Empirical Comparison of Hazard Models in Predicting SMEs Failure

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Abstract

This study aims to shed light on the debate concerning the choice between discrete-time and continuous-time hazard models in making bankruptcy or any binary prediction using interval censored data. Building on the theoretical suggestions from various disciplines, we empirically compare widely used discrete-time hazard models (with logit and clog-log links) and continuous-time Cox Proportional Hazards (CPH) model in predicting bankruptcy and financial distress of the United States Small and Medium-Sized Enterprises (SMEs). Consistent with the theoretical arguments, we report that discrete-time hazard models are superior to continuous-time CPH model in making binary predictions using interval censored data. Moreover, hazard models developed using failure definition based jointly on bankruptcy laws and firms' financial health exhibit superior goodness of fit and classification measures, in comparison to models that employ failure definition based either on bankruptcy laws or firms' financial health.

Keywords: Bankruptcy; SMEs; Discrete Hazard Models; Cox Proportional Hazard; Financial Distress

JEL Classification Codes: G33; C25; C41; C53

1. Introduction

Survival or *event history analysis* is the umbrella term for the set of statistical tools that are used to answer questions related to timing and the occurrence of an event of interest. It has traditionally been applied in the field of medical research where duration until death or duration until appearance or reappearance of a disease is usually the event of interest, hence the name Survival Analysis. Survival analysis has been prominently applied in other disciplines such as engineering (known as *reliability theory*), economics (known as *duration analysis* or *duration modelling*), sociology (known as *event history analysis*), and political science. The variable of primary interest in survival analysis is the time to an event, which in our application is the incorporation of a firm to bankruptcy filing or some other financial distress event. A firm is said to be at risk of the event (bankruptcy/financial distress) after the initial event (i.e. incorporation) has taken place. Alternatively, the response variable can be viewed as the duration of time that a firm spent in a healthy state before transition to a bankruptcy state occurs. Survival analysis demands special methods primarily due to *right-censoring*, where the time to the occurrence of an event is unknown for some subjects because the event of interest has not taken place by the end of the sampling or observation period. These statistical models examine the hazard rate, which is defined as the conditional probability that an event of interest occurs within a given time interval.

The growing popularity of the use of *hazard* models to predict corporate failure has motivated us to undertake this empirical study. Since the seminal work of Shumway (2001), the use of the hazard rate modelling technique has become a popular methodology in bankruptcy prediction studies (see among others Chava and Jarrow 2004; Campbell *et al.* 2008; Gupta *et al.* 2014). However, this growing popularity of hazard models in bankruptcy prediction seems to be trend or momentum driven, rather than based on a strong theoretical underpinning. Although the superiority of hazard models in predicting binary outcomes is well documented in the literature (see among others Beck *et al.* 1998; Shumway 2001; Allison 2014), its recent use in predicting corporate failure does not appropriately acknowledge fundamental concerns associated with survival analysis. This is because the vast majority of existing studies suffer from at least one of the following issues: (i) inappropriate or no explanation behind their choice of *discrete-time* or *continuous-time* hazard models (e.g. Bharath and Shumway 2008); (ii) inappropriate or no specification of baseline hazard rate while using discrete-time hazard models (e.g. Nam *et al.* 2008; Gupta *et al.* 2014); (iii) no test of proportional hazards assumption when using continuous-time *Extended Cox* models with time-independent covariates (e.g. Liang and Park 2010); (iv) no discussion on *frailty* and *recurrent* events (e.g. Shumway 2001); and (v) no explanation of how they dealt with the issue of *delayed entry* (e.g. Gupta, Wilson, *et al.* 2014a).

Thus, we contribute to the literature by presenting a review and analysis of popular hazard models in predicting corporate failure, taking into account the fundamental concerns discussed above. Since we often find continuous-time hazard models are being developed using discrete-time data (e.g. Bauer and Agarwal 2014), we also contribute to the literature by documenting empirical comparison of discrete-time and continuous-time hazard models. Multivariate hazard prediction models are developed using financial ratios obtained from income statements and balance sheets. The criteria for introducing covariates in multivariate models vary across scientific disciplines, and with underlying theoretical or atheoretical beliefs or assumptions. Traditionally, the vast majority of popular bankruptcy prediction studies report atheoretical approaches toward selection of covariates and developing multivariate prediction models (see among others Altman 1968; Ohlson 1980; Shumway 2001; Altman and Sabato 2007; Campbell *et al.* 2008; Korol 2013). The plausible theoretical angle that we may reason is the effect of any given covariate on firms' default likelihood. For instance, we may reason that a firm with a higher proportion of debt in its capital structure is more likely to default than an almost identical firm with a lower amount of debt in its capital structure. Thus an increasing value of the financial ratio debt/total assets enhances firms' default likelihood, and vice-versa. Similar analogies may be deduced for any possible covariate. However, with the vast number of financial ratios (or non-financial covariates) available, and no proper theory in place, scholars often select covariates that are either advocated by popular studies or that suit their empirical research. Thus, in line with the discussion in Hosmer Jr *et al.* (2013) on multivariate model building strategy, we propose an atheoretical econometric based model building strategy, based on covariates' *Average Marginal Effects* (AME) and their *inter-temporal discrimination* ability. The reasoning behind this approach is that a covariate with a higher value of AME induces higher change in the default probability, and thus should be given priority in the covariate selection process compared with one with a lower value of AME. In addition, the earlier the warning signals, the longer the preparation time period for the forthcoming crisis. Therefore, the covariate with forecasting ability over a longer horizon should be preferred to the covariate with the shorter horizon.

We also contribute to the fast growing literature on Small and Medium-Sized Enterprises (SMEs) bankruptcy by comparing SMEs failure prediction models developed using different definitions of default events. In particular, our comparison involves default definitions based on: (i) legal consequences (Chapter 7/11 bankruptcy filings in the United States); (ii) financial health, as discussed in Pindado *et al.* (2008) and Keasey *et al.* (2014); and (iii) both legal and financial health of an SME. Our legal definition classifies a firm as default when it files for bankruptcy under the legal bankruptcy law (*Event 1*), which is usually Chapter 7/11 in the United States. Our second definition follows the financial distress definition provided by Keasey *et al.* (2014) and classifies a firm as financially distressed if it reports earnings of less than its financial expenses for two consecutive years, has net worth/total debt less than one, and experiences negative growth in net worth for the same two consecutive time periods (*Event 2*). The definition of SMEs default that we propose combines *Event 1* and *Event 2*, and classifies a firm as default when it files for legal bankruptcy alongside financial distress (*Event 3*). The detailed analogy behind this default definition is discussed in Section 3. A recent study by Lin *et al.* (2012) on SMEs default prediction follows a similar line, but predicts SMEs default using different definitions of financial distress.

Our research differentiates itself from Lin *et al* (2012) in several respects. First, we present our analysis based on a sample of US SMEs, whereas their study employs sample of UK SMEs. They use static binary logistic regression to establish their empirical validations, while we use superior dynamic hazard models. Finally, they use flow-based (earnings/interest payable) and stock-based (1 – total liabilities/total assets) insolvency indicators to group the firms in their sample into four groups of financial health (which correspond to their four different definitions of financial distress), while our distress definitions are more realistic and arguably superior (see Tinoco and Wilson (2013) and Keasey *et al.* (2014) for relevant discussion).

Our test results, obtained by employing firm-year observations of the US SMEs, provide convincing evidence. To establish the empirical validation, we calculate a wide range of financial ratios to gauge a firm's performance from liquidity, solvency, profitability, leverage, activity, growth and financing dimensions. Then, following the suggestion of Hosmer Jr *et al.* (2013), we use appropriate strategies to narrow down our list of covariates, and develop multivariate models. First, in line with the theoretical arguments, the discrete-time duration-dependent hazard models that we develop with logit and complementary log-log (clog-log) links provide superior model fit compared with continuous-time Extended Cox models, as they have much lower Akaike Information Criterion (AIC) values than Cox models across all default definitions. However, all three econometric specifications lead to almost identical within sample and hold-out sample classification performance, thus one might be tempted to be indifferent to the choice of hazard specification. Moreover, if the event of interest is not duration dependent (i.e. some functional form of time or time dummies are not significant in the multivariate model), with hazard rates being invariant or vary mildly across different time periods, then the complications of hazard models may not be worthwhile considering the marginal gain one would obtain using such models. But in case of duration dependence, we suggest the use of discrete-time hazard model with logit link to model interval censored or discrete-time data since it produces minimum overall error of prediction models developed along with several other benefits. Like, it is well understood by researchers and thus does not require learning new statistical techniques; can be estimated with any statistical software package; and most importantly, can be extended easily in a variety of ways to suit one's purpose. While developing our multivariate models we find that, in the presence of financial covariates, about 90% of the time dummies that we use as baseline hazard specifications are insignificant, with very high values of standard errors. Thus we follow Shumway (2001) and use the natural logarithm of firms' annual age (variable AGE) as the baseline hazard specification. This specification is significant in most of our multivariate hazard models, but this objective can easily be achieved by developing regression models using panel logistic regression techniques that use some functional form of time to capture any duration dependency. Although Shumway (2001) argues that hazard models are superior to competing static models, variable AGE in his multivariate models are insignificant, so how can it be used reliably to predict duration specific hazard rates when this is primarily why hazard models are used? Unlike areas such as medicine or health economics, duration specific prediction of hazard rates is not common in bankruptcy or financial distress prediction, thus we do not see any real need for hazard models if similar objectives can be achieved using much simpler panel logistic regression that controls for any duration dependencies.

Second, the default definition that we propose (*Event 3*) performs best in classifying defaulted firms. A default definition based on firms' financial health is superior to default definition based on legal consequences, while a default definition that considers both legal consequence and firms' financial health is best. These differences in classification performance emphasise the fact that not all firms that file for legal bankruptcy do so purely due to financial difficulties - a significant number of firms consider it a planned exit strategy (Bates 2005). Furthermore, we also test the efficiency and stability of covariates suggested by the most popular study on US SMEs bankruptcy prediction, by Altman and Sabato (2007). Based on our test results, we conclude that the covariates they suggest fail to exhibit satisfactory discriminatory power across all default definitions and up to three lagged time periods, and find several other financial ratios which are better performers. Their suggestion might be biased due to their sample selection process, while our study employs near population data of US SMEs.

We expect this study to be a useful guide to academic scholars and practitioners interested in building hazard models for making binary predictions. The rest of this paper is organized as follows: Section 2 discusses common concerns regarding the use of hazard models and how they can be rectified; Section 3 discusses various default definitions that we consider in our study; Section 4 provides detailed discussion of our dataset, choice of covariates and methodology; in Section 5 we report and discuss our empirical findings and, finally, Section 6 concludes our findings.

2. Common Concerns of Hazard Models

2.1 Discrete-time vs Continuous-time Hazard Model

In bankruptcy studies, the *survival time*, which is the duration or time-to-event, is generally measured in quarterly or annual units, and the time scale used may be discrete or continuous. If the time of occurrence of an event is precisely known, *continuous-time* hazard models are employed, otherwise a *discrete-time* hazard model is an appropriate choice when the event takes place within a given time interval and the precise time is unknown (Rabe-Hesketh and Skrondal 2012). Thus, from a theoretical point of view discrete-time hazard models are an appropriate choice, as a firm may file for bankruptcy anytime within a quarter or a year. However, in both models the probability of occurrence of an event at time t is being modelled. The dependent variable in a continuous-time model is the *hazard rate*, but in a discrete-time model it is the *odds ratio* (if modelling is done using standard logit/probit models). However, recent studies do not provide appropriate explanation behind their choice between discrete-time (eg. Campbell *et al.* 2008; Gupta *et al.* 2014) and continuous-time model (eg. Bharath and Shumway 2008; Chen and Hill 2013). Furthermore, the required precision of the timing to an event is significantly dependent on the research question and data restrictions. Studies also suggest that results obtained from continuous-time and discrete-time methods are virtually identical in most models (Yamaguchi 1991; Allison 2014). Having said that, the performance of a bankruptcy prediction model is evaluated based on some non-parametric classification measures such as *misclassification matrix*, or area under *receiver operating characteristic* (ROC) *curve* (see Anderson (2007) for further details). Despite the theoretical differences between continuous-time and discrete-time models, if they lead to identical classification performance then this theoretical difference is of no practical relevance. Thus, we compare the classification performance of most widely used discrete-time duration-dependent hazard models (see among others Shumway 2001, Nam *et al.* 2008) with the most popular continuous-time duration-dependent *Cox* model (see among others Bharath and Shumway 2008; Chen and Hill 2013) to find any differences in their classification performance. If there are no differences, then the Cox model is a reasonable and convenient choice (although discrete-time hazard model is more appropriate), as it does not require any *baseline hazard* specification unlike discrete-time models (see Rabe-Hesketh and Skrondal 2012).

2.2 Specification of Baseline Hazard Rate

The final step before estimation of discrete-time hazard models is the specification of baseline hazard function, i.e. the hazard rate when all the covariates are set to zero. The baseline hazard can be estimated using time dummies (Beck *et al.* 1998) or some other functional form of time (see Jenkins (2005) for details). However, recent studies seem to distort this idea of baseline hazard and have established their own version of baseline hazard that includes, for instance, macroeconomic variables (Nam *et al.* 2008) or insolvency risk (Gupta *et al.* 2014) in the baseline hazard function (it is more appropriate to acknowledge them as control variables). While several studies do not report any baseline hazard function in their discrete hazard model (see among others Campbell *et al.* 2008; Bauer and Agarwal 2014). Omitting baseline specification (e.g. time dummies, ln(age) etc.) is equivalent to assuming that the baseline hazard is constant and that the model is duration-independent. In light of the basic theory of survival analysis, this is inappropriate.

We address this misleading concern in this study and show the steps to be followed in specifying the baseline hazard function while developing a discrete hazard model. This can be done by defining time-varying covariates that bear functional relationships with survival times. Popular specifications are log(survival time), polynomial in survival time, fully non-parametric and piece-wise constant (Jenkins 2005). Duration-interval-specific dummy variables need to be created for specifying a fully non-parametric baseline hazard. The number of dummy variables needs to be equals to one less than the maximum survival time in the dataset. For instance, if the maximum survival time is fifty years, then forty nine dummy variables are required for model estimation (e.g. Beck *et al.* 1998). However, this method becomes cumbersome if the maximum survival time in the dataset is very high, as in the case of bankruptcy databases. A reasonably convenient alternative way of specifying the baseline hazard function is to use the piece-wise constant method. In this, the survival times are split into different time intervals that are assumed to exhibit constant hazard rate. However, one must note that if there exist time intervals or time dummies with no events then one must drop the relevant firm-time observation with no event from the estimation, otherwise the duration specific hazard rates cannot be estimated for these time intervals/dummies (see Jenkins 2005; Rabe-Hesketh and Skrondal 2012). Considering the estimation convenience, one might be tempted to use the piece-wise constant specification of baseline hazard rate. However, if the hazard curve shows frequent and continuous steep rises and falls, then fully non-parametric baseline hazard specification might be an appropriate choice.

2.3 Proportional Hazards Assumption for Cox Model

Studies which employ continuous-time Cox models are mostly silent on the critical test of proportional hazards (PH) assumptions for time-independent covariates (e.g. Liang and Park 2010). The PH assumption implies that the hazard rate of any particular subject is a constant proportion of the hazard rate of any other subject across time (Mills 2011). The violation of this assumption might lead to overestimation (the covariate violates this assumption and exhibits an increasing hazard ratio over time) or underestimation (the covariate violates this assumption and exhibits a decreasing hazard ratio over time) of hazard risk (Mills 2011). It also results in incorrect standard errors and a decrease in the power of significance tests (Box-Steffensmeier and Zorn 2002). The violation of PH assumption is a frequent phenomenon, and thus it should always be checked and reported in studies. That said, Allison (2010) warns that it is necessary to worry not only about the violation of the PH assumption, but also about other basic requirements such as incorporation of relevant explanatory variables. He also asserts that the violation of PH assumption is often covariate specific and excessive. Although all the covariates that we employ in this study are time-dependent, if one also employs time-independent covariates, then one should take cognizance of this serious and neglected concern and use appropriate methods to test, report and rectify any violation of the proportional hazards assumptions[1].

2.4 Frailty and Recurrent Events

Another highly neglected area of concern is *frailty* and *recurrent* events. Correlation of event time occurs when firms experiencing a default event belong to a particular cluster or group such as industry, geographic location or, in the case of recurrent events, where a firm experiences a default event more than once in its lifetime. In the United States (US), the Bankruptcy Reform Act of 1978 (Bankruptcy Code) governs the legal processes involved in dealing with corporate financial distress. It provides firms facing financial distress with a liquidation process (Chapter 7) or a reorganization process (Chapter 11)[2]. Chapter 7 leads to permanent shutdown of a financially distressed firm, while Chapter 11 aims at rehabilitation of financially distressed but economically viable firms. Hotchkiss (1995) examines 197 publicly traded firms that filed for Chapter 11 protection from 1979 to 1988 and later recovered from Chapter 11 as publicly traded firms. He reports that 40% of the firms continue to experience operating losses and 32% either restructure their debt or re-enter bankruptcy in the three years following the acceptance of reorganization plans. Thus *a firm may witness multiple distress events in its lifetime*. Given that these issues of clustering and recurrent events are an integral part of the real-life environment, they should be made an essential and standard part of contemporary event history analysis (see Box-Steffensmeier and De Boef (2006) and Mills (2011) for advanced discussion). The solution is to introduce a *frailty* term in the hazard models. *Frailty* is an unobserved random proportionality factor that modifies the hazard function to account for random effects, association and unobserved heterogeneity in hazard models (Mills 2011). The exclusion of a frailty term implicitly assumes that all firms are homogeneous, which implies that all the firms are prone to experience default in the same way, with the duration of defaults considered to be independent from one another. However, in reality some firms are more '*frail*' and thus have a higher likelihood of experiencing default. Therefore, our empirical analysis also accounts for this neglected concern while developing hazard models.

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[1] See Kleinbaum and Klein (2012) for detailed understanding about various tests of proportional hazards assumption for time-independent covariates. A Cox model with time-dependent covariates does not need to satisfy the proportional hazards assumption and is called an *Extended Cox* model. However, if the model employs both time-dependent and time-independent covariates, then PH assumption for time-independent covariates must be satisfied.

[2] Although the law provides other provisions, we consider only Chapter 11 and Chapter 7 as the vast majority of the financially distressed firms file for either of these two.

