



**Please cite the Published Version**

Haid, Daniel, Foster, Leon, Hart, John, Greenwald, Richard, Allen, Tom , Sareh, Pooya and Duncan, Olly  (2023) Mechanical metamaterials for sports helmets: structural mechanics, design optimisation, and performance. *Smart Materials and Structures*, 32 (11). 113001 ISSN 0964-1726

**DOI:** <https://doi.org/10.1088/1361-665x/acfddf>

**Publisher:** IOP Publishing

**Version:** Published Version

**Downloaded from:** <https://e-space.mmu.ac.uk/632953/>

**Usage rights:**  [Creative Commons: Attribution 4.0](https://creativecommons.org/licenses/by/4.0/)

**Additional Information:** This is an open access article which originally appeared in *Smart Materials and Structures*, published by IOP Publishing

**Data Access Statement:** The data cannot be made publicly available upon publication because they are not available in a format that is sufficiently accessible or reusable by other researchers. The data that support the findings of this study are available upon reasonable request from the authors.

**Enquiries:**

If you have questions about this document, contact [openresearch@mmu.ac.uk](mailto:openresearch@mmu.ac.uk). Please include the URL of the record in e-space. If you believe that your, or a third party's rights have been compromised through this document please see our Take Down policy (available from <https://www.mmu.ac.uk/library/using-the-library/policies-and-guidelines>)

TOPICAL REVIEW • OPEN ACCESS

# Mechanical metamaterials for sports helmets: structural mechanics, design optimisation, and performance

To cite this article: Daniel Haid *et al* 2023 *Smart Mater. Struct.* **32** 113001

View the [article online](#) for updates and enhancements.

## You may also like

- [Eye tracking technology in sports-related concussion: a systematic review and meta-analysis](#)

N Snegireva, W Derman, J Patricios et al.

- [Review of wearable technologies and machine learning methodologies for systematic detection of mild traumatic brain injuries](#)

William Schmid, Yingying Fan, Taiyun Chi et al.

- [Blink duration is increased in concussed youth athletes: a validity study using eye tracking in male youth and adult athletes of selected contact sports](#)

Nadja Snegireva, Wayne Derman, Jon Patricios et al.

## Topical Review

# Mechanical metamaterials for sports helmets: structural mechanics, design optimisation, and performance

Daniel Haid<sup>1</sup>, Leon Foster<sup>1</sup> , John Hart<sup>1</sup>, Richard Greenwald<sup>2,3</sup>, Tom Allen<sup>4</sup> , Pooya Sareh<sup>5,6,\*</sup>  and Olly Duncan<sup>4</sup> 

<sup>1</sup> Advanced Wellbeing Research Centre, Sheffield Hallam University, Sheffield, United Kingdom

<sup>2</sup> Simbex, Lebanon, NH, United States of America

<sup>3</sup> Thayer School of Engineering, Dartmouth College, Hanover, NH, United States of America

<sup>4</sup> Department of Engineering, Manchester Metropolitan University, Manchester, United Kingdom

<sup>5</sup> School of Engineering, University of Liverpool, Liverpool, United Kingdom

<sup>6</sup> School of Engineering, Newcastle University, Newcastle upon Tyne, United Kingdom

E-mail: [pooya.sareh@liverpool.ac.uk](mailto:pooya.sareh@liverpool.ac.uk) and [pooya.sareh@newcastle.ac.uk](mailto:pooya.sareh@newcastle.ac.uk)

Received 18 May 2023, revised 24 August 2023

Accepted for publication 27 September 2023

Published 19 October 2023



CrossMark

## Abstract

Sports concussions are a public health concern. Improving helmet performance to reduce concussion risk is a key part of the research and development community response. Direct and oblique head impacts with compliant surfaces that cause long-duration moderate or high linear and rotational accelerations are associated with a high rate of clinical diagnoses of concussion. As engineered structures with unusual combinations of properties, mechanical metamaterials are being applied to sports helmets, with the goal of improving impact performance and reducing brain injury risk. Replacing established helmet material (i.e. foam) selection with a metamaterial design approach (structuring material to obtain desired properties) allows the development of near-optimal properties. Objective functions based on an up-to-date understanding of concussion, and helmet testing that is representative of actual sporting collisions and falls, could be applied to topology optimisation regimes, when designing mechanical metamaterials for helmets. Such regimes balance computational efficiency with predictive accuracy, both of which could be improved under high strains and strain rates to allow helmet modifications as knowledge of concussion develops. Researchers could also share mechanical metamaterial data, topologies, and computational models in open, homogenised repositories, to improve the efficiency of their development.

\* Author to whom any correspondence should be addressed.



Original content from this work may be used under the terms of the [Creative Commons Attribution 4.0 licence](https://creativecommons.org/licenses/by/4.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

Keywords: mechanical metamaterial, helmet performance, sports engineering, impact engineering

(Some figures may appear in colour only in the online journal)

## 1. Introduction

Sporting concussions are prevalent and recognised as a public health concern [1–5]. Mechanical metamaterials are engineered structures with combinations of mechanical properties that are not possible in the individual materials they are made from [6–15]. They are suggested as options to improve helmet impact performance (e.g. [16–33]). In helmets, mechanical metamaterials can be tailored to reduce linear and rotational acceleration, thought to be associated with the clinical diagnosis of concussion [16–33].

Helmets are an established mechanical intervention for reducing head injury risk. They are considered effective at preventing severe head injury (e.g. skull fracture), but less so for concussion [34–42]. While there are many mechanical metamaterials reviews [14, 43–49], including those on protective equipment [50, 51], as well as some on general helmet materials (e.g. [52]), there is not a published review of mechanical metamaterials for sports helmets. Here, we formulate the current challenges relating to helmet development, and summarise the breadth of relevant mechanical metamaterials research. Finally, we include perspectives on opportunities and requirements for helmet development, focusing on reducing concussion risk.

Concussions can cause short-term functional impairments and long-term health problems [53, 54]. They are typically considered a mild traumatic brain injury (TBI) [55–57], and are part of the larger family of TBI. Concussion injuries can be caused either by a direct blow to the head, or an impact to the body that causes head acceleration (e.g. via. whiplash) [2, 58]. Symptoms of concussion, such as those related to physical, cognitive, and emotional health, usually resolve within two weeks [2, 3, 53, 59–61], but can last longer [62, 63]. Concussions do not typically cause detectable structural damage to the brain [2, 62, 64], so they are challenging to diagnose and manage.

A history of concussions [3, 54, 62, 65] or repetitive sub-concussive head impacts [53, 62, 66] are associated with microstructural changes in the brain, and short or long-term functional, physiological, and neurological changes. Reported consequences include reduced quality of life [67], and increased risk of psychiatric disorders [4, 53, 60, 66, 68, 69], neurodegenerative disorders [53, 60, 66, 70], and suicide [71].

Rugby, American football, and ice hockey have the highest reported concussion rates in mainstream sports [41, 72]. Concussions are also of concern in other sports, including association football [73–76], lacrosse [72, 77], snow-sports [34, 78–82], cycling [27, 83–85], water-sports [86], and rock climbing [87]. Strategies to reduce concussion risk, such as rule changes and helmet developments, have been introduced to various sports, with limited success [35, 37, 38, 40, 42, 83,

88, 89]. There has also been notable investment by governmental agencies [90, 91], and charitable organisations [92, 93], in concussion research and related technology development over the past two decades.

Team sports are often played in environments that can be controlled and regulated [41, 72], unlike outdoor sports such as cycling, snow-sports, water-sports, and climbing [27, 34, 78–87]. In many mainstream sports, strategies such as promoting helmet use have had limited effects on concussion rates [34, 37, 39–42, 72, 79, 82, 94]. Factors affecting reported concussion rates are multifaceted, so identifying the effect of interventions is challenging.

Risk factors for concussion include impact surface shape and stiffness, and impact speed, energy, direction, and location [35, 37, 38, 40, 42, 83, 88, 89, 95]. Ice hockey presents an interesting case for helmet development, as it includes various diverse impact types (e.g. high-speed puck, rigid ice and boards, and collisions between players and their equipment—which are considered compliant) [96, 97]. The introduction and regulation of ice hockey helmets have helped to nearly eliminate serious head injuries, particularly skull fractures [37, 38]. Despite these developments, and as with other mainstream sports [37, 39–42, 72], concussion rates in ice hockey have been steadily increasing [37, 39–42, 72]. Most ice hockey concussions (93%) are caused by collisions between players (i.e. compliant surfaces) [37, 63, 88, 96–99], while the remaining 7% are from falls onto ice [97]. About two-thirds of players indicate they would continue to play even if they thought they had sustained a concussion [100]. This attitude to concussion likely results in underreporting [37, 39, 41, 101].

There is an ongoing debate over different possible concussion mechanisms [89, 102–105]. It is generally agreed that the clinical condition resulting from an injury associated with diagnosed concussion is caused by excessive, or overly rapid, tissue deformation [102]. Such tissue deformation can be caused by skull deformation [89, 106, 107], movement of the brain within it [102, 104, 106, 108], and by pressure gradients [106]. Most closed head injuries (non-fracture) follow head accelerations that damage brain tissue [104, 109–112]. During linear (radial) impacts, injury can be caused by the brain being forced against the faster-moving skull [89, 113, 114]. During head rotation, loose coupling can damage connective blood vessels and neurons [102, 104, 106, 114]. Linear and rotational head accelerations are likely to be present during head impacts [103, 110, 115, 116], and helmets should aim to limit both.

## 2. Measures of concussion risk

Helmets are typically designed to decrease the various measures thought to contribute to head injury risk. Peak linear acceleration (PLA) is thought to contribute to severe injuries

such as skull fractures, and concussions [109, 115, 117–121]. The Wayne state tolerance curve, derived from animal and cadaver tests, combined linear acceleration and duration when assessing injury risk [122, 123]. Further threshold curves (e.g. Gadd severity index [124] and head injury criterion [125]) integrate acceleration over a portion of the impact duration, with a weighting factor for high accelerations [95, 109, 125–142].

Peak rotational acceleration (and velocity) are commonly considered as measures of concussion risk [109, 120, 127, 137, 143–156]. Various measures of head injury risk use rotational kinematics (e.g. the rotational injury criterion [157] and brain injury criterion (BrIC) [158, 159]). The generalized acceleration model for brain injury tolerance [160] and head impact power [161–163] combine linear and rotational kinematics, while the weighted principal component score also includes impact location [55, 164].

Numerical brain trauma models have been developed (e.g. [114, 165–168]). These models use measured kinematics as input variables to predict brain deformation metrics, such as principal strain, cumulative strain damage, or pressure [169–172]. Modelling the material properties of the brain is challenging, and care must be taken to ensure meaningful results [173].

In-field measurements with sensors, following validation (typically against video footage), can detect and characterise actual sporting head impacts [73, 174]. These sensors can be attached to the skin [73, 175–178] or helmet [73, 176, 179–184] or embedded within mouthguards [73, 185–188]. Collected sensor data, along with subsequent clinical diagnosis, are helping to develop our understanding of concussion [73, 174], as are mechanical tests [189, 190], numerical simulations [173], and measurements from cadaver [70] and animal testing [191]. Findings from such work indicate benefits to (i) minimising peak linear and rotational accelerations; (ii) minimising the duration over which these values remain elevated; and (iii) shifting focus from PLA to also include rotational kinematics and duration. These measures, thought to increase concussion risk, are associated with impacts with compliant bodies, such as collisions between ice hockey players [37, 63, 88, 96–99]. Validated test methods, representative of conditions in the field of play, as well as brain models and biofidelic (similar to a biological system) headforms [192, 193], help further our understanding of concussion mechanisms.

### 3. Helmet testing

There are many reviews on helmet testing, and Whyte *et al.*'s is particularly comprehensive [95]. As such, only key points related to helmet development are summarised here. Helmets are typically fitted to a headform when tested [95]. Most helmets certification tests within standards include a drop test onto a fixed anvil [126–130, 132, 194–216], with some exceptions [126–130, 200]. None of these tests cover the full range of impact types a helmet may experience during use [95, 143, 190, 217, 218]. Certification tests within helmet standards

are typically designed to ensure a minimum level of protection from a severe head impact (e.g. skull fracture, rather than concussion) [95].

Standards typically use centric impact vectors that cause predominantly linear acceleration [126, 127, 195, 197, 199, 213–215, 219–221], and a rigid anvil [95, 217, 222]. Tests using non-centric impact vectors are more common in research publications than in standards, following growing recognition that few actual sporting collisions and falls cause centric impact vectors (e.g. [143–145, 170, 223–228]). Such non-centric impact vectors can be imparted using drop tests onto oblique anvils (e.g. [32, 179, 229, 230]), pneumatic rams [151, 190, 224, 231], pendulums and impulse hammers [77, 153, 226–228, 232, 233], or projectiles [234].

Energy is the typical metric used to classify impact tests and, for helmet testing, is usually between 18 and 150 J (depending on the sport [95]). Where rigid anvils are used for testing, energies may be lower than those expected during actual sporting collisions and falls, with a view to maintaining similar severities, and acceleration vs. time profiles, to the actual collisions and falls [190, 227, 229]. Wider ranges of impact velocities, energies, and anvil compliances are used in research studies than in standards [85, 95, 189, 229, 234]. Measures of magnitude, and sometimes duration, of linear and rotational acceleration, are used to define injury risk (to prevent varying helmet mass from affecting perceived performance). As covered in section 2, there is ongoing discussion around the acceleration magnitudes and time profiles that are associated with clinical diagnosis of concussion, which should be resolved before updating standards [95]. As such, metrics are often compared to in-field measures for actual sporting collisions and falls, and those collected with an un-helmeted headform [190, 226–229].

Various standardised headforms, with limited biofidelity [95, 234–241], are used to test helmets [95, 234–244]. Attempts have been made to use low friction covers to improve the biofidelity of the headform and helmet interface, with clear differences in rotational acceleration [32, 136, 245, 246]. Attempts have also been made to develop more biofidelic headforms [192, 193]. Neckforms [145, 153, 167, 170, 179, 222, 223, 239, 246–249], including biofidelic ones [95, 143, 146, 156, 218, 238, 250–254], are sometimes attached to headforms to achieve more realistic post-impact behaviour.

Sports concussions are typically caused by impacts with compliant surfaces [37, 63, 88, 96–99], inducing long-duration impacts (noted as high risk in section 2). These compliant surfaces may also increase friction, and rotational head acceleration, during oblique impacts [189, 190] thought to cause concussion. Such oblique impacts with compliant surfaces are not tested for in certification tests within standards and are rarely used in peer-reviewed studies on new helmet technologies. Further, headforms with low biofidelity may cause unrealistic coupling with the helmet, potentially introducing errors while measuring rotational kinematics in the laboratory [32, 136, 245, 246]. Mechanical metamaterials, offering greater control over effective properties than conventional materials, could be used in helmet development efforts focused on reducing measures of concussion risk, while maintaining protection against skull fracture.



**Figure 1.** (A) Vinyl nitrile (VN) foam, (B) Expanded polypropylene (EPP\_ foam and slip plane, and (C) a shear-thickening polymer (STP) pad as parts of an ice hockey helmet liner (authors' own images).

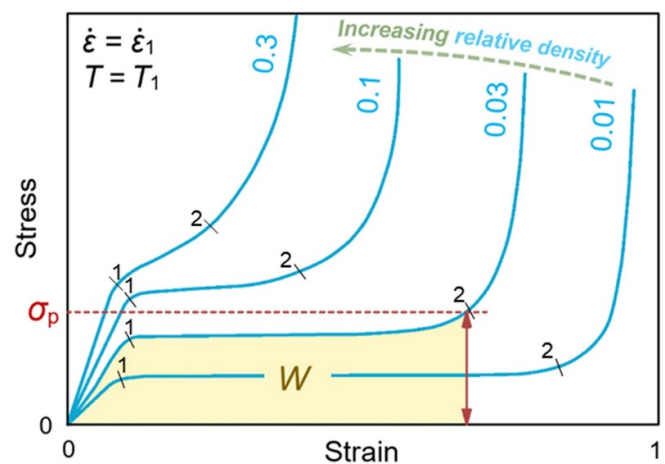
## 4. Helmet design

### 4.1. Established concepts

The idealised goal of impact protective equipment is to absorb induced impact energies without exceeding measures associated with injury risk. For a consistent impact scenario, like a certification test, the selection process for an energy-absorbing material is established. The challenge with helmets is the diverse variety of impact scenarios. There are various helmet designs available for different sports [35, 36, 217], with two main categories. The first category is single-impact helmets, which crush under impact and are designed to protect the head against one severe (high energy) impact. These include motorsports, cycling, and alpine sports helmets. After such an impact, these helmets should be replaced as they are damaged, and offer limited protection [35, 217]. The other category is multi-impact helmets, which are designed to maintain their impact performance over a (typically) long service life, e.g. several years. These are used for American Football, ice hockey, and lacrosse [35, 217], to name a few.

Helmets typically have at least three layers. A stiff (polymer or composite) outer shell (figure 1(A)) prevents penetration [35, 36, 84, 217, 255, 256], absorbs the initial shock [84, 257], and helps to hold the helmet together during or after an impact [84]. A compressible foam or lattice liner absorbs energy through deformation (figure 1, [217, 256]). Most single-impact helmets, particularly those for cycling, use crushable, expanded polystyrene (EPS) foam [36, 258]. Vinyl nitrile (VN) (figure 1(A)) and expanded polypropylene (EPP) (figure 1(B)), are often used in multi-impact helmets [35, 151, 259–261]. Many helmets also include a comfort liner, often a compliant foam [262], as shown in figure 1(A). New materials and components are also being added to helmets, generally intended to exceed minimum requirements in certification tests (e.g. [30, 31, 33, 263–269]).

Inspecting example compressive stress ( $\sigma$ ) vs. strain ( $\epsilon$ ) relationships (figure 2), the area under the curve is the energy absorbed per unit volume ( $W$  [270]). The compressive stress vs. strain relationships of cellular materials, such as foams, can often be divided into three sections: (i) linear elasticity, up to  $\sim 5\%$  strain; (ii) plateau, elastic or plastic buckling of the cell walls; and (iii) densification, where cell walls self-contact and the constituent material is compressed [270].

