Why Did Thomas Harriot Invent Binary?

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From the early eighteenth century onward, primacy for the invention of binary numeration and arithmetic was almost universally credited to the German polymath Gottfried Wilhelm Leibniz (1646–1716) (see, for example, [5, p. 335] and [10, p. 74]). Then, in 1922, Frank Vigor Morley (1899–1980) noted that an unpublished manuscript of the English mathematician, astronomer, and alchemist Thomas Harriot (1560–1621) contained the numbers 1 to 8 in binary. Morley's only comment was that this foray into binary was "certainly prior to the usual dates given for binary numeration" [6, p. 65]. Almost thirty years later, John William Shirley (1908–1988) published reproductions of two of Harriot's undated manuscript pages, which, he claimed, showed that Harriot had invented binary numeration "nearly a century before Leibniz's time" [7, p. 452]. But while Shirley correctly asserted that Harriot had invented binary numeration, he made no attempt to explain how or when Harriot had done so. Curiously, few since Shirley's time have attempted to answer these questions, despite their obvious importance. After all, Harriot was, as far as we know, the first to invent binary. Accordingly, answering the *how* and *when* questions about Harriot's invention of binary is the aim of this short paper.

The story begins with the weighing experiments Harriot conducted intermittently between 1601 and 1605. Some of these were simply experiments to determine the weights of different substances in a measuring glass, such as claret wine, seck (i.e., sack, a fortified wine), and canary wine (see [3, Harriot, Add. Mss. 6788, 176r]), while other experiments were intended to determine the specific gravity, that is, the relative density, of a variety of substances.[1]

Here are three results from Harriot's experiments [3, Harriot, Add. Mss. 6788, 176r]:

Claret wine	14Ꜿ	$\frac{1}{2}$	0	$\frac{1}{8}$	0	24g
Seck	14	$\frac{1}{2}$	0	$\frac{1}{8}$	$\frac{1}{16}$	6 gr.
Canary wine	14	$\frac{1}{2}$	$\frac{1}{4}$	0	0	24 gr.

Harriot's method of recording his measurements is the key to his invention of binary and so deserves some comment. Using the troy system of measurement, he recorded the weight of each substance by decomposing it into ounces (sometimes using the old symbol for ounces, Ꜿ, a variant of the more common ℥), then $\frac{1}{2}$ ounce, $\frac{1}{4}$ ounce, $\frac{1}{8}$ ounce, $\frac{1}{16}$ ounce, and finally grains. Since a troy ounce is composed of 480 grains, the various weights of his scale have the following grain values:

$$1\text{oz} = 480 \text{ grains}$$
$$\frac{1}{2}\text{ oz} = 240 \text{ grains}$$
$$\frac{1}{4}\text{ oz} = 120 \text{ grains}$$
$$\frac{1}{8}\text{ oz} = 60 \text{ grains}$$
$$\frac{1}{16}\text{ oz} = 30 \text{ grains}$$

Together, the four part-ounce weights are 30 grains shy of one ounce, and indeed, in all of Harriot's experiments, the measurement of grains never goes above 30. With this in mind, let us look again at his record of weighing claret wine:

Claret wine	14Ꜿ	$\frac{1}{2}$	0	$\frac{1}{8}$	0	24g

The first number (14) is ounces, the final number (24) grains, and the numbers in between refer to part ounces—the $\frac{1}{2}$ in the $\frac{1}{2}$ ounce position indicating that the $\frac{1}{2}$ ounce weight was used, the 0 in the $\frac{1}{4}$ ounce position indicating that the $\frac{1}{4}$ ounce weight was not used, etc.[2]

With regard to Harriot's invention of binary, of particular interest is one manuscript (reproduced below) that contains a record of a weighing experiment at the top, and examples of binary notation and arithmetic at the bottom. Here are the calculations from the weighing experiment, which was concerned with finding the difference in capacity between two measuring glasses [3, Harriot, Add. Mss. 6788, 244v]:

[1]In the latter case, Harriot works out the relative density of materials such as brown mortar, copper ore, and lapis calaminaris (calamine) by the Archimedean method of weighing them first in air and then in water, then working out the difference between the two weights before dividing the weight in air by the difference to determine the specific gravity (for more details on Harriot's experiments and specific gravity, see [2]).