2.5 Delayed Entry

In time-to-event studies the origin of time scale is an important consideration, as at this point in time a firm becomes at risk of experiencing the financial distress event. This is firms' incorporation date in bankruptcy studies. However, in cases where incorporation dates are unknown, firms' age or the earliest available date of information in the databases serves as useful proxy. A firm's incorporation date may differ from the start date of sampling period; as a result the time it becomes at risk does not coincide with the start of the sampling period. This leads to *delayed entry*, which means that a firm becomes at risk before entering the study. Thus the appropriate likelihood contribution under delayed entry is obtained by allowing the firm to start contributing observations from time period $t_k + 1$ and discarding prior time periods (see section 14.2.6 of Rabe-Hesketh and Skrondal 2012). Here, t_k is the time period for which a firm has already been at risk when it enters the research study.

3. Different Default Definitions for SMEs

Traditionally, the debate about financial distress has been rooted in the literature pertaining to firms' capital structure, with particular relevance to the cost of financial distress (see Altman and Hotchkiss (2006) for an overview). However, current studies also highlight its growing importance in the context of modelling firms' insolvency hazards (e.g. Keasey *et al.* 2014). Recent literature pertaining to firms' default prediction argue that a 'financial distress' based definition of the default contingent upon a firm's earnings and market value is more appropriate than a definition based on legal consequence (Pindado *et al.* 2008; Tinoco and Wilson 2013; Keasey *et al.* 2014). We see a range of definitions in the empirical literature that have been successfully used to define/proxy firms' default/distress risk. Most of the empirical models employ a definition of default that is in line with some legal consequence (e.g. Chapter 11/7 Bankruptcy Code in the United States; United Kingdom Insolvency Act), which lead to a well-defined and clearly separated population of bankrupt versus non-bankrupt firms. This remains the most widely used method of classifying financially distressed firms in the empirical literature that employs binary choice statistical models to predict firms' financial distress (see among others Altman 1968; Ohlson 1980; Hillegeist *et al.* 2004; Gupta *et al.* 2014a). However, legal definition of default may suffer from noteworthy issues. Since insolvency involves lengthy legal processes, there often exists a significant time gap between real/economic default date and legal default date. UK companies exhibit a significant time gap of up to 3 years (average of about 1.17 years) between the time they enter into the state of financial distress and the legal default dates (Tinoco and Wilson 2013), while companies in the US stop reporting their financial statements about two years before filing for bankruptcy (Theodossiou 1993). Recent changes to insolvency legislation (for instance, the Enterprise Act 2004 in the UK and Chapter 11 in the US) do consider this problem and suggest several stages of financial distress based upon the severances of financial distress.

Further, a financially distressed firm may go for a formal reorganization involving the court system or an informal reorganization through the market participants (e.g. Blazy *et al.* 2013). Debt restructuring, asset sale and infusion of new capital from external sources are the three most commonly used market-based or private methods of resolving financial distress (Senbet and Wang 2010). Debt restructuring allows a financially distressed firm to renegotiate the outstanding debt obligation or related credit terms with its creditors but is critically subject to whether the debt obligation is due to private or public entity. As an alternative to this, a distressed firm may sell-off some of its existing assets to reduce its outstanding liability, or may undertake new profitable investment opportunities, which may eventually help it to overcome its misery. Despite having profitable investment opportunities, a financially distressed firm might not be able to generate additional funding due to the high risk involved in financing distress firms and the "debt overhang" problem, as discussed by Myers (1977). As a consequence, infusion of new capital from external sources is rarely observed in the resolution of financial distress. Thus, we cannot rule out the possibility that a financially distressed firm may not file for Chapter 7 or Chapter 11 protection, and choose a private workout method of resolving financial distress. Gilson *et al.* (1990) and Gilson (1997) report that firms avoid legal bankruptcy processes by out of court negotiation with creditors. However, under the binary classification based on legal consequences, a financially distressed firm which has not filed for Chapter 7 or Chapter 11 is not considered to be a financially distressed firm. There is, therefore, a clear need for a mechanism to identify financially distressed firms beyond the legal definitions. In this context, we find the argument of Pindado *et al.* (2008) highly relevant, and thus explore the following definitions of SMEs' default events:

Event 1 – Any firm which files for bankruptcy under Chapter 7/11 is considered default and is said to have experienced *Event 1*. The vast majority of empirical literature on SMEs default prediction employs this kind of binary classification based on some legal consequences to classify a firm as healthy or bankrupt (see among others Altman and Sabato 2007; Gupta, *et al.* 2014b).

Event 2 – Here we follow the financial distress definition provided by Keasey *et al.* (2014) while classifying a SME as default under *Event 2*. In particular, we consider a firm to be financially distressed (have experienced *Event 2*) if its EBITDA (earnings before interest tax depreciation and amortization) is less than its financial expenses for two consecutive years, the net worth/total debt is less than one, and the net worth experienced negative growth between the two periods. Additionally, a firm is also recorded as financially distressed in the year immediately following these distress events.

Event 3 – The third default definition that we propose considers both legal and finance-based definitions of distress when classifying a firm as default. A firm is classified as default under *Event 3* if it satisfies the conditions of *Event 1* and *Event 2* simultaneously. That is, besides being financially distressed, it should also file for bankruptcy under Chapter 7/11. Thus a firm is said to experience *Event 3* in a given year if it experiences *Event 1* in that same year and *Event 2* the year earlier, the rationale being that not all business closures are due to financial difficulties. Many file for legal bankruptcies as part of their planned exit strategies (see among others Bates 2005). This definition can therefore identify firms which follow legal exit routes purely due to financial difficulties.

4. Empirical Methods

This section discusses the source and use of dataset, the selection of explanatory variables, and statistical models used in our study.

4.1 Dataset

To predict default events over the next one year horizon, our empirical analysis employs annual firm-level accounting data from the Compustat database. We consider a relatively long analysis period which includes all SMEs that entered the Compustat database after January 1950 but before April 2015. In line with the widely popular definition of SMEs provided by the European Union[3], we consider a firm as an SME if it has less than 250

[3] We are aware of the fact that the US Small Business Administration (SBA) defines SMEs differently. Broadly it considers a firm as an SME if it has less than 500 employees and annual turnover of less than $7.5 million. However, their precise definition varies across industrial sectors to reflect industry differences. For instance, a mining firm with less than 1000 employees, a general building and heavy construction firm with annual turnover of less than $36.5 million and a manufacturing firm with less than 1500 employees are all classified as small businesses as per SBA (https://www.sba.gov/contracting/getting-started-contractor/make-sure-you-meet-sba-

employees. In Compustat, a company with a “TL” footnote on status alert (data item STALT) indicates that the company is in bankruptcy or liquidation (i.e. Chapter 7/11). Generally, a company will have a "TL" footnote on status alert - quarterly (and annual) for the first and following quarters (and years) the company is in Chapter 11. An "AG" footnote will appear on Total Assets (AT_FN) – quarterly, on the quarter the company emerges from Chapter 11. Thus, within its lifetime, a firm may go for multiple bankruptcy filings in the form of Chapter 11, and may remain in the bankruptcy state until it emerges. Consequently, taking the bankruptcy filing date as the bankruptcy indicator ignores the possible subsequent bankruptcy states. Thus, our first definition (*Event 1*) considers a firm under *bankruptcy* when its status alert is “TL” and healthy otherwise. This classification is consistent with the basic notion of survival analysis in which a subject may remain in a given risky state for more than one time period, and thus experience an event of interest for more than one time period.

Table 1 reports age-wise distribution of censored and distressed firms under respective default events (see Section 3 for definitions of various default events). We proxy a firm’s age as the earliest year for which financial information for that firm is available in the Compustat database. In Compustat, 1950 is the earliest point in time for which financial information is available, therefore in order to get the complete financial history of a firm, we selected only those firms which entered the Compustat database after 1950. Further, firms belonging to the Transportation, Communications & Public Utilities; Finance; Insurance & Real Estate; and Public Administration industrial sectors have been excluded from our empirical analysis (see Table 2 for details). This is to ensure homogeneity within our sample, as financial firms have different asset-liability structures and the rest are heavily regulated by governments. It should be noted that the same firms might have multiple entries and exits in our database. For instance, when an existing SME reports a number of employees over 250, it exits our sample and returns only when its number of employees falls below 250. We also exclude subsidiary firms if the ‘stock ownership code’ (Compustat data item ‘stko’) is ‘1’ (subsidiary of a publicly traded company) or ‘2’ (subsidiary of a company that is not publicly traded) in the Compustat database.

size-standards/summary-size-standards-industry-sector; accessed on May 18, 2016). This may not be a convenient workable definition from the lender’s point of view. Many of these firms are too big to be called SMEs in the real sense, despite being classified as small firms as per SBA. They do this primarily to determine the eligibility of a firm for SBA financial assistance, or for its other programs. Thus we follow a more appropriate and popular definition of SMEs provided by the European Union for this study. The most popular study on predicting bankruptcy of US SMEs by Altman and Sabato (2007) also follows the European Union’s definition of SMEs. They consider firms as SMEs if they report sales revenue of less than $65 million (approximately €50 million, as suggested by the European Union).

As reported in Table 1, for any given age, the numbers of firms experiencing *Event 3* is significantly lower than the number of firms experiencing *Event 1*. This shows that legal bankruptcy filing due to financial distress is not a dominant exist strategy for US SMEs. Thus, vast majority of bankruptcy filings may be due to planned exit strategies (e.g. Bates 2005) which may not be triggered due to financial difficulties. One may also attribute this low frequency of *Event 3* to the fact that, vast majority of SMEs are unlevered and do not incur any interest expense. Thus, unlevered SMEs facing financial difficulties (in meeting their operating expenses or trade payables) cannot be classified as financially distressed as per Keasey *et al.* (2014)'s definition of financial distress, and thus they do not experience *Event 2*. We do acknowledge this as a form of shortcoming of our study, but the cost of bankruptcy or financial distress is highest when external debt is introduced into the capital structure. In case of default of a levered firm, significant portion of bankruptcy cost is borne by providers of debt capital. In this context, we, in line with similar earlier studies (e.g. Pindado *et al.* 2008, Keasey *et al.* 2014), find a definition of financial distress contingent upon a firms' ability to meet its financial expenses to be most appropriate for building default prediction models.

[Insert Table 1 Here]

[Insert Table 2 Here]

4.2 Selection of Variables

Dependent Variable: As discussed in Section 3, in this study we consider *Event 1*, *Event 2* and *Event 3* as dependent variables for the estimation of respective hazard models.

Independent Variables: To develop multivariate hazard models we employ a wide range of financial ratios that have an established reputation in predicting firms' default risk. Our choice of covariates reflects firms' performance from leverage, liquidity, solvency, activity, profitability and interest coverage dimensions. Specifically, we incorporate covariates from popular studies on SMEs bankruptcy, like Altman and Sabato (2007), Lin *et al.* (2012), Gupta *et al.* (2014), and a recent book on credit risk management by Joseph (2013).

Leverage – the level of leverage reflect financial position of firms, which in turn determines their capacity to raise new capital through borrowing and meet debt obligations. To measure the financial fragility of firms, we use several variables reported as useful proxy of leverage in earlier literature. A Higher value of total liabilities/tangible total assets (TLTA) and total liabilities/net worth (TLNW) signifies higher likelihood of failure. Capital employed divided by total liabilities (CETL) is used as an inverse measure of leverage and firms in financial distress are expected to be heavily dependent on borrowed funds and hence are expected to have a lower value of CETL. More importantly, inability to meet short-term debt financing immediately can be a trigger element in a firm's termination, therefore, short-term debt relative to equity book value (STDEBV) is expected to be critical immediately prior to failure.

Liquidity – during difficult time, firms may weaken their liquidity position in order to meet immediate payments (Zavgren 1985). We employ a number of measures linked to firms' liquidity that examine whether a company has adequate cash or cash equivalents to meet its current obligations without liquidating other non-cash assets such as stocks. We expect that a higher value of following proxies to have a negative effect on firms' failure probability; cash and short-term investment relative to total assets (CTA), cash and marketable securities to current liabilities (CHR), current assets relative to current liabilities (CR) and quick ratio (CHR).

Profitability – typically a firm approaching failure witness earnings deterioration. This is because when earnings are impaired, firms' liquidity position gets fragile and thus the default on debt service increases. Having said that, Taffler (1983) empirically show that short-term liquidity is less important in magnitude to liabilities and earning abilities. This is because, even when firms' liquidity position is weak, capital supplier are more inclined to provide funds to firms with high level of earning, therefore, a lower probability of default. To further explore this relationship, we initially employ a number of variables which reflect the strength of firms' profitability at different stages of its earning process: earnings before interest, tax, depreciation and amortization to total assets (EBITDATA), return on equity (ROE), operating profit to capital employed (OPEC), net income to sale (NIS) and operation profit to net income (OPNI). Healthy firms are characterized by higher value of these ratios than their distressed counterparts.

Financing – default occurs when a firm fails to pay its financial obligations. Hence, the probability of this incident increases monotonically with the level of financial claims on either the level of its assets, revenue stream or profitability/retained earnings. Following this statement, we explore the discriminatory power of financial expenses relative to total assets (FETA), financial expenses relative to sales (FES) and earnings before interest taxes, depreciation and amortisation over interest expenses (EBITDAIE). We also explore retained earnings to total assets (RETA) which indicates the cumulative profitability over a firm's life. Thus, a firm's age may have a direct impact on this variable, as younger firms may have a lower chance to generate a higher level of retained earnings relative to older firms and hence a higher chance of being bankrupt. However, in the real world younger firms are more likely to go bankrupt than older firms. Furthermore, all financing variables can be used as an indication of financial constraints. Given the fact that SMEs are normally classified as financially constrained firms, it is expected to observe that these variables contribute a higher magnitude to the failure of these firms.

Activity – the variables related to firms' activity evaluate how efficiently a manager is exploiting its assets which in turn affect firms' performance in the long run. Working capital to sale (WCS) and to total assets (WCTA) indicates whether any deficiency in financial management skills is shrinking current assets to total assets or sale. We also utilise other activity ratios related to debtors (DCP, debtor collection period) and creditors (TCP, trade creditor payment period) to examine the impact of firm's credit policies on its immediate ability to meet its financial obligations. The inclusion of these variables is important as Hudson (1986) states that many SMEs rely heavily on short-term financing through trade creditors. Additionally, the discriminatory power sale to total assets (STA) and stock holding period (SHP) has also been explored.

Growth – Previous studies find that accessing to finance depends on the firm size. Beck and Demirguc-Kunt (2006) repot that different size categories (micro, small, medium, and large) face different degree of burden in obtaining external financing, they further emphasise how this burden play a major obstacle toward their growth. Phillips and Kirchhoff (1989) report that survival rate is more than doubles for firms with higher growth rate. We include different variables to examine if the lower rate of growth contributes to the failure of SMEs. Sales growth (SAG) create the basic source of the bulk of a firm's income, they can also be an indicator of business risk and managers' capability in dealing with other competitors. Capital growth (CAG) and Earnings growth (ERG) are also included, as a higher rate of growth indicates a growing capacity to meet financial obligations and vice-versa.

The final covariate that we consider is the ratio between income taxes to total assets (TTA). A heathy profitable firm is expected to pay its tax obligations on time, and is also expected to pay higher amount of tax in comparison to an identical unhealthy less profitable firms. Thus, a higher value of TTA is expected to bring lower failure likelihood and vice-versa.

Table 3 lists all the covariates and their respective definitions. To eliminate the influence of outliers on our statistical estimates, we restrict the range of all our financial ratios between the 5th and 95th percentiles.

Control Variables: Considering the suggestion of Gupta, Gregoriou, *et al.* (2014) we control for the diversity between micro, small and medium firms by employing dummy variables for micro (less than 10 employees) and small (greater than 9 but less than 50 employees) firms in our multivariate models. To control the volatility in the macroeconomic environment affecting specific industrial sectors, we calculate an additional measure of industry risk (*RISK*) as the failure rate (number of firms experiencing the event of interest in the respective industrial sector in a given year/total number of firms in that industrial sector in that year) in each of the seven industrial sectors in a given year. Higher values indicate a higher risk of default, and vice versa.

[Insert Table 3 Here]

4.3 Hazard Model

4.3.1 Basic Hazard Model

Survival analysis deals with the analysis of the time to the occurrence of an event, which in this study is the time until a financial default event. Suppose T is a non-negative random variable which denotes the time to a distress event and *t* represents any specific value of interest for the random variable *T*. If, instead of referring to *T's* probability density function as $f(t)$ or its cumulative distribution function (CDF) as $F(t) = \Pr(T \leq t)$, we think of T's survivor function, $S(t)$ or its hazard function, $h(t)$, the understanding of survival analysis becomes much clearer (Cleves *et al.* 2010). The survivor function expresses the probability of survival beyond time *t*, which is simply the reverse CDF of *T*, i.e.:

$$S(t) = 1 - F(t) = \Pr(T > t) \quad (1)$$

At $t = 0$ the survivor function is equals to one, and moves toward zero as t approaches infinity. The relationship between survivor function and hazard function (also known as the conditional failure rate at a particular time t) is mathematically defined as follows:

$$h(t) = \lim_{\Delta t \to 0} \frac{\Pr(t + \Delta t > T > t | T > t)}{\Delta t} = \frac{f(t)}{S(t)} = \frac{-d \ln S(t)}{dt}; \quad (2)$$

In simple words, the hazard rate is the (limiting) probability that the failure event occurs within a given time interval, given that the subject has survived to the start of that time interval, divided by the width of the time interval. The hazard rate varies from zero to infinity and may be increasing, decreasing or constant over time. A hazard rate of zero signifies no risk of failure at that instant, while infinity signifies certainty of failure at that instant.

4.3.2 Extended Cox Model

An elegant and computationally feasible way to estimate the hazard function (2) is to use the semi-parametric Cox proportional hazards (CPH) model (Cox 1972, 1975) as shown in the following equation:

$$h_i(t) = h_0(t).\exp(x_i'\beta) \tag{3}$$

Here, x_i' is the transpose of covariates vector x_i, β is the vector of regression parameters and $h_0(t)$ is the arbitrary unspecified baseline hazard function (the hazard risk that the subject *i* faces in absence of covariates; i.e. $x = 0$). The regression parameters (βs) are estimated using the partial likelihood function, which takes into account censored survival times and eliminates the unspecified baseline hazard term $h_0(t)$. CPH model treats time as continuous, and is semi-parametric in the sense that the model does not make any assumptions related to the shape[4] of the hazard function over time.

