Accurate Approximation to Channel Distributions of Cascaded RIS-aided Systems with Phase Errors over Nakagami-m Channels

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Abstract

Reconfigurable intelligent surfaces (RISs) are a promising technology that is foreseen to drive the development of beyond-fifth-generation (B5G) wireless networks. This paper proposes a novel accurate approximation to the channel distribution of cascaded RIS-aided wireless networks, with a reasonable assumption of phase errors, over Nakagami-m fading channels. Using the approximation, the probability density function (PDF) and outage probability (OP) expressions are derived for the end-to-end cascaded RIS-aided communication. Obtained PDF and OP formulas are valid for any number of cascaded RISs, n, and reflecting elements per individual RIS, N. The approximation accuracy is validated using the Kolmogorov-Smirnov goodness-of-fit test. Analytical results are verified by Monte Carlo simulations under different system parameters. Finally, derived expressions have been used to illustrate the impact of various important system parameters on the outage performance of the proposed system.

Index Terms

Accurate approximation, channel distribution, reconfigurable intelligent surface (RIS), outage probability (OP).

I. Introduction

RECONFIGURABLE intelligent surfaces (RISs) have received significant attention for producing smart, energy efficient, non-complex, and adaptable wireless communication environments at a low cost [1], [2]. RIS is a planar metasurface composed of numerous inexpensive passive reflecting components that can be digitally manipulated to control the amplitudes and phases of the incident signals [3]. Unlike traditional active relaying, RIS passively reflects incident signals and operates in a full-duplex mode without antenna noise amplification or self-interference [4].

Many research findings have been published on RIS [3]–[6]. For instance, the authors in [5] presented a tutorial overview of the RIS-aided wireless communication, where issues related to the reflection optimization, channel estimation, and hardware

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architecture were addressed. In [6], the authors investigated the channel distribution of RIS-aided systems for two RIS-based transmission scenarios, where RIS acts as a relay and a transmitter source. Several works [7]–[9] studied the phase shifts introduced by RIS's reflecting elements and phase errors emerged due to the limited number of phase shifts. Although a single RIS-aided system with phase errors was studied in [9], the channel was modeled by Rayleigh fading. The RIS-aided networks possess the features of line-of-sight communication, which are preferably modeled by Nakagami-m fading model.

Although previous works have made valuable contributions to the field of RIS, they focused on the system models with single RIS [9]–[11]. At the same time, there are a few works considering the multi-hop multi-RIS implementation [12]–[16]. In [12], closed-form expressions for the outage probability (OP), ergodic capacity, and average symbol error probability were derived for cascaded RIS-aided systems, however, the phase errors were only considered at two edge-resided RISs while neglecting the signal scattering occurring between inner RISs. Furthermore, passive beamforming and beam routing were investigated for cascaded RIS-aided systems by considering the maximum signal-to-noise ratio (SNR) [13], [15]. In [14]–[16], the beamforming design, power scaling, and channel estimation were studied for double RIS-aided systems by assuming perfect phase adjustment. Thus, the PDF of the channel distribution of cascaded RIS-aided systems with practical phase shifting has not been investigated yet.

Motivated by the aforementioned discussion, in this letter, we consider the cascaded RIS-empowered systems with practical phase errors over Nakagami-m fading channels. We summarize our contributions as follows.

- A novel accurate closed-form Gamma approximation for the end-to-end (e2e) channel distribution is derived.
- The obtained approximation is valid for both small and large numbers of reflecting elements.
- The accuracy of this analytical expression is confirmed by using Kolmogorov-Smirnov (KS) goodness-of-fit test.
- The outage performance results demonstrate good agreement with Monte Carlo simulations.

II. System Model

The system model represents the cascaded RIS-aided communication network, as shown in Fig. 1, consisting of a transmitter, denoted by T, n cascades of RIS (denoted by $R_{t\in n}$ and equipped with N reflecting elements each), and an end-user, denoted by R. It is assumed that, due to the dense urban environment, no direct link exists between T and R. The channel entities related to the T$\rightarrow$ RIS$_1$, RIS$_1$ $\rightarrow$ RIS$_2$, ..., RIS$_{n-1}$ $\rightarrow$ RIS$_n$, and RIS$_n$ $\rightarrow$ R communication links[1] are respectively represented by $\mathbf{h} \in \mathbb{C}^{N\times 1} = \left\{|h_i|\, e^{j\phi_i}, \forall i \in N\right\}$, $\mathbf{G}_1 \in \mathbb{C}^{N\times N} = \left\{\left|g_{il}^{\mathbf{G}_1}\right| e^{j\theta_{il}^{\mathbf{G}_1}}, \forall i, j \in N\right\}$, ..., $\mathbf{G}_{n-1} \in \mathbb{C}^{N\times N}$, and $\mathbf{q} \in \mathbb{C}^{1\times N} = \left\{|q_k|\, e^{j\psi_k}, \forall k \in N\right\}$. The corresponding distances are denoted by $d_{\mathrm{T}R_1}$, $d_{R_1R_2}$, ..., $d_{R_{n-1}R_n}$ and $d_{R_n\mathrm{R}}$. τ is the path-loss coefficient assumed to be identical for all links. Each channel coefficient is drawn from the independent and identically distributed (i.i.d.) Nakagami-m distribution, with a probability density function (PDF) given by

$$f_X(x) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} e^{-\frac{m}{\Omega}x^2},\ x \geq 0, \tag{1}$$

where $\Gamma(\cdot)$ is the Gamma function [17, (8.310)]; $m \geq 1/2$ and $\Omega > 0$ are the shape and spread parameters, respectively.

[1]For the sake of tractability, it is assumed that RISs have the same number of reflecting elements, i.e., $N = N_1 = N_2 = \ldots = N_n$.