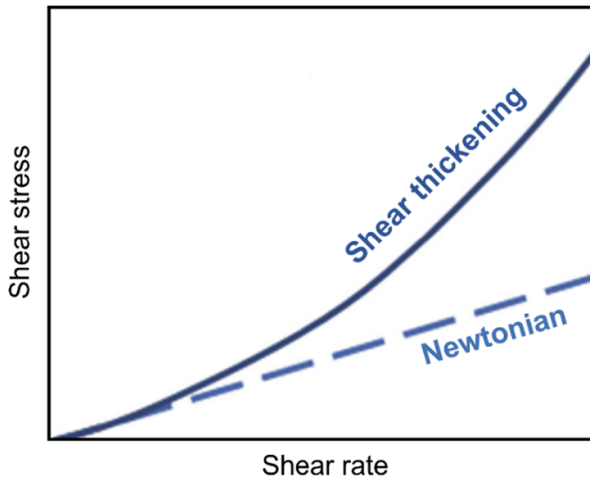


**Figure 2.** Example compression stress vs. strain for different relative densities of a foam at equal strain rates (arbitrarily using engineering strain values for simplicity). Area  $W$  under the curve illustrates the absorbed energy. The start (1) and end (2) of the stress plateau are marked for each foam. Reproduced with permission from [270].

Energy absorption efficiency ( $W/\sigma$ ) is highest during the plateau region [271], which can be tailored by modifying the constituent material or foam relative density [270]. An ideal foam for a given impact (e.g. curve 0.03 in figure 2) absorbs the induced energy during the plateau region, without densifying [35, 270]. Energy absorption before densification increases with liner thickness. However, overly large helmets are uncomfortable [35, 270] and can increase rotational accelerations by increasing torque applied to the head [35]. As such, helmet liner thickness is generally limited. So, combining layers of foam of varying relative density, and hence stiffness, may broaden the range of manageable impacts, but will give lower maximal efficiency [36, 272].

### 4.2. Emerging developments

Various approaches have been taken to make helmets more effective over a wider range of impacts. Shear-thickening fluids (STFs) and polymers (STPs) are non-Newtonian (figure 3). The viscosity of these materials increases with shear strain rate [273–276]. STFs include suspensions [273] and gels



**Figure 3.** Behaviour of Newtonian and shear-thickening fluids (where gradient increase with shear rate).

[277], while STPs (which are more commonly used in consumer products) are viscoelastic solids [275, 276, 278]. STPs adapt to impact severity [279, 280]; they can be flexible and elastic during normal use and minor impacts, or stiffen and increase damping during severe impacts [273, 281]. The viscosity change is reversible, providing an alternative to crushable foam over multiple impacts [282, 283], with slow recovery potentially reducing rebound, impact duration, and various measures of injury risk [125, 133–136]. So, foamed STPs are used in PPE [273, 275, 277, 279], including helmets liners (figure 1(C)) [34]. STPs can also be formed into a structure and used within helmets to reduce rotational kinematics [18, 25].

A low-friction layer, placed between the helmet's liner and shell (figure 2(B) [21, 284]), or between layers of foam [265], allows relative rotation between components. This relative rotation has been shown to reduce the rotational kinematics of headforms during oblique impact tests [32, 230, 245, 284–288]. A well-known example of this technology is the multi-directional impact protection system (MIPS) [264]. The inclusion of anisotropic helmet liners has also been shown to reduce rotational acceleration during certain oblique impacts [19–21, 289]. Such liners may have fibrous columns [21], or elongated cells [20], making them stiff through thickness but transversely compliant, lowering shear stiffness [290, 291]. Salomon's EPS 4D helmet liner uses a similar principal, whereby columns of EPS foam appear to be designed to shear during oblique impacts [268]. These examples are relatively standard, using conventional materials and manufacturing methods.

Patents have been filed featuring concepts related to the application of mechanical metamaterials in helmets. These include helmets, or helmet liners, based on structured polymers such as lattices (e.g. [292–297]), sprung/suspended inserts (e.g. [298]), modular/custom fit structured components (e.g. [299, 300]), foamed/structured shear thickening materials (e.g. [301, 302]), bulk shear thickening materials (e.g. [303]), and fluidic properties (e.g. [304, 305]). Many of

these innovations feature in commercially available helmets (e.g. [30, 31, 263, 269]).

## 5. Mechanical metamaterials

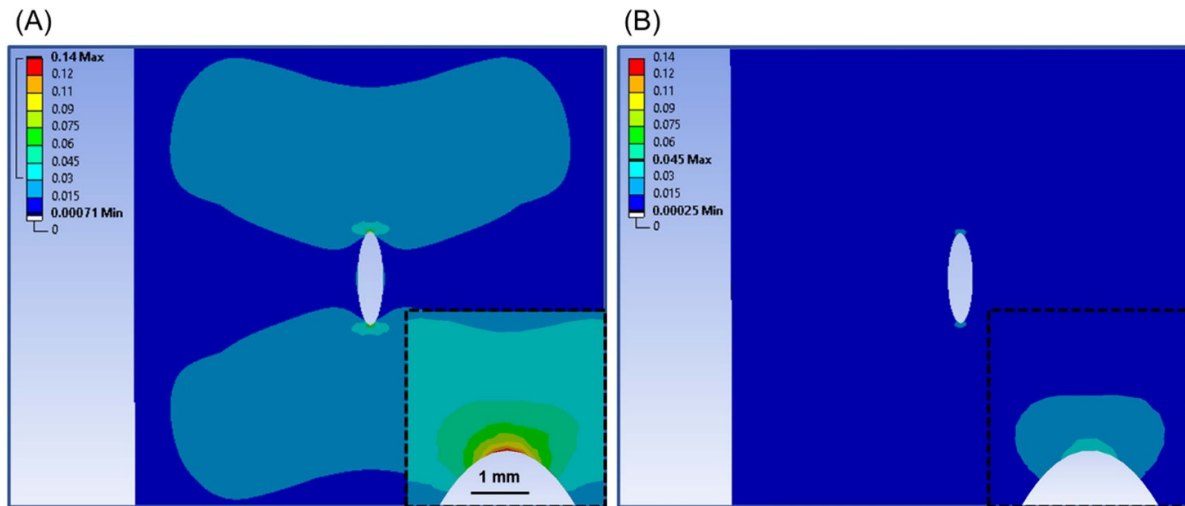
Mechanical metamaterials can be made in various ways and have unique properties that could improve sport helmet performance. They can be fabricated from conventional materials such as foam [306, 307] or textiles [51, 308], or designed as periodical/graded cellular structures [12–14, 309]. Mechanical metamaterials can also be made from sheets of material by folding (known as origami) [310–327], or by folding, cutting, and joining (known as kirigami) [17, 328–344]. With high levels of control over end properties—given the additional degrees of design freedom afforded by controlling topology and base material—mechanical metamaterials are well suited to addressing complex engineering problems, like impact protection [17, 50, 262]. The common forms of unusual mechanical properties are auxetic (negative Poisson's ratio) behaviour (covered extensively in various reviews [46, 50, 307, 309, 345, 346] and textbooks [308, 347, 348]), negative stiffness [349–356], shape morphing [337, 357–359], force/torque coupling [360–364], active/adaptive behaviour [351, 365–367], or programmable properties that are tuned to a specific application [17, 26, 28, 341, 361].

### 5.1. Auxetic metamaterials

Poisson's ratio is the negative product of the ratio of lateral to axial strain. Auxetic materials undergo transverse expansion when stretched axially, and contract transversely in compression [11, 285, 306]. So, they form a dome shape under bending, and are used in a helmet liner for this reason (i.e. flat sheets can fit into domed helmets [32, 263]).

Poisson's ratio is one of the basic elastic constants, and (with Young's modulus/stiffness) affects shear modulus, bulk modulus, and indentation resistance [368]. As detailed elsewhere [368], Poisson's ratio increases resistance to penetration by concentrated loads [369–373], and shear modulus [374–378]. The increased tendency of materials with a low or negative Poisson's ratio to deform volumetrically, rather than in shear (figure 4), may also increase strength. With lower shear strain, the likelihood of failure close to a crack tip or an ellipsoid reduces, according to Von-Mises, Tresca, and crack propagation theories [379, 380]. Without the presence of stress concentrations caused by a crack or ellipsoid, Von-Mises and Tresca criterion are unaffected [368]. So, auxetic helmet sections may fail less readily, reducing waste and severe head injury risk.

The re-entrant-like cellular structure of auxetic foams is imparted by compressing conventional foam to buckle cell ribs [306, 307, 381]. So, while there is some uncertainty over whether these foams meet the requirement for the precisely defined topology of some metamaterial definitions [7–9], they are still a related medium. Readers interested in auxetic foam manufacturing are referred to Jiang *et al's* review [307].



**Figure 4.** Contour plots of maximum shear (engineering) strain in  $100 \times 100$  mm thin plates loaded parallel to the short axis of a  $5 \times 20$  mm central ellipsoidal hole, with arbitrary, equal tensile loads and moduli, but Poisson's ratios of (A) 0.5 and (B)  $-0.5$ . Static structural simulations were undertaken in Ansys Mechanical to demonstrate this concept.

Auxetic foams have been shown to increase vibration damping [382, 383], and to exhibit peak impact forces up to ten times lower than their conventional counterparts [291, 384–388]. It should be noted that peak force during the typically stiff anvil and impactor impacts is not a scalar measure. Indeed, peak force increases exponentially as the foam densifies (see figure 2) and ‘bottoms out’ under impact. Further, auxetic foams made from expanded foam, as typically used in helmet liners, have not been reported.

Auxetics may provide benefits in helmet liners: (i) The ability to adapt to the shape of impacting bodies—remaining soft when impacting a relatively flat surface, but effectively stiffening under concentrated loads. (ii) High vibration damping, redistributing vibrations transversely. (iii) A tendency to bulk over shear deformation, reducing the likelihood of failure. Conversely, the early densification strain caused by the tendency to bulk deformation may shorten the stress vs. strain plateau in cellular solids—causing densification at lower strain ([291], figure 2). With the ability to include a stiff shell, the benefit of the high indentation resistance possible for auxetics is unclear and has not been empirically demonstrated in helmets. Flexible shell helmets (e.g. [227, 228]) may, however, benefit from the increased indentation resistance of auxetic materials. Further, the increase in shear modulus with negative Poisson's ratio goes against the broad strategy of reducing liner shear stiffness to reduce rotational acceleration [25, 32, 263, 264, 284, 286–288]. So, the application of auxetic materials in helmet liners requires careful design based on justifiable benefits, such as increasing indentation resistance to facilitate lower stiffness liners.

An unstudied, potentially useful topic is auxetic helmet shells. Fibre-polymer composites can be auxetic, with the negative Poisson's ratio achieved by fibre alignment [389, 390]. Due to the use of conventional fibres and pre-preg, auxetic fibre-polymer composites can be made with standard composite manufacturing methods [391]. These auxetic composites have been shown to resist back face damage under impacts

[392]. Such increased resistance to back face damage could increase the lifespan of multi-impact helmets featuring composite shells, particularly those with flat sections that are more susceptible to back face damage.

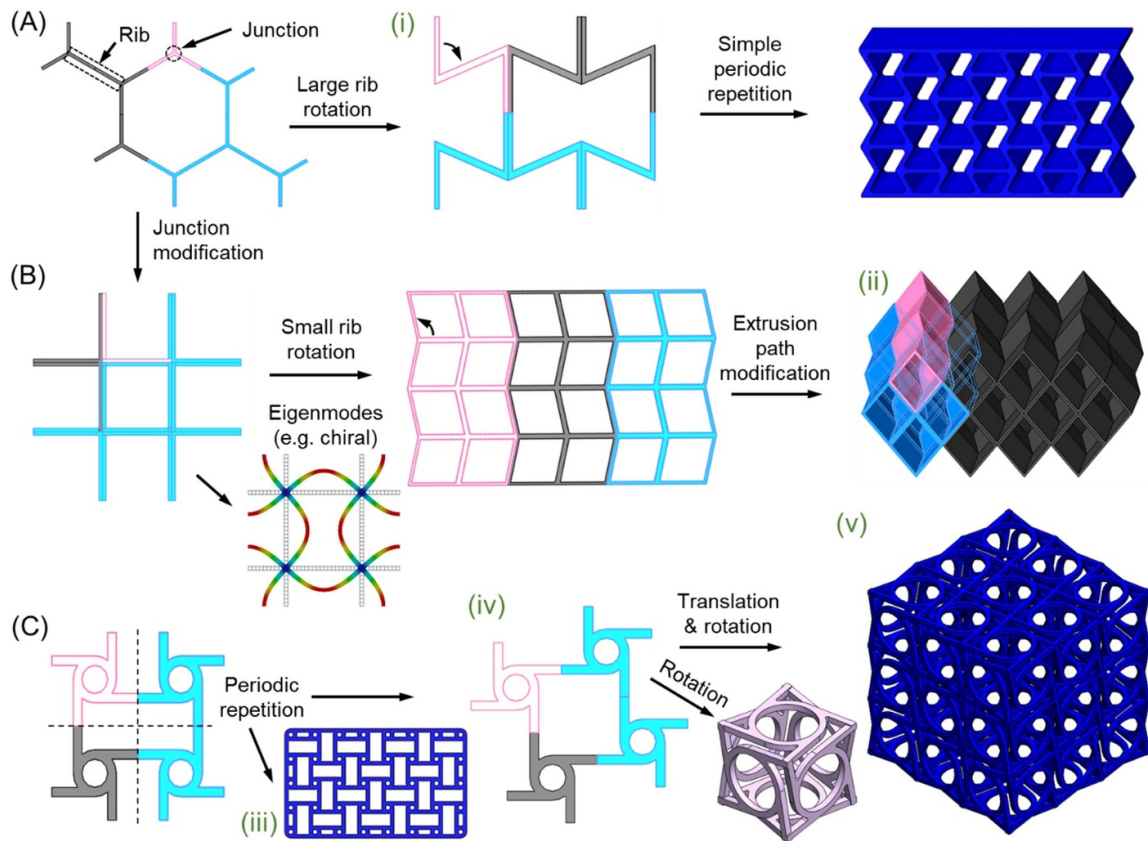
## 5.2. Periodic structures

Advances in additive manufacturing, and moulding methods [25], have allowed mass production of lattice and honeycomb mechanical metamaterials (e.g. [31, 33, 263, 393, 394]). These methods allow precise manufacturing of complex geometries [16, 395–397], expanding the range of available properties to meet complex requirements, such as those seen in helmets. 2D extruded cellular structures, such as honeycombs (figure 5(A)), repeat periodically in two directions [398]. Honeycomb and tubular structures are studied frequently as energy-absorbing elements in sports equipment and helmets [23, 399–403], such as in Koroyd's helmet liner [31]. 3D periodic cellular structures, such as lattices, consist of unit cells repeating in three directions, increasing degrees of freedom during design, but also increasing manufacturing complexity and hence costs [395, 397, 398, 404–409].

Exemplary unit cell designs include hexagonal/re-entrant (figure 5(A)) [410, 411], square/cubic (figure 5(B)) [412], or chiral/antichiral (figure 5(C)) [413–417]. Unit cell design degrees of freedom include; varying rib orientation (figures 5(A) and (B)), length, or thickness—slender ribs are often less stiff, varying rib form (figure 5(B))—Eigenmode example); varying the number of ribs (figures 5(A) and (B)), or adding and combining shapes and features (figure 5(C)). Unit cell patterning also affects topology, and so metamaterial properties; unit cells can be mirrored (figures 5(A-i) and (C)), linearly repeated (figures 5(A), (B) and (C-iii)), or rotated (figure 5 (C-v)).

When creating honeycombs, some variation can be applied in the extruded direction, as is the case of Miura-ori-inspired structures (figure 5 (B-ii)) [418, 419]. Gradient metamaterials





**Figure 5.** Some notable mechanical metamaterial design degrees of freedom: (A) Honeycombs, with large angle rib modifications, becoming (i) re-entrant. (B) Quadrilateral honeycomb, with Eigenmode rib tessellation to form an auxetic unit cell, or small rib rotations and extrusion path modification, forming a (ii) Miura-ori inspired metamaterial. (C) Various periodic rotational and translational repetitions of a chiral unit cell, forming (iii) antitetra chiral, (iv) 2D chiral and (v) 3D chiral metamaterials. Pink wireframe notes the simplest repeating unit cell, subsequent repetitions are shown in grey or blue.

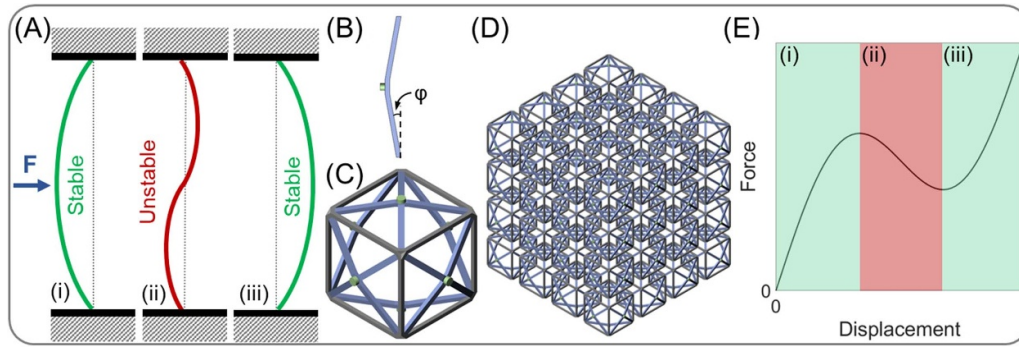
can also be developed by spatially varying unit cell parameters and properties [402, 403, 420, 421]. These variations can be continuous or discrete (i.e. gradual or abrupt change) [422–424]. 3D periodic cellular structures are being used in helmets (e.g. ice hockey [33] and American football [393]). Such liners, or inserts, can also feature some through thickness variation, and can be made from STPs [30].