Some of the factors (leverage ratio, profitability ratio, volatility, etc.) affecting firms' survival vary with time, but the fixed CPH model as highlighted in Equation (3) does not allow for time-varying covariates. However, inclusion of time-varying covariates in the CPH framework is relatively easy and thus enables us to predict dynamic survival probability over the life of the firm. The CPH model can be generalized to allow for the covariate vector x to be time-varying as follows:

$$h_i(t) = h_0(t).\exp(x(t)_i'\beta) \tag{4}$$

where $x(t)$ is the covariate vector at time t. The rate of change of time-varying covariates is different for different subjects, and hence the estimated hazard ratio does not remain constant over time. However, the inclusion of time-varying covariates is not problematic for the partial

[4] It could be increasing, decreasing, and then increasing or any shape we may imagine. But it assumes that whatever the general shape of the hazard function, it is same for all subjects.

likelihood estimation (Allison 2010), and hence the CPH model can be easily improved to allow for non-proportional hazard risks. It implies that a general hazard model which does not have the restrictive assumption of constant proportional hazard ratio can be generalized by inclusion of both duration-dependent and duration-independent covariates in the same model. However, a CPH model with time-varying covariates is no longer a proportional hazards model, and a CPH model with time-varying covariates is appropriately called the *Extended Cox* model (see Kleinbaum and Klein 2012). Additionally, the time-varying covariates do not need to satisfy the proportional hazards assumption. However, if the model also includes time-independent covariates, then appropriate test of proportionality is suggested (see Kleinbaum and Klein 2012). One major advantage of the Cox method is that it easily addresses the problem of right censoring, but it suffers from a major disadvantage of proportional hazards assumption if time-independent covariates are also included in the model. One may test this restrictive proportional hazard assumption that is being neglected in most empirical studies by using the *scaled Schoenfeld residual* (Grambsch and Therneau 1994) rather than the *Schoenfeld residual* (Schoenfeld 1982). While estimating our empirical model we also control for *unobserved heterogeneity* and *recurrent events* by including a *shared frailty* term into our model via a multiplicative scaling factor α_i (see Cleves *et al.* 2010). These signify group-level frailty and are unobservable positive values assumed to follow the *Gamma distribution* with mean 1 and variance θ to be estimated using the development sample (see Jenkins 2005). Also, the time at which the distress event occurs is not really relevant for hazard risk analysis using the Cox method, but the ordering of the distress event is critically important. In situations where multiple firms experience the event of interest at the same time, the exact ordering of distress events is difficult to determine. Thus we use *Efron's*[5] (1977) method to handle cases of tied failure times.

Recent empirical literature highlights the use of CPH in default prediction studies (see among others Bharath and Shumway 2008; Chen and Hill 2013) but it is inappropriate to use the CPH model in discrete-time frameworks for the reasons discussed earlier, and in the following section. Both Bharath and Shumway (2008) and Chen and Hill (2013) are silent on issues pertaining to shared frailty and tied failure times, which we consider to be important aspects and should be addressed in empirical studies if one chooses to use the CPH modelling technique.

[5] In our analysis the risk set keeps decreasing with successive failures. Efron's (1977) method reduces the weight of contributions to the risk set from the subjects which exhibit tied event times in successive risk sets.

4.3.3 Discrete Hazard Model

When an event may be experienced at any instant in continuous-time (exact censoring and survival times are recorded in relatively fine time scales such as seconds, hours or days) and there are no *tied* survival time periods, then the continuous-time survival model is an appropriate choice (Rabe-Hesketh and Skrondal 2012). However, if the data has relatively few censoring or survival times with *tied* survival time periods, then the discrete-time survival model is more appropriate, where coarse time-scales are generally used, for instance, expressing time to an event in weeks, months or years (Rabe-Hesketh and Skrondal 2012). Interval-censoring[6] leads to discrete-time data, which is the case with our database. Here, the beginning and end of each time interval is same for all of the SMEs in analysis time, as the information is recorded on an annual basis. Thus, the event of interest may take place at any time within the year but it cannot be known until the information is provided at the end of the year. Hence, considering the discussion above we also estimate our hazard models in discrete-time framework with *random effects* (α_i), thus controlling for *unobserved heterogeneity* or *shared frailty*.

The discrete-time representation of the continuous-time proportional hazard model with time-varying covariates leads to a generalized linear model with *complementary log-log* (Grilli 2005; Jenkins 2005; Rabe-Hesketh and Skrondal 2012) link, specified as follows:

$$cloglog\big(h_i(t)\big) \equiv \ln\{-\ln(1 - h_i(t))\} = \beta x(t)'_i + \lambda_t \qquad (5)$$

Here, λ_t is a time-specific constant which is estimated freely for each time period *t*, thus making no assumption about the baseline hazard function within the specified time interval.

However, in most empirical studies logit link is used over complementary log-log (clog-log) link as specified in Equation 6:

$$P_{i,t}(Y = 1) = \frac{e^{\alpha(t) + x(t)'_i\beta}}{1 + e^{\alpha(t) + x(t)'_i\beta}} \qquad (6)$$

where *α(t)* captures the baseline hazard rate and $P_{i,t}(Y = 1)$ is the probability of experiencing the event of interest by subject *i* at time *t*. This will produce very similar results as long as the time intervals are small (Rabe-Hesketh and Skrondal 2012) and sample bad rate (% of failed to non-failed) is small (Jenkins 2005). One may also choose probit link function, if one

[6] The event is experienced in continuous-time but we only record the time interval within which the event takes place.

strongly believes that the underlying distribution of the process being modelled is normal, or if the event under study is not a binary outcome but a proportion (e.g. proportion of population at different income levels). While these specifications will generally yield results that are quite similar, there are significant differences in terms of non-proportionality (see Sueyoshi (1995) for detailed discussion). Thus, we estimate our discrete hazard models with clog-log and logit links, and analyse any differences in the magnitude of coefficients and classification performance of multivariate models developed.

4.4 Performance Evaluation

To gauge the classification performance of the models developed to identify distressed firms, we estimate area under the Receiver Operating Characteristic (ROC) curve (AUROC). This curve originates from the signal detection theory, which shows how the receiver detects existence of signal in presence of noise. It is obtained by plotting the probability of detecting true-positive (sensitivity) (a firm actually defaults and the model classifies it as *expected default*) and false-negative (1 – specificity) (a firm actually defaults but the model classifies it as *expected non-default*) for an entire range of possible *cutpoints* (these are probability values). *Cutpoint*, *c*, is defined to obtain a derived binary variable by comparing each estimated probability with *c*. If the estimated probability is greater than *c*, the value of the derived binary variable equals to 1, or 0 otherwise. AUROC is now considered to be the most popular non-parametric method for evaluating a fitted prediction model's ability to assign, in general, higher probabilities of the *event of interest* to the subgroup which develops the event of interest (dependent variable = 1) than it does to the subgroup which do not develop the event of interest (dependent variable = 0). The AUROC provides a measure of the prediction model's ability to discriminate between those firms which experience the event of interest, versus those who do not. Its value ranges from 0.5 to 1.0, which encapsulates the classification performance of the model developed. AUROC of 1 denotes a model with perfect prediction accuracy, and 0.5 suggests no discrimination ability. In general there is no 'golden rule' regarding the value of AUROC, however anything between 0.7 and 0.8 is acceptable, while above 0.8 is considered to be excellent (Hosmer Jr *et al.* 2013).

5. Results and Discussion

We begin this section with analysis of key measures of descriptive statistics of our covariates, along with relevant discussion pertaining to correlation among them. We perform univariate regression analysis of each covariate in turn using respective event definition and respective econometric specification, to understand any unexpected behaviour in their discriminatory performance. We then discuss development and performance evaluation of multivariate discrete-time hazard models developed using logit and clog-log links. Finally, we develop multivariate extended Cox models and provide a comparative discussion on the performance of multivariate models developed using different default definitions. To gauge any temporal variation in the explanatory power of our covariates, we perform our regression analysis using covariates that are lagged by $T – 1$, $T – 2$ and $T – 3$ time periods.

5.1 Descriptive Statistics and Correlation

Inspection of descriptive statistics gives us an initial understanding about the variability of covariates and the potential biasness that may arise in the multivariate setup due to any unexpected extreme variability. We expect the mean of covariates that exhibit positive relationships with the insolvency/distress hazard to be higher for distressed groups (status indicator = 1) than for their healthy or censored counterparts (e.g. STDEBV in Table 4). On the contrary, the mean of covariates that show negative relationships with the insolvency hazard is expected to be lower for distressed groups than for their healthy counterparts (e.g. CTA in Table 4). A closer look at Table 4 reveals that the mean, median and standard deviation of most of the covariates under respective event definitions are as we expect without any extreme variability. However, EBITDAIE and STDEBV raise some serious concerns. The mean of EBITDAIE is very high, as most of the firms in our sample do not incur (or incur very little) interest expenses[7]. This leads to a very high difference between its mean and median values, resulting in a highly skewed distribution and very high value of standard deviation. Although STDEBV and TLNW are positively related to firms' default probability, the mean of the default group is significantly lower than the censored group under *Event 3*, which is quite surprising as we expect otherwise. We also observe that the mean of respective covariates across different default definitions in Table 4 reveal very little variation in value. This signals little variation in the classification performance of multivariate models developed, and is confirmed by our results in Section 5.

[Insert Table 4 Here]

The correlation matrix in Table 5 shows that some of the covariates exhibit moderate to strong correlation with other covariates. RETA shows a strong positive correlation of

[7] While calculating the ratio EBITDAIE, zero interest expense (IE) for all firm-year observations is replaced with $1 to avoid missing values.

approximately 0.65 with EBITDATA, supporting the belief that SMEs primarily rely on internal sources for their funding requirements, thus they end up retaining a significant portion of their income. Issues associated with multicollinearity therefore need to be addressed carefully when developing multivariate models. Section 5.3.2 on model building strategy discusses how we address this issue in this study.

[Insert Table 5 Here]

5.2 Univariate Regression and Average Marginal Effects

It is always advisable to do some univariate analysis before proceeding to estimation of multivariate models. In survival analysis the standard approach is to initially look at Kaplan-Meier survival curves of all categorical covariates to get an insight into the shape of survival functions and proportionality of each group[8]. Popular non-parametric tests of equality of survival functions, like the log-rank test and the Wilcoxon–Breslow–Gehan test (see Cleves *et al.* 2010), are also reported. However, it is not feasible to calculate Kaplan-Meier curves or conduct these non-parametric tests for continuous predictors as continuous predictors have too many different levels[9]. But, Nam *et al.* (2008) report the log-rank test and the Wilcoxon–Breslow–Gehan test for their continuous predictor, which, to the best of our knowledge, is inappropriate. Considering this constraint, we perform univariate regression of each covariate in turn, for an initial insight into their effects on respective default events.

In order to narrow down our list of covariates, at first we obtain univariate regression estimates using *Event 2* as the dependent variable and Equation (6) as the regression methodology (discrete-time hazard model with logit link). Here we use the financial distress based definition rather than legal bankruptcy, with the presumption that it is the primary reason behind bankruptcy and always precedes the bankruptcy filing event. Further, filing for legal bankruptcy is the least efficient exit strategy for SMEs (Balcaen *et al.* 2012). Additionally, to gauge the temporal variation in the explanatory power of covariates we obtain regression estimates for $T-1$, $T-2$ and $T-3$ lagged time periods (see Table 6). At this stage we exclude covariates from further empirical analysis that (i) are not significant in all three time periods (this ensures that the selected covariates are consistent predictors of firms' financial health over a sufficiently long time interval to allow for developing a reasonable *early warning system*), or (ii) are significant but exhibit *Average Marginal*

[8] See Cleves *et al.* (2010) for a detailed description of Kaplan-Meier curves.
[9] See for example http://www.ats.ucla.edu/STAT/stata/seminars/stata_survival/default.htm (accessed May 18, 2016). Also see Cleves *et al.* (2010) for a more thorough understanding.

Effects[10] (AME) of less than 5% in all three time periods. The rationale is that a unit change in the value of significant covariates must induce sufficient change in the magnitude of the outcome probability to clearly distinguish between distressed and healthy firms. An interesting observation in Table 6 is the AME of Altman and Sabato's (2007) covariates that are widely employed in modelling default risk of SMEs. Out of the five covariates that they suggest, three (STDEBV, EBITDAIE and RETA) exhibit AME of less than 5% with AME of EBITDAIE being almost zero. The other two covariates CTA and EBITDATA have AME values less than 17%. This suggests that, although these covariates are significant predictors, a unit change in their value does not transmit significant change in the probability of outcome variable. Although three of Altman and Sabato's (2007) covariates have AME less than 5%, we include them for further empirical analysis to gain greater understanding of their explanatory power in the multivariate setup. Furthermore, the AME of FETA and TTA are highest among all covariates suggesting that financial expense and tax are dominant signals to identify financially distressed firms. From 27 variables, this helps us to narrow down to 16 variables that we use for further empirical analysis. Table 7 reports the final list of covariates that we use for further univariate and multivariate regression analysis.

[Insert Table 6 Here]

[Insert Table 7 Here]

Univariate Regression of Event 1: Section A of Table 8 reports the univariate regression estimates for *Event 1* using discrete and continuous-time hazard models. Magnitude of coefficients of respective covariates (β in Table 8) obtained using discrete-time hazard specification with logit and cloglog links, and the extended Cox model, are close to each other with some variation for covariates TLTA, FETA, CAG, SAG, TTA and RETA. However, logit and cloglog estimates exhibit almost identical model fit as their AIC[11] values are almost identical, but they are about three times higher for Cox estimates. This suggests

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[10] In non-linear regression analysis, Marginal Effects are a useful way to examine the effect of changes in a given covariate on changes in the outcome variable, holding other covariates constant. These can be computed as marginal change (it is the partial derivative for continuous predictors) when a covariate changes by an infinitely small quantity and discrete change (for factor variables) when a covariate changes by a fixed quantity. Average Marginal Effects (AME) of a given covariate is the average of its marginal effects computed for each observation at its observed values. Alternatively, AME can be interpreted as the change in the outcome (financial distress = 1, in our case) probabilities due to unit change in the value of a given covariate, provided other covariates are held constant. See Long and Freese (2014) for detailed discussion on this topic.

[11] Akaike Information Criterion (AIC) is defined as: $\text{AIC} = -2 \times L + 2 \times (p + 1)$ where L is the log-likelihood of the fitted model and p is the number of regression coefficients estimated for non-constant covariates. In general, models with lower values of AIC are preferred to larger ones.

that discrete-time model with logit/cloglog links offer better model fit than extended Cox models[12]. We also see that CAG and SAG are significant in all time periods when estimated using discrete hazard model, but becomes insignificant (in *T – 2* and *T – 3*) when estimated using Cox model. Further, the statistical significance of TLTA, NIS and RETA also varies with their econometric specification. Altman and Sabato's (2007) covariate EBITDATA loses its statistical significance beyond *T – 1*; STDEBV shows unstable explanatory power (sign is opposite to expectation for *T – 1* logit and cloglog estimates), is insignificant in *T – 1* but significant in *T – 2* and *T – 3*; EBITDAIE is significant but its coefficients are almost 0; RETA is significant in *T – 1* but is insignificant in *T – 2* and *T – 3* when estimated using discrete-hazard specification. Only CTA shows consistent and reliable explanatory power in all three time periods across all econometric specifications. *Event 1* is the same event definition that they use in their default prediction study, however our results do not approve the covariates suggested by them. Their suggestion might be biased due to their sample selection process, while we use near-population data to establish our empirical validation.

Univariate Regression of Event 2: Section B of Table 8 reports univariate regression estimates obtained using *Event 2* as a dependent variable. All covariates are significant across all econometric specifications for all lagged time periods. However, the AIC values of logit and cloglog estimates are about three to six times lower than values obtained using Cox specification. This asserts that discrete hazard models offer better model fit than their continuous counterparts. Additionally, in *T – 3*, FETA and WCTA fail to remain significant when estimated using Cox specification. In this case, all of Altman and Sabato's (2007) covariates are significant with expected sign across all econometric specification for respective lagged time periods except RETA, for Cox estimate at *T – 3*. All of their covariates also have reasonable magnitude of respective coefficients except EBITDAIE, which is again almost 0. However, the real litmus test of their covariates will be performed in the multivariate section.

Univariate Regression of Event 3: Section C of Table 8 reports univariate regression estimates obtained using *Event 3* as the dependent variable. Unlike *Event 2* estimates, many of the covariates (OPCE, NIS, CAG, STDEBV, EBITDAIE and RETA) show varying

[12] For most covariates and their respective time lags in Table 8, absolute values of coefficients are highest for logit estimates, followed by cloglog estimates, and least for Cox estimates (i.e. |*logit*| > |*cloglog*| > |*Cox*|). However, based on this it shall be inappropriate to conclude that, for a unit change in the value of a covariate logit estimates lead to highest change in the outcome probability than its alternative counterparts. This generalization may only be valid if their Average Marginal Effects (AME) or other similar estimate also show this pattern.

(insignificant) explanatory power across different time periods. Two of Altman and Sabato's (2007) covariates (STDEBV and EBITDAIE) also fail miserably to discriminate distressed and censored firms across $T - 1$ and $T - 2$ lagged time periods.

[Insert Table 8 Here]

5.3 Developing Multivariate Hazard Models

In this section, we develop and discuss multivariate hazard models for the respective default definitions discussed in Section 3. We begin with our choice for specification of the baseline hazard rate, which is required for developing discrete-time duration-dependent hazard models, then develop and discuss the multivariate discrete-time and continuous-time hazard models.

5.3.1 Detection of Baseline Hazard Rate

Before developing multivariate discrete-time hazard models it is important to choose a baseline specification for the hazard rate. Figure 1 shows the table of hazard curves[13] estimated using the Kaplan-Meier (KM) estimator for different default events (Kaplan and Meier 1958). KM estimator also known as product limit estimator is the most prolific and classic non-parametric technique of survival analysis. It is primarily used to produce useful visual plots of survival/life tables, survival curves and hazard curves. This KM estimator estimates the survival function at time t, denoted by $\hat{S}(t)$, which is the probability of survival time being greater than t. The formula for KM survival probability at failure time t_j gives the probability of surviving past the previous failure time t_{j-1}, multiplied by the conditional probability of surviving past time t_j, given survival to at least time t_j.

$$\hat{S}(t_j) = \hat{S}(t_{j-1}) \times \Pr(T > t_j | T \geq t_j) \tag{7}$$

[13] Table 1 shows that the earliest age that a firm can experience a distress event under all three default definitions is one year. However, the hazard curves start from somewhere around five years. This difference is due to the fact that the "sts graph" command in Stata performs an adjustment of the smoothed hazard near the boundaries. In case of the default kernel function of -sts graph- (Epanechnikov kernel), the plotting range of the smoothed hazard function is restricted to within one bandwidth of each endpoint. The same is true for other kernels, except the epan2, biweight, and rectangular kernels, in which case the adjustment is performed using boundary kernels. If we wish to plot an estimate of the hazard for the entire range, we could use a kernel without a boundary correction. Alternatively, we can use the -noboundary- option, but this will produce an estimate that is biased near the edges. See "help sts graph" in Stata and Silverman (1986) for further details. This will not affect the empirical analysis if one uses a fully non-parametric method of baseline hazard specification. However, one needs to be careful while using piecewise-constant specification.