[2]Clucas claims that Harriot's "weighing is done to the highest degree of accuracy in ounces, drachms, scruples and grains" [1, p. 124]. But this is clearly not the case. In the troy system, one ounce is equivalent to 8 drachms, and each drachm in turn equivalent to 3 scruples (with each scruple worth 20 grains). Yet Harriot's measurements divide the ounce into 16, not 8 (drachms) or 24 (scruples), indicating that the weights he was using were simply $\frac{1}{2}$ ounce, $\frac{1}{4}$ ounce, $\frac{1}{8}$ ounce, etc.

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	troz.					
A. Rounde measuring glasse weyeth dry	3℥	$\frac{1}{2}$	0	$\frac{1}{8}$	$\frac{1}{16}$	+ 21 gr.
B. The other rounde measure	3℥	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	+5 gr.
A. Glasse & water	11℥	0	0	$\frac{1}{8}$	0	+ 28 gr.
	3	$\frac{1}{2}$	0	$\frac{1}{8}$	$\frac{1}{16}$	+ 21
Water	7	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	+ 7 gr.
B. Glasse & water	10℥	$\frac{1}{2}$	0	0	$\frac{1}{16}$	+ 10 gr.
	3	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	+ 5
Water	7	0	0	$\frac{1}{8}$	0	5
diff.			$\frac{1}{4}$	0	$\frac{1}{16}$	+ 2 gr.

Note here that "troz" stands for "troy ounce." Underneath all this, Harriot sketched a table of the decimal numbers 1 to 16 in binary notation and worked out three examples of multiplication in binary: 109 × 109 = 11881, 13 × 13 = 169, and 13 × 3 = 39; see Figure 1.

So far as I know, the only person who has attempted to explain Harriot's transition from weighing experiments to the invention of binary is Donald E. Knuth, who writes:

> Clearly he [Harriot] was using a balance scale with half-pound, quarter-pound, etc., weights; such a subtraction was undoubtedly a natural thing to do. Now comes the flash of insight: he realized that he was essentially doing a calculation with radix 2, and he abstracted the situation [4, p. 241].

While Knuth is mistaken about the size of weights used, apparently missing the abbreviation "troz" (= troy ounce) and taking the glyph ℥ to refer to pound rather than ounce, his suggestion regarding Harriot's "flash of insight" looks plausible. But it is possible to go further, because it is unlikely that Harriot hit upon binary notation simply because he was using weights in a power-of-2 ratio, something that was a well-established practice at the time. Equally if not more important was the fact that he *recorded the measurements made with these weights in a power-of-2 ratio too*. For when recording the weights of the various part-ounce measures, Harriot used a rudimentary form of positional notation, in which for every position he put down either the full place value or 0, depending on whether or not the weight in question had been used. Hence when weighing the first "glass and water," Harriot's result is equivalent to:

Position:	Ounces	$\frac{1}{2}$ ounces	$\frac{1}{4}$ ounces	$\frac{1}{8}$ ounces	$\frac{1}{16}$ ounces	Grains
Harriot's measurement:	11	0	0	$\frac{1}{8}$	0	28

Or indeed, if we just focus on the part-ounces and express them as powers of 2:

2^{-1} ounce	2^{-2} ounce	2^{-3} ounce	2^{-4} ounce
0	0	2^{-3} ounce	0

From such a method of recording weights in a power-of-2 ratio, it is but a very small step to binary notation, in which, instead of noting in each position either 0 or the full place value, one simply puts down either 0 or 1 depending on whether or not the weight in question was needed. Harriot's invention of binary therefore owed at least as much to his own idiosyncratic form of positional notation for recording part-ounce weights as it did to his use of those weights.

One oddity with Harriot's "flash of insight" is that it did not lead him to binary expansions of reciprocals, which is what his notation is closest to. That is, he did not represent $\frac{1}{2}$ ounce as [0].1, $\frac{1}{4}$ ounce as [0].01, $\frac{1}{8}$ ounce as [0.]001, or $\frac{1}{16}$ ounce as [0].0001. Instead, he continued to use decimal fractions to record the part-ounce weights in his weighing experiments. So although binary was an outgrowth of Harriot's idiosyncratic method of recording part-ounce weights, at no point did he use binary to record these weights. From that we may surmise that he did not think binary notation offered greater convenience or clarity than his own method of recording part-ounce weights.