Concerning some common topologies, hexagonal honeycombs are relatively stiff, with low density, for compression parallel to their extruded dimension [402, 425–428]. During impacts in the extruded direction, cell walls crumple and buckle [31, 426], with densification occurring at  $\sim 75\%$  compression [402]. When impacted or compressed perpendicular to their extruded dimensions, honeycomb stiffness is lower [28, 402, 427, 429, 430]. Re-entrant hexagonal honeycombs can be more compliant than equivalent density regular hexagonal ones (at low strains), due to the extra junction for rib hinging to occur around [423, 429, 431, 432]. Buckling may not occur with these re-entrant unit cells, causing an almost linear compressive stress vs. strain response, i.e. a less pronounced plateau and densification region [369, 423, 429]. Such re-entrant unit cells may not be optimal within a target impact severity (e.g. as in figure 2), but are less prone to stark peak force increases during severe impacts [29, 291, 385]. It

is possible to design tall, slender stiff re-entrant cells that undergo buckling [271, 422].

Structures made of solid rotating shapes are often stiff in compression, as the internal shapes undergo self-contact/densification at low strains, making them less suited for sporting impacts [433–436]. These rotating shapes have been made as lattices with hollow cells [437], or cut from foams, to tune out of plane properties while relying on foam characteristics through thickness [438, 439]. High energy absorption, low initial crushing peak forces, large densification strain, and low strain rate sensitivity can be achieved with folded (kirigami or origami) structures [332, 333, 335, 440, 441]. By including such folds through the thickness of these structures, it is possible to tune buckling regions, and the length of the compressive stress plateau [17] (figure 2). Kirigami structures made from paper have been used in a recyclable cycling helmet [442].

For application in helmets, periodic structures must be patterned to fill an often complex/nearly spherical space. Such patterning, using conventional computer-aided design software, can be time-consuming and inefficient. Algorithmic-based design software, such as those marketed by nTopology [443], Hyperganic [444], or Rhino3d [445] can make the process of generating such geometries more efficient. Noting



**Figure 6.** (A) Stages of snap through in a buckling beam. (B) Snap through element. (C) Inclusion in a cubic unit cell and (D) metamaterial, recreated from ref [353]. (E) Example force displacement (arbitrary values), including stages from (A). Reprinted from [353], © 2019 Elsevier Ltd.

that impact vectors are usually non-centric, further challenges arise. Where there are enough unit cells, the response at various angles can be calculated based on orthotropic, strain-dependent properties, using standard elasticity tensor transformation [423, 446, 447]. So, response to off-axis impacts can be designed by tuning the out-of-plane properties. Where there are too few unit cells to homogenise the material properties, as is often the case for periodic structures, the off-axis response must be obtained by higher order material approximations [448], microstructurally faithful simulations [25], or experimentally [25, 32].

### 5.3. Force torque coupling

Advances in fabrication methods have allowed realisation of a wide range of unusual and potentially beneficial properties. Mechanical metamaterials with force–torque coupling (known herein as twist) have seen increasing interest, since their rational design was shown in 2017 [363]. Like (negative) Poisson’s ratio, twist translates axial deformation to transverse deformation—increasing resistance to indentation [360]. So, twisting mechanical metamaterials may resist penetration by concentrated loads, without shortening the stress plateau under compression (as seen for auxetics—see section 5.1).

The development of these metamaterials could also facilitate more efficient analysis and design of lattices. The twisting response is not included in classical (Cauchy) continuum mechanics, but it is in micro-morphic continuum mechanics (where a uniform load causes internal strain gradients [448]). Eringen presented some special cases in micro-morphic theories [448]. These include micropolar—where the gradients occur by rotation of rigid unit cells (sand provides an intuitive example), and micro-stretch—where the gradients occur by unit cell volume change, without shape change (picture the bronchi). Each of these allows some simplification (over the micro-morphic continuum)—reducing the amount of information required to approximate the response of a metamaterial—but may also cause some loss in precision.

Clearly, in the case of foams and lattices, which have relatively large internal features (when compared to the scale of external loads), micro-morphic continuum theories often apply [360, 449, 450]. Where classical continuum theories

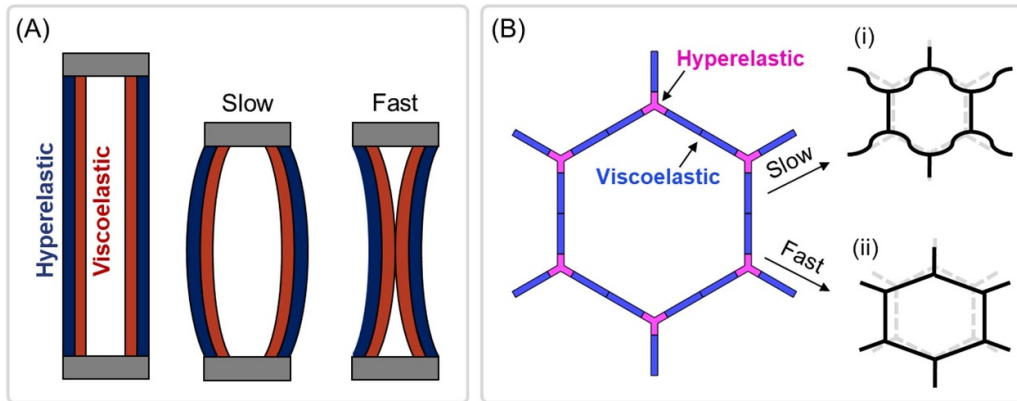
cannot be used, typical approaches to designing mechanical metamaterials for impact protection are experimental, or by using microstructurally faithful numerical models [17, 26, 28, 29, 415, 416, 419, 451], which are less efficient. So, developing and applying these micro-morphic continuum theories, including their viscoelastic and visco-plastic formulations, could facilitate more efficient mechanical metamaterial analysis, design, and application in single or multi-impact helmets [448].

### 5.4. Negative stiffness

Snap-through elements cause negative stiffness behaviour, corresponding to a drop in force as applied deformation increases [349–355]. Negative stiffness can be achieved by the buckling of an end constrained/preconditioned beam (figure 6). The beam snaps from one state of equilibrium to the next following the application of a perpendicular load (often via a connecting rib) [349, 350, 355]. The negative stiffness region is present for a segment of the force vs. compression relationship, corresponding to when the beam snaps through (figure 6(E)). Increasing the diagonal angle of the symmetric beam ( $\varphi$ , figure 6(B)) increases the onset, and length, of the region [353]. Making the beam less slender increases the critical buckling load and hence the magnitude of both positive and negative stiffness [353]. Negative stiffness has been shown to improve protection during impacts [452], balancing the positive stiffness of neighbouring unit cells to flatten and elongate the stress plateau (figure 2). Designing and manufacturing relatively unstable negative stiffness inclusions within helmets could bring added complexity, and further work applying these concepts to helmets is needed.

### 5.5. Topology optimisation

Precise control over topology allows the design of a desired response to various loading conditions. An efficient approach to topology optimisation is to optimise a unit cell, and homogenise (expand to an effective bulk material) using a set of boundary conditions (i.e. periodic symmetry [451, 453–458]). Readers interested in homogenisation theory could refer to refs [459–461]. Such an approach is not widely applied during



**Figure 7.** (A) Single bi-beam design and buckling direction for different axial compressive strain rates. (B) Multi-material hexagonal cell causing a switch in the dominant deformation mode.

sporting impacts, causing high strains and strain rates, meaning degrees of freedom surrounding unit cell boundaries are influential and challenging to predict. For example, end loading for buckling beams, such as cell ribs, can vary with neighbouring cell deformation, meaning a multi-cell optimisation is needed [462–464]. Instead, whole metamaterial samples are often optimised or iteratively improved [17, 22, 26, 32]. Developing and applying micro-morphic theories [448] to lattices under impact may facilitate the prediction of rib constraints, and efficient (unit-cell) topology optimisation [465]. Open data approaches (such as the meta-genome [466]) could help develop these new homogenisation methods.

### 5.6. Adaptive metamaterials

While shear thickening polymers can adapt their stiffness to various collision types, more degrees of design freedom, such as deformation mode switching, are possible. Deformation mode switching can be achieved by including an adaptive material, such as a viscoelastic one, that activates a topological instability [14]. For example, a beam's buckling mode (e.g., direction) can be designed to switch at a given strain rate. When two laterally connected beams of different stiffness (i.e. bi-beams) are axially compressed (figure 7(A)), the stiffer one drives the buckling direction while the other provides support, causing buckling towards the stiffer side. So, in figure 7(A), the deformation direction will switch at the strain rate when the viscoelastic beam becomes stiffer than the hyperelastic one. Bi-beams positioned like those in figure 7(A) will buckle away from each other at low compression rates, and towards each other, causing stiffening via self-contact, at high compression rates. Negative effective viscoelasticity can also be achieved with such a system of bi-beams [365], if the order in figure 7(A) is reversed, so bi-beams effectively soften by buckling away from each other at high compression rates. Obtaining negative effective viscoelasticity with highly viscoelastic materials demonstrates the level of control available via topology that could be useful when designing helmets. The response of these bi-beams, while previously shown to be retained for off-axis deformation angles of  $\sim 10^\circ$  [365], is unknown during oblique impacts. Flexing of the beams may

provide a desirable low shear modulus, as in similar tests of long-cell anisotropic foam liners [20, 289], and should be studied further.

More stable adaptive metamaterial systems can also be designed. The dominant deformation mode in hexagonal honeycombs and lattices is cell rib flexure [429, 447] (figure 7(B-i)). Such flexure reduces the distance to the neighbouring junction, respectively reducing and increasing the magnitude of positive and negative compressive Poisson's ratios. Placing viscoelastic material in the cell ribs could switch the dominant mode, increasing the magnitude of positive Poisson's ratio, and hardness [368], during more severe impacts (figure 7(B-ii)). Conversely, placing viscoelastic material in the junction of auxetic, re-entrant honeycombs or lattices could have a similar effect, by amplifying the dominance of rib flexure to draw neighbouring junctions inward. These concepts have been demonstrated using dual materials of different stiffness, but not viscoelastic ones, and present notable options for future research, particularly related to applications in helmets [467, 468]. Interestingly, such switching mechanisms and changes to Poisson's ratio can be achieved with one material, based on local changes in strain rate and stiffness; shifting the point of deformation [469–471].

The use of embedded electronics as active adaptive mechanisms is emerging [14, 472, 473]. These include piezoelectric inclusions, which stiffen when an electrical field is applied [472], or embedded electromagnets [352]. Such systems allow a controlled response, with the potential for embedded electronics to sense environmental changes, like impact severity or strain rate, fall initiation, temperature or relative humidity, and micro-controllers to define a programmed response or adaption [14, 472, 473]. Challenges to application are associated with the manufacture of sufficiently small and robust components [14, 352].

## 6. Discussion

Some commercial, or mid- to high- technology readiness level mechanical metamaterials, show promise to reduce concussion risk when applied to helmets

**Table 1.** Summary of key metamaterial types, properties, benefits, and challenges for application in helmets.

Metamaterial	Potential benefit	Potential application	Challenges
Periodic structures	Tuneable response	Compliant/crushable liners	Efficient design and manufacturing, particularly during non-centric impacts
Auxetics	Domed curvature	Helmet liner manufacturing solution	Already established
	High indentation resistance	Compliant/crushable liners, particularly of soft-shell helmets	Early densification, and increased shear modulus
	High toughness	Compliant/crushable liners Helmet shells, particularly fibre-polymer composites	Making crushable auxetic foam Demonstrating the requirement
Force–torque coupling	High indentation resistance	Compliant/crushable liners, particularly of soft-shell helmets	Cannot be simulated as bulk solids using the Cauchy continua
Negative stiffness	Extension of stress vs. strain plateau region	Compliant liners	Cost-effective design of such unstable mechanisms
Adaptive metamaterials	Improved performance across various impact types	Compliant liners	Cost-effective manufacture and design—often featuring multiple materials deformation mechanisms

[18, 21, 25, 30, 32, 263–265, 393]. Separately, these metamaterials appear to have sufficient degrees of freedom to reduce shear and compressive response, and rotational and linear measures (e.g. [18, 21, 25, 30, 32, 263–265, 268, 284, 393]), reduce the duration of high accelerations via crushable or viscoelastic liners (e.g. [18, 25, 30–32, 263, 393]), and adapt to surface compliance, or impact severity, via rate dependency (e.g. [18, 30, 266]). For commercial helmets [30–33, 263–266, 268, 284], independent, peer-reviewed, analysis of such functions is rarely published. These systems, along with [18], have demonstrated commercial viability of mechanical metamaterials (e.g. use of single material injection moulding or additive manufacturing). So, they reduce barriers to entry and raise awareness, facilitating continuous improvements and further application of mechanical metamaterials. Table 1 summarises the key mechanical metamaterial types covered in this review.

With the greater degrees of freedom afforded when designing mechanical metamaterials come some additional challenges. Firstly, metamaterials can be more expensive to manufacture than established helmet materials, so currently tend to only be used in high-end helmets [30, 31, 33, 263, 269]. Secondly, the effect of increasing, or changing, the pool of materials used in helmet design needs to be considered; particularly susceptibility to environmental considerations such as temperature, relative humidity, contaminants, and ultra-violet radiation [474]. Conversely, the greater design affordances associated with mechanical metamaterials could reduce susceptibility to environmental effects; with potential to achieve the required performance using only materials that resist degradation due to environmental conditions. The sports market may again be an important early adopter; providing long term, in-field (user) testing in variable environments before uptake by more conservative sectors such as aerospace.

Peer-reviewed publications noting tests of new helmet technologies rarely use biofidelic anthropomorphic test dummy heads or necks, nor impacts onto compliant anvils. Unrealistic coupling between helmet and head may affect rotational accelerations [32, 136, 245, 246], while impacts with compliant surfaces are amongst the most common causes of sporting concussion [37, 63, 88, 96–99]. Mechanical metamaterial design streams that reflect these high injury risk scenarios could be developed, to ensure mechanical metamaterials are designed and implemented in helmets based on up-to-date measures of concussion risk. An extensive range of mechanical metamaterial properties has been demonstrated (e.g. [18, 21, 25, 32, 284]), so such design streams appear feasible. Funding calls, challenges, and open data approaches that promote collaborations and knowledge exchange between groups with state-of-the-art test methods, and those developing metamaterials and helmets, could be beneficial. Such initiatives could be sport specific (e.g. [475]), or broader (e.g. [90–93, 466]).

A metamaterials approach to helmet design: unit cell optimisation and patterning, based on an objective function of measures of concussion risk and manufacturing constraints, could be used to improve helmet impact performance [456, 462]. Such an approach requires some form of rate dependency or adaptation, justifying research developing ranges of viscoelastic materials for additive manufacturing. We note two forms of unusual property that are of prominent interest in helmet development: (1) negative stiffness inclusions, to extend or flatten the stress plateau (figure 6), and (2) adaptive metamaterials (figure 7). Switching of deformation mechanisms may provide options to increase rate dependence without the presence of extreme viscoelasticity. Efficient topology optimisation for such systems also requires some development, to apply periodic constraints that reflect rib buckling with single/minimal

unit cells [456, 462]. With such methods, helmet manufacturers could adapt to developing knowledge of concussions, or design affordances offered by new manufacturing methods.

## 7. Conclusions

Mechanical metamaterial design affords degrees of freedom that could allow helmets with an impact response that adapts between severe impacts that cause skull fracture and those that might lead to clinically diagnosed concussion. The objective functions that mechanical metamaterial helmet liners are designed or optimised for could be modified, by testing and refining the designed helmets on biofidelic headforms under representative test conditions to better understand the required effective properties. As such, efforts encouraging collaborations between those developing helmets, mechanical metamaterials, and test methods, could improve helmets. Epidemiological studies may help identify the effect of such interventions over time. Improving the efficiency and availability of topology optimisation tools at high strains and strain rates, would allow helmets to be updated as knowledge of concussion improves. Here, open data and opensource software initiatives will be beneficial. To increase options for mechanisms of adaption to impact severity, researchers could focus on increasing options to print or mould viscoelastic materials or developing topological approaches to tune effective viscoelasticity.

## Data availability statement

The data cannot be made publicly available upon publication because they are not available in a format that is sufficiently accessible or reusable by other researchers. The data that support the findings of this study are available upon reasonable request from the authors.

## Conflict of interest

Dr Greenwald is the founder of Simbex, Lebanon, New Hampshire, USA, and is involved in the development of head impact exposure monitoring technology used commercially by Riddell, Inc. All other authors declare no conflict of interest.

## Author contributions

Mr Haid drafted the review, while Drs Duncan, Allen, Sareh, and Greenwald edited it. Drs Hart and Foster provided feedback.