As shown below, Equation (7) can also be written as product limit of the survivor function if we substitute for survival probabilities $\hat{S}(t_{j-1})$ with the product of all fractions that estimate the conditional probabilities for failure time t_{j-1} and earlier.

$$\hat{S}(t_j) = \prod_{i=1}^{j-1} \Pr(T > t_i | T \geq t_i) \quad (8)$$

Comparison of survival curves is useful when comparison is made between survival patterns of two or more categories. In our case, we do not have the same event for multiple categories, but multiple events for the same category or sample. Thus, analysis of hazard rate/curves that use information from Equation (8), and as stated in Equation (2) is more relevant in this context. This also helps us in defining baseline hazard rate of multivariate models.

As reported in Figure 1, hazard curves of all three events exhibit fairly different functional relationships with firms' age. The hazard curves of all three events show increasing and decreasing relationships with firms' age, and the shape of hazard curves of *Event 1* and *Event 3* are quite similar. From the surface it might seem that the default events are highly duration-dependent. However, one might turn sceptic after looking at the magnitude of hazard rates on the vertical axis. For *Event 1* it ranges approximately between 0.006 and 0.013; *Event 2* between 0.05 and 0.13; and *Event 3* between 0.00175 and 0.00325. Considering these tight intervals of hazard rates, piece-wise specification of baseline hazards might fail to reflect the differences in the hazard rates between respective age groups. Additionally, all three hazard curves show steep rises and falls with some flatness in a couple of time intervals, thus it is inappropriate to assume that the hazard rates are constant for any defined age group. In this situation it may be appropriate to use a fully non-parametric baseline hazard specification, thus age specific dummy variables to specify the baseline hazard rate. To statistically test our intuition, we estimated multivariate discrete hazard models (with logit link) with *Event 1*, *Event 2* and *Event 3* respectively as dependent variables and only age dummies as independent variables. Regression results[14] confirm that about 90% of age dummies are significant (p-value < 0.05) in explaining respective outcome of interest. However, when supplemented with financial covariates, only about 10% of age dummies remain statistically significant with large values of standard errors of their coefficients. This suggests that in presence of financial covariates, temporal dummies fail to capture duration dependence of

[14] These results are not reported in this paper; however, it may be made available from the authors.

hazard rates. Additionally, one also needs to consider that too many variables may make the multivariate model numerically unstable. Thus, following Shumway (2001) we re-estimate these models using the natural logarithm of firms' annual age (variable AGE in Table 9) as the baseline hazard specification. In contrast to Shumway's (2001) results, variable AGE is significant in most of our multivariate hazard models. In light of this discussion we use the natural logarithm of firms' age (AGE) to proxy the baseline hazard rate for all our multivariate models developed.

[Insert Figure 1 Here]

5.3.2 Model-Building Strategy

The criteria for including a covariate in the multivariate model often vary across scientific disciplines, but they all strive to develop the 'best' model that is numerically stable and can be easily adapted for real life applications. The standard error of a model increases with the increase in the number of covariates, and this also makes the model more dependent on the observed data. Thus the objective should be to employ a minimum number of covariates for a desired accuracy level. A good start is to perform univariate regressions of each covariate in turn, and consider covariates with p-values of less than 0.25 for developing multivariate models (see chapter 4 of Hosmer Jr *et al.* 2013). Another school of thought suggests inclusion of all theoretically motivated covariates in the multivariate model irrespective of their significance level in the univariate analysis. Some studies exclude insignificant predictors (p-value > 0.05) from their multivariate models, yet insignificant predictors may explain some of the variation of the dependent variable. Multicollinearity can be a serious issue that may make the model unstable if not addressed effectively. Thus, at first we rank the covariates in Table 7 based on the magnitude of their AME (the covariate with the highest value of |AME| is ranked 1, and so on) and then introduce each covariate in turn into the multivariate setup, starting with the covariate with the highest ranking (rank = 1 in Table 7). The rationale is that the higher the value of AME, the higher the change in the predicted probability due to unit changes in the covariate's value. Thus a covariate with a higher value of AME (e.g. FETA in Table 7) is more efficient in discriminating between distressed and censored firms than a covariate with lower value of AME (e.g. TLTA in Table 7). Furthermore, we exclude a covariate from the multivariate model if, when introduced: (i) it affects the sign[15] of any previously added covariate; (ii) it bears the opposite sign to that

[15] Coefficients with a negative sign become positive and vice versa.

expected; (iii) it bears the expected sign but has a p-value greater than 0.25; and (iv) it makes a previously added covariate insignificant with a p-value greater than 0.25. These scenarios may primarily arise due to multicollinearity among covariates, thus our screening mechanism seems to be a reasonable choice. Moreover, we believe that this method of covariate introduction while developing multivariate models reasonably addresses the multicollinearity problem, and leaves us with a 'best' set of covariates that explain the variance of the dependent variable. Using the discrete hazard model with logit link, this process is applied to *Event 1*, *Event 2* and *Event 3* respectively for all three (*T – 1*, *T – 2* and *T – 3*) respective lagged time periods. Then, multivariate hazard models with cloglog link and extended Cox are estimated using the same set of covariates selected using logit link to see any differences that may arise due to different estimation methods.

The final set of multivariate hazard models reported in Table 9 are estimated using all observations available to us covering the entire sampling period, thus we do not have separate test and hold-out samples. In order to assess within-sample classification performance of the models developed we estimate area under ROC (AUROC) curves for respective models using the full estimation sample. For out-of-sample validation, we first estimate multivariate hazard models using observations until the year 2011, and using these estimates we predict the default probabilities for the year 2012; we then include 2012 in the estimation sample and predict default probabilities for 2013 and so on, until the year 2015. We then use these predicted probabilities from the year 2012 through 2015 to estimate out-of-sample AUROC for respective multivariate hazard models.

[Insert Table 9 Here]

5.3.3 Hazard Models for Event 1

The binary dependent variable used is *Event 1*, i.e. firms that filed for legal bankruptcy proceedings and are therefore considered to have experienced the default event and censored otherwise (please see Section 3 for detailed discussion). Section A of Table 9 reports multivariate hazard models estimated for *T – 1*, *T – 2* and *T – 3* lagged time periods developed using respective econometric specification. As we can see, the logit estimates of factors affecting the outcome probability of *Event 1* vary considerably across time periods, except FETA. However, the control variables Micro, Small, RISK1 and AGE are strongly significant across all time periods. Among of five Altman and Sabato's (2007) covariates, EBITDATA and RETA fail to find a place in our multivariate models for all three time periods. Additionally, STDEBV, CTA and EBITDAIE do not show consistency in their explanatory power. As seen in univariate regression, here too the coefficients of EBITDAIE are almost 0. This clearly shows the inefficiency of Altman and Sabato's (2007) covariate in predicting corporate bankruptcies for SMEs. The statistical significance of most of the covariates does not vary considerably when estimated using cloglog and Cox specifications except STDEBV, CAG and SAG. However, AIC values of logit and cloglog estimates are almost identical and are about half that of Cox estimates. This clearly suggests that discrete-time hazard models offer much superior model fit than the continuous extended Cox model. However, the within-sample AUROC for all econometric specifications are almost identical, with slight variation among estimates of the hold-out sample (see Figure 2). This suggests no significant loss in the classification performance if one uses Cox specification over discrete-time. Additionally, the AUROC of all our multivariate models developed are around 0.8 or higher, which is considered to be excellent. However, shapes of ROC curves of hold-out sample estimates are steps rather than concave due to very low number of outcome events in out-of-sample validation[16].

[Insert Figure 2 Here]

5.3.4 Hazard Models for Event 2

Unlike *Event 1*, multivariate models developed for *Event 2* using logit specification show consistent explanatory power of most covariates over all three lagged time periods (see Section B of Table 9). However, the statistical significance of SAG ($T - 1$) and AGE ($T - 1$ and $T - 3$) varies with the estimation technique. All control variables (Small, Medium and RISK2) are also highly significant across all lagged time periods. Among Altman and Sabato's (2007) covariates, STDEBV, EBITDATA and EBITDAIE exhibit significant explanatory power across all lagged time periods and econometric specifications. However, the coefficient of EBITDAIE is almost 0 here as well. The variable CTA finds place only in the models developed for $T - 2$ time periods, while RETA fails to meet our screening criteria for inclusion in the multivariate model. Thus, Altman and Sabato's (2007) covariates are not efficient predictors of financial distress, unlike some of the other financial ratios reported in Section B of Table 9. Here too the AIC values of discrete hazard models are about three to four times lower than Cox models, thus discrete-time hazard models offer a superior model fit compared with their continuous counterpart. The within sample and hold-out sample

[16] This might result in misleading estimates of AUROC. Thus one needs to be careful when drawing inferences regarding out-of-sample predictive ability of the forecasting model.

AUROC estimated for different multivariate models are around, or higher than, 0.80, suggesting excellent classification performance of our multivariate models across all time periods and econometric specifications (see Figure 2).

5.3.5 Hazard Models for Event 3

The final set of hazard models that we estimate is based on the default definition (*Event 3*) that we propose in this study, which considers both legal bankruptcy filing and firms' financial health while classifying SMEs as default (please see Section 3 for details). Section C of Table 9 reports multivariate regression estimates for *Event 3* across all three lagged time periods and respective econometric specifications. A look at the results reveals that factors affecting outcome probability vary reasonably across time periods. Even the statistical significance of six covariates (STDEBV, OPCE, RETA, CAG, SAG and TTA) is sensitive to estimation technique. Among Altman and Sabato's (2007) covariates STDEBV finds place in $T-2$ and $T-3$, while RETA finds a place in $T-1$ only. EBITDATA, CTA and EBITDAIE fail to meet our inclusion criteria into the multivariate setup. This reinforces the inefficiency of covariates suggested by Altman and Sabato (2007) in predicting SMEs financial distress. Here too the AIC values are in favour of discrete-time models, which are about 0.8 times lower than continuous Cox estimates. Both within sample and hold-out sample classification of all multivariate models across all time periods and econometric specifications are close to or above 0.9, which is superior to *Event 1* and *Event 2* models' classification performance (see Figure 2).

5.3.6 Comparative Performance of Hazard Models

As reported in Table 9, the extended Cox model performs almost identically to discrete-time models with logit and clog-log links as it shows almost identical classification performance across all default definitions. Thus one might be indifferent in her choice of hazard specification. But, if the event of interest is not duration dependent (i.e. some functional form of time or time dummies are not significant in the multivariate model), with the hazard rates being invariant or varying mildly across different time periods, then getting involved in the complications of hazard models is not rewarding considering the marginal gain one would obtain using such models. As reported earlier, in the presence of other financial covariates about 90% of time dummies that we use as baseline hazard specification are insignificant, with very high values of standard errors. Thus we use natural logarithm of firms' annual age as baseline hazard specification. However, such objective can easily be achieved by developing regression models using a panel logistic regression technique that uses some functional form of time to capture any duration dependency. Although Shumway (2001) argues that hazard models are superior to competing static models but AGE variables in his multivariate models are insignificant, how can it be used to reliably predict duration specific hazard rate, which is why hazard models are primarily used? Unlike other scientific disciplines such as medicine or health economics, duration specific prediction of hazard rates is not a common practice in bankruptcy/financial distress prediction studies, thus we do not see any real need for hazard models if similar objective can be achieved using much simpler logistic regression that controls for any duration dependencies, as both involve identical statistical estimation methods. Another interesting observation is the classification performance measures across different default definitions. Based on the AUROC measures, *Event 1* is the weakest definition of default while *Event 3* is the strongest, as it has the highest values of AUROC across all time periods. Also, the AIC measure of *Event 3* models is the lowest among the three default definitions, which indicates that the *Event 3* default definition provides a vastly improved fit compared to other two competing default definitions.

6. Conclusion

The use of hazard models in estimating bankruptcy prediction is gathering momentum in financial academic literature. Unfortunately, the vast majority of previous studies suffer from at least one of the following shortcomings: (i) insufficient reasoning behind their choice of *discrete-time* or *continuous-time* hazard models; (ii) inappropriate specification of baseline hazard rate; (iii) no test of proportional hazards assumption when using the *extended Cox* model with time-independent covariates; (iv) ignores *frailty* and *recurrent* events; or (v) insufficient explanation of how they dealt with the issues of *delayed entry*.

We contribute to the literature by acknowledging all of these commonly neglected concerns in our research. To our knowledge we are the first academic paper to report a performance comparison of popular hazard models (discrete hazard models with logit and clog-log links and the extended Cox models) used in the recent literature (e.g. Campbell *et al.* 2008; Chen and Hill 2013). We also contribute to the literature by undertaking an empirical investigation which compares various default definitions of the US SMEs. Three default definitions that we compare are based on legal bankruptcy laws (*Event 1*), firms' financial health (*Event 2*), and the third definition (*Event 3*) proposed in this study that considers both legal bankruptcy and firms' financial health. Considering the suggestion of Hosmer Jr *et al.* (2013) on multivariate model building strategy, we propose an atheoretical econometric based multivariate model building strategy based on covariates' *Average Marginal Effects* (AME) and their *inter-temporal discrimination* ability. Finally, we further contribute to the field by examining the efficiency of covariates, suggested by the most popular study on SMEs' bankruptcy by Altman and Sabato (2007), in predicting SMEs bankruptcy across varying default definitions and lagged time periods.

Our findings show almost identical classification performance of both discrete-time and continuous-time hazard model across all three default definitions, suggesting insignificant variance of classification performance to econometric specification. Based on comparison of AIC measures, discrete-time hazard models provide considerably superior fit than continuous-time Cox models. However, AIC measures for both discrete-time hazard models (logit and clog-log links) are almost identical; hence the choice between them is left to the personal preference of the users. Also, Altman and Sabato's (2007) covariates are unstable and inefficient in predicting event outcome across different default definitions and lagged time periods in comparison to other competing financial ratios. Furthermore, based on the classification performance and AIC values of models developed using different default definitions, we understand that the default definition that we propose performs best in identifying distressed firms.

Given the importance of hazard models in predicting bankruptcy, and the robustness of our results in dealing with neglected econometric issues in most previous empirical research in bankruptcy related survival analysis, we believe this paper makes a significant contribution to SMEs and corporate failure literature.

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Table and Figures

Table 1: Survival Table

Age	Event 1			Event 2			Event 3		
	1	0	% 1	1	0	% 1	1	0	% 1
1	8	3,931	0.20	316	3,623	8.02	2	3,937	0.05
2	13	6,373	0.20	0	6,386	0.00	1	6,385	0.02
3	28	6,749	0.41	607	6,170	8.96	1	6,776	0.01
4	41	6,412	0.64	894	5,559	13.85	6	6,447	0.09
5	52	6,242	0.83	831	5,463	13.20	12	6,282	0.19
6	47	5,791	0.81	910	4,928	15.59	12	5,826	0.21
7	56	4,941	1.12	812	4,185	16.25	16	4,981	0.32
8	48	4,277	1.11	716	3,609	16.55	20	4,305	0.46
9	54	3,776	1.41	697	3,133	18.20	17	3,813	0.45
10	50	3,283	1.50	618	2,715	18.54	14	3,319	0.42
11	35	2,564	1.35	476	2,123	18.31	11	2,588	0.43
12	25	2,233	1.11	371	1,887	16.43	11	2,247	0.49
13	24	2,039	1.16	358	1,705	17.35	6	2,057	0.29
14	19	1,817	1.03	323	1,513	17.59	5	1,831	0.27
15	15	1,625	0.91	273	1,367	16.65	6	1,634	0.37
16	10	1,460	0.68	248	1,222	16.87	3	1,467	0.20
17	11	1,285	0.85	224	1,072	17.28	2	1,294	0.15
18	7	1,128	0.62	195	940	17.18	3	1,132	0.27
19	8	1,017	0.78	199	826	19.41	3	1,022	0.29
20	12	900	1.32	157	755	17.21	4	908	0.44
21	10	786	1.26	132	664	16.58	1	795	0.13
22	6	715	0.83	123	598	17.06	0	721	0.00
23	6	642	0.93	111	537	17.13	1	647	0.15
24	6	573	1.04	107	472	18.48	0	579	0.00
25	9	483	1.83	95	397	19.31	3	489	0.61
26	10	445	2.20	93	362	20.44	3	452	0.66
27	6	411	1.44	74	343	17.75	5	412	1.21
28	4	379	1.04	62	321	16.19	0	383	0.00
29	4	329	1.20	62	271	18.62	1	332	0.30
30	5	271	1.81	50	226	18.12	2	274	0.73
31	5	235	2.08	41	199	17.08	1	239	0.42
32	5	201	2.43	38	168	18.45	1	205	0.49
33	4	178	2.20	23	159	12.64	1	181	0.55
34	4	163	2.40	20	147	11.98	1	166	0.60
35	4	145	2.68	15	134	10.07	0	149	0.00
36	3	127	2.31	16	114	12.31	0	130	0.00
37	2	115	1.71	16	101	13.68	1	116	0.86
38	1	111	0.89	11	101	9.82	0	112	0.00
39	0	102	0.00	13	89	12.75	0	102	0.00
40	0	91	0.00	9	82	9.89	0	91	0.00
41	1	69	1.43	6	64	8.57	0	70	0.00
42	0	45	0.00	0	45	0.00	0	45	0.00
43	0	46	0.00	3	43	6.52	0	46	0.00
44	0	41	0.00	3	38	7.32	0	41	0.00
45	0	36	0.00	2	34	5.56	0	36	0.00
46	0	30	0.00	3	27	10.00	0	30	0.00
47	0	27	0.00	2	25	7.41	0	27	0.00
48	0	23	0.00	0	23	0.00	0	23	0.00
49	0	23	0.00	1	22	4.35	0	23	0.00
50	0	20	0.00	1	19	5.00	0	20	0.00
51	0	19	0.00	1	18	5.26	0	19	0.00
52	0	14	0.00	2	12	14.29	0	14	0.00
53	0	11	0.00	0	11	0.00	0	11	0.00
54	0	8	0.00	0	8	0.00	0	8	0.00
55	0	6	0.00	1	5	16.67	0	6	0.00
56	0	0	0.00	0	0	0.00	0	0	0.00
57	0	0	0.00	0	0	0.00	0	0	0.00
58	0	1	0.00	0	1	0.00	0	1	0.00

Notes: This table shows the age wise distribution of firm-year observations for respective default events discussed in Section 3. Numeric '0' signifies censorship and '1' signifies that a firm has experienced the respective default event.

Table 2: Sample Industrial Classification

Industry Code	SIC Code	Industry	Included/Excluded
1	< 1000	Agriculture, Forestry, Fishing	Included
2	1000 to < 1500	Mining	Included
3	1500 to < 1800	Construction	Included
4	2000 to < 4000	Manufacturing	Included
5	5000 to < 5200	Wholesale Trade	Included
6	5200 to < 6000	Retail Trade	Included
7	7000 to < 8900	Services	Included
Excluded	4000 to < 5000	Transportation, Communications & Public Utilities	Excluded
Excluded	6000 to < 6800	Finance, Insurance & Real Estate	Excluded
Excluded	9100 to < 10000	Public Administration	Excluded

Notes: This table reports Standard Industrial Classification (SIC) of US firms. SIC Code is a four digit code that represents given industrial sectors. The last column reports the industrial sectors that we included or excluded from our sample.