Yet Harriot was sufficiently intrigued by his new number system to explore it over a further four manuscript pages, working out how to do three of the four basic arithmetic operations (all but division) in binary notation. On one sheet, Harriot wrote examples of binary addition (equivalent to 59 + 119 = 178 and 55 + 114 = 169) and subtraction (equivalent to 178 − 59 = 119 and 169 − 55 = 114) and the same example of multiplication in binary (109 × 109) as above, this time solved in two different ways (Harriot, Add. Mss. 6786, 347r). On a different sheet, he converted 1101101_2 to 109, calling the process "reduction," and then worked through the reciprocal process, called "conversion," of 109 to 1101101_2 (Harriot, Add. Mss. 6786, 346v). On yet another sheet, he jotted down a table of 0 to 16 in binary, a simple binary sum: 100000 + [0]1[00]1[0] = 110010 (i.e., 32 + 19 = 51), and another example of multiplication, 101 × 111 = 100011 (i.e., 5 × 7 = 35) (Harriot, Add. Mss. 6782, 247r). And on a different sheet again (reproduced below), he drew a table of 0 to 16 in binary, another with the binary equivalents of 1, 2, 4, 8, 16, 32, and 64, gave several examples of multiplication in binary (equivalent to 3 × 3 = 9; 7 × 7 = 49; and 45 × 11 = 495), and produced a simple algebraic representation of the first few terms of the powers of 2 geometric sequence (see Figure 2):

b.	a.	$\frac{aa}{b}$	$\frac{aaa}{bb}$	$\frac{aaaa}{bbb}$
1.	2.	4.	8.	16.
1	2	$\frac{2[\times]2}{1}$	$\frac{2[\times]2[\times]2}{1[\times]1}$	$\frac{2[\times]2[\times]2[\times]2}{1[\times]1[\times]1}$

And on a further sheet, Harriot employed a form of binary reckoning using repeated squaring, combining this with floating-point interval arithmetic, in order to calculate the upper and lower bounds of 2^{28262} [3, Harriot, Add. Mss. 6786, 243v]; for further details see [4, pp. 242–243]). The whole of Harriot's work on binary is captured on the handful of manuscript pages described in this paper.