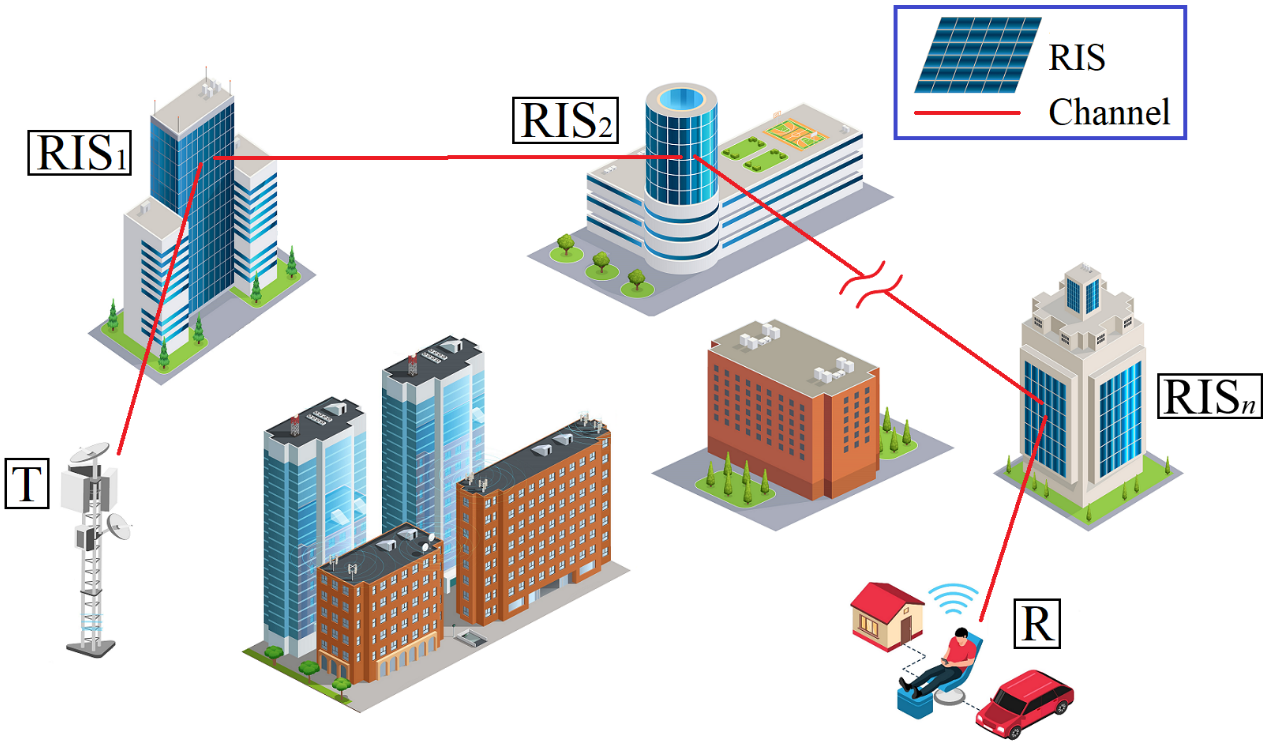

Fig. 1: Schematic of a cascaded RIS-assisted network.

The effective signal received at R can be written as

$$y_{\mathrm{R}}=\sum_{i=1}^{N}\sum_{l=1}^{N}\cdots\sum_{j=1}^{N}\sum_{k=1}^{N}\sqrt{\frac{P}{d_{\mathrm{T}R_1}^{\tau}d_{R_1R_2}^{\tau}\cdots d_{R_{n-1}R_n}^{\tau}d_{R_n\mathrm{R}}^{\tau}}}\left|h_i\right|\left|\eta_i^{R_1}\right|\left|g_{il}^{\mathbf{G}_1}\right|\cdots\left|g_{jk}^{\mathbf{G}n-1}\right|\left|\eta_k^{R_n}\right|\left|q_k\right|$$
$$\times\, e^{j\left(\phi_i-\alpha_i^{R_1}+\theta_{il}^{\mathbf{G}_1}+\cdots+\theta_{jk}^{\mathbf{G}_{n-1}}-\alpha_k^{R_n}+\psi_k\right)}\chi+n_{\mathrm{R}}, \tag{2}$$

where P, χ, and n_{R} denote the transmit power at T, transmitted message, and additive white Gaussian noise, with zero mean and variance σ_{R}^2, accordingly. At the same time, every particular element ω of RIS t contributes to the received signal by $\left|\eta_\omega^{R_t}\right|e^{-j\alpha_\omega^{R_t}}$, where the magnitude[2] $\left|\eta_\omega^{R_t}\right|$ and phase $\alpha_\omega^{R_t}$ range in the intervals $[0,1]$ and $[0,2\pi)$. Assuming $\left|\eta_\omega^{R_t}\right|=1,\ \omega\in N,\ t\in n$, SNR at R can be given by

$$\gamma_{\mathrm{R}}=\frac{\left|\bar{P}\sum_{i=1}^{N}\sum_{l=1}^{N}\cdots\sum_{j=1}^{N}\sum_{k=1}^{N}X_{il\ldots jk}e^{j\delta_{il\ldots jk}}\right|^2}{\sigma_{\mathrm{R}}^2}, \tag{3}$$

where $\bar{P}=\sqrt{\frac{P}{d_{\mathrm{T}R_1}^{\tau}d_{R_1R_2}^{\tau}\cdots d_{R_{n-1}R_n}^{\tau}d_{R_n\mathrm{R}}^{\tau}}}$, $X_{il\ldots jk}=\left|h_i\right|\left|g_{il}^{\mathbf{G}_1}\right|\cdots\left|g_{jk}^{\mathbf{G}n-1}\right|\left|q_k\right|$, and $\delta_{il\ldots jk}=\phi_i-\alpha_i^{R_1}+\theta_{il}^{\mathbf{G}_1}+\cdots+\theta_{jk}^{\mathbf{G}_{n-1}}-\alpha_k^{R_n}+\psi_k$ is the phase error, which is found to be unavoidable in the cascaded RIS-aided systems for the following two reasons. First, although, the phase shift produced by a channel can be estimated perfectly by the vth element of RIS, it can only be adjusted towards a single e2e channel path. In the cascaded system with identical RIS, N^n channel paths are formed, while