Table 3: List of Covariates

Category	Variable	Definition	Compustat Data Item
Leverage	STDEBV	Short term debt/equity book value	DLC/SEQ
	TLTA	Total liabilities/tangible total assets	LT/(AT – INTAN)
	TLNW	Total liabilities/net worth	LT/(AT - LT)
	CETL	Capital employed/total liabilities	(AT – LCT)/LT
Liquidity	CTA	Cash and short-term investments/total assets	CHE/AT
	CR	Current Ratio; current assets/current liabilities	ACT/LCT
	QR	Quick Ratio; (current assets – stocks - prepayments)/current liabilities	(ACT – INVT – XPP)/LCT
	CHR	Cash Ratio; (cash + bank + marketable securities)/current liabilities	CHE/LCT
Financing	FETA	Financial expenses/total assets	XINT/AT
	FES	Financial expenses/sales	XINT/SALE
	RETA	Retained earnings/total assets	RE/AT
	EBITDAIE	Earnings before interest taxes depreciation and amortization/interest expense	EBITDA/XINT
Profitability	EBITDATA	Earnings before interest taxes depreciation and amortization/total assets	EBITDA/AT
	OPCE	Operating profit/capital employed	EBIT/(AT - LCT)
	ROE	Return on equity; Net profit/equity	NI/SEQ
	NIS	Net income/sales	NI/SALE
	OPNI	Operating profit/net income	EBIT/NI
Activity	SHP	Stock holding period; (stock × 365)/sales	(INVT × 365)/SALE
	DCP	Debtor collection period; (trade debtors × 365)/sales	(RECTR × 365)/SALE
	TCP	Trade creditors payment period; (trade creditors × 365)/sales	(AP × 365)/SALE
	WCTA	Working capital/total assets	WCAP/AT
	WCS	Working capital/sales	WCAP/SALE
	STA	Sales/tangible assets	SALE/(AT – INTAN)
Growth	CAG	Capital growth; calculated as ($Capital_t$ / $Capital_{t-1}$) - 1	(AT - LCT)
	SAG	Sales growth; calculated as ($Sale_t$ / $Sale_{t-1}$) - 1	SALE
	ERG	Earnings growth; calculated as ($EBIT_t$ / $EBIT_{t-1}$) - 1	EBIT
Other	TTA	Income taxes/total assets	TXT/AT
Control	Micro	No. of employees < 10	
	Small	10 =< No. of employees < 50	
	RISK	Event rate in a given industrial sector in a given year (calculated separately for different *Event* definition)	

Notes: This table lists the set of covariates, along with their respective definition, that we use for the empirical analysis. The last column lists the specific Compustat data items that we use to calculate the financial covariates.

Table 4: Descriptive Statistics

Variable	Status Indicator	Event 1			Event 2			Event 3		
		Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
STDEBV	0	0.1889	0.0144	0.4235	0.1843	0.0176	0.3977	0.1893	0.0143	0.4245
	1	0.1967	0.0000	0.5262	0.2163	0.0000	0.5558	0.0442	-0.0571	0.4291
OPNI	0	1.1200	1.0058	1.0524	1.1707	1.0338	1.0748	1.1177	1.0050	1.0533
	1	0.6870	0.6729	1.0861	0.7976	0.8270	0.8516	0.5416	0.4644	0.9635
TLTA	0	0.6390	0.4978	0.5124	0.5481	0.4325	0.4430	0.6404	0.4993	0.5130
	1	0.9685	0.8343	0.5879	1.1870	1.0742	0.5578	1.2802	1.3848	0.5885
TLNW	0	1.0663	0.4885	2.4897	1.0672	0.4964	2.2490	1.0681	0.4890	2.4963
	1	1.0459	0.3762	3.3770	1.0595	0.1858	3.6364	0.2452	-1.1327	3.4305
CETL	0	2.5593	1.5343	2.5867	2.8603	1.8361	2.6255	2.5520	1.5274	2.5852
	1	1.1824	0.5146	1.8534	0.7110	0.3667	1.2149	0.6954	0.1984	1.2239
CTA	0	0.2346	0.1357	0.2351	0.2425	0.1475	0.2362	0.2343	0.1352	0.2352
	1	0.1856	0.0660	0.2335	0.1853	0.0747	0.2230	0.1681	0.0578	0.2272
CR	0	2.8370	1.9516	2.4008	3.0807	2.2228	2.4266	2.8311	1.9443	2.4002
	1	1.6869	0.9452	1.9695	1.3366	0.8283	1.5489	1.3542	0.6025	1.6870
QR	0	2.1085	1.2028	2.1818	2.3227	1.3980	2.2275	2.1027	1.1982	2.1804
	1	1.1411	0.4881	1.6776	0.8220	0.3966	1.2752	0.9004	0.3045	1.4141
CHR	0	1.4273	0.4502	1.9092	1.5658	0.5720	1.9647	1.4240	0.4478	1.9081
	1	0.8188	0.0984	1.5851	0.5769	0.0900	1.2250	0.6384	0.0572	1.4420
FETA	0	0.0294	0.0172	0.0324	0.0244	0.0136	0.0285	0.0295	0.0173	0.0324
	1	0.0507	0.0474	0.0381	0.0583	0.0591	0.0375	0.0602	0.0689	0.0402
FES	0	0.0611	0.0192	0.0953	0.0509	0.0157	0.0854	0.0613	0.0193	0.0954
	1	0.0982	0.0510	0.1087	0.1227	0.0624	0.1237	0.1235	0.0608	0.1222
EBITDAIE	0	99.092	-0.0423	537.480	120.948	1.2368	576.057	98.612	-0.0524	536.500
	1	22.388	-0.7447	333.325	-24.179	-3.723	166.108	20.568	-0.6260	312.090
EBITDATA	0	-0.2248	-0.0043	0.6142	-0.1498	0.0266	0.5294	-0.2250	-0.0045	0.6141
	1	-0.3060	-0.0438	0.6628	-0.6531	-0.2799	0.8446	-0.4394	-0.0765	0.7822
OPCE	0	-0.1001	-0.0052	0.4090	-0.0905	0.0025	0.3881	-0.1003	-0.0057	0.4091
	1	-0.1207	-0.0373	0.4296	-0.1573	-0.1130	0.5120	-0.0597	-0.0003	0.4276
ROE	0	-0.1204	0.0105	0.5801	-0.1293	0.0080	0.5361	-0.1202	0.0104	0.5809
	1	-0.0163	0.0411	0.6791	-0.0624	0.0754	0.7918	0.1536	0.1829	0.6011
NIS	0	-0.4445	-0.0374	0.7868	-0.3881	-0.0069	0.7603	-0.4451	-0.0382	0.7866
	1	-0.5511	-0.2287	0.7520	-0.7995	-0.4063	0.8510	-0.5537	-0.2568	0.7432
RETA	0	-1.5313	-0.4904	2.2310	-1.2142	-0.3045	2.0203	-1.5314	-0.4929	2.2298
	1	-2.0233	-1.0498	2.2862	-3.3941	-3.1440	2.4774	-3.2022	-2.7933	2.4772
SHP	0	50.132	35.532	51.986	49.777	35.764	51.378	50.134	35.536	51.986
	1	49.579	31.523	52.770	52.287	33.687	55.590	46.837	22.446	54.655
DCP	0	63.907	55.077	49.284	64.987	56.112	48.985	63.873	55.017	49.304
	1	59.123	46.274	53.136	57.293	47.770	50.741	60.949	49.949	55.460
TCP	0	168.668	35.971	532.046	146.338	33.444	485.965	168.499	35.982	531.456
	1	187.043	40.940	558.262	306.849	58.592	742.050	308.623	50.766	801.227
WCTA	0	0.2370	0.2774	0.3594	0.2857	0.3245	0.3327	0.2357	0.2760	0.3598
	1	-0.0152	-0.0288	0.3790	-0.0645	-0.0845	0.3707	-0.1111	-0.2199	0.3810
WCS	0	0.4157	0.2371	0.6494	0.4689	0.2735	0.6404	0.4140	0.2357	0.6496
	1	0.0812	-0.0269	0.5831	0.0657	-0.0388	0.5965	-0.0504	-0.1886	0.4929
STA	0	1.0735	0.8915	0.9189	1.0633	0.8915	0.8993	1.0737	0.8912	0.9192
	1	1.1171	0.8864	0.9830	1.1347	0.8914	1.0262	1.1383	0.9604	1.0089
CAG	0	0.1639	0.0292	0.6464	0.1924	0.0469	0.6229	0.1621	0.0283	0.6465
	1	-0.0889	-0.2126	0.6427	0.0053	-0.2042	0.7368	-0.0191	-0.1314	0.7247
SAG	0	0.1788	0.0815	0.4423	0.1906	0.0955	0.4327	0.1776	0.0804	0.4424

	1	-0.0170	-0.1309	0.4161	0.1033	-0.0326	0.4852	-0.0466	-0.1897	0.4138
ERG	0	-0.0394	-0.0681	1.3143	-0.0439	-0.0420	1.3478	-0.0407	-0.0695	1.3150
	1	-0.2671	-0.3645	1.3192	-0.0295	-0.1764	1.1325	-0.3475	-0.4978	1.1091
TTA	0	0.0130	0.0000	0.0282	0.0148	0.0000	0.0296	0.0130	0.0000	0.0282
	1	0.0061	0.0000	0.0222	0.0020	0.0000	0.0140	0.0037	0.0000	0.0167

Notes: This table reports mean, median and standard deviation for healthy (censored; status indicator = 0) and unhealthy (firms which experienced default event; status indicator = 1) groups of firms for respective covariates under different definitions of default events as discussed in Section 3.

Table 5: Correlation Matrix

Variable		1	2	3	4	5	6	7	8
STDEBV	1	1							
TLTA	2	0.0780	1						
CETL	3	-0.2986	-0.6772	1					
CTA	4	-0.2670	-0.3276	0.4829	1				
FETA	5	0.1943	0.7093	-0.5344	-0.3343	1			
FES	6	0.0084	0.4339	-0.2462	-0.0563	0.6230	1		
EBITDAIE	7	-0.1004	-0.1983	0.2729	0.1330	-0.2071	-0.1508	1	
EBITDATA	8	0.1278	-0.4223	0.1548	-0.1694	-0.2899	-0.3963	0.2129	1
OPCE	9	-0.1097	0.0783	-0.0390	-0.2278	0.0482	-0.1084	0.2402	0.3127
NIS	10	0.1112	-0.2128	-0.0345	-0.2941	-0.172	-0.5105	0.2517	0.6920
RETA	11	0.1292	-0.5031	0.2299	-0.1590	-0.3409	-0.3353	0.1872	0.6588
WCTA	12	-0.1851	-0.7413	0.5870	0.5451	-0.5782	-0.4074	0.1994	0.3148
WCS	13	-0.2144	-0.5619	0.6392	0.7005	-0.4397	-0.1252	0.0593	0.0360
CAG	14	-0.0992	-0.1505	0.1613	0.1336	-0.1323	-0.0293	0.0540	0.1490
SAG	15	-0.0117	-0.0651	0.0448	0.0833	-0.0868	-0.0430	0.0225	0.0375
TTA	16	-0.0680	-0.1870	0.0894	-0.0085	-0.1813	-0.2153	0.2993	0.3199
		9	10	11	12	13	14	15	16
OPCE	9	1							
NIS	10	0.4705	1						
RETA	11	0.3057	0.5371	1					
WCTA	12	-0.0633	0.1543	0.3499	1				
WCS	13	-0.2574	-0.2318	0.1047	0.7403	1			
CAG	14	0.2161	0.0921	0.1658	0.1969	0.1783	1		
SAG	15	0.0424	0.0061	0.0497	0.0554	0.0676	0.2730	1	
TTA	16	0.4175	0.3232	0.2985	0.2158	-0.0259	0.1345	0.0971	1

Table 6: Event 2 Univariate Regression

Variable	*Sign*	*T – 1*			*T – 2*			*T – 3*		
		β	*SE*	*AME* %	*β*	*SE*	*AME* %	*β*	*SE*	*AME* %
STDEBV	+	0.1700[a]	0.0298	1.05[a]	0.3928[a]	0.0312	2.84[a]	0.3225[a]	0.0329	2.72[a]
OPNI	-	-0.3627[a]	0.0147	-2.32[a]	-0.3323[a]	0.0150	-2.48[a]	-0.1608[a]	0.0146	-1.36[a]
TLTA	+	2.4762[a a]	0.0332	16.59 [a]	2.2925[a]	0.0340	17.77[a]	0.6454[a]	0.0317	5.78[a]
TLNW	+	0.0096[b]	0.0048	0.05[b]	0.0349[a]	0.0050	0.25[a]	0.0294[a]	0.0053	0.24[a]
CETL	-	-0.9832[a]	0.0175	-7.60[a]	-0.8332[a]	0.0155	-7.20[a]	-0.2073[a]	0.0076	-1.87[a]
CTA	-	-1.9079[a]	0.0753	-11.67[a]	-2.2780[a]	0.0882	-16.20[a]	-0.5436[a]	0.0770	-4.55[a]
CR	-	-0.5862[a]	0.0113	-4.09[a]	-0.5718[a]	0.0113	-4.60[a]	-0.1559[a]	0.0077	-1.35[a]
QR	-	-0.6774[a]	0.0179	-4.69[a]	-0.6328[a]	0.0171	-4.86[a]	-0.1790[a]	0.0113	-1.51[a]
CHR	-	-0.5532[a]	0.0132	-3.55[a]	-0.5760[a]	0.0138	-4.30[a]	-0.1436[a]	0.0098	-1.22[a]
FETA	+	32.1570[a]	0.4631	248.01[a]	24.0993[a]	0.4610	219.22[a]	8.8175[a]	0.4786	84.55[a]
FES	+	7.2175[a]	0.1558	50.52[a]	6.3882[a]	0.1665	51.69[a]	3.7784[a]	0.1733	33.55[a]
EBITDAIE	-	-0.0007[a]	0.0000	-0.00[a]	-0.0020[a]	0.0001	-0.02[a]	-0.0014[a]	0.0000	-0.01[a]
EBITDATA	-	-1.0990[a]	0.0233	-7.98[a]	-1.5416[a]	0.0289	-11.39[a]	-0.8150[a]	0.0276	-7.22[a]
OPCE	-	-0.1528[a]	0.0320	-0.94[a]	-1.0082[a]	0.0351	-7.50[a]	-1.1616[a]	0.0373	-9.96[a]
ROE	-	0.2635[a]	0.0210	1.64[a]	-0.2591[a]	0.0219	-1.90[a]	-0.5588[a]	0.0238	-4.75[a]
NIS	-	-0.5768[a]	0.0200	-3.66[a]	-1.0970[a]	0.0237	-7.89[a]	-0.7226[a]	0.0229	-5.97[a]
RETA	-	-0.4931[a]	0.0078	-3.68[a]	-0.4340[a]	0.0079	-3.76[a]	-0.1911[a]	0.0077	-1.81[a]
SHP	+	-0.0002	0.0000	-0.00	0.0034[a]	0.0003	0.02[a]	0.0048[a]	0.0003	0.04[a]
DCP	+	-0.0039[a]	0.0003	-0.02[a]	-0.0023[a]	0.0003	-0.02[a]	0.0005	0.0003	0.00
TCP	+	0.0003[a]	0.0000	0.00[a]	0.0003[a]	0.0000	0.00[a]	0.001[a]	0.0000	0.00[a]
WCTA	-	-3.2430[a]	0.0500	-22.62[a]	-3.1762[a]	0.0517	-25.47[a]	-0.8462[a]	0.0462	-7.44[a]
WCS	-	-1.5370[a]	0.0335	-9.60[a]	-1.4041[a]	0.0336	-10.04[a]	-0.1892[a]	0.0276	-1.50[a]
STA	+	0.2387[a]	0.0184	1.44[a]	0.0406[b]	0.0199	0.28[b]	-0.2551[a]	0.0215	-2.10[a]
CAG	-	-0.4236[a]	0.0210	-3.12[a]	-1.1145[a]	0.0271	-9.68[a]	-0.4648[a]	0.0240	-3.78[a]
SAG	-	-0.4975[a]	0.0321	-3.42[a]	-0.8480	0.0351	-6.76[a]	-0.3526[a]	0.0356	-2.70[a]
ERG	-	-0.0145	0.0104	-0.10	-0.0009	0.0108	-0.00	-0.0331[a]	0.0116	-0.26[a]
TTA	-	-24.5294[a]	0.8286	-166.40[a]	-46.0535[a]	1.2150	-370.58[a]	-28.3887[a]	0.9206	-255.68[a]

Notes: a (b) [c] significant at the 1 % (5 %) [10 %] level (two-sided test). This table reports univariate regression estimates of respective covariates at respective lagged time periods, estimated using a discrete-time hazard model with logit link and *Event 2* = 1 as outcome event. 'Sign' represents expected sign of regression coefficients, *β* is the regression coefficient, SE is standard error and AME is Average Marginal Effects in percentage.