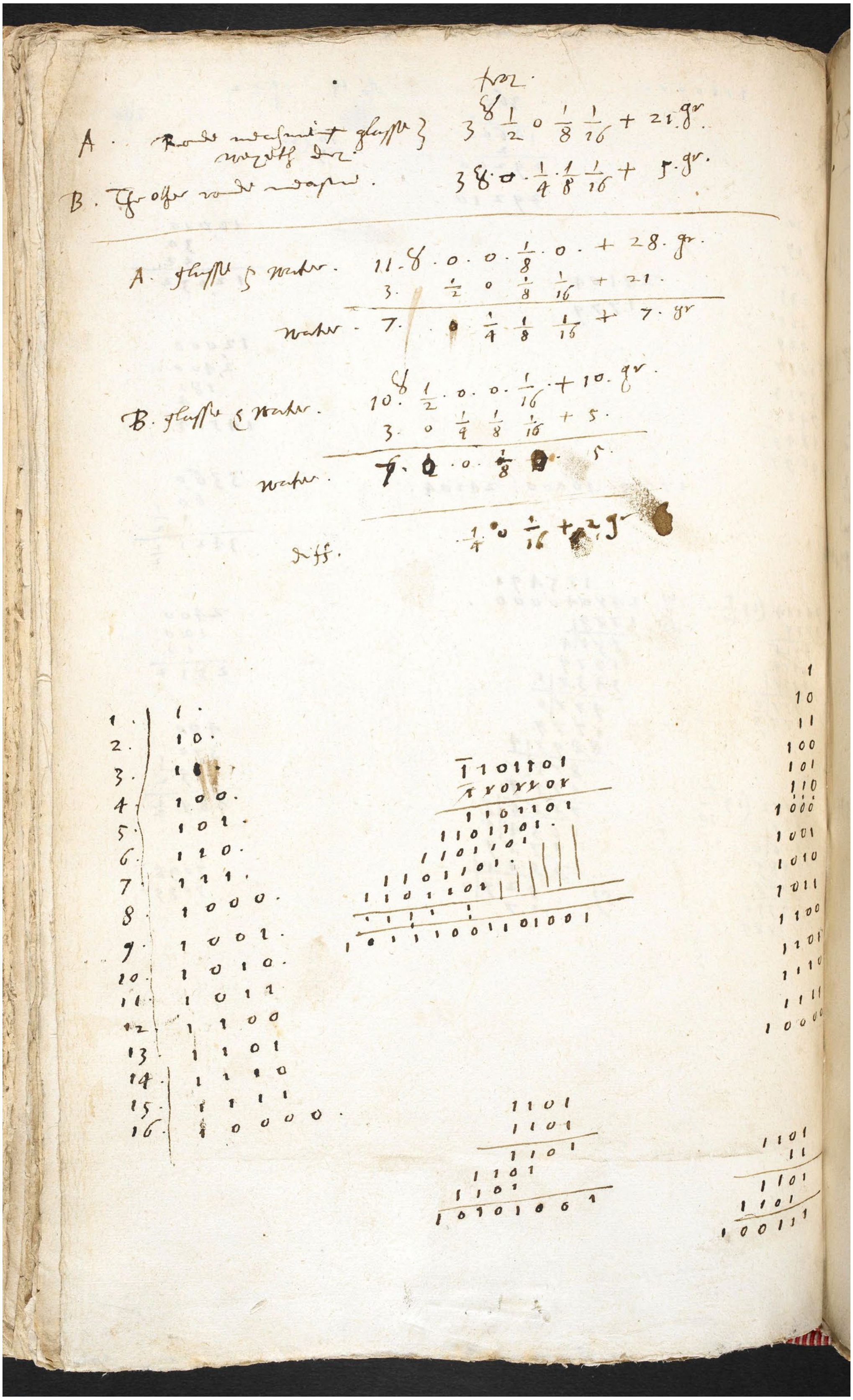

Figure 1 Thomas Harriot's binary multiplication [3, Harriot, Add. Mss. 6788, 244v]. Courtesy of the British Library Board.

