frac{1}{x_l - x_r} \int_{x_l}^{x_r} v(x, t) \, dx - \dot{\zeta}_2(t). \tag{9}$$

where $\dot{\zeta}_2(t)$ is the vertical velocity of the hulls, v(x,t) is the time- and space-dependent vertical velocity in the space-fixed coordinate system along the horizontal entrance of the moonpool, and x_l and x_r are the left and right edges of the moonpool entrance attached to the twin hulls. In this work, V_A is calculated according to the averaged value of $\tilde{v}(t)$ between 80 s - 120 s, which is in accordance with the calculation of the free surface amplitude A_{mp} and the heave motion amplitude ξ_2 in Sec. 4.

The calculated KC Numbers at the first and second peak frequencies are illustrated in Tabs. 4 and 5, for two sets of incident wave heights and six types of vertical stiffness. For the purpose of comparison, the

values of the free surface amplitude A_{mp}/A_i and the heave motion amplitude ξ_2/A_i at the first and second 275 peak frequencies $\omega_p^{(1)}$ and $\omega_p^{(2)}$ with various vertical stiffnesses k from 0 to ∞ are also included in these 276 tables. For a specific vertical stiffness, the values of KC Number at the incident wave height $H_i = 0.03$ m 277 are always larger than those at $H_i = 0.01$ m at both the first and second peak frequencies $\omega_p^{(1)}$ and $\omega_p^{(2)}$. This 278 confirms the increased influence of the fluid viscosity and flow rotation with the increase of the incident wave 279 amplitude. It is the essential reason for the decreased normalized amplitudes of the free surface oscillation 280 and heave motion response around the first and second peak frequencies in Fig. 8, where the viscous fluid 281 flow results deviate more from the potential flow solutions with the increase of the incident wave amplitude. 282 Moreover, the influence of the vertical stiffness on the KC number can be quantified in these tables. 283 As shown in Tab. 4, the KC number increases with the increase of the vertical stiffness at the first peak 284 frequency $\omega_p^{(1)}$, indicating the energy dissipation plays a more important role. This is in accordance with 285 the increased discrepancy between the potential flow and viscous fluid flow results with the increase of the 286 vertical stiffness at the first peak frequency $\omega_p^{(1)}$ in Fig. 7. In fact, the above variation of the KC number can 287 also be speculated based on Eq. (9) and Figs. 9 and 10. That is, the values of v(x,t) and $\dot{\zeta}_2(t)$ always have 288 the same sign due to the in-phase relationship at the first peak frequencies. On the occasion, the increase of 289 the vertical stiffness would increase the relative motion V_A between the heave motion response and the free 290 surface oscillation, leading to the increased KC number at the first peak frequency. Oppositely, as illustrated 291 in Tab. 5, the KC number roughly decreases with the increase of the vertical stiffness at the second peak 292 frequency $\omega_p^{(2)}$. The values of v(x,t) and $\dot{\zeta}_2(t)$ always have the opposite sign because of the out-of-phase 293 relationship at the second peak frequency. However, it is not a strict decrease in the KC number at the 294 second peak frequency, which is attributed to the complex variation of the free surface oscillations in the 295 moonpool with the vertical stiffness. 296

Finally, the KC numbers with various vertical stiffnesses at the first and second peak frequencies are 297 compared. It can be observed that the effect of the vertical stiffness affects the KC number more significantly 298 at the first peak frequency $\omega_p^{(1)}$ than that at the second peak frequency $\omega_p^{(2)}$. As an example, when the incident 299 wave amplitude is $H_i = 0.03$ m, the KC number is from 0.460 to 3.253 with the increase of the vertical stiffness 300 from k = 0 to k = 14000 N/m at the first peak frequency; while it ranges from 2.353 to 1.430 at the second 301 peak frequency. In Tab. 4, the values of A_{mp}/A_i and ξ_2/A_i increase and decrease with the increase of the 302 vertical stiffness at the first peak frequency. Based on Eq. (9) and the in-phase relationship, it is found 303 that the variation of KC number comes from the contribution of both A_{mp}/A_i and ξ_2/A_i . At the second 304 peak frequency, although the decreased values of A_{mp}/A_i can be observed, the insensitivity of ξ_2/A_i with 305 the vertical stiffness appears in Tab. 5. It indicates the variation of KC number is only due to the change of 306 A_{mp}/A_i . The above analysis also indicates that the energy dissipation plays a more important role with the 307 increase of the vertical stiffness at the first peak frequency; while the influence of the vertical stiffness on the 308 energy dissipation is not very sensitive at the second peak frequency. 309