Table 7: Final Set of Covariates

Variable	*Sign*	*T – 1*			*T – 2*			*T – 3*		
		β	*Rank*	*AME %*	*β*	*Rank*	*AME %*	*β*	*Rank*	*AME %*
STDEBV	+	0.1700[a]	14	1.05[a]	0.3928[a]	15	2.84[a]	0.3225[a]	11	2.72[a]
TLTA	+	2.4762[a]	5	16.59[a]	2.2925[a]	5	17.77[a]	0.6454[a]	8	5.78[a]
CETL	-	-0.9832[a]	9	-7.60[a]	-0.8332[a]	12	-7.20[a]	-0.2073[a]	13	-1.87[a]
CTA	-	-1.9079[a]	6	-11.67[a]	-2.2780[a]	6	-16.20[a]	-0.5436[a]	9	-4.55[a]
FETA	+	32.1570[a]	1	248.01[a]	24.0993[a]	2	219.22[a]	8.8175[a]	2	84.55[a]
FES	+	7.2175[a]	3	50.52[a]	6.3882[a]	3	51.69[a]	3.7784[a]	3	33.55[a]
EBITDAIE	-	-0.0007[a]	16	-0.00[a]	-0.0020[a]	16	-0.02[a]	-0.0014[a]	16	-0.01[a]
EBITDATA	-	-1.0990[a]	8	-7.98[a]	-1.5416[a]	7	-11.39[a]	-0.8150[a]	6	-7.22[a]
OPCE	-	-0.1528[a]	15	-0.94[a]	-1.0082[a]	11	-7.50[a]	-1.1616[a]	4	-9.96[a]
NIS	-	-0.5768[a]	11	-3.66[a]	-1.0970[a]	10	-7.89[a]	-0.7226[a]	7	-5.97[a]
RETA	-	-0.4931[a]	10	-3.68[a]	-0.4340[a]	14	-3.76[a]	-0.1911[a]	14	-1.81[a]
WCTA	-	-3.2430[a]	4	-22.62[a]	-3.1762[a]	4	-25.47[a]	-0.8462[a]	5	-7.44[a]
WCS	-	-1.5370[a]	7	-9.60[a]	-1.4041[a]	8	-10.04[a]	-0.1892[a]	15	-1.50[a]
CAG	-	-0.4236[a]	13	-3.12[a]	-1.1145[a]	9	-9.68[a]	-0.4648[a]	10	-3.78[a]
SAG	-	-0.4975[a]	12	-3.42[a]	-0.8480	13	-6.76[a]	-0.3526[a]	12	-2.70[a]
TTA	-	-24.5294[a]	2	-166.40[a]	-46.0535[a]	1	-370.58[a]	-28.3887[a]	1	-255.68[a]

Notes: a (b) [c] significant at the 1 % (5 %) [10 %] level (two-sided test). This table reports the final set of covariates that we use for multivariate hazard analysis. This excludes covariates reported in Table 6 that are not significant in all three time periods or are significant but exhibit Average Marginal Effects (AME) of less than 5% in all three time periods. It also includes all covariates of Altman and Sabato's (2007) study irrespective of their significance or AME values. 'Sign' represents expected sign of regression coefficients, β is the regression coefficient, SE is standard error and AME is Average Marginal Effects in percentage. *Rank* is based on the absolute values of AME, where highest value gets 1, second highest get 2 and so on.

Table 8: Univariate Regression

Section A: Event 1									
Variable	logit			clog-log			Cox		
	T – 1	*T – 2*	*T – 3*	*T – 1*	*T – 2*	*T – 3*	*T – 1*	*T – 2*	*T – 3*
TLTA									
β	1.6496[a]	0.9402[a]	0.5690[a]	1.4897[a]	0.8714[a]	0.5411[a]	1.2130[a]	0.5792[a]	0.2296[c]
SE	0.1164	0.1183	0.1230	0.1031	0.1060	0.1168	0.1111	0.1153	0.1312
AIC	4866.94	4711.79	4394.56	4865.63	4713.43	4398.85	16902.86	16511.26	15864.87
CETL									
β	-0.4373[a]	-0.3022[a]	-0.2104[a]	-0.4068[a]	-0.2801[a]	-0.1931[a]	-0.3974[a]	-0.2629[a]	-0.1569[a]
SE	0.0424	0.0371	0.0346	0.0393	0.0340	0.0316	0.0396	0.0324	0.0337
AIC	5125.48	4966.65	4650.35	5125.60	4971.34	4663.57	17225.83	17114.84	16301.24
CTA									
β	-1.5618[a]	-2.4716[a]	-2.0885[a]	-1.3537[a]	-2.0737[a]	-1.7839[a]	-1.6333[a]	-2.3980[a]	-1.9371[a]
SE	0.2933	0.3325	0.3420	0.2617	0.2903	0.3031	0.2818	0.3004	0.3151
AIC	5546.80	5240.32	4859.45	5552.50	5253.27	4870.86	18900.08	18084.53	16885.00
FETA									
β	21.9980[a]	16.2405[a]	12.2258[a]	19.737[a]	14.708[a]	11.454[a]	17.330[a]	11.251[a]	7.526[a]
SE	1.6891	1.7580	1.9064	1.5101	1.5475	1.6837	1.572	1.598	1.753
AIC	5028.53	4975.84	4634.43	5033.99	4976.34	4636.87	16649.44	16779.55	16034.27
FES									
β	4.6120[a]	4.2092[a]	3.4310[a]	4.2783[a]	3.9209[a]	3.3885[a]	4.0081[a]	3.4730[a]	2.6573[a]

SE	0.5815	0.6150	0.6842	0.5280	0.5521	0.6161	0.601	0.6159	0.6763
AIC	4907.00	4804.39	4458.37	4913.86	4812.60	4466.04	16158.28	16259.79	15558.52
EBITDATA									
β	-0.4419[a]	-0.1463	-0.0290	-0.4109[a]	-0.1540	-0.0349	-0.3719[a]	-0.0523	0.0821
SE	0.0952	0.1106	0.1293	0.0878	0.1021	0.1184	0.0987	0.1137	0.1354
AIC	4943.13	4525.44	3924.66	4944.80	4531.46	3928.27	16900.65	15599.45	13849.46
OPCE									
β	-0.3015[a]	-0.5390[a]	-0.0830	-0.2527[b]	-0.4796[a]	-0.1038	-0.2982[b]	-0.4521[a]	-0.0438
SE	0.1351	0.1428	0.1560	0.1236	0.1302	0.1389	0.1304	0.1338	0.1444
AIC	5180.19	5010.50	4688.66	5185.94	5020.43	4698.28	17676.46	17420.71	16541.07
NIS									
β	-0.4056[a]	-0.3281[a]	-0.1338	-0.3923[a]	-0.3229[a]	-0.1613[b]	-0.4466[a]	-0.3397[a]	-0.1647[b]
SE	0.0759	0.0809	0.0887	0.0696	0.0732	0.0795	0.0752	0.0779	0.0841
AIC	5130.54	5021.24	4681.91	5138.88	5032.81	4696.54	17108.88	17131.33	16317.38
WCTA									
β	-2.3753[a]	-1.5757[a]	-1.0011[a]	-2.1715[a]	-1.4630[a]	-0.9621[a]	-2.0940[a]	-1.2861[a]	-0.6302[a]
SE	0.1790	0.1790	0.1188	0.1616	0.1615	0.1717	0.1738	0.1723	0.1805
AIC	5004.96	4924.59	4641.70	5012.94	4929.73	4649.67	16388.07	16688.53	16071.58
WCS									
β	-1.1173[a]	-0.7756[a]	-0.4955[a]	-1.0632[a]	-0.7286[a]	-0.4557[a]	-1.1180[a]	-0.7139[a]	-0.3625[a]
SE	0.1230	0.1185	0.1190	0.1155	0.1083	0.1089	0.1262	0.1190	0.1210
AIC	4827.48	4722.42	4436.58	4834.08	4730.81	4450.76	15642.71	15998.45	15491.13
CAG									
β	-0.7359[a]	-0.5442[a]	-0.2251[b]	-0.6653[a]	-0.5101[a]	-0.2557[a]	-0.4378[a]	-0.2648[a]	-0.0032
SE	0.0991	0.1015	0.0986	0.0910	0.0914	0.0883	0.0863	0.0840	0.0826
AIC	4939.88	4600.72	4203.41	4947.41	4606.36	4208.70	17167.37	16333.60	15133.20
SAG									
β	-1.2002[a]	-0.7332[a]	-0.4815[a]	-1.1010[a]	-0.6863[a]	-0.4584[a]	-0.5733[a]	-0.1759	0.1238
SE	0.1472	0.1444	0.1465	0.1348	0.1289	0.1290	0.1312	0.1249	0.1245
AIC	4850.35	4613.32	4226.02	4857.08	4624.41	4237.87	16259.93	15966.82	15100.79
TTA									
β	-15.279[a]	-19.145[a]	-14.104[a]	-12.914[a]	-16.382[a]	-12.404[a]	-9.151[a]	-10.854[a]	-5.7572[a]
SE	2.6195	2.8173	2.7590	2.3253	2.5141	2.455	2.396	2.433	2.3351
AIC	5364.37	5199.73	4846.44	5373.75	5214.21	4854.49	18084.87	17845.73	16942.59
STDEBV									
β	-0.1305	0.3249[a]	0.2026[c]	-0.1293	0.2497[b]	0.1838[c]	0.0581	0.3164[a]	0.1792[c]
SE	0.1211	0.1127	0.1226	0.1075	0.0984	0.1070	0.1023	0.0949	0.0972
AIC	5536.37	5259.03	4875.63	5540.88	5267.90	4883.93	18894.16	18123.03	16986.19
EBITDAIE									
β	-0.0005[a]	-0.0007[a]	-0.0007[a]	-0.0005[a]	-0.0006[a]	-0.0007[a]	-0.0005[a]	-0.0007[a]	-0.0007[a]
SE	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0001	0.0001	0.0002
AIC	5116.50	4967.77	4579.99	5119.08	4972.20	4587.21	17237.08	17021.55	15996.35
RETA									
β	-0.2179[a]	-0.0403	0.0579	-0.2123[a]	-0.0545[c]	0.0342	-0.0963[a]	0.0828[b]	0.2082[a]
SE	0.0285	0.0313	0.0363	0.0259	0.0285	0.0326	0.0292	0.0328	0.0377
AIC	5276.77	5164.54	4789.04	5277.01	5173.25	4802.39	17632.39	17494.17	16569.14
Section B: Event 2									
TLTA									
β	2.4763[a]	2.2925[a]	0.6454[a]	1.9131[a]	1.7385[a]	0.4722[a]	1.5491[a]	1.2181[a]	-0.0936[a]

	SE	0.0332	0.0340	0.0317	0.02433	0.0249	0.0254	0.0269	0.0263	0.0268
	AIC	39819.01	38592.44	39138.22	40351.88	39033.42	39367.36	169228.88	169676.96	151787.27
CETL										
	β	-0.9832[a]	-0.8333[a]	-0.2073[a]	-0.8996[a]	-0.7392[a]	-0.1744[a]	-0.7573[a]	-0.5932[a]	-0.0674[a]
	SE	0.0175	0.0155	0.0076	0.0154	0.0135	0.0067	0.0153	0.0128	0.0071
	AIC	42936.67	40933.98	41487.03	42838.21	40938.75	41690.80	178868.12	179742.98	163152.60
CTA										
	β	-1.9080[a]	-2.2780[a]	-0.5437[a]	-1.5666[a]	-1.8406[a]	-0.4203[a]	-1.3751[a]	-1.5422[a]	-0.1930[a]
	SE	0.0753	0.0821	0.0770	0.0626	0.0667	0.0632	0.0692	0.0689	0.0691
	AIC	50466.9	47135.76	43106.02	50713.44	47333.51	43285.52	187157.31	186678.96	166773.82
FETA										
	β	32.157[a]	24.0993[a]	8.8175[a]	25.545[a]	18.576[a]	6.5496[a]	19.732[a]	11.720[a]	0.3837
	SE	0.4632	0.4610	0.4786	0.3534	0.3537	0.3826	0.3847	0.3743	0.3990
	AIC	44019.66	43860.67	41314.35	44370.10	44184.51	41531.49	181845.91	185102.80	163025.31
FES										
	β	7.2176[a]	6.3882[a]	3.7784[a]	5.5920[a]	4.7768[a]	2.8066[a]	4.8881[a]	3.7060[a]	1.8270[a]
	SE	0.1558	0.1665	0.1733	0.1181	0.1252	0.1355	0.1368	0.1371	0.1472
	AIC	42745.25	40838.36	37532.89	43045.40	41128.05	37750.06	161702.51	162744.67	144887.85
EBITDATA										
	β	-1.0991[a]	-1.5416[a]	-0.8150[a]	-0.8326[a]	-1.1046[a]	-0.5830[a]	-0.6991[a]	-0.8538[a]	-0.3375[a]
	SE	0.0233	0.0288	0.0275	0.0173	0.0200	0.0206	0.0195	0.0203	0.0221
	AIC	46126.68	39795.64	36760.93	46499.05	40255.74	37028.58	177955.61	165151.81	143739.48
OPCE										
	β	-0.1529[a]	-1.0081[a]	-1.1616[a]	-0.0848[a]	-0.7633[a]	-0.8937[a]	-0.1837[a]	-0.6900[a]	-0.7763[a]
	SE	0.0320	0.0351	0.0374	0.0268	0.0284	0.0296	0.0269	0.0274	0.0291
	AIC	49958.13	46026.44	41098.81	50193.82	46327.37	41345.69	183139.45	182568.16	161874.04
NIS										
	β	-0.5768[a]	-1.0970[a]	-0.7226[a]	-0.4656[a]	-0.8509[a]	-0.5663[a]	-0.5472[a]	-0.9142[a]	-0.5837[a]
	SE	0.0201	0.0237	0.0229	0.0165	0.0181	0.0181	0.0192	0.0197	0.0203
	AIC	45352.45	40878.51	38124.25	45592.83	41159.19	38350.81	164494.07	162248.50	146980.02
WCTA										
	β	-3.2430[a]	-3.1762[a]	-0.8462[a]	-2.6079[a]	-2.4827[a]	-0.6387[a]	-2.1781[a]	-1.8792[a]	-0.0260
	SE	0.0490	0.0516	0.0461	0.0379	0.0391	0.0376	0.0413	0.0404	0.0403
	AIC	44180.67	41937.58	41577.89	44532.85	42262.79	41793.63	178733.60	179140.35	161417.71
WCS										
	β	-1.5368[a]	-1.4041[a]	-0.1892[a]	-1.3067[a]	-1.1611[a]	-0.1411[a]	-0.9721[a]	-0.7765[a]	0.1827[a]
	SE	0.0336	0.0336	0.0276	0.0282	0.0276	0.0227	0.0286	0.0270	0.0245
	AIC	41981.14	39752.62	37887.54	42170.25	39923.35	38056.91	158354.53	158024.93	142668.54
CAG										
	β	-0.4236[a]	-1.1145[a]	-0.4648[a]	-0.3359[a]	-0.9067[a]	-0.3640[a]	-0.1901[a]	-0.6554[a]	-0.1948[a]
	SE	0.0209	0.0271	0.0240	0.0177	0.0230	0.0199	0.0170	0.0218	0.0194
	AIC	46446.14	40032.58	36510.56	46667.55	40334.54	36707.40	182447.86	161571.4	142418.85
SAG										
	β	-0.4975[a]	-0.8480[a]	-0.3526[a]	-0.4018[a]	-0.6950[a]	-0.2773[a]	-0.0007	-0.2608[a]	0.0784[a]
	SE	0.0321	0.0351	0.0356	0.0266	0.0292	0.0291	0.0263	0.0286	0.0288
	AIC	42940.24	38469.84	34404.26	43119.91	38641.54	34566.08	163908.56	147657.20	130515.75
TTA										
	β	-24.529[a]	-46.053[a]	-28.388[a]	-21.440[a]	-39.567[a]	-25.191[a]	-16.111[a]	-32.700[a]	-17.570[a]
	SE	0.8287	1.2150	0.9206	0.7345	1.0230	0.8199	0.7956	1.0520	0.8362

AIC	49823.67	45309.29	41671.94	50062.04	45472.27	41817.08	186556.15	185582.80	165637.98
STDEBV									
β	0.1701[a]	0.3928[a]	0.3226[a]	0.1142[a]	0.2924[a]	0.2537[a]	0.2214[a]	0.3352[a]	0.3159[a]
SE	0.0299	0.0312	0.0329	0.0250	0.0252	0.0261	0.0248	0.0243	0.0261
AIC	50832.49	47532.22	42826.97	51068.9	47749.17	43001.94	186510.11	185977.43	165771.45
EBITDAIE									
β	-0.0007[a]	-0.0020[a]	-0.0014[a]	-0.0006[a]	-0.0016[a]	-0.0013[a]	-0.0008[a]	-0.0021[a]	-0.0014[a]
SE	0.0001	0.0001	0.0001	0.00004	0.0001	0.0001	0.0001	0.0001	0.0001
AIC	49315.06	45838.27	41007.13	49550	46036.04	41161.93	184043.20	185078.73	162333.72
RETA									
β	-0.4932[a]	-0.4340[a]	-0.1911[a]	-0.3929[a]	-0.3403[a]	-0.1501[a]	-0.2244[a]	-0.1383[a]	0.0965[a]
SE	0.0078	0.0079	0.0077	0.0059	0.0061	0.0062	0.0068	0.0068	0.0072
AIC	45778.64	44205.43	42160.60	46152.58	44510.55	42364.95	185095.29	185404.33	165212.43

Section C: Event 3									
TLTA									
β	2.2613[a]	2.2702[a]	1.7594[a]	2.0836[a]	2.0658[a]	1.6174[a]	1.9450[a]	1.7950[a]	1.0670[a]
SE	0.2104	0.2194	0.2057	0.1874	0.1927	0.1835	0.2118	0.2594	0.2529
AIC	1672.94	1605.55	1607.53	1678.16	1606.31	1612.21	8793.33	11036.43	11439.28
CETL									
β	-0.8100[a]	-1.3677[a]	-1.0503[a]	-0.7819[a]	-1.2917[a]	-0.9926[a]	-0.5795[a]	-1.1940[a]	-0.8392[a]
SE	0.1256	0.1756	0.1501	0.1204	0.1596	0.1352	0.1162	0.1653	0.1334
AIC	1807.22	1713.77	1705.97	1812.53	1715.99	1709.98	11896.03	8558.01	8911.28
CTA									
β	-1.2774[b]	-2.7134[a]	-3.1083[a]	-1.0860[b]	-2.3802[a]	-2.7352[a]	-1.3870[a]	-2.8200[a]	-2.7380[a]
SE	0.4999	0.5819	0.6234	0.4575	0.5207	0.5522	0.5584	0.6180	0.6329
AIC	1956.25	1879.62	1829.70	1964.16	1889.12	1839.38	13023.02	12597.72	12360.73
FETA									
β	24.262[a]	29.334[a]	28.935[a]	21.696[a]	27.067[a]	26.671[a]	20.470[a]	24.470[a]	21.970[a]
SE	2.8578	3.0460	3.2041	2.5710	2.7135	2.8383	3.0910	3.2460	3.3690
AIC	1782.76	1768.11	1734.50	1792.13	1772.51	1737.39	11905.15	11712.32	11686.16
FES									
β	5.9683[a]	5.9914[a]	6.2774[a]	5.6128[a]	5.5103[a]	5.8787[a]	5.8460[a]	5.7780[a]	5.9680[a]
SE	0.9425	0.9578	1.0459	0.8731	0.8526	0.9317	1.2450	1.2390	1.2920
AIC	1683.05	1720.33	1663.62	1688.37	1726.73	1670.39	11989.54	12111.67	11787.72
EBITDATA									
β	-0.6521[a]	-0.7622[a]	-0.8135[a]	-0.6212[a]	-0.6576[a]	-0.7441[a]	-0.4909[a]	-0.4434[a]	-0.7848[a]
SE	0.1534	0.1749	0.1794	0.1428	0.1497	0.1628	0.1799	0.1988	0.2319
AIC	1719.03	1598.20	1437.74	1724.62	1599.91	1444.79	12248.42	11576.53	10713.39
OPCE									
β	0.4351[c]	-0.0031	-0.5827[b]	0.4065[c]	-0.0273	-0.5849[a]	0.2023	-0.0342	-0.6290[c]
SE	0.2456	0.2434	0.2499	0.2257	0.2239	0.2246	0.2418	0.2353	0.2484
AIC	1842.11	1840.29	1798.55	1848.99	1848.20	1807.13	12810.22	12672.58	12406.50
NIS									
β	-0.1690	-0.5182[a]	-0.6758[a]	-0.1570	-0.4651[a]	-0.6240[a]	-0.2152	-0.5197[a]	-0.7127[a]
SE	0.1341	0.1321	0.1387	0.1235	0.1172	0.1219	0.1566	0.1499	0.1585
AIC	1748.85	1759.28	1689.10	1757.05	1766.78	1697.26	12644.75	12542.26	12144.18
WCTA									
β	-2.9201[a]	-3.4000[a]	-2.8521[a]	-2.7048[a]	-3.1075[a]	-2.6474[a]	-2.1300[a]	-2.5091[a]	-1.6840[a]