[2]For analytical purposes, this paper assumes an ideal reflection amplitude for all RISs, i.e., $\left|\eta_\omega^{R_t}\right|=1$. Please refer to [18] and [19] for more details.

$$\mathbb{E}[Z^4] = \frac{(N^n-1)}{N^{-n}}\left(\frac{\Omega}{m}\right)^{2(n+1)}\Bigg(\frac{\Gamma^{n+1}(m+2)}{\Gamma^{n+1}(m)(N^n-1)} + \frac{\Gamma^{2(n+1)}(m+1)}{\Gamma^{2(n+1)}(m)} + 4\frac{\Gamma^{n+1}(m+\frac{3}{2})}{\Gamma^{n+1}(m)}\frac{\Gamma^{n+1}(m+\frac{1}{2})}{\Gamma^{n+1}(m)}\frac{Q^2\sin^2(\frac{\pi}{Q})}{\pi^2}$$
$$+ 2(N^n-2)\frac{\Gamma^{n+1}(m+1)}{\Gamma^{n+1}(m)}\frac{\Gamma^{2(n+1)}(m+\frac{1}{2})}{\Gamma^{2(n+1)}(m)}\frac{Q^2\sin^2(\frac{\pi}{Q})}{\pi^2} + 2\frac{\Gamma^{2(n+1)}(m+1)}{\Gamma^{2(n+1)}(m)}\frac{8\pi^2+Q^2-Q^2\cos(\frac{4\pi}{Q})}{16\pi^2} + 4(N^n-2)$$
$$\times\frac{\Gamma^{n+1}(m+1)}{\Gamma^{n+1}(m)}\frac{\Gamma^{2(n+1)}(m+\frac{1}{2})}{\Gamma^{2(n+1)}(m)}\frac{Q^2\sin^2\left(\frac{\pi}{Q}\right)}{4\pi^3\left(2\pi+Q\sin\left(\frac{2\pi}{Q}\right)\right)^{-1}} + (N^n-2)(N^n-3)\frac{\Gamma^{4(n+1)}(m+\frac{1}{2})}{\Gamma^{4(n+1)}(m)}\frac{Q^4\sin^4(\frac{\pi}{Q})}{\pi^4}\Bigg) \quad (10)$$

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the maximum number of phases that can be adjusted equals $N+(n-1)(N-1)$. In general, $(N-1)\left(\sum_{i=0}^{n-1}N^i-n\right)$ paths experience the phase errors. Second, indeed, the desired phases cannot be accurately generated by RIS once it has a limited set of discrete phases, denoted by $Q=2^b$, where b is the number of quantization bits. Thus, the phase errors, in general, are inevitable when a phase shift produced by the channel path does not coincide with any available phase from the phases' set introduced by RIS. In this work, phase errors are modeled by the uniform distribution [9] with PDF given by $f_{\delta_{il\dots jk}}(\delta)=\frac{Q}{2\pi}$, if $-\frac{\pi}{Q}\le\delta\le\frac{\pi}{Q}$, and equal to 0, otherwise.

III. Novel Approximation

This paper presents novel PDF and OP expressions for cascaded RIS-aided systems with practical phase errors. The OP metric is defined as the probability of achievable SNR of message, χ, being below predefined SNR, ξ, and can be expressed as [20]

$$P_{\text{out}}(\xi)=\Pr\left[\gamma_{\text{R}}<\xi\right]=\Pr\left[Z<T\right]=\int_0^T f_Z(z)\ \mathrm{d}z, \quad (4)$$

where $f_Z(z)$ is PDF of a random variable (RV) $Z=\left|\sum_{i=1}^{N}\sum_{l=1}^{N}\cdots\sum_{j=1}^{N}\sum_{k=1}^{N}X_{il\dots jk}e^{j\delta_{il\dots jk}}\right|$, $T=\sqrt{\frac{\xi\sigma_{\text{R}}^2}{P^2}}$, and $\xi=2^{R_{\text{th}}}-1$, where R_{th} is the desired rate threshold.

Remark 1: *It is pertinent to note that Z is RV that characterizes the random process of the e2e communication in cascaded RIS-aided systems.* It is equal to the absolute value of the sum of correlated $Z_{il\dots jk}$ RVs, where $Z_{il\dots jk}=X_{il\dots jk}e^{j\delta_{il\dots jk}}$. Thus, this work presents novel and exact analytical expressions that describe the density of Z. Empirical research results show that PDF of Z can be well approximated by the Gamma distribution defined as [21]

$$f_Z(z)=\frac{z^{\alpha-1}\beta^{\alpha}}{\Gamma(\alpha)}e^{-\beta z},\ z>0,\alpha>0,\beta>0, \quad (5)$$

where α and β are the shape and scale parameters, respectively. Consequently, the closed-form OP of (4) can be expressed as

$$P_{\text{out}}(T)=\frac{\gamma\left(\alpha,\beta T\right)}{\Gamma(\alpha)}, \quad (6)$$