## ORCID iDs

Leon Foster  <https://orcid.org/0000-0002-1551-0316>  
 Tom Allen  <https://orcid.org/0000-0003-4910-9149>  
 Pooya Sareh  <https://orcid.org/0000-0003-1836-2598>  
 Olly Duncan  <https://orcid.org/0000-0001-9503-1464>

## References

- [1] 2021 Concussion in sport—inquiry (available at: <https://committees.parliament.uk/work/977/concussion-in-sport/>) (Accessed 6 April 2023)
- [2] McCrory P *et al* 2017 Consensus statement on concussion in sport—the 5th international conference on concussion in sport held in Berlin, October 2016 *Br. J. Sports Med.* **51** 838–47
- [3] Harmon K G *et al* 2019 American medical society for sports medicine position statement on concussion in sport *Clin. J. Sport Med.* **29** 87–100
- [4] Guskiewicz K M, Marshall S W, Bailes J, Mccrea M, Harding H P, Matthews A, Mihalik J R and Cantu R C 2007 Recurrent concussion and risk of depression in retired professional football players *Med. Sci. Sports Exerc.* **39** 903–9
- [5] Moreland G and Barkley L C 2021 Concussion in sport *Curr. Sports Med. Rep.* **20** 181–2
- [6] Dancer C 2023 What are metamaterials? (available at: <https://metamaterials.network/what-are-metamaterials/>) (Accessed 27 March 2023)
- [7] Nature Mechanical metamaterials collection 2022 (available at: [www.nature.com/collections/iebdfeffdc/](http://www.nature.com/collections/iebdfeffdc/)) (Accessed 2 August 2022)
- [8] Boardman A 2011 Pioneers in metamaterials: John Pendry and Victor Veselago *J. Opt.* **13** 020401
- [9] Pendry J 2006 Beyond metamaterials *Nat. Mater.* **5** 763–4
- [10] Ball P 2018 Bending the laws of optics with metamaterials: an interview with John Pendry *Natl Sci. Rev.* **5** 200–2
- [11] Pendry J B 2000 Negative refraction makes a perfect lens *Phys. Rev. Lett.* **85** 3966–9
- [12] Barchiesi E, Spagnuolo M and Placidi L 2019 Mechanical metamaterials: a state of the art *Math. Mech. Solids* **24** 212–34
- [13] Surjadi J U, Gao L, Du H, Li X, Xiong X, Fang N X and Lu Y 2019 Mechanical metamaterials and their engineering applications *Adv. Eng. Mater.* **21** 1–37
- [14] Zadpoor A A 2016 Mechanical meta-materials *Mater. Horiz.* **3** 371–81
- [15] Lee J H, Singer J P and Thomas E L 2012 Micro-/nanostructured mechanical metamaterials *Adv. Mater.* **24** 4782–810
- [16] Soe S, Ryan M, McShane G and Theobald P 2015 Energy absorption characteristics of additively manufactured TPE cellular structures *Second Int. Conf. on Sustainable Design and Manufacturing, (Seville, Spain 12–14 April 2015) 2nd Int. Conf. on Advanced Materials and Additive Manufacturing* pp 145–58 (available at: <http://nimbusvault.net/publications/koala/inimpact/papers/sdm15-052.pdf>)
- [17] Townsend S, Adams R, Robinson M, Hanna B and Theobald P 2020 3D printed origami honeycombs with tailored out-of-plane energy absorption behavior *Mater. Des.* **195** 108930
- [18] La Fauci G, Parisi M, Nanni A, Crosetta L, Pugno N M and Colonna M 2023 Design and proof-of-concept of an advanced protective system for the dissipation of tangential impact energy in helmets, based on non-Newtonian fluids *Smart Mater. Struct.* **32** 044004
- [19] Mosleh Y, Cajka M, Depreitere B, Vander Sloten J and Ivens J 2018 Designing safer composite helmets to reduce rotational accelerations during oblique impacts *Proc. Inst. Mech. Eng. H* **232** 479–91
- [20] Vanden Bosche K, Mosleh Y, Depreitere B, Vander Sloten J, Verpoest I and Ivens J 2017 Anisotropic polyethersulfone foam for bicycle helmet liners to reduce rotational acceleration during oblique impact *Proc. Inst. Mech. Eng. H* **231** 1–11

- [21] Mosleh Y, Cajka M, Depreitere B, Ivens J and Vander Sloten J 2023 Smart material and design solutions for protective headgear in linear and oblique impacts: column/matrix composite liner to mitigate rotational accelerations *Smart Mater. Struct.* **32** 014001
- [22] Krishnan B R, Biswas A N, Kumar K A and Sreekanth P R 2022 Auxetic structure metamaterial for crash safety of sports helmet *Mater. Today: Proc.* **56** 1043–9
- [23] Khosroshahi S F, Tsampas S A and Galvanetto U 2018 Feasibility study on the use of a hierarchical lattice architecture for helmet liners *Mater. Today Commun.* **14** 312–23
- [24] Farajzadeh Khosroshahi S, Yin X, Donat C K, McGarry A, Yanez Lopez M, Baxan N, J. Sharp D, Sastre M and Ghajari M 2021 Multiscale modelling of cerebrovascular injury reveals the role of vascular anatomy and parenchymal shear stresses *Sci. Rep.* **11** 1–13
- [25] Siegkas P, Sharp D J and Ghajari M 2019 The traumatic brain injury mitigation effects of a new viscoelastic add-on liner *Sci. Rep.* **9** 1–10
- [26] Hanna B, Adams R, Townsend S, Robinson M, Soe S, Stewart M, Burek R and Theobald P 2021 Auxetic metamaterial optimisation for head impact mitigation in American football *Int. J. Impact Eng.* **157** 103991
- [27] Soe S P, Martin P, Jones M, Robinson M and Theobald P 2015 Feasibility of optimising bicycle helmet design safety through the use of additive manufactured TPE cellular structures *Int. J. Adv. Manuf. Technol.* **79** 1975–82
- [28] Adams R, Townsend S, Soe S and Theobald P 2022 Finite element-based optimisation of an elastomeric honeycomb for impact mitigation in helmet liners *Int. J. Mech. Sci.* **214** 106920
- [29] Shepherd T, Winwood K, Venkatraman P, Alderson A and Allen T 2020 Validation of a finite element modelling process for auxetic structures under impact *Phys. Status Solidi b* **257** 1900197
- [30] Rheon and Xenith (available at: <https://rheonlabs.com/rheon-technology-products/shadow-xr/>) (Accessed 6 April 2023)
- [31] Koroyd 2020 Koroyd (available at: <https://koroyd.com/>) (Accessed 05 October 2023)
- [32] Bliven E, Rouhier A, Tsai S, Willinger R, Bourdet N, Deck C, Madey S M and Bottlang M 2019 Evaluation of a novel bicycle helmet concept in oblique impact testing *Accid. Anal. Prev.* **124** 58–65
- [33] Carbon and CCM Carbon3D CCM super tacks X 2020 (available at: [www.carbon3d.com/resources/blog/ccm-super-tacks-xl/](http://www.carbon3d.com/resources/blog/ccm-super-tacks-xl/)) (Accessed 13 March 2023)
- [34] Bailly N, Laporte J D, Afquir S, Masson C, Donnadiou T, Delay J B and Arnoux P-J 2018 Effect of helmet use on traumatic brain injuries and other head injuries in alpine sport *Wilderness Environ. Med.* **29** 151–8
- [35] Hoshizaki T B and Brien S E 2004 The science and design of head protection in sport *Neurosurgery* **55** 956–67
- [36] Piland S G, Gould T E, Jesunathadas M, Wiggins J S, McNair O and Caswell S V 2019 Protective helmets in sports *Materials in Sports Equipment 2* (Woodhead Publishing Series in Composites Science and Engineering) pp 71–121
- [37] Pauelsen M, Nyberg G, Tegner C and Tegner Y 2017 Concussion in ice hockey—a cohort study across 29 seasons *Clin. J. Sport Med.* **27** 283–7
- [38] Beaver W 2018 Concussion in the NHL: a case study *J. Contemp. Athl.* **12** 123–38
- [39] Adams R *et al* 2018 Modifying factors for concussion incidence and severity in the 2013–2017 National Hockey League seasons *Cureus* **10** e3530
- [40] Prien A, Grafe A, Rössler R, Junge A and Verhagen E 2018 Epidemiology of head injuries focusing on concussions in team contact sports: a systematic review *Sports Med.* **48** 953–69
- [41] Nezwiek T A and Lee C S 2016 Concussion in the NHL: where do we stand? *J. Orthop. Res. Ther.* **2016**(2) 1–3
- [42] Kuhn A W and Solomon G S 2016 Concussion in the National Hockey League: a systematic review of the literature *Concussion* **1** 30202546
- [43] Lakes R S 2017 Negative-Poisson's-ratio materials: auxetic solids *Annu. Rev. Mater. Res.* **47** 63–81
- [44] Ren X, Das R, Tran P, Ngo T D and Xie Y M 2018 Auxetic metamaterials and structures: a review *Smart Mater. Struct.* **27** 023001
- [45] Wu Y, Lai Y and Zhang Z Q 2011 Elastic metamaterials with simultaneously negative effective shear modulus and mass density *Phys. Rev. Lett.* **107** 1–5
- [46] Evans K E and Alderson A 2000 Auxetic materials: functional materials and structures from lateral thinking! *Adv. Mater.* **12** 617–28
- [47] Kelkar P U, Kim H S, Cho K H, Kwak J Y, Kang C Y and Song H C 2020 Cellular auxetic structures for mechanical metamaterials: a review *Sensors* **20** 1–26
- [48] Lakes R S 2020 *Composites and Metamaterials* (World Scientific)
- [49] Lim T-C 2020 *Mechanics of Metamaterials with Negative Parameters* 1st edn (Springer) (<https://doi.org/10.1007/978-981-15-6446-8>)
- [50] Duncan O, Shepherd T, Moroney C, Foster L, Venkatraman P D, Winwood K, Allen T and Alderson A 2018 Review of auxetic materials for sports applications: expanding options in comfort and protection *Appl. Sci.* **8** 941
- [51] Tahir D, Zhang M and Hu H 2022 Auxetic materials for personal protection: a review *Phys. Status Solidi* **259** 1–13
- [52] Singh O and Kumar Behera B 2023 Review: a developmental perspective on protective helmets *J. Mater. Sci.* **58** 6444–73
- [53] McAllister T and McCrea M 2017 Long-term cognitive and neuropsychiatric consequences of repetitive concussion and head-impact exposure *J. Athl. Train* **52** 309–17
- [54] Bailes J E, Petraglia A L, Omalu B I, Nauman E and Talavage T 2013 Role of subconcussion in repetitive mild brain injury *J. Neurosurg.* **119** 1235–45
- [55] Greenwald R M, Gwin J T, Chu J J and Crisco J J 2008 Head impact severity measures for evaluating mild traumatic brain injury risk exposure *Neurosurgery* **62** 789–98
- [56] Rowson S and Duma S M 2013 Brain injury prediction: assessing the combined probability of concussion using linear and rotational head acceleration *Ann. Biomed. Eng.* **41** 873–82
- [57] Patton D A, McIntosh A S and Kleiven S 2015 The biomechanical determinants of concussion: finite element simulations to investigate brain tissue deformations during sporting impacts to the unprotected head *J. Appl. Biomech.* **31** 264–8
- [58] Carroll L J, Cassidy J D, Holm L, Kraus J and Coronado V G 2004 Methodological issues and research recommendations for mild traumatic brain injury: the WHO collaborating centre task force on mild traumatic brain injury *J. Rehabil. Med. Suppl.* **43** 113–25
- [59] Ferry B and Alexei D 2022 *Concussion* (StatPearls Publishing) pp 1–8
- [60] Jordan B D 2013 The clinical spectrum of sport-related traumatic brain injury *Nat. Rev. Neurol.* **9** 222–30
- [61] Marshall C M 2012 Sports-related concussion: a narrative review of the literature *J. Can. Chiropr. Assoc.* **56** 299–310
- [62] Choe M C 2016 The pathophysiology of concussion *Curr. Pain Headache Rep.* **20** 1–10

- [63] Tator C H, Blanchet V and Ma J 2022 Persisting concussion symptoms from bodychecking: unrecognized toll in boys' Ice Hockey *Can. J. Neurol. Sci.* **50** 1–9
- [64] Signoretti S, Lazzarino G, Tavazzi B and Vagnozzi R 2011 The pathophysiology of concussion *Phys. Med. Rehabil.* **3** 359–68
- [65] Mainwaring L, Ferdinand Pennock K M, Mylabathula S and Alavie B Z 2018 Subconcussive head impacts in sport: a systematic review of the evidence *Int. J. Psychophysiol.* **132** 39–54
- [66] Manley G *et al* 2017 A systematic review of potential long-term effects of sport-related concussion *Br. J. Sports Med.* **51** 969–77
- [67] Gard A, Lehto N, Engström Å, Shahim P, Zetterberg H, Blennow K, Marklund N and Tegner Y 2020 Quality of life of ice hockey players after retirement due to concussions *Concussion* **5** CNC78
- [68] Gouttebauge V and Kerkhoffs G M M J 2021 Sports career-related concussion and mental health symptoms in former elite athletes *Neurochirurgie* **67** 280–2
- [69] Chrisman S P D and Richardson L P 2014 Prevalence of diagnosed depression in adolescents with history of concussion *J. Adolesc. Health* **54** 582–6
- [70] Mez J *et al* 2017 Clinicopathological evaluation of chronic traumatic encephalopathy in players of American football *JAMA* **318** 360–70
- [71] Fralick M, Thiruchelvam D, Tien H and Redelmeier D 2016 Risk of suicide after a concussion *Can. Med. Assoc. J.* **188** 497–504
- [72] Van Pelt K L, Puetz T, Swallow J, Lapointe A P and Broglio S P 2021 Data-driven risk classification of concussion rates: a systematic review and meta-analysis *Sports Med.* **51** 1227–44
- [73] O'Connor K L, Rowson S, Duma S M and Broglio S P 2017 Head-impact-measurement devices: a systematic review *J. Athl. Train* **52** 206–27
- [74] Kawata K, Tierney R, Phillips J and Jeka J J 2016 Effect of repetitive sub-concussive head impacts on Ocular near point of convergence *Int. J. Sports Med.* **37** 405–10
- [75] Dunn M, Davies D and Hart J 2020 Effect of football size and mass in youth football head impacts *Proc. 13th Conf. Int. Sport. Eng. Assoc* vol 49
- [76] Parr J V V, Uiga L, Marshall B and Wood G 2023 Soccer heading immediately alters brain function and brain-muscle communication *Front. Hum. Neurosci.* **17** 1–10
- [77] Oeur R A, Zanetti K and Hoshizaki T B 2014 Angular acceleration responses of American football, lacrosse and ice hockey helmets subject to low-energy impacts *IRCOBI Conf. 2014 2014 IRCOBI Conf. Proc.—Int. Research Council on the Biomechanics of Injury* pp 81–92 (available at: [http://www.ircobi.org/wordpress/downloads/irc14/pdf\\_files/18.pdf](http://www.ircobi.org/wordpress/downloads/irc14/pdf_files/18.pdf))
- [78] Dickson T J, Trathen S, Waddington G, Terwiel F A and Baltis D 2016 A human factors approach to snowsport safety: novel research on pediatric participants' behaviors and head injury risk *Appl. Ergon.* **53** 79–86
- [79] Dickson T J, Trathen S, Terwiel F A, Waddington G and Adams R 2017 Head injury trends and helmet use in skiers and snowboarders in Western Canada, 2008–2009–2012–2013: an ecological study *Scand. J. Med. Sci. Sports* **27** 236–44
- [80] Dickson T J, Waddington G and Terwiel F A 2018 Snowsport experience, expertise, lower limb injury and somatosensory ability *J. Sci. Med. Sports* **24** 6–10
- [81] Dickson T J, Forsdyke S and James S 2021 Terrain park participants' perceptions of contributing factors in injury events and risk management suggestions *J. Outdoor Recreat. Tour.* **35** 100416
- [82] Dickson T J and Terwiel F A 2020 Head injury and helmet usage trends for alpine skiers and snowboard in western Canada during the decade 2008–9–2017–18 *J. Sci. Med. Sport* **1004–9**
- [83] Shaw A J 2019 A review of the incidence of head injuries in football, baseball, ice hockey, and cycling *Am. J. Sports Sci.* **7** 1
- [84] Leng B, Ruan D and Tse K M 2022 Recent bicycle helmet designs and directions for future research: a comprehensive review from material and structural mechanics aspects *Int. J. Impact Eng.* **168** 104317
- [85] Bland M L, McNally C and Rowson S 2018 Differences in impact performance of bicycle helmets during oblique impacts *J. Biomech. Eng.* **140** 20–23
- [86] Scher I S, Stepan L L and Hoover R W 2020 Head and neck injury potential during water sports falls: examining the effects of helmets *Sports Eng.* **23** 1–10
- [87] Begonia M, Rowson B, Scicli B and Goff J E 2023 Laboratory evaluation of climbing helmets: assessment of linear acceleration *Smart Mater. Struct.* **32** 034003
- [88] Anderson G R, Melugin H P and Stuart M J 2019 Epidemiology of injuries in ice hockey *Sports Health* **11** 514–9
- [89] Hardy W N *et al* 2007 A study of the response of the human cadaver head to impact *Stapp Car Crash J.* **51** 17–80
- [90] Mott M and Koroshetz W 2016 Concussion research at the national institutes of health: an update from the national institute of neurological disorders and stroke *Concussion* **1** 8–11
- [91] Hicks K Department of defense warfighter brain health initiative (available at: <https://media.defense.gov/2022/Aug/24/2003063181/-1/-1/0/DOD-WARFIGHTER-BRAIN-HEALTH-INITIATIVE-STRATEGY-AND-ACTION-PLAN.PDF>) (Accessed 5 May 2023)
- [92] The Concussion Foundation 2022 (available at: <https://theconcussionfoundation.org/>) (Accessed 19 April 2023)
- [93] Brain Research UK 2023 (available at: [www.brainresearchuk.org.uk/research/apply](http://www.brainresearchuk.org.uk/research/apply)) (Accessed 20 April 2023)
- [94] O'Reilly M, Mahon S, Reid D, Hume P, Hardaker N and Theadom A 2020 Knowledge, attitudes, and behavior toward concussion in adult cyclists *Brain Inj.* **34** 1175–82
- [95] Whyte T, Stuart C A, Mallory A, Ghajari M, Plant D J, Siegmund G P and Crompton P A 2019 A review of impact testing methods for headgear in sports: considerations for improved prevention of head injury through research and standards *J. Biomech. Eng.* **141** 070803
- [96] Hutchison M G, Comper P, Meeuwisse W H and Echemendia R J 2015 A systematic video analysis of National Hockey League (NHL) concussions, part I: who, when, where and what? *Br. J. Sports Med.* **49** 547–51
- [97] Hutchison M G, Comper P, Meeuwisse W H and Echemendia R J 2015 A systematic video analysis of National Hockey League (NHL) concussions, part II: how concussions occur in the NHL *Br. J. Sports Med.* **49** 552–5
- [98] Robidoux M A, Kendall M, Laflamme Y, Post A, Karton C and Hoshizaki T B 2020 Comparing concussion rates as reported by hockey Canada with head contact events as observed across minor ice-hockey age categories *J. Concussion* **4** 205970022091128
- [99] Van Pelt K L *et al* 2021 Detailed description of division I ice hockey concussions: findings from the NCAA and department of defense CARE consortium *J Sport Health Sci.* **10** 162–71
- [100] Hutchinson S, Ellison P, Levy A and Marchant D 2019 Knowledge and attitudes towards concussion in UK-based male ice hockey players: a need for attitude change? *Int. J. Sport. Sci. Coach.* **14** 153–61