SE	0.3286	0.3533	0.3375	0.2987	0.3107	0.3000	0.3571	0.3645	0.3442
AIC	1745.31	1701.38	1700.15	1751.42	1706.69	1705.59	12114.67	11805.99	11934.96
WCS									
β	-1.8870^{a}	-1.8229^{a}	-1.6438^{a}	-1.7958^{a}	-1.7241^{a}	-1.5782^{a}	-1.5530^{a}	-1.3580^{a}	-1.0540^{a}
SE	0.2770	0.2724	0.2729	0.2609	0.2501	0.2482	0.2890	0.2652	0.2595
AIC	1638.45	1658.85	1613.76	1643.09	1664.47	1620.71	11665.17	11757.54	11502.82
CAG									
β	-0.2799^{c}	-0.5790^{a}	-1.2346^{a}	-0.2719^{b}	-0.5426^{a}	-1.1636^{a}	-0.0643	-0.2673^{c}	-0.7301^{a}
SE	0.1512	0.1731	0.2203	0.1390	0.1586	0.2026	0.1342	0.1440	0.1791
AIC	1830.94	1775.43	1610.59	1838.31	1783.90	1616.51	12647.23	12316.50	11161.17
SAG									
β	-1.1656^{a}	-1.6190^{a}	-1.7849^{a}	-1.1151^{a}	-1.4808^{a}	-1.4904^{a}	-0.5766^{b}	-0.9741^{a}	-0.7436^{a}
SE	0.2665	0.2951	0.3119	0.2478	0.2693	0.2711	0.2661	0.2680	0.2664
AIC	1719.37	1666.91	1556.88	1725.12	1675.49	1566.20	12345.43	11927.15	11210.35
TTA									
β	-20.974^{a}	-20.448^{a}	-48.177^{a}	-17.538^{a}	-17.843^{a}	-42.345^{a}	-17.620^{a}	-16.050^{a}	-45.040^{a}
SE	5.5795	5.5359	8.4749	4.8925	4.9486	7.4022	5.6440	5.4830	9.1990
AIC	1890.55	1877.57	1799.09	1898.95	1886.24	1808.33	12943.40	12792.58	12340.86
STDEBV									
β	-1.7364^{a}	-0.0330	0.5541^{a}	-1.5598^{a}	-0.0460	0.5348^{a}	-0.8312^{a}	0.0242	0.4216^{b}
SE	0.3239	0.1968	0.1824	0.3018	0.1769	0.1596	0.2529	0.1814	0.1770
AIC	1901.23	1897.58	1846.94	1908.93	1905.26	1854.31	12948.38	12805.56	12503.92
EBITDAIE									
β	-0.0002	-0.0003	-0.0009^{b}	-0.0002	-0.0003	-0.0008^{b}	-0.0003	-0.0003	-0.0009^{b}
SE	0.0002	0.0003	0.0004	0.0002	0.0003	0.0004	0.0003	0.0003	0.0004
AIC	1827.81	1851.02	1791.41	1833.82	1858.40	1799.65	12565.97	12561.62	12210.71
RETA									
β	-0.4576^{a}	-0.3484^{a}	-0.2025^{a}	-0.4209^{a}	-0.3201^{a}	-0.1846^{a}	-0.2789^{a}	-0.1585^{a}	0.0303
SE	0.0511	0.0484	0.0493	0.0453	0.0431	0.0441	0.0598	0.0596	0.0627
AIC	1788.77	1813.79	1811.38	1793.14	1819.79	1820.26	12422.61	12528.49	12529.59

Notes: a (b) [c] significant at the 1 % (5 %) [10 %] level (two-sided test). This table reports univariate regression estimates of Event 1, Event 2 and Event 3 using respective hazard models and lagged time periods. Section A reports regression estimates of Event 1, Section B reports Event 2, and Section C reports Event 3.

Table 9: Multivariate Regression

Section A: Event 1

Variable	logit			clog-log			Cox		
	T – 1	*T – 2*	*T – 3*	*T – 1*	*T – 2*	*T – 3*	*T – 1*	*T – 2*	*T – 3*
STDEBV									
β		0.3008^{b}			0.2010^{c}			0.3709^{a}	
SE		0.1295			0.1201			0.0884	
p-value		0.0200			0.0940			0.0000	
TLTA									
β	0.9948^{a}			0.8597^{a}			0.7369^{a}		
SE	0.2226			0.1913			0.1384		

p-value	0.0000			0.0000			0.0000		
CETL									
β	-0.2530[a]			-0.2388[a]			-0.2259[a]		
SE	0.0700			0.0618			0.0448		
p-value	0.0000			0.0000			0.0000		
CTA									
β	-0.3425		-2.2829[a]	-0.2665		-1.9289[a]	-0.2804		-1.7416[a]
SE	0.4572		0.4935	0.3970		0.4305	0.2786		0.2893
p-value	0.2440		0.0000	0.2320		0.0000	0.2100		0.0000
FETA									
β	7.3617[a]	12.369[a]	6.1982[b]	6.0372[a]	11.740[a]	6.4750[a]	3.0878[a]	9.1737[a]	5.5547[a]
SE	2.7061	2.3270	2.5504	2.3321	2.2416	2.2326	1.8124	1.6206	1.5421
p-value	0.0070	0.0000	0.0150	0.0100	0.0000	0.0040	0.0000	0.0000	0.0030
FES									
β									
SE									
p-value									
EBITDAIE									
β		-0.0005[b]	-0.0006[a]		-0.0005[b]	-0.0006[a]		-0.0003[b]	-0.0005[a]
SE		0.0002	0.0002		0.0002	0.0002		0.0001	0.0001
p-value		0.0200	0.0090		0.0120	0.0090		0.0420	0.0019
EBITDATA									
β									
SE									
p-value									
OPCE									
β	-0.5270[a]	-0.2243		-0.4234[a]	-0.2546		-0.2847[b]	-0.1002	
SE	0.1812	0.1809		0.1550	0.1697		0.1185	0.1277	
p-value	0.0040	0.2150		0.0060	0.1340		0.0160	0.2300	
NIS									
β									
SE									
p-value									
RETA									
β									
SE									
p-value									
WCTA									
β		-0.5958[b]			-0.6578[a]			-0.5090[a]	
SE		0.2543			0.2462			0.1748	
p-value		0.0190			0.0080			0.0036	
WCS									
β									
SE									
p-value									
CAG									
β	-0.2328[c]	-0.1335		-0.2025[c]	-0.1082		-0.3474[a]	-0.2175[b]	
SE	0.1206	0.1120		0.1063	0.1039		0.0913	0.0862	

p-value	0.0540	0.2330		0.0570	0.2480		0.0000	0.0120	
SAG									
β	-0.5326[a]	-0.2226	-0.2023	-0.4826[a]	-0.1808	-0.2368[c]	-0.8411[a]	-0.4764[a]	-0.2778[b]
-SE	0.1748	0.1528	0.1643	0.1551	0.1438	0.1415	0.1351	0.1200	0.1155
p-value	0.0020	0.1450	0.2180	0.0020	0.2090	0.0940	0.0000	0.0000	0.0160
TTA									
β	-4.1461	-10.394[a]	-10.428[a]	-3.4981	-9.5294[a]	-8.9473[a]	-1.1812	-7.6240[a]	-7.0481[a]
SE	3.5669	3.6154	3.5802	3.1214	3.3587	3.0952	2.4601	2.5793	2.3600
p-value	0.2450	0.0040	0.0040	0.2420	0.0050	0.0040	0.6300	0.0031	0.0000
Micro									
β	2.0176[a]	1.9982[a]	2.5646[a]	1.7140[a]	2.0089[a]	2.2413[a]	0.7910[a]	1.1579[a]	1.5554[a]
SE	0.2419	0.2119	0.2412	0.2078	0.2038	0.2116	0.1295	0.1254	0.1257
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Small									
β	0.7552[a]	0.8696[a]	1.2075[a]	0.6548[a]	0.8792[a]	1.0438[a]	0.1142	0.3240[a]	0.5422[a]
SE	0.2064	0.1899	0.2060	0.1829	0.1802	0.1861	0.1266	0.1225	0.1275
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3700	0.0084	0.0000
AGE									
	0.4964[a]	0.5408[a]	0.7246[a]	0.3949[a]	0.5447[a]	0.6130[a]	-33.253[a]	-36.575[a]	-43.231[a]
β	0.1452	0.1451	0.1835	0.1266	0.1435	0.1631	1.0312	1.2182	1.2320
SE	0.0010	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000	0.0000	0.0000
p-value									
RISK1	90.495[a]	77.879[a]	91.854[a]	77.607[a]	78.677[a]	78.7463[a]	46.871[a]	44.689[a]	43.998[a]
β	7.3345	6.1372	7.3431	6.2999	6.1021	6.4905	3.0780	2.9346	3.3759
SE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
p-value									
Model's Goodness of Fit and Performance Measure									
Chi2	403.06[a]	321.8[a]	311.8[a]	411.2[a]	377.8[a]	311.9[a]	2785[a]	2874[a]	3222[a]
likelihood	-1746.9	-1837.2	-1791.8	-1748.6	-1838.5	-1790.4	-3051.2	-3291.5	-3283.1
AIC	3521.9	3702.3	3605.6	3525.3	3705.1	3602.9	6294.5	6753.4	7237.08
N	46927	44400	40882	46927	44400	40882	46927	44400	40882
Event	433	464	469	433	464	469	433	464	469
AUROC-W	0.8209	0.7936	0.7827	0.8212	0.7929	0.7831	0.8192	0.7912	0.7797
AUROC-H	0.7943	0.8339	0.9242	0.7890	0.8937	0.9233	0.7572	0.8784	0.9112
Section B: Event 2									
STDEBV									
β	0.1743[a]	0.2511[a]	0.2533[a]	0.1544[a]	0.2151[a]	0.1998[a]	0.2003[a]	0.2046[a]	0.1974[a]
SE	0.0388	0.0453	0.0456	0.0290	0.0328	0.0348	0.0233	0.0249	0.0290
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TLTA									
β	1.9886[a]	2.2478[a]	0.3296[a]	1.4622[a]	1.5668[a]	0.2143[a]	1.1641[a]	1.1266[a]	0.4912[a]
SE	0.0654	0.0777	0.0723	0.0481	0.0548	0.0549	0.0347	0.0370	0.0418
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
CETL									
β									
SE									
p-value									
CTA									
β		-1.3447[a]			-0.9909[a]			-0.5477[a]	

SE		0.1462			0.1082			0.0769	
p-value		0.0000			0.0000			0.0000	
FETA									
β	15.786[a]	2.6571[a]	3.3366[a]	11.798[a]	2.1859[a]	2.6998[a]	8.6179[a]	3.6960[a]	4.5587[a]
SE	0.7083	0.9955	0.9549	0.5354	0.7093	0.7338	0.4185	0.5306	0.6053
p-value	0.0000	0.0080	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000	0.0000
FES									
β		0.3141	0.8577[a]		0.0920	0.5806[a]		-0.3210[b]	0.1233
SE		0.2911	0.2675		0.2110	0.2061		0.1461	0.1607
p-value		0.2410	0.0010		0.6630	0.0050		0.0280	0.4400
EBITDAIE									
β	-0.0003[a]	-0.0004[a]	-0.0006[a]	-0.0004[a]	-0.0004[a]	-0.0006[a]	-0.0005[a]	-0.0004[a]	-0.0006[a]
SE	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
p-value	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
EBITDATA									
β	-0.1624[a]	-0.7619[a]	-0.3114[a]	-0.0850[a]	-0.5110[a]	-0.2281[a]	-0.0860[a]	-0.3205[a]	-0.1783[a]
SE	0.0380	0.0467	0.0422	0.0264	0.0312	0.0312	0.0177	0.0205	0.0236
p-value	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000
OPCE									
β	-0.9026[a]	-1.4723[a]	-0.8999[a]	-0.6398[a]	-1.0036[a]	-0.6650[a]	-0.5802[a]	-0.7744[a]	-0.6558[a]
SE	0.0471	0.0580	0.0538	0.0336	0.0395	0.0409	0.0260	0.0289	0.0341
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
NIS									
β									
SE									
p-value									
RETA									
β									
SE									
p-value									
WCTA									
β	-0.5875[a]	-0.2946[a]	-0.1058	-0.5189[a]	-0.3310[a]	-0.1064	-0.3858[a]	-0.2745[a]	-0.0722
SE	0.0881	0.1130	0.0948	0.0668	0.0832	0.0733	0.0501	0.0595	0.0566
p-value	0.0000	0.0090	0.2440	0.0000	0.0000	0.1470	0.0000	0.0000	0.2000
WCS									
β									
SE									
p-value									
CAG									
β		-0.7094[a]	-0.1534[a]		-0.4575[a]	-0.1116[a]		-0.4140[a]	-0.1096[a]
SE		0.0373	0.0300		0.0278	0.0240		0.0239	0.0221
p-value		0.0000	0.0000		0.0000	0.0000		0.0000	0.0000
SAG									
β	-0.0569	-0.3881[a]		-0.0652[b]	-0.2993[a]		-0.1152[a]	-0.2601[a]	
SE	0.0395	0.0461		0.0300	0.0342		0.0252	0.0284	
p-value	0.1500	0.0000		0.0300	0.0000		0.0000	0.0000	
TTA									
β	-11.374[a]	-28.651[a]	-14.413[a]	-10.596[a]	-25.191[a]	-13.489[a]	-11.846[a]	-24.508[a]	-14.586[a]

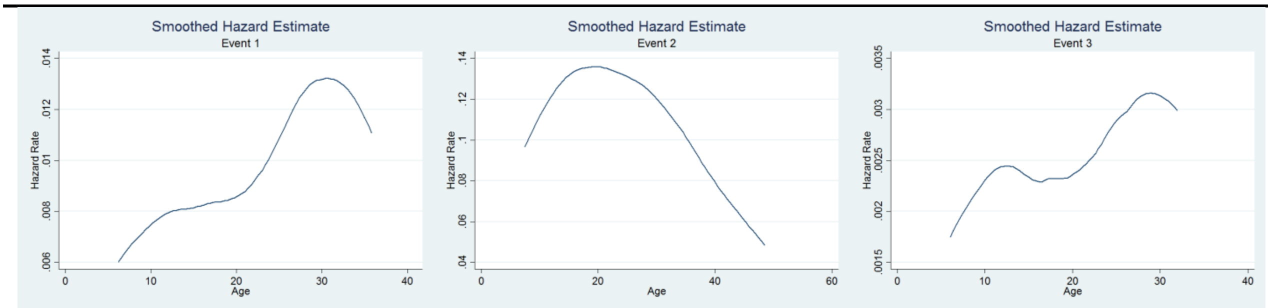
SE	1.0783	1.5922	1.1580	0.9030	1.3307	1.0279	0.7909	1.0302	0.9033
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Micro									
β	0.4154[a]	0.5147[a]	1.0577[a]	0.2928[a]	0.3762[a]	0.8451[a]	0.1853[a]	0.2582[a]	0.6144[a]
SE	0.0662	0.0742	0.0683	0.0504	0.0556	0.0536	0.0342	0.0366	0.0377
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Small									
β	0.2786[a]	0.3265[a]	0.6749[a]	0.2094[a]	0.2675[a]	0.5466[a]	0.1471[a]	0.1845[a]	0.4122[a]
SE	0.0488	0.0551	0.0521	0.0381	0.0423	0.0421	0.0279	0.0301	0.0316
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AGE									
β	-0.0285	-0.1323[a]	-0.0029	-0.0732[a]	-0.1535[a]	-0.0137	-24.865[a]	-27.661[a]	-33.798[a]
SE	0.0353	0.0445	0.0453	0.0277	0.0343	0.0366	0.2734	0.3437	0.4217
p-value	0.4190	0.0030	0.9490	0.0080	0.0000	0.7090	0.0000	0.0000	0.0000
RISK2									
β	5.4673[a]	4.0201[a]	6.5065[a]	4.1230[a]	3.2239[a]	5.2226[a]	2.4035[a]	1.7983[a]	3.3036[a]
SE	0.4117	0.4662	0.4423	0.3152	0.3513	0.3475	0.2266	0.2531	0.2688
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Model's Goodness of Fit and Performance Measure									
Chi2	5135.9[a]	4381.9[a]	2228.2[a]	5818.2[a]	5039.8[a]	2307.3[a]	42073[a]	37046[a]	29048[a]
likelihood	-13550.1	-10534.2	-12115.9	-13807.9	-10862.8	-12225.9	-52703.6	-42792.4	-39919.4
AIC	27130.1	21104.3	24263.6	27645.9	21761.6	24483.8	105433.1	85616.7	79866.8
N	44740	36907	33396	44740	36907	33396	44740	36907	33396
Event	7553	6390	5721	7553	6390	5721	7553	6390	5721
AUROC-W	0.8739	0.9015	0.7794	0.8721	0.8991	0.7767	0.8699	0.8969	0.7865
AUROC-H	0.8436	0.8714	0.7783	0.8425	0.8686	0.7745	0.8414	0.8705	0.7803
Section C: Event 3									
STDEBV									
β		0.4912[c]	0.6417[b]		0.4266[c]	0.4680[c]		0.6404[a]	0.6278[a]
SE		0.2715	0.2861		0.2250	0.2401		0.1721	0.1593
p-value		0.0700	0.0250		0.0580	0.0510		0.0000	0.0000
TLTA									
β		1.7594[a]	1.5434[a]		1.6420[a]	1.3451[a]		1.4437[a]	1.0166[a]
SE		0.4712	0.4767		0.3943	0.4140		0.2787	0.2551
p-value		0.0000	0.0010		0.0000	0.0010		0.0000	0.0000
CETL									
β	-0.4543[a]			-0.4430[a]			-1.0176[a]		
SE	0.1747			0.1682			0.1237		
p-value	0.0090			0.0080			0.0000		
CTA									
β			-1.3311			-1.0743			-0.9048
SE			1.1703			1.0090			0.6390
p-value			0.2450			0.2870			0.1600
FETA									
β	12.022[a]	16.656[a]	15.631[a]	9.5936[a]	13.686[a]	14.313[a]	5.7686[b]	10.272[a]	7.3060[b]
SE	4.1089	4.9550	5.7427	3.5011	4.1907	4.9884	2.8568	3.3580	3.5285
p-value	0.0030	0.0010	0.0060	0.0060	0.0010	0.0040	0.0440	0.0000	0.0380
FES									
β									