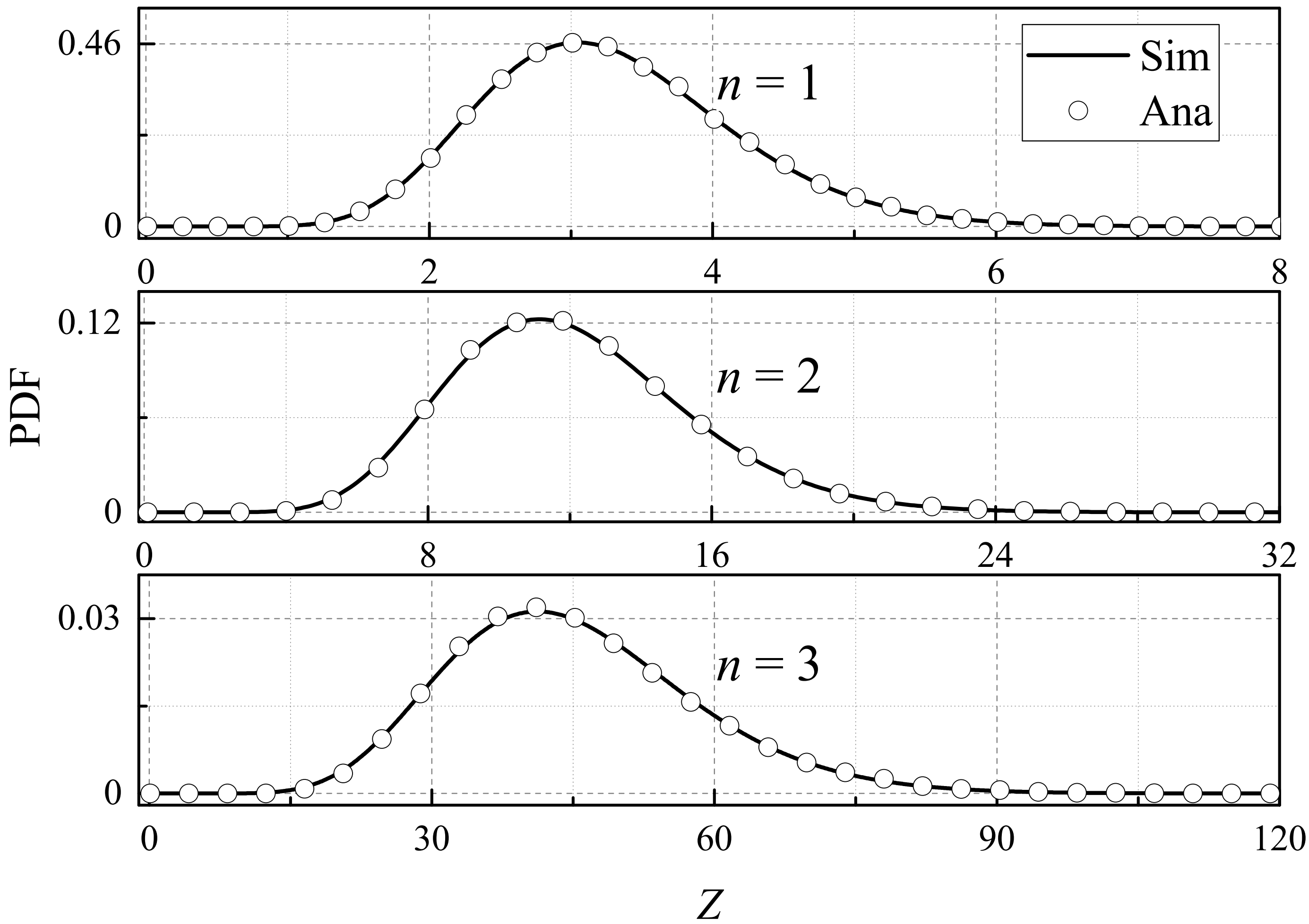

Fig. 2: PDF for $n \in \{1, 2, 3\}$, $m = 2$, and $N = 4$.

where $\gamma(.,.)$ is the lower incomplete Gamma function [17, (8.350.1)]. Using the moments defined in [22, (6)], the 2nd and 4th moments of Z are defined, as in (7) and (10), as

$$\mathbb{E}[Z^2] = N^n \left(\frac{\Omega}{m}\right)^{n+1} \left(\frac{\Gamma^{n+1}(m+1)}{\Gamma^{n+1}(m)} + (N^n - 1)\,\frac{\Gamma^{2(n+1)}(m+\frac{1}{2})}{\Gamma^{2(n+1)}(m)}\,\frac{Q^2 \sin^2(\frac{\pi}{Q})}{\pi^2}\right). \tag{7}$$

From these moments, the α and β parameters of the Gamma distribution are given by [9]

$$\alpha = \frac{-\mathbb{E}[Z^4] + 5\mathbb{E}[Z^2]^2 + U}{2(\mathbb{E}[Z^4] - \mathbb{E}[Z^2]^2)}, \tag{8}$$

$$\beta = \sqrt{\frac{-U + 2\mathbb{E}[Z^4] + 2\mathbb{E}[Z^2]^2)}{6\mathbb{E}[Z^2]}}, \tag{9}$$

where $U = \sqrt{\mathbb{E}[Z^4]^2 + 14\mathbb{E}[Z^4]\mathbb{E}[Z^2]^2 + \mathbb{E}[Z^2]^4}$.

Remark 2: *This approximation is only valid for systems that are free from correlated channels.*

However, in cascaded RIS-aided systems, the correlation is unavoidable, thus, additional approximation steps should be considered to quantify the correlation between the $Z_{il\ldots jk}$ terms. From obtained α and β, α^* and β^* can be found, which approximate the sum of correlated $n{*}$Nakagami-m RVs, by using the mean and variance of the Gamma distribution. The first moments of the sum of correlated and uncorrelated $n*$Nakagami-m RVs are equal as

$$\mathbb{E}[Z] = \alpha\beta = \alpha^*\beta^*. \tag{11}$$

The variance of the sum of correlated RVs is dependent on that of uncorrelated RVs by the following relation

$$\mathrm{var}\left(\sum_{i=1}^{N^n} Z_{i,corr}\right) = \mathrm{var}\left(\sum_{i=1}^{N^n} Z_{i,uncorr}\right) + \sum_{i=1}^{N^n}\sum_{j=1, i\neq j}^{N^n} \mathrm{cov}\left(Z_i, Z_j\right), \tag{12}$$

where $\mathrm{cov}(Z_i, Z_j) = \mathbb{E}[Z_i Z_j] - \mathbb{E}[Z_i]\mathbb{E}[Z_j]$ is the corresponding covariance. In terms of Gamma parameters, we get

$$\alpha^*\beta^{*2} = \alpha\beta^2 + \sum_{i=1}^{n-1} N^n \frac{(i+1)}{(N-1)^{i-n}} \sin^2\left(\frac{\pi}{Q}\right)\left(\frac{\Omega}{m}\right)^{n+1}\left(\frac{Q}{\pi}\right)^2 \frac{\Gamma^{2(n+1-i)}(m+\frac{1}{2})}{\Gamma^{2(n+1-i)}(m)}\left[\frac{\Gamma^i(m+1)}{\Gamma^i(m)} - \frac{\Gamma^{2i}(m+\frac{1}{2})}{\Gamma^{2i}(m)}\right]. \tag{13}$$