- [101] Williamson I J S and Goodman D 2006 Converging evidence for the under-reporting of concussions in youth ice hockey *Br. J. Sports Med.* **40** 128–32
- [102] Post A and Hoshizaki T B 2012 Mechanisms of brain impact injuries and their prediction: a review *Trauma* **14** 327–49
- [103] King A I, Yang K H, Zhang L and Hardy W 2003 Is head injury caused by linear or angular acceleration? *Proc. Int. Research Conf. Biomechanics Impacts* pp 1–12
- [104] Bayly P V, Cohen T S, Leister E P, Ajo D, Leuthardt E C and Genin G M 2005 Deformation of the human brain induced by mild acceleration *J. Neurotrauma* **22** 845–56
- [105] Zhang L, Yang K H and King A I 2001 Biomechanics of neurotrauma *Neurol. Res.* **23** 144–56
- [106] Hardy W N, Khalil T B and King A I 1994 Literature review of head injury biomechanics *Int. J. Impact Eng.* **15** 561–86
- [107] Thomas L M, Roberts V L and Gurdjian E S 1967 Impact-induced pressure gradients along three orthogonal axes in the human skull *J. Neurosurg.* **26** 316–21
- [108] Kleiven S 2003 Influence of impact direction on the human head in prediction of subdural hematoma *J. Neurotrauma* **20** 365–79
- [109] Pellman E J, Viano D C, Withnall C, Shewchenko N, Bir C A and Halstead P D 2006 Concussion in professional football: helmet testing to assess impact performance—part 11 *Neurosurgery* **58** 78–95
- [110] Zhang J, Yoganandan N, Pintar F A and Gennarelli T A 2006 Role of translational and rotational accelerations on brain strain in lateral head impact *Biomed. Sci. Instrum.* **42** 501–6
- [111] Thomas L M, Roberts V L and Gurdjian E S 1966 Experimental intracranial pressure gradients in the human skull *J. Neurol. Neurosurg. Psychiatry* **29** 404–11
- [112] Post A, Hoshizaki T B, Kartton C, Clark J M, Dawson L, Cournoyer J, Taylor K, Oeur R A, Gilchrist M D and Cusimano M D 2019 The biomechanics of concussion for ice hockey head impact events *Comput. Methods Biomech. Biomed. Eng.* **22** 631–43
- [113] Hardy W N, Foster C D, Mason M J, Yang K H, King A and Tashman S 2001 Investigation of head injury mechanisms using neutral density technology and high-speed biplanar x-ray *Stapp Car Crash J.* **45** 337–68
- [114] Gilchrist M D and O'Donoghue D 2000 Simulation of the development of frontal head impact injury *Comput. Mech.* **26** 229–35
- [115] Zhang L, Yang K H and King A I 2004 A proposed injury threshold for mild traumatic brain injury *J. Biomech. Eng.* **126** 226–36
- [116] Bandak F A and Eppinger R H 1994 A three-dimensional finite element analysis of the human brain under combined rotational and translational accelerations *SAE Tech. Pap* vol 103 pp 1708–26
- [117] Funk J R, Duma S M, Manoogian S J and Rowson S 2007 Biomechanical risk estimates for mild traumatic brain injury *51st Annual Proc. Association Advances in Automotive Medicine* vol 51 pp 343–61
- [118] Denny-Brown D and Russell R 1941 Experimental cerebral concussion *Brain* **64** 93–164
- [119] Gurdjian E S, Roberts V L and Thomas M 1966 Tolerance curves of acceleration and intracranial pressure and protective index in experimental head injury *J. Trauma* **6** 600–4
- [120] Ono K, Kikuchi A, Nakamura M, Kobayashi H and Nakamura N 1980 Human head tolerance to sagittal impact reliable estimation deduced from experimental head injury using subhuman primates and human cadaver skulls *SAE Tech. Pap.* pp 101–60
- [121] Ommaya A K, Hirsch A E, Flamm E S and Mahone R H 1966 Cerebral concussion in the monkey: an experimental model *Science* **153** 211–3
- [122] Lissner H R, Lebow M and Evans F G 1960 Experimental studies on the relation between acceleration and intracranial pressure changes in man *Surg. Gynecol. Obstet.* **111** 329–38
- [123] Gurdjian E S, Lissner H R, Latimer F R, Haddad B F and Webster J E 1953 Quantitative determination of acceleration and intracranial pressure in experimental head injury: preliminary report *Neurology* **3** 417–23
- [124] Gadd C W 1966 Use of a weighted-impulse criterion for estimating injury hazard *SAE Technical Paper No. 660793* pp 95–100
- [125] Hutchinson J, Kaiser M J and Lankarani H M 1998 The head injury criterion (HIC) functional *Appl. Math. Comput.* **96** 1–16
- [126] NOCSAE DOC (ND) 030-11m16 2016 *Standard Performance Specification for Newly Manufactured Ice Hockey Helmets* (National Operating Committee on Standards for Athletic Equipment)
- [127] NOCSAE DOC (ND) 002-17m19a 2019 *Standard Performance Specification for Newly Manufactured Football Helmets* (National Operating Committee on Standards for Athletic Equipment)
- [128] NOCSAE DOC (ND) 022-21 2021 *Standard Performance Specification for Newly Manufactured Baseball/Softball Batter's Helmets* (National Operating Committee on Standards for Athletic Equipment)
- [129] NOCSAE DOC (ND) 024-21 *Standard Performance Specification for Newly Manufactured Baseball/Softball Catcher's Helmets with Faceguard* (National Operating Committee on Standards for Athletic Equipment)
- [130] NOCSAE DOC (ND) 029-21 2021 *Standard Performance Specification for Newly Manufactured Baseball/Softball Fielder's Headgear* (National Operating Committee on Standards for Athletic Equipment)
- [131] NOCSAE DOC (ND) 050-11m19 2019 *Standard Performance Specification for Newly Manufactured Polo Helmets* (National Operating Committee on Standards for Athletic Equipment)
- [132] NOCSAE DOC (ND) 041-15m18 2018 *Standard Performance Specification for Newly Manufactures Lacrosse Helmets With Faceguard* (National Operating Committee on Standards for Athletic Equipment)
- [133] Versace J 1971 A review of the severity index *SAE Technical Paper—15th Stapp Car Crash Conf.* (<https://doi.org/10.4271/710881>)
- [134] Jorgensen J K *et al* 2017 Interpreting oblique impact data from an accelerometer-instrumented ice hockey helmet *Proc. Inst. Mech. Eng.* P **231** 307–16
- [135] Richards D, Ivarsson B J, Scher I, Hoover R, Rodowicz K and Cripton P 2016 Ice hockey shoulder pad design and the effect on head response during shoulder-to-head impacts *Sports Biomech.* **15** 385–96
- [136] Pellman E J, Viano D C, Tucker A M, Casson I R and Waeckerle J F 2003 Concussion in professional football: reconstruction of game impacts and injuries *Neurosurgery* **53** 799–814
- [137] Rousseau P and Hoshizaki T B 2009 The influence of deflection and neck compliance on the impact dynamics of a Hybrid III headform *Proc. Inst. Mech. Eng.* P **223** 89–97
- [138] Carlson S, Zerpa C, Przysucha E, Liu M, Sanzo P and Bay T 2019 Energy measures across hockey helmet impact locations *ISBS Proc. Archive* vol 371 pp 443–6
- [139] Johnson G I 2003 A comparison of results on helmet impact testing *J. Test. Eval.* **31** 79–90
- [140] Pennock B, Kivi D and Zerpa C 2021 Effect of neck strength on simulated head impacts during falls in female ice hockey players *Int. J. Exerc. Sci.* **14** 446–61
- [141] Zerpa C, Carlson S, Przysucha E, Liu M and Sanzo P 2021 Evaluating the performance of a hockey helmet in



- mitigating concussion risk using measures of acceleration and energy during simulated free fall *Int. J. Extreme Autom. Connect. Healthcare* **3** 33–50
- [142] NOCSAE DOC (ND) 022-20 2020 *Standard Performance Specification for Newly Manufactured Baseball/Sofball Batter's Helmets* (National Operating Committee on Standards for Athletic Equipment)
- [143] Clark J M, Post A, Hoshizaki T B and Gilchrist M D 2016 Protective capacity of ice hockey helmets against different impact events *Ann. Biomed. Eng.* **44** 3693–704
- [144] de Grau S, Post A, Hoshizaki T B and Gilchrist M D 2019 Effects of surface compliance on the dynamic response and strains sustained by a player's helmeted head during ice hockey impacts *Proc. Inst. Mech. Eng. P* **234** 98–106
- [145] Kendall M, Post A, Rousseau P, Oeur A, Gilchrist M D and Hoshizaki B, 2012 A comparison of dynamic impact response and brain deformation metrics within the cerebrum of head impact reconstructions representing three mechanisms of head injury in ice hockey *IRCOBI Conf. 2012* pp 12–4
- [146] Michio Clark J, Post A, Blaine Hoshizaki T and Gilchrist M D 2018 Distribution of brain strain in the cerebrum for laboratory impacts to ice hockey goaltender masks *J. Biomech. Eng.* **140** 1–10
- [147] Ommaya A K, Hirsch A E and Martinez J L 1966 The role of whiplash in cerebral concussion *SAE Technical Paper*
- [148] Gennarelli T A and Thibault L E 1982 Biomechanics of acute subdural hematoma *J. Trauma* **22** 680–6
- [149] Unterharnscheidt F J 1971 Translational versus rotational acceleration-animal experiments with measured input *Scand. J. Rehabil. Med.* (<https://doi.org/10.4271/710880>)
- [150] Pincemaille Y, Trosseille X, MacK P, Tarière C, Breton F and Renault B 1989 Some new data related to human tolerance obtained from volunteer boxers *SAE Technical Paper* vol 98 pp 1752–65
- [151] Rousseau P, Post A and Hoshizaki T B 2009 The effects of impact management materials in ice hockey helmets on head injury criteria *Proc. Inst. Mech. Eng. P* **223** 159–65
- [152] Rousseau P, Hoshizaki T B and Gilchrist M D 2014 For ASTM F-08: protective capacity of ice hockey player helmets against puck impacts *Mech. Concussion Sport* pp 196–207
- [153] Giacomazzi A, Smith T, Kersey R, Greenwald R, Ashare A and Dean S W 2009 Analysis of the impact performance of ICE hockey helmets using two different test methodologies *J. ASTM Int.* **6** 1–7
- [154] Kendall M, Post A, Rousseau P and Hoshizaki T B 2014 (<https://doi.org/10.1520/STP155220120150>) The effect of shoulder pad design on reducing peak resultant linear and rotational acceleration in shoulder-to-head impacts *Mech. Concussion Sport* pp 142–52
- [155] Post A, Karton C, Hoshizaki T B and Gilchrist M D 2014 Analysis of the protective capacity of ice hockey helmets in a concussion injury reconstruction *2014 IRCOBI Conf. Proc.—International Research Council on Biomechanics of Injury* pp 72–80
- [156] Clark J M, Post A, Hoshizaki T B and Gilchrist M D 2015 Determining the relationship between linear and rotational acceleration and MPS for different magnitudes of classified brain injury risk in ice hockey *2015 IRCOBI Conf. Proc.* (<https://doi.org/10.1002/osp4.6>)
- [157] Kimpara H and Iwamoto M 2012 Mild traumatic brain injury predictors based on angular accelerations during impacts *Ann. Biomed. Eng.* **40** 114–26
- [158] Takhounts E G, Hasija V, Ridella S A, Rowson S and Duma S M 2011 Kinematic rotational brain injury criterion (BRIC) *Proc. 22nd Enhanced Safety of Vehicles Conference Paper No 11–0263* (available at: <http://www-esv.nhtsa.dot.gov/Proceedings/22/isv7/main.htm>)
- [159] Takhounts E G, Craig M J, Moorhouse K, McFadden J and Hasija V 2013 Development of brain injury criteria (BrIC) *Stapp Car Crash J.* **57** pp 243–66
- [160] Newman J A 1986 A generalized acceleration model for brain injury threshold (GAMBIT) *Proc. Int. IRCOBI Conf.*
- [161] Newman J A, Barr C, Beusenberg M, Fournier E, Shewchenko N, Welbourne E and Withnall C 2000 A new biomechanical assessment of mild traumatic brain injury. Part 2: results and conclusions *IR CO BI Conf. IR CO BI Conf.* vol 108 pp 223–33
- [162] Newman J A and Shewchenko N 2000 A proposed new biomechanical head injury assessment function—the maximum power index *44th Stapp Car Crash Conference* (SAE Technical Papers)
- [163] Fréchède B and McIntosh A S 2009 Numerical reconstruction of real-life concussive football impacts *Med. Sci. Sports Exerc.* **41** 390–8
- [164] Beckwith J, Chu J, Crisco J, Mcallister T W, Duma S, Brolinson P and Greenwald R M 2009 Severity of head impacts resulting in mild traumatic brain injury *Proceedings of the American Society of Biomechanics Annual Meeting* pp 4–5 (available at: [www.asbweb.org/conferences/2009/pdf/1144.pdf](http://www.asbweb.org/conferences/2009/pdf/1144.pdf))
- [165] Gilchrist M D, O'Donoghue D and Horgan T J 2001 A two-dimensional analysis of the biomechanics of frontal and occipital head impact injuries *Int. J. Crashworthiness* **6** 253–62
- [166] Gilchrist M D 2003 Modelling and accident reconstruction of head impact injuries *Key Eng. Mater.* **245–246** 417–30
- [167] Knowles B M and Dennison C R 2017 Predicting cumulative and maximum brain strain measures from hybrid head kinematics: a combined laboratory study and post-hoc regression analysis *Ann. Biomed. Eng.* **45** 2146–58
- [168] Ouckama R and Pearsall D J 2014 Projectile impact testing of ice hockey helmets: headform kinematics and dynamic measurement of localized pressure distribution *IRCOBI Conf. 2014 2014 IRCOBI Conf. Proc.—Int. Research Council On Biomechanics of Injury* pp 62–71 (available at: [http://www.ircobi.org/wordpress/downloads/irc14/pdf\\_files/16.pdf](http://www.ircobi.org/wordpress/downloads/irc14/pdf_files/16.pdf))
- [169] Willinger R, Deck C, Halldin P and Otte D 2014 Towards advanced bicycle helmet test methods *International Cycling Safety Conf. 2014 Int. Cycling Safety Conf.* pp 1–11
- [170] Post A, Oeur A, Hoshizaki B and Gilchrist M D 2013 Examination of the relationship between peak linear and angular accelerations to brain deformation metrics in hockey helmet impacts *Comput. Methods Biomech. Biomed. Eng.* **16** 511–9
- [171] Post A, Dawson L, Hoshizaki T B, Gilchrist M D and Cusimano M D 2019 The influence of impact source on variables associated with strain for impacts in ice hockey *Comput. Methods Biomech. Biomed. Eng.* **22** 713–26
- [172] Gabler L F, Crandall J R and Panzer M B 2019 Development of a second-order system for rapid estimation of maximum brain strain *Ann. Biomed. Eng.* **47** 1971–81
- [173] Ji S, Ghajari M, Mao H, Kraft R H, Hajiaghameh M, Panzer M B, Willinger R, Gilchrist M D, Kleiven S and Stitzel J D 2022 Use of brain biomechanical models for monitoring impact exposure in contact sports *Ann. Biomed. Eng.* **50** 1389–408
- [174] Patton D A, Huber C M, Jain D, Myers R K, McDonald C C, Margulies S S, Master C L and Arbogast K B 2020 Head impact sensor studies in sports: a systematic review of exposure confirmation methods *Ann. Biomed. Eng.* **48** 2497–507