Parameters		K = 0	K = 1000	K = 4000	K = 7000	K = 14000	$K = \infty$
$H_i = 0.01 \text{ m}$	A_{mp}/A_i	1.530	1.862	2.680	3.166	3.606	4.134
	ξ_2/A_i	1.040	0.998	0.729	0.570	0.345	0.000
	KC Number	0.171	0.302	0.681	0.906	1.138	1.443
$H_i = 0.03 \text{ m}$	A_{mp}/A_i	1.459	1.701	2.405	2.835	3.096	3.107
	ξ_2/A_i	1.020	0.914	0.647	0.496	0.296	0.000
	KC Number	0.460	0.824	1.842	2.450	2.932	3.253

Table 4: Variation of KC Number with the vertical mooring stiffness under two sets of incident wave heights at the first peak frequency $\omega_p^{(1)}$.

Table 5: Variation of *KC* Number with the vertical mooring stiffness under two sets of incident wave heights at the second peak frequency $\omega_p^{(2)}$.

Parameters		K = 0	K = 1000	K = 4000	K = 7000	K = 14000	$K = \infty$
	A_{mp}/A_i	2.660	2.520	2.087	1.826	1.182	
$H_i = 0.01 \text{ m}$	ξ_2/A_i	0.712	0.714	0.892	1.022	0.811	0.000
	KC Number	1.177	1.129	1.040	0.994	0.696	
	A_{mp}/A_i	1.718	1.777	1.533	1.207	0.805	
$H_i=0.03~{\rm m}$	ξ_2/A_i	0.529	0.551	0.688	0.690	0.561	0.000
	KC Number	2.353	2.439	2.326	1.987	1.430	

6. Flow pattern analysis

The flow pattern in the vicinity of the moonpool is investigated for demonstrating the hydrodynamic 311 behavior involved in the fluid resonance, as illustrated in Figs. 11 and 12 (Multimedia available online). The 312 above geometry with the vertical stiffness k = 0 and 7000 N/m under the incident wave height $H_i = 0.03$ 313 m at the corresponding first and second peak frequencies are selected. The vortex contours at eight time 314 instants during a stable period T beginning from the up zero-crossing point of the free surface oscillation in 315 the moonpool are considered. It should be noted that the free surface oscillation in the moonpool and the 316 heave motion response of the hulls are in-phase and out-of-phase at the first and second peak frequencies, 317 respectively, as illustrated in Figs. 9 and 10. This would play an important role in understanding the behavior 318 of flow patterns affected by the vertical stiffness at two peak frequencies. 319

Fig. 11 (Multimedia available online) shows the flow patterns at the first peak frequency $\omega_p^{(1)}$ with the vertical stiffness k = 0 and 7000 N/m, where the free surface oscillation and the heave motion response are



Figure 11: Vortex contour in the near region of the moonpool entrance during one period of free surface oscillation at the first peak frequency. 1st row: $\omega = 4.50 \text{ rad/s}$, k = 0; 2nd row: $\omega = 5.00 \text{ rad/s}$, k = 7000. (a) 0, (b) T/8, (c) 2T/8, (d) 3T/8, (e) 4T/8, (f) 5T/8, (g) 6T/8, (h) 7T/8. (Multimedia available online)

in-phase. When the vertical stiffness is k = 0, the amplitudes of the free surface motion and heave motion 322 response are very close. Correspondingly, the relative velocity between the fluid flow along the moonpool 323 bottom and the heave motion of the hulls is very small. Therefore, it is hard to find the shear shedding and 324 vortex structure in the first row of the figure. It implies the negligible influence of the energy dissipation 325 on the hydrodynamic behavior of the moonpool resonance coupling with the heave motion response. In the 326 second row of the figure, the relative velocity between the fluid flow and heave motion becomes large due to 327 the effect of the vertical stiffness k = 7000 N/m. It can generate significant shear shedding and produce clear 328 vortex structures, leading to the significant effect of energy dissipation. The above phenomena are essentially 329 revealed by the KC number in Sec. 5, where the KC numbers increase with the increase of the vertical 330 stiffness in Tab. 4. It also coincides with the results in Figs. 7 and 8, where the increased influence of the 331 energy dissipation can be observed with the increase of the vertical stiffness at the first peak frequency. 332