From (13), β^* can be found as

$$\beta^* = \beta + \frac{1}{\alpha\beta}\sum_{i=1}^{n-1} N^n \frac{(i+1)}{(N-1)^{i-n}} \sin^2\left(\frac{\pi}{Q}\right)\left(\frac{\Omega}{m}\right)^{n+1}\left(\frac{Q}{\pi}\right)^2 \frac{\Gamma^{2(n+1-i)}(m+\frac{1}{2})}{\Gamma^{2(n+1-i)}(m)}\left[\frac{\Gamma^i(m+1)}{\Gamma^i(m)} - \frac{\Gamma^{2i}(m+\frac{1}{2})}{\Gamma^{2i}(m)}\right], \tag{14}$$

while α^* is obtained using (11).

Fig. 2 shows the distribution of Z (i.e., “Sim”) and its Gamma approximation (i.e., “Ana”) for $n \in \{1, 2, 3\}$. All analytical and simulation results are in good agreement.

A. *KS goodness-of-fit test*

The KS goodness-of-fit test is used to evaluate the accuracy of the approximation by finding the maximum deviation

$$D \triangleq \max\left|F_Z(z) - F_{\hat{Z}}(z)\right|, \tag{15}$$

where $F_Z(z)$ and $F_{\hat{Z}}(z)$ are empirical and approximated cumulative distribution functions (CDFs) of RV Z, respectively. The hypothesis H_0 indicates that Z follows Gamma distribution. H_0 is true only when the critical value $D_{\max}$ is greater than D, i.e., $D_{\max} > D$, defined as

$$D_{\max} \approx \sqrt{-\frac{\ln\frac{\alpha}{2}}{2L}}, \tag{16}$$

where L is a sample size and α is a confidence interval [23].

Example: Consider the system with two RISs and four reflecting elements per RIS. Using (11) and (14), Z is approximated by the Gamma distribution. The KS test statistic, D, for this system model is equal to 0.8×10^{-3}. In the simulation, we set $v = 10^6$ and $\alpha = 0.05$. Using (16), our hypothesis H_0 is accepted because $D_{\max} = 1.4 \times 10^{-3} > D$.

IV. Results Discussion

This section presents some analytical and simulation results for the considered cascaded RIS-aided system over Nakagami-m fading channels. The adopted system parameters are as follows: $n \in \{1, 2, 3\}$, $m \in \{2, 3, 4, 5, 6\}$, $\Omega \in \{1, 2, 3, 4, 5, 6\}$,