- [175] Wu L C, Nangia V, Bui K, Hammor B, Kurt M, Hernandez F, Kuo C and Camarillo D B 2016 *In vivo* evaluation of wearable head impact sensors *Ann. Biomed. Eng.* **44** 1234–45
- [176] Miyashita T, Diakogeorgiou E, Marrie K and Danaher R 2016 Frequency and location of head impacts in division I men's lacrosse players *Athl. Train. Sports Health Care* **8** 202–8
- [177] Press J N and Rowson S 2017 Quantifying head impact exposure in collegiate women's soccer *Clin. J. Sport Med.* **27** 104–10
- [178] McIntosh A S, Willmott C, Patton D A, Mitra B, Brennan J H, Dimech-Betancourt B, Howard T S and Rosenfeld J V 2019 An assessment of the utility and functionality of wearable head impact sensors in Australian Football *J. Sci. Med. Sport* **22** 784–9
- [179] Allison M A, Kang Y S, Bolte I V J H, Maltese M R and Arbogast K B 2014 Validation of a Helmet-based system to measure head impact biomechanics in ice hockey *Med. Sci. Sports Exerc.* **46** 115–23
- [180] Simbex A helmet that detects hard hits (available at: <https://simbex.com/helmet-detects-hard-hits/>) (Accessed 6 April 2020)
- [181] Cummiskey B, Schiffmiller D, Talavage T M, Leverenz L, Meyer J J, Adams D and Nauman E A 2017 Reliability and accuracy of helmet-mounted and head-mounted devices used to measure head accelerations *Proc. Inst. Mech. Eng. P* **231** 144–53
- [182] Caswell S V, Kelshaw P, Lincoln A E, Hepburn L, Dunn R and Cortes N 2019 Game-related impacts in high school boys' Lacrosse *Orthop. J. Sport. Med.* **7** 1–8
- [183] Cortes N, Lincoln A E, Myer G D, Hepburn L, Higgins M, Putukian M and Caswell S V 2017 Video analysis verification of head impact events measured by wearable sensors *Am. J. Sports Med.* **45** 2379–87
- [184] Duma S M, Manoogian S J, Bussone W R, Brolinson P G, Goforth M W, Donnenwerth J J, Greenwald R M, Chu J J and Crisco J J 2005 Analysis of real-time head accelerations in collegiate football players *Clin. J. Sport Med.* **15** 3–8
- [185] Bartsch A J, Hedin D S, Gibson P L, Miele V J, Benzel E C, Alberts J L, Samorezov S, Shah A, Stemper B S and McCrea M M 2019 Laboratory and on-field data collected by a head impact monitoring mouthguard *Proc. Annu. Int. Conf. IEEE Eng. Med. Biol. Soc. EMBS (IMM)* pp 2068–72
- [186] Greybe D G, Jones C M, Brown M R and Williams E 2020 Comparison of head impact measurements via an instrumented mouthguard and an anthropometric testing device *Sports Eng.* **23** 1–11
- [187] Wu L C, Kuo C, Loza J, Kurt M, Laksari K, Yanez L Z and Camarillo D B 2018 Detection of American Football head impacts using biomechanical features and support vector machine classification *Sci. Rep.* **8** 1–14
- [188] Kuo C, Wu L, Loza J, Senif D, Anderson S C, Camarillo D B and Janigro D 2018 Comparison of video-based and sensor-based head impact exposure *PLoS One* **13** 1–19
- [189] Michio Clark J, Connor T A, Post A, Blaine Hoshizaki T, Ní Annaidh A and Gilchrist M D 2020 Could a compliant foam anvil characterize the biofidelic impact response of equestrian helmets? *J. Biomech. Eng.* **142** 1–9
- [190] Meehan A, Post A, Hoshizaki T B and Gilchrist M D 2022 Investigation of an Ice Hockey helmet test protocol representing three concussion event types *J. Test. Eval.* **50** 20200436
- [191] Wojnarowicz M W, Fisher A M, Minaeva O and Goldstein L E 2017 Considerations for experimental animal models of concussion, traumatic brain injury, and chronic traumatic encephalopathy-these matters matter *Front. Neurol.* **8** 1–14
- [192] Petrone N, Carraro G, Dal Castello S, Broggio L, Koptuyg A and Backstrom M 2018 A novel instrumented human head surrogate for the impact evaluation of helmets *Proc. 12th Conf. Sports Engineering ISEA* pp 1–7
- [193] Stone B, Mitchell S, Miyazaki Y, Peirce N and Harland A 2021 A destructible headform for the assessment of sports impacts *Proc. Inst. Mech. Eng. P* **237** 7–18
- [194] ASTM Standard F1447 2018 Standard specification for helmets used in recreational bicycling or roller skating (ASTM International) (<https://doi.org/10.1520/F1447-18>)
- [195] ASTM Standard F1952-22 2015 Standard specification for helmets used for downhill mountain bicycle racing (ASTM International) (<https://doi.org/10.1520/F1952-22>)
- [196] 16 CFR Part 1203 1998 Safety standard for bicycle helmets; final rule (Consumer Product Safety Commission)
- [197] EN 1078:2012+A1:2012 2014 Helmets for pedal cyclists and for users of skateboards and roller skates *DIN Deutsches Institut für Normung e. V.*
- [198] Snell B-95A 1998 1995 standard for protective headgear for use in bicycling (Snell Memorial Foundation Inc)
- [199] ASTM Standard F1446-20 2020 Standard test methods for equipment and procedures used in evaluating the performance characteristics of protective headgear (ASTM International) (<https://doi.org/10.1520/F1446-20>)
- [200] BS 7928:2013+A1 2019 Specification for head protectors for cricketers (BSI Standards Publication)
- [201] Snell E2016 2016 Standard for protective headgear for use in horseback riding (Snell Memorial Foundation Inc)
- [202] ASTM Standard F1163-15 2015 Standard specification for protective headgear used in horse sports and horseback riding (ASTM International) (<https://doi.org/10.1520/F1163-15>)
- [203] BS EN 1384:2017 2017 Helmets for equestrian activities
- [204] Snell foundation S.P.F.H. Snell SA 2015 2020 Standard for protective headgear for use in competitive automotive sports (Snell Memorial Foundation Inc.)
- [205] Snell SA20 2019 2020 special applications standard for protective headgear (Snell Memorial Foundation Inc)
- [206] Snell EA2016 2016 Standard for protective headgear for use in elite automotive sports (Snell Memorial Foundation Inc)
- [207] Federation Internationale de l'Automobile 2018 Norme Fia 8860-2018 Et 8860-2018-Abp Fia standard 8860-2018 and 8860-2018-Abp Casque Haute performance
- [208] Used P. ASTM Standard F1492-22 2001 Standard specification for helmets used in skateboarding and trick roller skating (ASTM International) (<https://doi.org/10.1520/F1492-22>) pp 2–4
- [209] ASTM Standard F2040, 2018 2003 Standard specification for helmets used for recreational snow sports (ASTM International) (<https://doi.org/10.1520/F2040-18>)
- [210] Snell RS-98 1998 1998 standard for protective headgear for use in recreational skiing and snowboarding (Snell Memorial Foundation Inc)
- [211] Snell S-98 1998 1998 standard for protective headgear for skiing and other winter activities (Snell Memorial Foundation Inc)
- [212] Deutsche Norm. EN 1077:2007 2007 Helmets for alpine skiers and snowboarders (Deutsches Institut für Normung)
- [213] ASTM Standard F1045-22 2022 Standard performance specification for ice hockey helmets (ASTM International) (<https://doi.org/10.1520/F1045-22>)
- [214] EN ISO 10256-2:2018 2018 Protective equipment for use in ice hockey—part 2: head protection for skaters (BSI Standards Publication)
- [215] Canadian Standards Association 2009 Casques de hockey sur glace (CAN/CSA Standard No. Z262.1-09)

- [216] Snell B-90A B-90C 1998 Standard for protective headgear for use in bicycling (Snell Memorial Foundation Inc)
- [217] Hoshizaki T B, Post A, Oeur R A and Brien S E 2014 Current and future concepts in helmet and sports injury prevention *Neurosurgery* **75** s136–48
- [218] Post A, Dawson L, Hoshizaki T B, Gilchrist M D and Cusimano M D 2020 Development of a test method for adult ice hockey helmet evaluation *Comput. Methods Biomech. Biomed. Eng.* **23** 690–702
- [219] NOCSAE DOC (ND) 001–17m19, 2019, “Standard test method and equipment used in evaluating the performance characteristics of headgear/equipment (National Operating Committee on Standards for Athletic Equipment)
- [220] EN 966:2012+A1:2012 2013 Helmets for airborne sports (DIN Deutsches Institut für Normung e. V.)
- [221] NOCSAE DOC (ND) 081–18am19a 2019 Standard Pneumatic Ram Test method and equipment used in evaluating the performance characteristics of protective headgear and face guards (National Operating Committee on Standards for Athletic Equipment)
- [222] Oeur R A and Hoshizaki T B 2016 The effect of impact compliance, velocity, and location in predicting brain trauma for falls in sport 2016 *IRCOBI Conf. Proc.—Int. Res. Counc. Biomech. Inj.* pp 228–38
- [223] Walsh E S, Post A, Rousseau P, Kendall M, Karton C, Oeur A, Foreman S and Hoshizaki T B 2012 Dynamic impact response characteristics of a helmeted Hybrid III headform using a centric and non-centric impact protocol *Proc. Inst. Mech. Eng.* P **226** 220–5
- [224] Rousseau P, Post A, Hoshizaki T B, Greenwald R, Ashare A and Dean S W 2009 A comparison of peak linear and angular headform accelerations using ice hockey helmets *J. ASTM Int.* **6** 101877
- [225] Levy Y, Gallone M B, Bian K, McDougall K, Ouckama R and Mao H 2020 Using a strain-based computational approach for ice hockey helmet performance evaluation *IRCOBI conf. 2020* pp 569–80
- [226] Cummiskey B, Sankaran G N, McIver K G, Shyu D, Markel J, Talavage T M, Leverenz L, Meyer J J, Adams D, Nauman E A 2019 Quantitative evaluation of impact attenuation by football helmets using a modal impulse hammer *Proc. Inst. Mech. Eng.* P **233** 301–11
- [227] McIver K G, Lee P, Bucherl S, Talavage T M, Myer G D and Nauman E A 2023 Design considerations for the attenuation of translational and rotational accelerations in American Football helmets *J. Biomech. Eng.* **145** 1–9
- [228] McIver K G, Sankaran G N, Lee P, Bucherl S, Leiva N, Talavage T M, Leverenz L and Nauman E A 2019 Impact attenuation of male and female lacrosse helmets using a modal impulse hammer *J. Biomech.* **95** 109313
- [229] Haid D, Duncan O, Hart J and Foster L 2023 Free fall drop test with interchangeable surfaces to recreate concussive ice hockey head impacts *Sports Eng.* **26** 1–11
- [230] Halldin P, Gilchrist A and Mills N J 2001 A new oblique impact test for motorcycle helmets *Int. J. Crashworthiness* **6** 53–64
- [231] Jeffries L, Zerpa C, Przysucha E, Sanzo P and Carlson S 2017 The use of a pneumatic horizontal impact system for helmet testing *J. Saf. Eng.* **6** 8–13
- [232] Schmitt K U, Muser M H, Thueler H and Bruegger O 2018 Crash-test dummy and pendulum impact tests of ice hockey boards: greater displacement does not reduce impact *Br. J. Sports Med.* **52** 41–46
- [233] Tyson A M and Rowson S 2018 Adult Football STAR methodology (Virginia Tech Helmet Lab) pp 3–6
- [234] McIntosh A S and Janda D 2003 Evaluation of cricket helmet performance and comparison with baseball and ice hockey helmets *Br. J. Sports Med.* **37** 325–30
- [235] Cobb B R, Macalister A, Young T J, Kemper A R, Rowson S and Duma S M 2015 Quantitative comparison of hybrid III and National Operating Committee on standards for athletic equipment headform shape characteristics and implications on football helmet fit *Proc. Inst. Mech. Eng.* P **229** 39–46
- [236] Post A, Oeur A, Hoshizaki B and Gilchrist M D 2013 An examination of American football helmets using brain deformation metrics associated with concussion *Mater. Des.* **45** 653–62
- [237] MacAlister A 2013 Surrogate head forms for the evaluation of head injury risk *Brain Inj. Biomech. Symp.*
- [238] Chen W, Post A, Karton C, Gilchrist M D, Robidoux M and Hoshizaki T B 2020 A comparison of frequency and magnitude of head impacts between Pee Wee And Bantam youth ice hockey *Sports Biomech.* **22** 1–24
- [239] Post A, Karton C, Robidoux M, Gilchrist M D and Hoshizaki T B 2019 An examination of the brain trauma in Novice and Midget ice hockey: implications for helmet innovation *J. Biomed. Opt.* **24** 1–4
- [240] Post A, De Grau S, Ignacy T, Meehan A, Zemek R, Hoshizaki B and Gilchrist M D 2016 Comparison of helmeted head impact in youth and adult ice hockey *IRCOBI Conf. Proc.—Int. Res. Counc. Biomech. Inj.* pp 194–204
- [241] Kendall M, Walsh E S and Hoshizaki T B 2012 Comparison between Hybrid III and Hodgson-WSU headforms by linear and angular dynamic impact response *Proc. Inst. Mech. Eng.* P **226** 260–5
- [242] ASTM Standard F2220-2015 2015 Standard specification for headforms (ASTM International) (<https://doi.org/10.1520/F2220-15>)
- [243] Humanetics 2020 Hybrid III 50th Male
- [244] British Standards Institution 2006 BS EN 960:2006, Headforms for use in the testing of protective helmets
- [245] Bottlang M, Rouhier A, Tsai S, Gregoire J and Madey S M 2020 Impact performance comparison of advanced bicycle helmets with dedicated rotation-damping systems *Ann. Biomed. Eng.* **48** 68–78
- [246] Allison M A, Kang Y S, Maltese M R, Bolte J H and Arbogast K B 2015 Measurement of hybrid III head impact kinematics using an accelerometer and gyroscope system in Ice Hockey helmets *Ann. Biomed. Eng.* **43** 1896–906
- [247] Walsh E S and Hoshizaki T B 2010 Poster Session III, July 15th 2010—abstracts sensitivity analysis of a hybrid III head- and neckform to impact angle variations *Proc. Eng.* **2** 3487
- [248] Rousseau P and Hoshizaki T B 2015 Defining the effective impact mass of elbow and shoulder strikes in ice hockey *Sports Biomech.* **14** 57–67
- [249] McIntosh A S, Lai A and Schilter E 2013 Bicycle helmets: head impact dynamics in helmeted and unhelmeted oblique impact tests *Traffic Inj. Prev.* **14** 501–8
- [250] Farmer J, Mitchell S, Sherratt P and Miyazaki Y 2022 A human surrogate neck for traumatic brain injury research *Front. Bioeng. Biotechnol.* **10** 1–19
- [251] Clark J M, Connor T A, Post A, Hoshizaki T B and Gilchrist M D 2019 The influence of impact surface on head kinematics and brain tissue response during impacts with equestrian helmets *Sports Biomech.* **20** 1–14
- [252] Clark J M, Taylor K, Post A, Hoshizaki T B and Gilchrist M D 2018 Comparison of Ice Hockey goaltender helmets for concussion type impacts *Ann. Biomed. Eng.* **46** 986–1000
- [253] Clark J M, Hoshizaki T B and Gilchrist M D 2017 Protective capacity of an ice hockey goaltender helmet for three events associated with concussion *Comput. Methods Biomech. Biomed. Eng.* **20** 1299–311

- [254] Post A, Clark J M, Robertson D G E, Hoshizaki T B and Gilchrist M D 2017 The effect of acceleration signal processing for head impact numeric simulations *Sports Eng.* **20** 111–9
- [255] Bhudolia S K, Gohel G, Subramanyam E S B, Leong K F and Gerard P 2021 Enhanced impact energy absorption and failure characteristics of novel fully thermoplastic and hybrid composite bicycle helmet shells *Mater. Des.* **209** 110003
- [256] Di Landro L, Sala G and Olivieri D 2002 Deformation mechanisms and energy absorption of polystyrene foams for protective helmets *Polym. Test.* **21** 217–28
- [257] Pinnoji P K and Mahajan P 2010 Analysis of impact-induced damage and delamination in the composite shell of a helmet *Mater. Des.* **31** 3716–23
- [258] Coelho R M, Alves de Sousa R J, Fernandes F A O and Teixeira-Dias F 2013 New composite liners for energy absorption purposes *Mater. Des.* **43** 384–92
- [259] Andena L, Caimmi F, Leonardi L, Ghisi A, Mariani S and Braghin F 2016 Towards safer helmets: characterisation, modelling and monitoring *Proc. Eng.* **147** 478–83
- [260] McGillivray K, Przysucha E, Sanzo P, Liu M and Zerpa C 2022 Comparison of hockey helmet lining technologies in mitigating concussion risk during simulated horizontal head collisions *Int. J. Extreme Autom. Connect. Healthcare* **4** 1–17
- [261] Gimbel G and Hoshizaki T 2008 Compressive properties of helmet materials subjected to dynamic impact loading of various energies *Eur. J. Sport Sci.* **8** 341–9
- [262] Foster L, Peketi P, Allen T, Senior T, Duncan O and Alderson A 2018 Application of auxetic foam in sports helmets *Appl. Sci.* **8** 1–12
- [263] Wavecел 2023 Botranger (available at: <https://wavecел.com/>) (Accessed 05 October 2023)
- [264] MIPS. Mipsprotection (available at: <https://mipsprotection.com/>) (Accessed 5 April 2023)
- [265] MIPS 2023 Mips Integra (available at: <https://mipsprotection.com/product-range/mips-integra/>) (Accessed 5 April 2023)
- [266] D3O 2020 D3O Protection (available at: <https://www.d3o.com/our-products/defence/>) (Accessed 05 October 2023)
- [267] POC. POC SPIN pad kit (available at: [www.pocsports.com/products/omne-air-spin-pad-kit?variant=36219708604568](http://www.pocsports.com/products/omne-air-spin-pad-kit?variant=36219708604568)) (Accessed 5 May 2023)
- [268] Salomon 2016 Salomon EPS 4D (available at: <https://youtu.be/YcmUFMUyjsE>) (Accessed 13 April 2023)
- [269] Vicis 2023 Vicis matrix technology (available at: <https://vicis.com/pages/zero2-matrix>) (Accessed 14 August 2023)
- [270] Gibson L J and Ashby M F 1997 Energy absorption in cellular materials *Cellular Solids Structure and Properties* 2nd edn (Cambridge University Press) pp 309–44
- [271] Bailly N, Petit Y, Desrosier J M, Laperriere O, Langlois S and Wagnac E 2020- Strain rate dependant behaviour of Vinyl Nitrile helmet foam in compression and combined compression and shear *Appl. Sci.* **10** 8286
- [272] Mustafa H, Pang T Y, Ellena T and Nasir S H 2019 Impact attenuation of user-centred bicycle helmet design with different foam densities *J. Phys.: Conf. Ser.* **1150** 012043
- [273] Wei M, Lin K and Sun L 2022 Shear thickening fluids and their applications *Mater. Des.* **216** 110570
- [274] Li S, Wang J, Zhao S, Cai W, Wang Z and Wang S 2017 Giant rheological effect of shear thickening suspension comprising silica nanoparticles with no aggregation *J. Mater. Sci. Technol.* **33** 261–5
- [275] Cossa K N 2019 Basic concepts on rheology and application of shear-thickening fluids in protective gear *SN Appl. Sci.* **1** 1–6
- [276] LeMaitre J 2001 *Introduction to Elasticity and Viscoelasticity* (Academic) pp 71–74
- [277] Zhao C, Gong X, Wang S, Jiang W and Xuan S 2020 Shear stiffening gels for intelligent anti-impact applications *Cell Rep. Phys. Sci.* **1** 100266
- [278] Coussot P 2012 Introduction to the rheology of complex fluids *Understanding the Rheology of Concrete* (Woodhead Publishing Series in Civil and Structural Engineering) 3–22
- [279] Fowler J N, Pallanta A A, Swanik C B and Wagner N J 2015 The use of shear thickening nanocomposites in impact resistant materials *J. Biomech. Eng.* **137** 054504
- [280] Soutrenon M and Michaud V 2014 Impact properties of shear thickening fluid impregnated foams *Smart Mater. Struct.* **23** 035022
- [281] Nakonieczna P, Wierzbicki Ł, Śladowska B, Leonowicz M, Lisiecki J and Nowakowski D 2017 Composites with impact absorption ability based on shear thickening fluids and auxetic foams *Compos. Theory Pract.* **17** 67–72
- [282] Jachowicz M and Owczarek G 2020 Analysis of selected mechanical parameters for foamed materials with non-Newtonian liquid characteristics in terms of their use in aspects of protective helmets *Int. J. Occup. Saf. Ergon.* **26** 617–23
- [283] Waitukaitis S R and Jaeger H M 2012 Impact-activated solidification of dense suspensions via dynamic jamming fronts *Nature* **487** 205–9
- [284] Halldin P, Aare M, Kleiven S and von Holst H 2003 Improved helmet design and test methods to reduce rotational induced brain injuries *RTO Specialist Meeting, the NATO's Research and Technology Organization*
- [285] Evans K E 1991 Auxetic polymers: a new range of materials *Endeavour* **15** 170–4
- [286] Aare M and Halldin P 2003 A new laboratory rig for evaluating helmets subject to oblique impacts *Traffic Inj. Prev.* **4** 240–8
- [287] Holst V 2004 *United States Patent* US 6,658,671 vol 2 pp 0–5
- [288] Jacques Durocher S-J 2018 *United States Patent* US 9,961,952 B2 vol 2
- [289] Mosleh Y, Vanden Bosche K, Depreitere B, Vander Sloten J, Verpoest I and Ivens J 2018 Effect of polymer foam anisotropy on energy absorption during combined shear-compression loading *J. Cell. Plast.* **54** 597–613
- [290] Huber M T 1923 The theory of crosswise reinforced ferroconcrete slabs and its application to various important constructional problems involving rectangular slabs *Der Bauingenieur* **4** 354–60
- [291] Parisi M F, Allen T, Cologna M, Pugno N and Duncan O 2023 Indentation and impact response of conventional, auxetic, and shear gel thickening infused auxetic closed cell foam *Under Rev. Int. J. Impact Eng.* **32** 074004
- [292] Stone A, Alferness A P, Czerski M, Neubauer J and Frank A 2018 Laterally supported filaments *US Patent* US 2018/0184745 A1
- [293] Bologna V, Gillogly M and Ide T M 2021 Football helmet with components additively manufactured to manage impact forces vol 2 *U.S. Patent* US 11,167,198 B2
- [294] Posner J D, Dardis J T, Leonard P C and Reinhall P G 2020 Protective helmets including non—linearly deforming elements *US Patent* US 10,813,402 B2
- [295] Bottlang M, 2022 Energy—absorbing structure with defined multi—phasic crush properties *US Patent* US 2022/0324194 A1 (<https://doi.org/10.1016/j.heliyon.2022.e09962>)
- [296] Chilson J A, Lloyd J, Rogers J and Storey P 2020 Helmet with shock absorbing inserts *US Patent* US 10,736,373 B2
- [297] Baracco S 2020 *Protective Helmet European Patent* EP003130243B1
- [298] Suddaby L S 2017 Helmet with multiple protective zones *US Patent* US 9,795, 178 B2