Flow patterns at the second peak frequency $\omega_p^{(2)}$ with the vertical stiffness k = 0 and 7000 N/m are 333 illustrated in Fig. 12 (Multimedia available online), where the free surface oscillation and the heave motion 334 response are out-of-phase. The out-of-phase relationship means that the relative velocity between the fluid 335 flow and the heave motion is always large at the frequency. Therefore, the evident shear shedding and clear 336 vortex structures can be found in the first and second rows of the figures for both the vertical stiffness k337 = 0 and 7000 N/m. This implies that energy dissipation can always play an important role at the second 338 peak frequency. In difference to the results at the first peak frequency, the influence of the vertical stiffness 339 on the flow pattern is not very remarkable at the second peak frequency. This is in accordance with the 340 conclusion that the variation of the KC number against the vertical stiffness at the second peak frequency 341 $\omega_p^{(2)}$ is weaker than that at the first peak frequency $\omega_p^{(1)}$. According to the flow pattern analysis in Figs. 11 342 and 12 (Multimedia available online), it is confirmed that the conclusions about the local KC number are 343 reasonable and supported by the physical mechanism. 344



Figure 12: Vortex contour in the near region of the moonpool entrance during one period of free surface oscillation at the second peak frequency. 1st row: $\omega = 6.00 \text{ rad/s}$, k = 0; 2nd row: $\omega = 7.20 \text{ rad/s}$, k = 7000. (a) 0, (b) T/8, (c) 2T/8, (d) 3T/8, (e) 4T/8, (f) 5T/8, (g) 6T/8, (h) 7T/8. (Multimedia available online)

345 7. Discussion

This study mainly focuses on the fundamental physics of coupling actions between the fluid resonance in 346 the moonpool and the heave motion response of the hulls with vertical stiffness. The findings are potentially 347 relevant to marine engineering applications. The vertical stiffness can significantly affect the two peak values 348 of the heave responses and the free surface oscillations, including the peak amplitudes and peak frequencies. 349 Around the two peak frequencies, the comparisons between the potential flow and viscous fluid flow models 350 indicate that the influence of energy dissipation plays an important role. A local Keulegan-Carpenter (KC)351 number is defined for describing the energy dissipation effect on the fluid resonance and the heave motion 352 response. The increase of the vertical stiffness can increase the KC number at the first peak frequency, 353 indicating the increased effect of the energy dissipation with the increase of the vertical stiffness. At the 354 second peak frequency, the variation of the KC number is not sensitive to the vertical stiffness, implying the 355 influence of the energy dissipation is not much dependent on the vertical stiffness. The above quantitative 356 analysis based on the local KC number is helpful in revealing the mechanism of energy dissipation in coupling 357 actions. 358

In this study, numerical work is conducted in a two-dimensional domain, and only the piston-mode 359 resonance and the heave motion response are discussed. The piston-mode features with free surface heaving up 360 and down like a solid body, which has an important effect on heave motion responses, and a two-dimensional 361 simulation is able to reveal the basic characteristics of the piston-mode resonance. However, there is another 362 resonant mode, that is, sloshing-mode resonance in the moonpool, where the free surface is similar to the 363 fluid motion inside a tank. It would cause more complex coupling actions with the motion response of the 364 hulls. Therefore, a three-dimensional and six-degree-of-freedom work carried out in the future can generate 365 more interesting information for practical engineering. 366

367 8. Conclusion

The highlight of this study is the coupling actions between the free surface oscillation in the moonpool 368 and the heave motion response of the hulls with the vertical mooring stiffness. The free surface oscillation in 369 the moonpool shows a two-peak variation with the incident wave frequency, which are defined as the first and 370 second peak frequencies. However, only one peak value around the second peak frequency can be observed 371 in the results of the heave motion response. The difference between the potential flow and viscous fluid flow 372 results can be observed at the first and second peak frequencies. At the first peak frequency, the increase of 373 the vertical stiffness is able to increase the difference between the two numerical models; while the insensitive 374 variation of the difference with the vertical stiffness appears at the second peak frequency. The comparison of 375 time histories shows that the free surface oscillation and heave motion response are in-phase and out-of-phase 376 at the first and second peak frequencies, respectively, which plays an important role in understanding the 377 influence of the vertical stiffness at two peak frequencies. 378

To better understand the essential mechanism behind the phenomena, a local Keulegan-Carpenter (KC)379 number is developed for quantifying the influence of the fluid viscosity and flow rotation. At the first peak 380 frequency with the in-phase relationship, the increase of the vertical stiffness is able to increase the free 381 surface oscillation and decrease the heave motion response, respectively. In addition, both the fluid motion 382 and the heave motion are able to increase the KC number with the increase of the vertical stiffness. At the 383 second peak frequency with the out-of-phase relationship, the increase of the vertical stiffness can increase 384 the free surface amplitude but has little effect on the heave motion amplitude. Therefore, the influence of 385 the vertical stiffness on the KC number at the second peak frequency is weaker than that at the first peak 386 frequency. The above phenomena can be confirmed by the flow pattern analysis, where strong shear shedding 387 and clear vortex structure can be observed in the cases with high KC numbers. 388

389 Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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