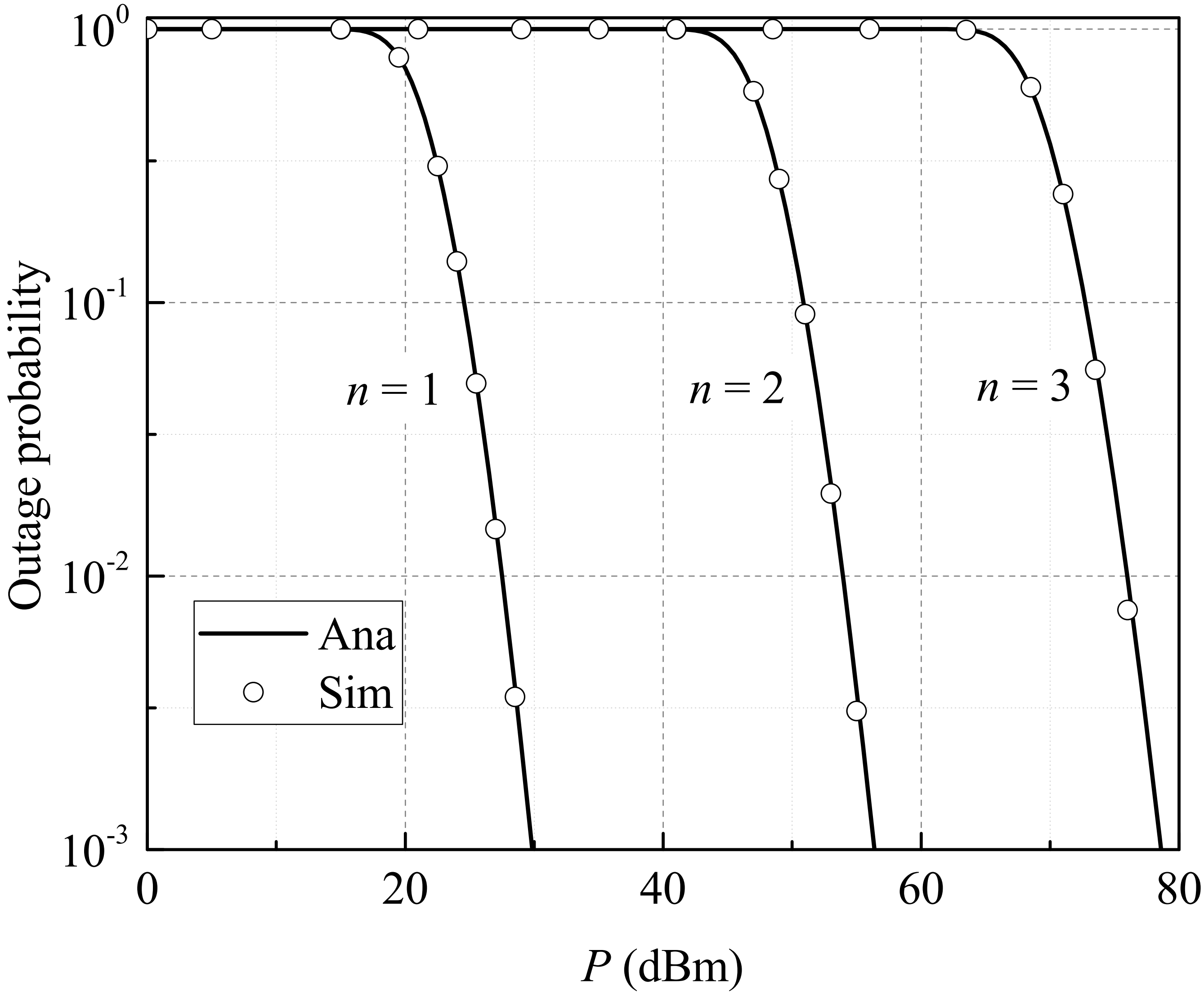

Fig. 3: OP for $m = 2$, $\Omega = 1$, and $N = 4$.

Table I: Obtained total errors for the results in Figs. 3, 4, and 6.

No. of RIS, n	**Total error**, Δ_n	**Shape parameter**, m	**Total error**, Δ_m	**No. of elements**, N	**Total error**, Δ_N
1	0.0186	2	0.0161	4	0.0171
		3	0.0117	8	0.0083
2	0.0529	4	0.0085	16	0.0044
		5	0.0058	32	0.0028
3	0.0919	6	0.0047	64	0.0025

$N \in \{4, 8, 16, 32, 64\}$, $\left|\eta_{\omega}^{R_t}\right| = 1$ [18], $\tau = 3$, $R_{\text{th}} = 2$ bits/s/Hz, $\sigma_R^2 = -70$ dBm, and $Q = 4$. The total e2e distance is evenly[3] divided with respect to the given number of RISs. It is important to highlight that all presented figures demonstrate a good agreement between the analytical and numerical results.

Fig. 3 depicts the OP results obtained for the system with different numbers of RISs between T and R. In the proposed system

[3]Please note that the RIS localization problem is out of scope of this paper. However, the considered system model is still practical since, in dense urban areas, the direct link through single RIS and near-end RIS placement are not always possible, thus, cascaded RISs are used to bypass the obstacles.

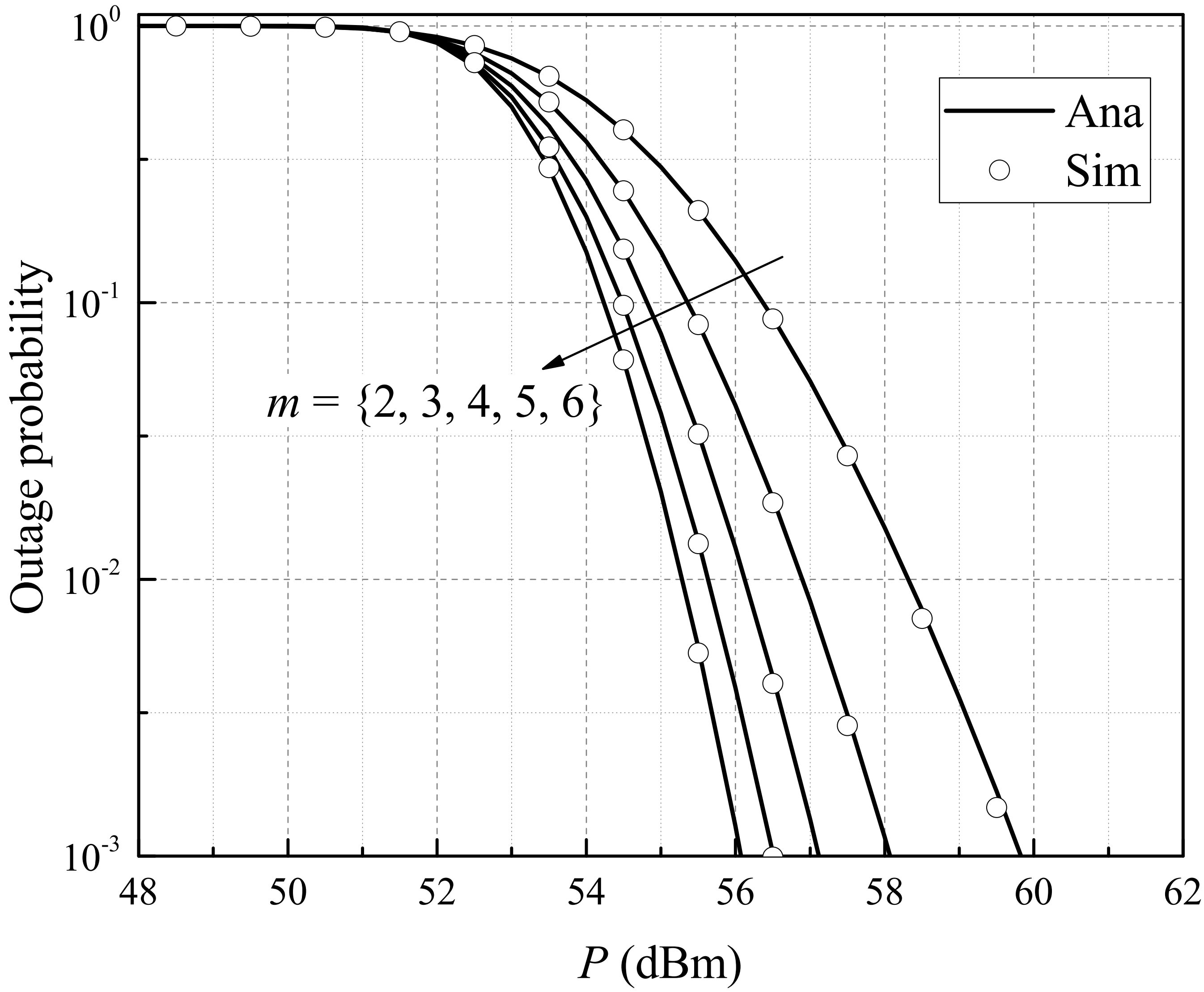

Fig. 4: OP for $n = 2$, $N = 8$, and $\Omega = 1$.