- [299] Vanhoutin L A, Long V R, Loucks N and Groff R 2021 Sports helmet with custom—fit liner *US Patent US* 11,026,466 B2
- [300] Fischer K, Fukuda K, Czarski M, Frank A and Santiago C 2019 Modular liner system for protective helmets *US Patent US* 10,342, 281 B2
- [301] Plant D J 2020 Energy absorbing system *US Patent US* 2020/0040958 A1
- [302] Plant D J 2009 Flexible energy absorbing material and methods of manufacture thereof *US Patent US* 7.608,314 B2
- [303] Morgan J T and Morgan G E 2022 Helmet with non—Newtonian fluid liner system *US Patent US* 11,219,263 B2
- [304] Warmouth C, VanHoutin L A and Long V R 2017 Sports helmet with liner system *US Patent US* 2017/0056750 A1
- [305] Kirshon J E 2020 IMPACT—dissipating liners and methods of fabricating impact—dissipating liners *US Patent US* 2020/0205502 A1
- [306] Lakes R Lakes Roderic 1987 Foam structures with a negative Poisson's ratio *Science* **235** 1038–40
- [307] Jiang W, Ren X, Wang S L, Zhang X G, Zhang X Y, Luo C, Xie Y M, Scarpa F, Alderson A and Evans K E 2022 Manufacturing, characteristics and applications of auxetic foams: a state-of-the-art review *Composites B* **235** 109733
- [308] Hu H, Zhang M and Yanping L 2019 *Auxetic Textiles* (Woodhead Publishing Ltd.)
- [309] Novak N, Vesenjok M and Ren Z 2016 Auxetic cellular materials—a review *Stroj. Vestn./J. Mech. Eng.* **62** 485–93
- [310] Miura K 1994 Map fold a la Miura style, its physical characteristics and application to the space science *Research of Pattern Formation* ed R Takaki (KTK Scientific Publishers) pp 77–90
- [311] Miura K 1980 Method of packaging and deployment of large membranes in space *Congress of International Astronautical Federation*
- [312] Sareh P and Guest S D 2015 Design of non-isomorphic symmetric descendants of the Miura-ori *Smart Mater. Struct.* **24** 85002
- [313] Sareh P 2019 The least symmetric crystallographic derivative of the developable double corrugation surface: computational design using underlying conic and cubic curves *Mater. Des.* **183** 108128
- [314] Chen Y, Lu C, Fan W, Feng J and Sareh P 2023 Data-driven design and morphological analysis of conical six-fold origami structures *Thin-Walled Struct.* **185** 110626
- [315] Chen Y, Xu R, Lu C, Liu K, Feng J and Sareh P 2023 Multi-stability of the hexagonal origami hyper based on group theory and symmetry breaking *Int. J. Mech. Sci.* **247** 108196
- [316] Hunt G W and Ario I 2005 Twist buckling and the foldable cylinder: an exercise in origami *Int. J. Non-Linear Mech.* **40** 833–43
- [317] Chen Y, Shi P, Bai Y, Li J, Feng J and Sareh P 2023 Engineered origami crease perforations for optimal mechanical performance and fatigue life *Thin-Walled Struct.* **185** 110572
- [318] Zhang Z, Ma W, Wu H, Wu H, Jiang S and Chai G 2018 A rigid thick Miura-Ori structure driven by bistable carbon fibre-reinforced polymer cylindrical shell *Compos. Sci. Technol.* **167** 411–20
- [319] Chen Y, Lu C, Yan J, Feng J and Sareh P 2022 Intelligent computational design of scalene-faceted flat-foldable tessellations *J. Comput. Des. Eng.* **9** 1765–74
- [320] Miura K 2009 The science of Miura-Ori: a review *Origami 4* 1st edn (A K Peters/CRC Press) p 14
- [321] Miura K 2009 Triangles and quadrangles in space *Symposium of the International Association for Shell and Spatial Structures*
- [322] Nojima T 2002 Modelling of folding patterns in flat membranes and cylinders by origami *JSME Int. J. C* **45** 364–70
- [323] Chen Y, Yan J, Feng J and Sareh P 2021 Particle swarm optimization-based metaheuristic design generation of non-trivial flat-foldable origami tessellations with degree-4 vertices *J. Mech. Des.* **143** 011703
- [324] Chen Y, Ye W, Shi P, He R, Liang J, Feng J and Sareh P 2023 Computational parametric analysis of cellular solids with the Miuraori metamaterial geometry under quasistatic compressive loads *Adv. Eng. Mater.* **25** 2201762
- [325] Sareh P and Chen Y 2020 Intrinsic non-flat-foldability of two-tile DDC surfaces composed of glide-reflected irregular quadrilaterals *Int. J. Mech. Sci.* **185** 105881
- [326] Lu C, Chen Y, Yan J, Feng J and Sareh P 2024 Algorithmic spatial form-finding of four-fold origami structures based on mountain-valley assignments *J. Mech. Robot.* **16** 031001
- [327] Sareh P and Guest S D 2015 Design of isomorphic symmetric descendants of the Miura-ori *Smart Mater. Struct.* **24** 85001
- [328] Tang Y, Lin G, Yang S, Yi Y K, Kamien R D and Yin J 2017 Programmable Kiri-Kirigami metamaterials *Adv. Mater.* **29** 1604262
- [329] Castle T, Cho Y, Gong X, Jung E, Sussman D M, Yang S and Kamien R D 2014 Making the cut: lattice kirigami rules *Phys. Rev. Lett.* **113** 1–5
- [330] Heimbs S, Cichosz J, Klaus M, Kilchert S and Johnson A F 2010 Sandwich structures with textile-reinforced composite foldcores under impact loads *Compos. Struct.* **92** 1485–97
- [331] Heimbs S, Middendorf P, Kilchert S, Johnson A F and Maier M 2007 Experimental and numerical analysis of composite folded sandwich core structures under compression *Appl. Compos. Mater.* **14** 363–77
- [332] Li Z, Chen W and Hao H 2018 Crushing behaviours of folded kirigami structure with square dome shape *Int. J. Impact Eng.* **115** 94–105
- [333] Li Z, Chen W and Hao H 2017 Numerical study of folded dome shape aluminium structure against flatwise crushing *12th Int. Conference on Shock & Impact Loads on Structures*
- [334] Teng T L, Liang C C and Nguyen V H 2014 Innovative design of bicycle helmet liners *Proc. Inst. Mech. Eng. L* **228** 341–51
- [335] Li Z, Chen W, Hao H, Cui J and Shi Y 2019 Experimental study of multi-layer folded truncated structures under dynamic crushing *Int. J. Impact Eng.* **131** 111–22
- [336] Viconic Defense 2020 Viconic (available at: <https://www.viconicdefense.com/>) (Accessed 05 October 2023)
- [337] Neville R M, Scarpa F and Pirrera A 2016 Shape morphing Kirigami mechanical metamaterials *Sci. Rep.* **6** 1–12
- [338] Tang Y and Yin J 2017 Design of cut unit geometry in hierarchical kirigami-based auxetic metamaterials for high stretchability and compressibility *Extrem. Mech. Lett.* **12** 77–85
- [339] Alderete N A, Medina L, Lamberti L, Sciammarella C and Espinosa H D 2021 Programmable 3D structures via Kirigami engineering and controlled stretching *Extrem. Mech. Lett.* **43** 101146
- [340] Hou Y, Neville R, Scarpa F, Remillat C, Gu B and Ruzzene M 2014 Graded conventional-auxetic Kirigami sandwich structures: flatwise compression and edgewise loading *Composites B* **59** 33–42
- [341] Chen Y, Ye W, Xu R, Sun Y, Feng J and Sareh P 2023 A programmable auxetic metamaterial with tunable crystal symmetry *Int. J. Mech. Sci.* **249** 108249

- [342] Jalali E, Soltanizadeh H, Chen Y, Xie Y M and Sareh P 2022 Selective hinge removal strategy for architecting hierarchical auxetic metamaterials *Commun. Mater.* **3** 97
- [343] Hwang D-G, Trent K and Bartlett M D 2018 Kirigami-inspired structures for smart adhesion *ACS Appl. Mater. Interfaces* **10** 6747–54
- [344] Sun Y, Ye W, Chen Y, Fan W, Feng J and Sareh P 2021 Geometric design classification of kirigami-inspired metastructures and metamaterials *Structures* **33** 3633–43
- [345] Saxena K K, Das R and Calius E P 2016 Three decades of auxetics research – materials with negative Poisson's ratio: a review *Adv. Eng. Mater.* **18** 1847–70
- [346] Prawoto Y 2012 Seeing auxetic materials from the mechanics point of view: a structural review on the negative Poisson's ratio *Comput. Mater. Sci.* **58** 140–53
- [347] Lim T C 2014 Micromechanical models for auxetic materials *Auxetic Materials and Structures* (Springer) pp 45–105
- [348] Lim T C 2014 *Auxetic Materials and Structures* (Springer)
- [349] Platus D L Technology M.K. 1999 Negative stiffness mechanism *Proc. SPIE* **3786** 98–105
- [350] Lakes R S and Drugan W J 2002 Dramatically stiffer elastic composite materials due to a negative stiffness phase? *J. Mech. Phys. Solids* **50** 979–1009
- [351] Churchill C B, Shahan D W, Smith S P, Keefe A C and McKnight G P 2016 Dynamically variable negative stiffness structures *Sci. Adv.* **2** 1–7
- [352] Hewage T A M, Alderson K L, Alderson A and Scarpa F 2016 Double-negative mechanical metamaterials displaying simultaneous negative stiffness and negative Poisson's ratio properties *Adv. Mater.* **28** 10323–32
- [353] Ha C S, Lakes R S and Plesha M E 2019 Cubic negative stiffness lattice structure for energy absorption: numerical and experimental studies *Int. J. Solids Struct.* **178–179** 127–35
- [354] Zhang K, Qi L, Zhao P, Zhao C and Deng Z 2023 Buckling induced negative stiffness mechanical metamaterial for bandgap tuning *Compos. Struct.* **304** 116421
- [355] Goldsberry B M and Haberman M R 2018 Negative stiffness honeycombs as tunable elastic metamaterials *J. Appl. Phys.* **123** 091711
- [356] Mehreganian N, Fallah A S and Sareh P 2021 Structural mechanics of negative stiffness honeycomb metamaterials *J. Appl. Mech.* **88** 051006
- [357] Zhang Z and Krushynska A O 2022 Programmable shape-morphing of rose-shaped mechanical metamaterials *APL Mater.* **10** 080701
- [358] Dudek K K, Iglesias Martinez J A I, Ulliac G and Kadic M 2022 Micro-scale auxetic Hierarchical mechanical metamaterials for shape morphing *Adv. Mater.* **34** 2110115
- [359] de Jong P, Schwab A, Mirzaali M J and Zadpoor A A 2023 A multibody kinematic system approach for the design of shape-morphing mechanism-based metamaterials *Res. Sq.* (<https://doi.org/10.21203/rs.3.rs-2510931/v1>)
- [360] Duncan O, Chester M, Wang W, Alderson A and Allen T 2023 Effect of twist on indentation resistance *Mater. Today Commun.* **35** 105616
- [361] Goswami D, Zhang Y, Liu S, Abdalla O A, Zavattieri P D and Martinez R V 2021 Mechanical metamaterials with programmable compression-twist coupling *Smart Mater. Struct.* **30** 015005
- [362] Frenzel T, Kadic M and Wegener M 2017 Three-dimensional mechanical metamaterials with a twist *Science* **358** 1072–4
- [363] Fernandez-corbaton I, Rockstuhl C, Ziemke P, Gumbsch P, Albiez A, Schwaiger R, Frenzel T, Kadic M and Wegener M 2019 New twists of 3D chiral metamaterials *Adv. Mater.* **31** 1–7
- [364] Canejo J P, Borges J P, Godinho M H, Brogueira P, Teixeira P I C and Terentjev E M 2008 Helical twisting of electrospun liquid crystalline cellulose micro- and nanofibers *Adv. Mater.* **20** 4821–5
- [365] Janbaz S, Narooei K, Van Manen T and Zadpoor A A 2020 Strain rate-dependent mechanical metamaterials *Sci. Adv.* **6** eaba0616
- [366] Wu R, Roberts P C E, Lyu S, Zheng F, Soutis C, Diver C, Zhou D, Li L and Deng Z 2020 Lightweight self-forming super-elastic mechanical metamaterials with adaptive stiffness *Adv. Funct. Mater.* **31** 2008252
- [367] Zhu R, Chen Y Y, Barnhart M V, Hu G K, Sun C T and Huang G L 2016 Experimental study of an adaptive elastic metamaterial controlled by electric circuits *Appl. Phys. Lett.* **108** 011905
- [368] Timoshenko S P and Goodier J N 1970 *Theory of Elasticity* 3rd edn (McGraw-Hill)
- [369] Duncan O, Foster L, Allen T and Alderson A 2023 Effect of Poisson's ratio on the indentation of open cell foam *Eur. J. Mech. A* **99** 1–9
- [370] Lakes R S and Elms K 1993 Indentability of conventional and negative Poisson's ratio foams *J. Compos. Mater.* **27** 1193–202
- [371] Chan N and Evans K E 1998 Indentation resilience of conventional and auxetic foams *J. Cell. Plast.* **34** 231–60
- [372] Allen T, Duncan O, Foster L, Senior T, Zampieri D, Edeh V and Alderson A 2016 Auxetic foam for snow-sport safety devices *Snow Sports Trauma and Safety* 1st edn (Springer) pp 145–59
- [373] Alderson K L, Pickles A P, Neale P J and Evans K E 1994 Auxetic polyethylene: the effect of a negative Poisson's ratio on hardness *Acta Metall. Mater.* **42** 2261–6
- [374] Novak N, Duncan O, Allen T, Alderson A, Vesenjajk M and Ren Z 2021 Shear modulus of conventional and auxetic open-cell foam *Mech. Mater.* **257** 104743
- [375] Novak N, Krstulović-Opara L, Ren Z and Vesenjajk M 2020 Compression and shear behaviour of graded chiral auxetic structures *Nejc Mech. Mater.* **148** 103524
- [376] Scarpa F and Tomlinson G 2000 Theoretical characteristics of the vibration of sandwich plates with in-plane negative Poisson's ratio values *J. Sound Vib.* **230** 45–67
- [377] Scarpa F and Tomlin P J 2000 On the transverse shear modulus of negative Poisson's ratio honeycomb structures *Fatigue Fract. Eng. Mater. Struct.* **23** 717–20
- [378] Chun Cheon H, Scarpa F, Hallak Panzera T, Farrow I and Peng H-X 2018 Shear stiffness and energy absorption of auxetic open cell foams as sandwich cores *Phys. Status Solidi* **256** 1–9
- [379] Kwon K and Phan A V 2015 Symmetric-Galerkin boundary element analysis of the dynamic T-stress for the interaction of a crack with an auxetic inclusion *Mech. Res. Commun.* **69** 91–96
- [380] Adam M M, Berger J R and Martin P A 2013 Singularities in auxetic elastic bimetals *Mech. Res. Commun.* **47** 102–5
- [381] Chan N and Evans K E 1997 Fabrication methods for auxetic foams *J. Mater. Sci.* **32** 5945–53
- [382] Scarpa F, Giacomini J, Zhang Y and Pastorino P 2005 Mechanical performance of auxetic polyurethane foam for antivibration glove applications *Cell. Polym.* **24** 253–68
- [383] Bianchi M and Scarpa F 2013 Vibration transmissibility and damping behaviour for auxetic and conventional foams under linear and nonlinear regimes *Smart Mater. Struct.* **22** 084010
- [384] Duncan O, Foster L, Senior T, Alderson A and Allen T 2016 Quasi-static characterisation and impact testing of auxetic foam for sports safety applications *Smart Mater. Struct.* **25** 054014
- [385] Allen T, Shepherd J, Hewage T A M, Senior T, Foster L and Alderson A 2015 Low-kinetic energy impact response of auxetic and conventional open-cell polyurethane foams *Phys. Status Solidi* **252** 1631–9