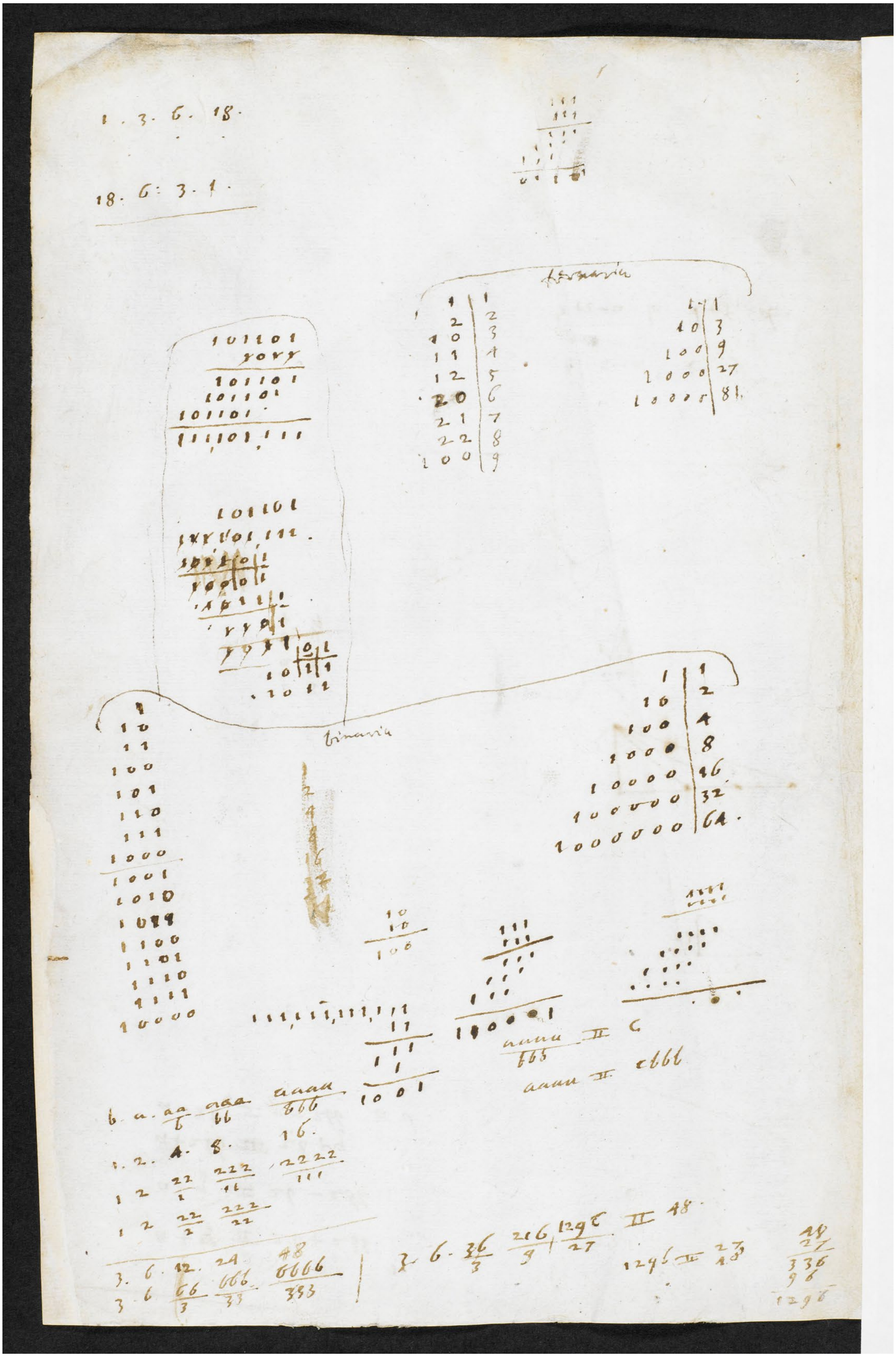

Figure 2 A page of Thomas Harriot's calculations. In the bottom left-hand corner can be seen the calculation of first few terms of the powers of two geometric series reproduced in the text [3, Harriot, Add. Mss. 6786, 516v]. Courtesy of the British Library Board.

Now that we know how Harriot arrived at binary, it remains to ask when he did so. Although Harriot often recorded the date on his manuscripts, unfortunately he did not do so on any of the manuscript pages featuring binary numeration. As such, it is not possible to determine the exact date of his invention, though it can be narrowed down, as we shall see. Knuth conjectured that "Harriot invented binary arithmetic one day in 1604 or 1605" on the grounds that the manuscript containing a weighing experiment together with binary numeration and arithmetic is catalogued between one dated June 1605 and another dated July 1604 [4, p. 241].

Yet as Knuth concedes, Harriot's manuscripts are not in order (as should be clear enough from the fact that one dated July 1604 follows one dated June 1605), so affixing a date to one manuscript based on its position in the catalogue is problematic. As noted at the outset, Harriot's weighing experiments began in 1601, indeed on September 22, 1601, and already in manuscripts from that year he was using his idiosyncratic method of recording part-ounce weights (see [3, Harriot, Add. Mss. 6788 172r] and [176r]) that led to his thinking of binary, so it cannot be ruled out that binary was invented as early as September 1601. The latest date for Harriot's invention of binary is probably November 1605, at which time Harriot's patron, Henry Percy, 9th Earl of Northumberland (1564–1632), was imprisoned in connection with the Gunpowder Plot.

Around this time, Harriot, too, fell under suspicion of being involved in the plot and was imprisoned for a number of weeks before successfully pleading for his freedom. After his release, he did not resume his weighing experiments or, we may suppose, the investigations into binary that arose from them. This is perhaps unsurprising. Whereas Leibniz saw a practical advantage in using binary notation to illustrate problems and theorems involving the powers of 2 geometric sequence (see [8]), Harriot appears to have treated binary as little more than a curiosity with no practical value.

Nevertheless, Harriot's invention of binary is a startling achievement when you realize that the idea of exploring nondecimal number bases, as opposed to tallying systems, was not commonplace in the seventeenth century. While counting in fives, twelves, or twenties was well understood and widely practiced, the idea of numbering in bases other than 10 was not. The modern idea of a base for a positional numbering system was still coalescing, but it was conceived by a few, with Harriot perhaps the first. Unfortunately, despite his great insight, Harriot did not publish any of his work on binary, and his manuscripts remained unpublished until quite recently, being scanned and put online in 2012–2015. Although Harriot rightly deserves the accolade of inventing binary many decades before Leibniz, his work on it remained unknown until 1922, and so did not influence Leibniz or anyone else, nor did it play any part in the adoption of binary as computer arithmetic in the 1930s (see [9]). That is one accolade that still belongs to Leibniz.

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