model, we set $m = 2$, $\Omega = 1$, $N = 4$, and $d_{\mathrm{TR}} = 120$ m. One can observe that more transmit power is required for the case when $n = 3$ to achieve the same outage performance of 10^{-3} achievable for $n = 1$ and $n = 2$, i.e., 78.6 dBm vs. 29.9 dBm and 56.4 dBm, respectively. It can be explained by the fact that increasing the number of RISs between T and R, increases the path loss; thus, more power should be allocated to avoid an outage of message χ.

Fig. 4 illustrates the OP results for the m parameter of the Nakagami-m distribution by considering the following system specifications: $n = 2$, $N = 8$ and $\Omega = 1$. The OP of 10^{-3} is within $[56, 60]$ dBm interval for all m values. With increasing m, the outage performance improves by acquiring less power, i.e., $m = 4$, which is a better choice compared with the case when $m = 2$. This phenomenon is explained by the fact that the range of Nakagami-m distribution rises with increasing m parameter, which models the scenario with less power usage.

Fig. 5 shows the OP results plotted versus the rate threshold, R_{th}, for different values of Ω, with $n = 2$, $m = 2$, $N = 8$, and $P = 50$ dBm. From the results, it can be observed that the OP metric deteriorates as the rate threshold increases for all Ω, as

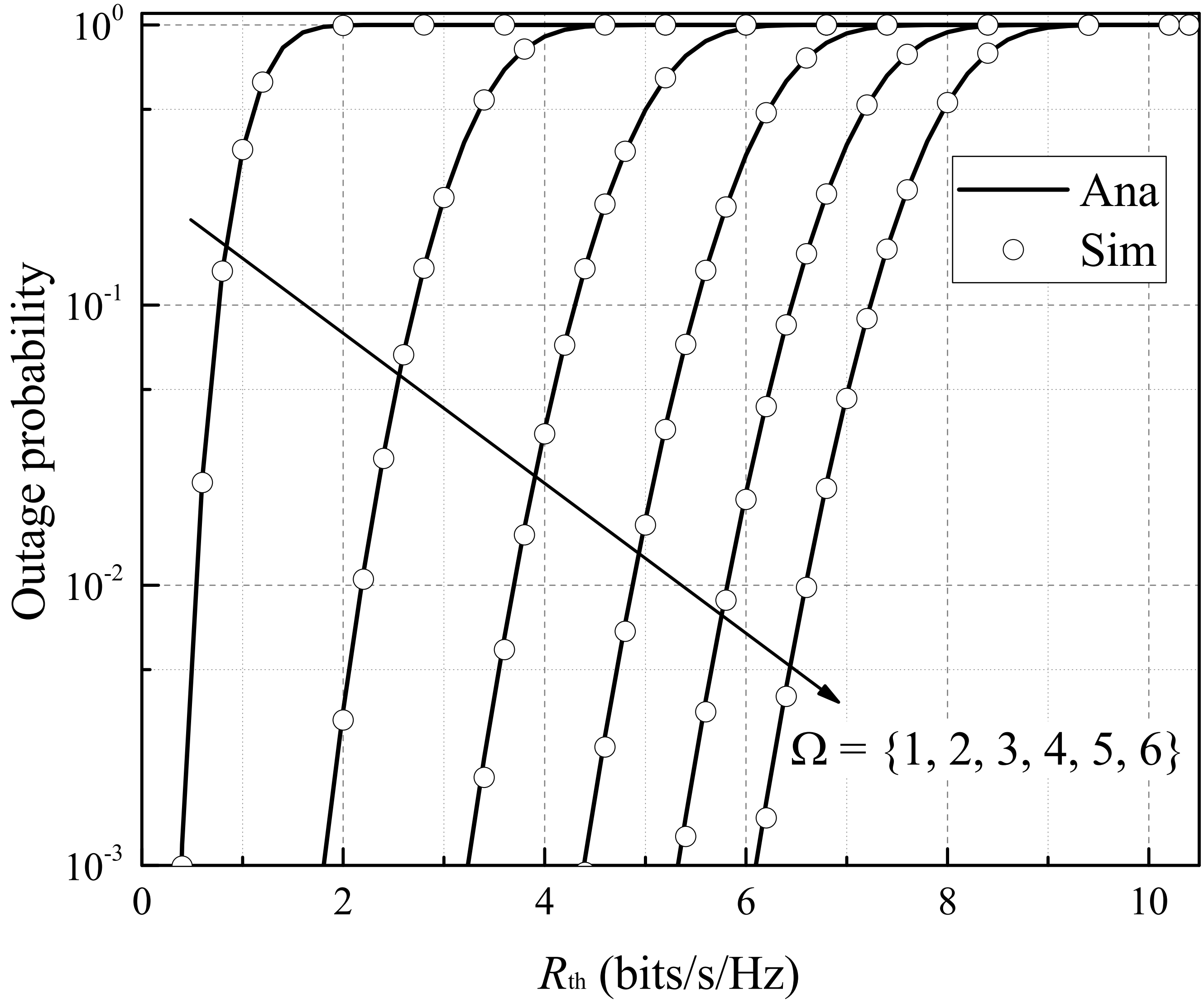

Fig. 5: OP for $n = 1$, $N = 8$, and $m = 2$ when $P = 50$ dBm.

expected. The full outage events for $\Omega = 1$ and $\Omega = 6$ are obtained at 1.8 bits/s/Hz and 9.2 bits/s/Hz, which means that the increase of Ω leads to better OP, which can be explained by the fact that Ω is responsible for the average channel power.