- [386] Ge C 2013 A comparative study between felted and triaxial compressed polymer foams on cushion performance *J. Cell. Plast.* **49** 521–33
- [387] Lisiecki J, Błaziejewicz T, Kłysz S, Gmurczyk G, Reymer P and Mikulowski G 2013 Tests of polyurethane foams with negative Poisson's ratio *Phys. Status Solidi* **250** 1988–95
- [388] Zhang Q, Scarpa F, Barton D, Zhu Y, Lang Z, Zhang D and Peng H-X 2022 Impact properties of uniaxially thermoformed auxetic foams *Int. J. Impact Eng.* **163** 104176
- [389] Evans K E, Donoghue J and Alderson K 2004 The design, matching and manufacture of auxetic carbon fibre laminates *J. Compos. Mater.* **38** 95–106
- [390] Alderson K L, Simkins V R, Coenen V L, Davies P J, Alderson A and Evans K E 2005 How to make auxetic fibre reinforced composites *Phys. Status Solidi* **242** 509–18
- [391] HEAD 2021 Auxetic—the science behind the sensational feel (available at: [www.head.com/de\\_CH/tennis/all-about-tennis/auxetic-the-science-behind-the-sensational-feel](http://www.head.com/de_CH/tennis/all-about-tennis/auxetic-the-science-behind-the-sensational-feel)) (Accessed 12 May 2022)
- [392] Alderson K L and Coenen V L 2008 The low velocity impact response of auxetic carbon fibre laminates *Phys. Status Solidi* **496** 489–96
- [393] Carbon and Riddell 2020 Carbon3d (available at: <https://www.carbon3d.com/news/press-releases/riddell-carbon-produce-football-helmet>) (Accessed 05 October 2023)
- [394] Adidas 4D shoes (available at: [www.adidas.co.uk/4d-shoes](http://www.adidas.co.uk/4d-shoes))
- [395] Tancogne-Dejean T, Spierings A B and Mohr D 2016 Additively-manufactured metallic micro-lattice materials for high specific energy absorption under static and dynamic loading *Acta Mater.* **116** 14–28
- [396] Qiu W, Lu F, Wang G, Huang G, Zhang H, Zhang Z and Gong C 2019 Evaluation of mechanical performance and optimization design for lattice girders *Tunn. Undergr. Space Technol.* **87** 100–11
- [397] Zheng X *et al* 2014 Ultralight, ultrastiff mechanical metamaterials *Science* **344** 1373–7
- [398] Gümriük R and Mines R A W 2013 Compressive behaviour of stainless steel micro-lattice structures *Int. J. Mech. Sci.* **68** 125–39
- [399] Fernandes F A O, de Sousa R J A, Ptak M and Migueis G 2019 Helmet design based on the optimization of biocomposite energy-absorbing liners under multi-impact loading *Appl. Sci.* **9** 1–26
- [400] Hansen K, Dau N, Feist F, Deck C, Willinger R, Madey S M and Bottlang M 2013 Angular impact mitigation system for bicycle helmets to reduce head acceleration and risk of traumatic brain injury *Accid. Anal. Prev.* **59** 109–17
- [401] Caserta G D, Iannucci L and Galvanetto U 2011 Shock absorption performance of a motorbike helmet with honeycomb reinforced liner *Compos. Struct.* **93** 2748–59
- [402] Kholoosi F and Galehdari S A 2019 Design, optimisation and analysis of a helmet made with graded honeycomb structure under impact load *Int. J. Crashworthiness* **24** 645–55
- [403] Kholoosi F and Galehdari S A 2017 Design and analysis of a helmet equipped with graded honeycomb structure under impact of flat and hemi-spherical anvils *Proc. Eng.* **173** 1299–306
- [404] Ozdemir Z, Hernandez-Nava E, Tyas A, Warren J A, Fay S D, Goodall R, Todd I and Askes H 2016 Energy absorption in lattice structures in dynamics: experiments *Int. J. Impact Eng.* **89** 49–61
- [405] Fleck N A, Deshpande V S and Ashby M F 2010 Micro-architected materials: past, present and future *Proc. R. Soc. A* **466** 2495–516
- [406] Song J, Wang Y, Zhou W, Fan R, Yu B, Lu Y and Li L 2019 Topology optimization-guided lattice composites and their mechanical characterizations *Composites B* **160** 402–11
- [407] Schaedler T A, Ro C J, Sorensen A E, Eckel Z, Yang S S, Carter W B and Jacobsen A J 2014 Designing metallic microlattices for energy absorber applications *Adv. Eng. Mater.* **16** 276–83
- [408] Du Y, Li H, Luo Z and Tian Q 2017 Topological design optimization of lattice structures to maximize shear stiffness *Adv. Eng. Softw.* **112** 211–21
- [409] Ashby M F 2006 The properties of foams and lattices *Phil. Trans. R. Soc. A* **364** 15–30
- [410] Liu Z, Liu J, Liu J, Zeng W and Huang W 2023 The impact responses and failure mechanism of composite gradient reentrant honeycomb structure *Thin-Walled Struct.* **182** 110228
- [411] Özen İ, Çava K, Gedikli H, Alver Ü and Aslan M 2020 Low-energy impact response of composite sandwich panels with thermoplastic honeycomb and reentrant cores *Thin-Walled Struct.* **156** 106989
- [412] Wang S, Xu Y and Zhang W 2019 Low-velocity impact response of 3D-printed lattice sandwich panels *IOP Conf. Ser.: Mater. Sci. Eng.* **531** 012056
- [413] Gao D, Wang S, Zhang M and Zhang C 2021 Experimental and numerical investigation on in-plane impact behaviour of chiral auxetic structure *Compos. Struct.* **267** 113922
- [414] Lu Q, Qi D, Li Y, Xiao D and Wu W 2019 Impact energy absorption performances of ordinary and hierarchical chiral structures *Thin-Walled Struct.* **140** 495–505
- [415] Wu W, Hu W, Qian G, Liao H, Xu X and Berto F 2019 Mechanical design and multifunctional applications of chiral mechanical metamaterials: a review *Mater. Des.* **180** 107950
- [416] Ye M, Gao L, Wang F and Li H 2021 A novel design method for energy absorption property of chiral mechanical metamaterials *Materials* **14** 1–21
- [417] Kai L, Xiaofei C, Peng Z, WenWang W and Ying L 2022 Dynamic mechanical performances of enhanced anti-tetra-chiral structure with rolled cross-section ligaments under impact loading *Int. J. Impact Eng.* **166** 104204
- [418] Xiang X, Qiang W, Hou B, Tran P and Lu G 2020 Quasi-static and dynamic mechanical properties of Miura-ori metamaterials *Thin-Walled Struct.* **157** 106993
- [419] Xiang X, Fu Z, Zhang S, Lu G, Ha N S, Liang Y and Zhang X 2021 The mechanical characteristics of graded Miura-ori metamaterials *Mater. Des.* **211** 110173
- [420] Galehdari S A, Khodarahmi H and Atrian A 2017 Design and analysis of graded honeycomb shock absorber for increasing the safety of passengers in armored vehicles exposed to mine explosion *J. Solid Mech.* **9** 370–83
- [421] Sun G, Jiang H, Fang J, Li G and Li Q 2016 Crashworthiness of vertex based hierarchical honeycombs in out-of-plane impact *Mater. Des.* **110** 705–19
- [422] Duncan O, Alderson A and Allen T 2021 Fabrication, characterization and analytical modeling of gradient auxetic closed cell foams *Smart Mater. Struct.* **30** 035014
- [423] Duncan O, Allen T, Foster L, Senior T and Alderson A 2017 Fabrication, characterisation and modelling of uniform and gradient auxetic foam sheets *Acta Mater.* **126** 426–37
- [424] Sanami M, Alderson A, Alderson K L, McDonald S A, Mottershead B and Withers P J 2014 The production and characterization of topologically and mechanically gradient open-cell thermoplastic foams *Smart Mater. Struct.* **23** 055016
- [425] Evans A G, Hutchinson J W, Fleck N A, Ashby M F and Wadley H N G 2001 The topological design of multifunctional cellular metals *Prog. Mater. Sci.* **46** 309–27

- [426] Caccese V, Ferguson J R and Edgecomb M A 2013 Optimal design of honeycomb material used to mitigate head impact *Compos. Struct.* **100** 404–12
- [427] Zhang Y, Lu M, Wang C H, Sun G and Li G 2016 Out-of-plane crashworthiness of bio-inspired self-similar regular hierarchical honeycombs *Compos. Struct.* **144** 1–13
- [428] Liu Y, Schaedler T A and Chen X 2014 Dynamic energy absorption characteristics of hollow microlattice structures *Mech. Mater.* **77** 1–13
- [429] Gibson L J and Ashby M F 1997 *Cellular Solids. Structure and Properties* (Press Syndicate of the University of Cambridge) pp 4., 67, 103, 106, 167–169, 176–183, 259–264, 286, 3p
- [430] Robinson M, Soe S, Johnston R, Adams R, Hanna B, Burek R, McShane G, Celeghini R, Alves M and Theobald P 2019 Mechanical characterisation of additively manufactured elastomeric structures for variable strain rate applications *Addit. Manuf.* **27** 398–407
- [431] Wan H, Ohtaki H, Kotosaka S and Hu G 2004 A study of negative Poisson's ratios in auxetic honeycombs based on a large deflection model *Eur. J. Mech. A* **23** 95–106
- [432] Yang S, Qi C, Guo D M and Wang D 2012 Energy absorption of an re-entrant honeycombs with negative Poisson's ratio *Appl. Mech. Mater.* **148** 992–5
- [433] Dobnik Dubrovski P, Novak N, Borovinšek M, Vesenj M and Ren Z 2019 In-plane behavior of auxetic nonwoven fabrics based on rotating square unit geometry under tensile load *Polymers* **11** 1–13
- [434] Grima J N and Evans K E 2000 Auxetic behavior from rotating squares *J. Mater. Sci. Lett.* **19** 1563–5
- [435] Grima J N and Evans K E 2006 Auxetic behavior from rotating triangles *J. Mater. Sci.* **41** 3193–6
- [436] Dudek K K, Drzewiński A and Kadic M 2021 Self-rotating 3D mechanical metamaterials *Proc. R. Soc. A* **477** 20200825
- [437] Gao Y, Wei X, Han X, Zhou Z and Xiong J 2021 Novel 3D auxetic lattice structures developed based on the rotating rigid mechanism *Int. J. Solids Struct.* **233** 111232
- [438] Cross T M, Hoffer K W, Jones D P, Kirschner P B and Meschter J C 2015 Auxetic structures and footwear with soles having auxetic structures US 2015/0075034 A1 vol 1
- [439] Moroney C 2021 The application of auxetic structures for rugby shoulder padding *PhD Thesis* Manchester Metropolitan University
- [440] Li Z, Chen W and Hao H 2018 Numerical study of open-top truncated pyramid folded structures with interconnected side walls against flatwise crushing *Thin-Walled Struct.* **132** 537–48
- [441] Li Z, Chen W and Hao H 2018 Blast mitigation performance of cladding using square dome-shape kirigami folded structure as core *Int. J. Mech. Sci.* **145** 83–95
- [442] Shiffer I, Hertz K, Tu D and Heller L Ecohelmet 2017 (available at: [www.ecohelmet.com/](http://www.ecohelmet.com/)) (Accessed 14 April 2023)
- [443] nTopology 2023 (available at: [www.ntop.com/](http://www.ntop.com/))
- [444] Hyperganic 2023 (available at: [www.hyperganic.com/](http://www.hyperganic.com/)) (Accessed 14 August 2023)
- [445] Rhino 2023 (available at: [www.rhino3d.com/6/new/grasshopper/](http://www.rhino3d.com/6/new/grasshopper/)) (Accessed 14 August 2023)
- [446] Hearmon R F 1962 *An Introduction to Applied Aniso-tropic Elasticity* (Oxford University Press) p 12
- [447] Masters I G and Evans K E 1996 Models for the elastic deformation of honeycombs *Compos. Struct.* **35** 403–22
- [448] Eringen A C 1999 *Microcontinuum Field Theories* 1st edn (Springer Science+Business Media)
- [449] Lakes R 1991 Experimental micro mechanics methods for conventional and negative Poisson's ratio cellular solids as cosserat continua *J. Eng. Mater. Technol. Trans. ASME* **113** 148–55
- [450] Lakes R and Drugan W J 2015 Bending of a cosserat elastic bar of square cross section: theory and experiment *Trans. ASME, J. Appl. Mech.* **82** 1–16
- [451] Nightingale M, Hewson R and Santer M 2021 Multiscale optimisation of resonant frequencies for lattice-based additive manufactured structures *Struct. Multidiscip. Optim.* **63** 1187–201
- [452] Pan F, Li Y, Li Z, Yang J, Liu B and Chen Y 2019 3D pixel mechanical metamaterials *Adv. Mater.* **31** 1–8
- [453] Wang Y, Groen J P and Sigmund O 2019 Simple optimal lattice structures for arbitrary loadings *Extrem. Mech. Lett.* **29** 100447
- [454] Andreassen E, Clausen A, Schevenels M, Lazarov B S and Sigmund O 2011 Efficient topology optimization in MATLAB using 88 lines of code *Struct. Multidiscip. Optim.* **43** 1–16
- [455] Sigmund O 2001 A 99 line topology optimization code written in Matlab *Struct. Multidiscip. Optim.* **21** 120–7
- [456] Murphy R, Imediogwu C, Hewson R and Santer M 2021 Multiscale structural optimization with concurrent coupling between scales *Struct. Multidiscip. Optim.* **63** 1721–41
- [457] Mehreganian N, Fallah A S and Sareh P 2023 Impact response of negative stiffness curved-beam-architected metastructures *Int. J. Solids Struct.* **179** 112389
- [458] Chen Y, Shi J, He R, Lu C, Shi P, Feng J and Sareh P 2023 A unified inverse design and optimization workflow for the Miura-oRing metastructure *J. Mech. Des.* **145** 091704
- [459] Pinho-da-Cruz J, Oliveira J A and Teixeira-Dias F 2009 Asymptotic homogenisation in linear elasticity. Part I: mathematical formulation and finite element modelling *Comput. Mater. Sci.* **45** 1073–80
- [460] Oliveira J A, Pinho-da-Cruz J and Teixeira-Dias F 2009 Asymptotic homogenisation in linear elasticity. Part II: finite element procedures and multiscale applications *Comput. Mater. Sci.* **45** 1081–96
- [461] Cioranescu D and Donato P 1999 *Introduction to Homogenization* (Oxford University Press)
- [462] Carstensen J V, Lotfi R, Chen W, Szyniszewski S, Gaitanaros S, Schroers J and Guest J K 2022 Topology-optimized bulk metallic glass cellular materials for energy absorption *Scr. Mater.* **208** 114361
- [463] Carstensen J V, Lotfi R and Guest J K 2015 Topology optimization of cellular materials for properties governed by nonlinear mechanics *11th World Congress on Structural and Multidisciplinary Optimization* pp 1–6
- [464] Carstensen J V, Guest J K and Lotfi R 2016 Topology optimization of nonlinear cellular materials *17th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conf.* pp 1–10
- [465] Wu L, Mustafa M, Segurado J and Noels L 2023 Second-order computational homogenisation enhanced with non-uniform body forces for non-linear cellular materials and metamaterials *Comput. Methods Appl. Mech. Eng.* **407** 115931
- [466] Earnshaw J *et al* 2023 Meta-genome (available at: <https://meta-genome.org/>) (Accessed 27 March 2023)
- [467] Johnston R and Kazancı Z 2021 Analysis of additively manufactured (3D printed) dual-material auxetic structures under compression *Addit. Manuf.* **38** 101783
- [468] Wang K, Chang Y H, Chen Y, Zhang C and Wang B 2015 Designable dual-material auxetic metamaterials using three-dimensional printing *Mater. Des.* **67** 159–64
- [469] Gao D, Wang B, Gao H, Ren F, Guo C, Ma S, Cao T, Xia Y and Wu Y 2021 Strain rate effect on mechanical properties



- of the 3D-printed metamaterial foams with tunable negative Poisson's ratio *Front. Mater.* **8** 1–12
- [470] Cervinek O, Pettermann H, Todt M, Koutny D and Vaverka O 2022 Non-linear dynamic finite element analysis of micro-strut lattice structures made by laser powder bed fusion *J. Mater. Res. Technol.* **18** 3684–99
- [471] Mauko A, Fíla T, Falta J, Koudelka P, Rada V, Neuhäuserová M, Zlamal P, Vesenjāk M, Jiroušek O and Ren Z 2021 Dynamic deformation behaviour of chiral auxetic lattices at low and high strain-rates *Metals* **11** 1–15
- [472] Qi J *et al* 2022 Recent progress in active mechanical metamaterials and construction principles *Adv. Sci.* **9** 1–27
- [473] Levine D J, Turner K T and Pikul J H 2021 Materials with electroprogrammable stiffness *Adv. Mater.* **33** 1–26
- [474] Mills N J 2007 *Polymer Foams Handbook: Engineering and Biomechanics Applications and Design Guide* (Elsevier)
- [475] HeadHealthTECH Helmet challenge grants 2020 (available at: [www.nfl.com/playerhealthandsafety/equipment-and-innovation/headhealthtech/headhealthtech-challenges](http://www.nfl.com/playerhealthandsafety/equipment-and-innovation/headhealthtech/headhealthtech-challenges)) (Accessed 6 April 2023)