In Fig. 6, the OP performance is built as a function of the transmit power P for different values of N, when $n = 1$, $m = 2$, and $\Omega = 1$. From the figure, it is seen that the OP performance improves by increasing the number of passive elements per RIS, N. This is because the more passive elements are involved in the reflection, the more signal replicas get directed toward the destination, resulting in the amplification gain; thus, the transmission power needed for data transfer can be decreased. The OP of 10^{-3} is achieved at $P = \{4, 11, 18, 26, 35\}$ dBm for $N = \{4, 8, 16, 32, 64\}$, respectively, with approximately 8 dBm gap.

The KS goodness-of-fit test confirms the validity of the approximation method presented in this work by accepting the hypothesis that PDF of RV Z follows Gamma distribution. To double check the validity of the approximation, total errors for results obtained in Figs. 3, 4, and 6 are calculated and illustrated in Table I. It is seen from the table that increasing n leads to the growth of the total approximation error. This is because the number of RVs that are needed to be approximated is directly

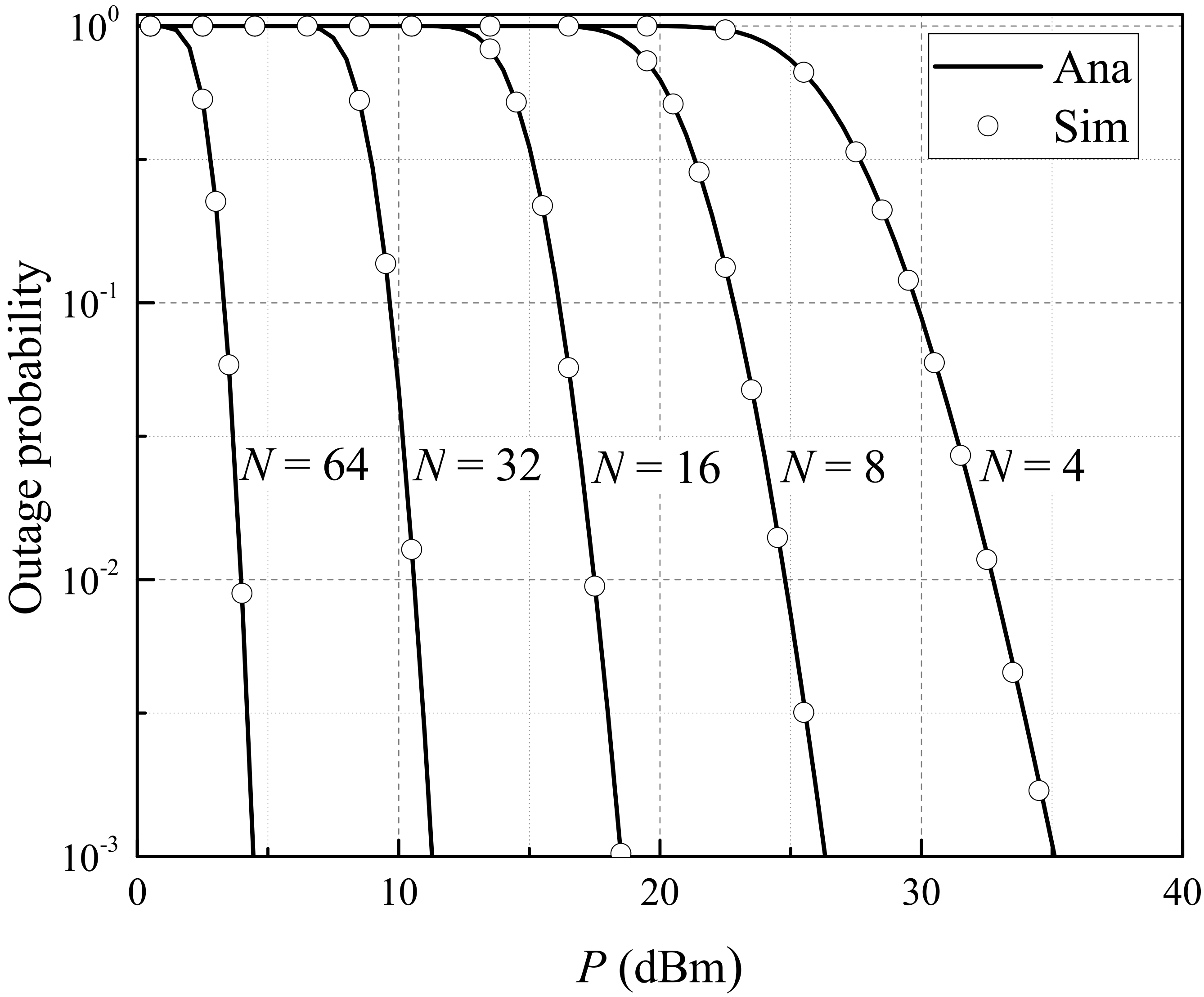

Fig. 6: OP for $n = 1$, $m = 2$, and $\Omega = 1$.

proportional to the number of RISs. With increasing m, the approximation error is decreased from 1.6×10^{-2} to 0.5×10^{-2}. Furthermore, the approximation errors decrease from 1.7×10^{-2} to 0.25×10^{-2} with increasing N. Among all these three parameters, n has the highest approximation error equal to 9×10^{-2} when $n = 3$. This is because the correlation between RVs appears for $n > 1$, which needs an additional approximation stage to acquire the Gamma parameters.

V. Conclusion

This paper presented a novel approximation method to evaluate the channel performance of cascaded RIS-aided wireless networks with phase errors over Nakagami-m fading channels. The obtained analytical results show consistency with the corresponding Monte Carlo simulations, which validates the correctness of the derived closed-form OP expressions. It was deduced that increasing m, Ω, and N parameters improves the outage performance; however, increasing n leads to the degradation of the system performance. The KS goodness-of-fit test and total approximation errors show that the Gamma approximation well fits the actual distribution of the end-to-end channel of the system.

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