SE									
p-value									
EBITDAIE									
β									
SE									
p-value									
EBITDATA									
β									
SE									
p-value									
OPCE									
β		-0.4816	-0.3651		-0.4501	-0.2209		-0.5053[b]	-0.3928[c]
SE		0.3382	0.3815		0.2829	0.3328		0.2123	0.2202
p-value		0.1540	0.2390		0.1120	0.5070		0.0170	0.0750
NIS									
β									
SE									
p-value									
RETA									
β	-0.2000[a]			-0.1957[a]			0.0104		
SE	0.0751			0.0659			0.0451		
p-value	0.0080			0.0030			0.8199		
WCTA									
β	-0.6195	-0.8440		-0.5472	-0.5571		0.0368	-0.4660	
SE	0.5165	0.6515		0.4475	0.5384		0.3657	0.4109	
p-value	0.2300	0.1950		0.2210	0.3010		0.9200	0.2600	
WCS									
β			-0.4132			-0.3643			-0.2302
SE			0.4486			0.3830			0.2504
p-value			0.2470			0.3410			0.3600
CAG									
β			-0.5432[b]			-0.4291[c]			-0.4133[b]
SE			0.2587			0.2286			0.1692
p-value			0.0360			0.0610			0.0150
SAG									
β	-0.2695	-0.7311[b]	-0.8411[b]	-0.1909	-0.6557[b]	-0.7206[b]	-0.7072[a]	-1.0009[a]	-0.6980[a]
SE	0.2873	0.3311	0.3575	0.2571	0.2863	0.3126	0.2307	0.2441	0.2369
p-value	0.2480	0.0270	0.0190	0.4580	0.0220	0.0210	0.0022	0.0004	0.0032
TTA									
β	-15.152[c]	-3.948	-36.756[a]	-14.043[c]	-3.288	-30.773[a]	-12.173[b]	-3.463	-21.901[a]
SE	8.458	8.164	11.596	7.407	6.970	10.121	5.458	5.151	7.195
p-value	0.0730	0.6290	0.0020	0.0580	0.6370	0.0020	0.0260	0.5000	0.0023
Micro									
β	1.9486[a]	2.4476[a]	2.7400[a]	1.6619[a]	1.9905[a]	2.2511[a]	1.3615[a]	1.3692[a]	1.5166[a]
SE	0.4272	0.4541	0.4979	0.3718	0.3700	0.4144	0.2614	0.2485	0.2334
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Small									
β	0.4747	0.4216	0.6411	0.3976	0.3934	0.5285	0.4086[c]	0.2318	0.2639
SE	0.3809	0.4366	0.4742	0.3376	0.3681	0.4114	0.2418	0.2565	0.2572

p-value	0.2130	0.2340	0.1760	0.2390	0.2850	0.1990	0.0910	0.3700	0.3000
AGE									
β	0.9837[a]	0.7653[b]	0.4242	0.8869[a]	0.6001[b]	0.3010	-42.888[a]	-47.477[a]	-35.227[a]
SE	0.2714	0.3363	0.3910	0.2386	0.2799	0.3439	2.4175	2.0754	3.2681
p-value	0.0000	0.0230	0.2480	0.0000	0.0320	0.3810	0.0000	0.0000	0.0000
RISK3									
β	219.806[a]	226.564[a]	239.428[a]	204.698[a]	189.601[a]	211.302[a]	94.989[a]	87.859[a]	79.818[a]
SE	22.477	24.405	29.131	16.901	17.913	21.444	11.357	11.824	10.728
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Model's Goodness of Fit and Performance Measure									
Chi2	170.1[a]	155.2[a]	132.6[a]	236.5[a]	202.6[a]	167.3[a]	1149[a]	1112[a]	1084[a]
likelihood	-657.02	-590.5	-529.3	-660.6	-592.1	-530.7	-938.6	-881.6	-808.5
AIC	1338.1	1207.1	1088.6	1345.2	1210.3	1091.4	2117.9	2048.1	1808.8
Censored	50126	40639	35327	50126	40639	35327	50126	40639	35327
Event	143	136	131	143	136	131	143	136	131
AUROC-W	0.8840	0.9019	0.9020	0.8840	0.9031	0.9015	0.8783	0.8955	0.8964
AUROC-H	0.9249	0.8924	0.9668	0.9317	0.9019	0.9214	0.9447	0.8653	0.9556

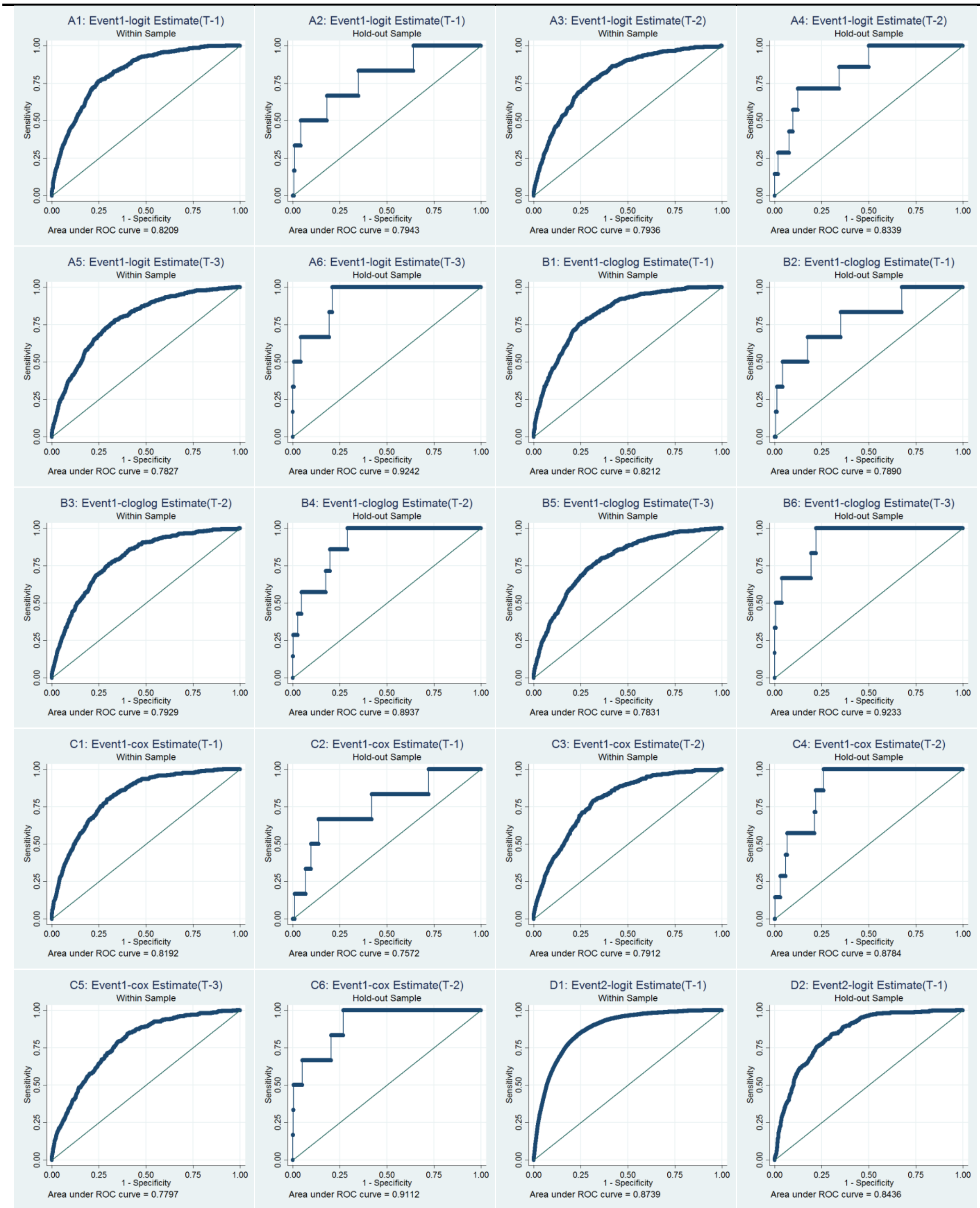
Notes: a (b) [c] significant at the 1 % (5 %) [10 %] level (two-sided test). This table reports multivariate regression estimates of Event 1, Event 2 and Event 3 using respective hazard models and lagged time periods. Section A reports regression estimates of Event 1, Section B reports Event 2, and Section C reports Event 3. The Chi2 values reported for logit and cloglog estimates are obtained using the Wald test, while for Cox regression it is obtained using likelihood ratio test. AUROC-W represents within sample and AUROC-H represents hold-out sample area under ROC curves. 'Event' reports total number of observations with dependent variable = 1 and 'censored' reports total number of observations with dependent variable = 0. Additionally, missing values of *β*, *SE* and *p-value* for any covariate implies that it has been excluded from the multivariate model due to its non-compliance with our model building strategy discussed in Section 5.3.2.

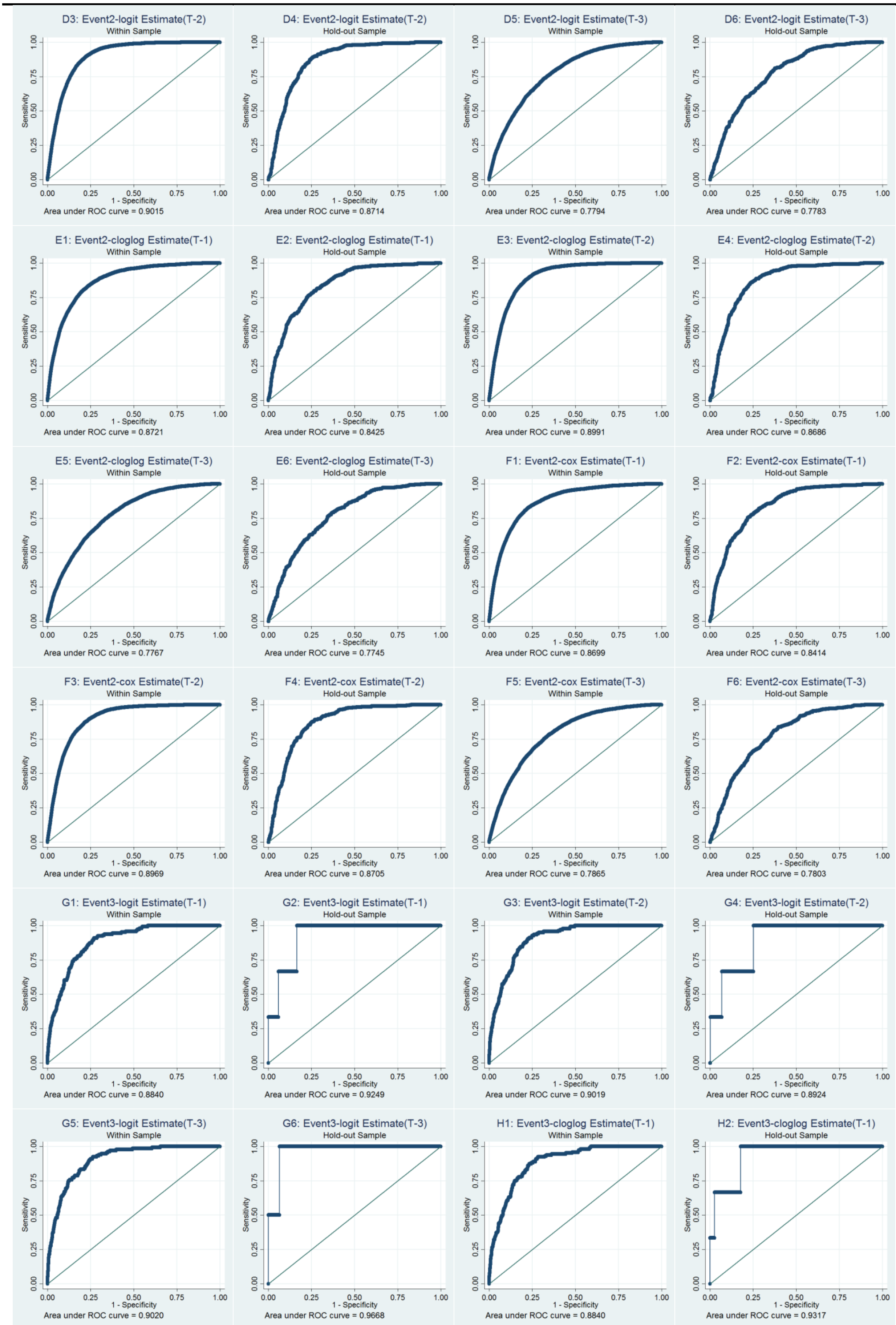
Figure 1: Table of Hazard Curves

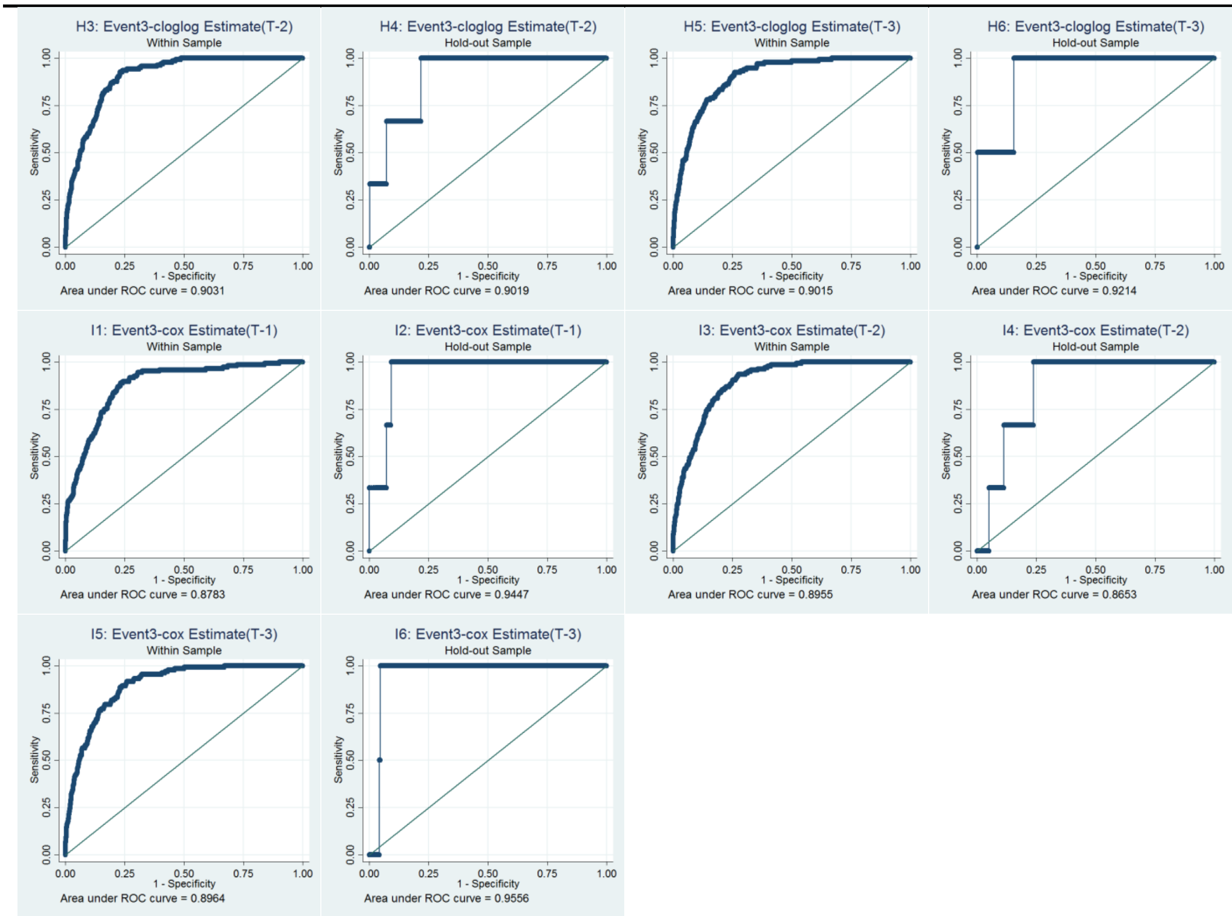


Notes: This table reports smoothed hazard curves estimated using the development sample for different definitions of financial distress events as discussed in Section 3. Here 'Age' represents the age of firms in years.

Figure 2: Table of Area under ROC curves




D3: Event2-logit Estimate(T-2)
Within Sample
Area under ROC curve = 0.9015
D4: Event2-logit Estimate(T-2)
Hold-out Sample
Area under ROC curve = 0.8714
D5: Event2-logit Estimate(T-3)
Within Sample
Area under ROC curve = 0.7794
D6: Event2-logit Estimate(T-3)
Hold-out Sample
Area under ROC curve = 0.7783
E1: Event2-cloglog Estimate(T-1)
Within Sample
Area under ROC curve = 0.8721
E2: Event2-cloglog Estimate(T-1)
Hold-out Sample
Area under ROC curve = 0.8425
E3: Event2-cloglog Estimate(T-2)
Within Sample
Area under ROC curve = 0.8991
E4: Event2-cloglog Estimate(T-2)
Hold-out Sample
Area under ROC curve = 0.8686
E5: Event2-cloglog Estimate(T-3)
Within Sample
Area under ROC curve = 0.7767
E6: Event2-cloglog Estimate(T-3)
Hold-out Sample
Area under ROC curve = 0.7745
F1: Event2-cox Estimate(T-1)
Within Sample
Area under ROC curve = 0.8699
F2: Event2-cox Estimate(T-1)
Hold-out Sample
Area under ROC curve = 0.8414
F3: Event2-cox Estimate(T-2)
Within Sample
Area under ROC curve = 0.8969
F4: Event2-cox Estimate(T-2)
Hold-out Sample
Area under ROC curve = 0.8705
F5: Event2-cox Estimate(T-3)
Within Sample
Area under ROC curve = 0.7865
F6: Event2-cox Estimate(T-3)
Hold-out Sample
Area under ROC curve = 0.7803
G1: Event3-logit Estimate(T-1)
Within Sample
Area under ROC curve = 0.8840
G2: Event3-logit Estimate(T-1)
Hold-out Sample
Area under ROC curve = 0.9249
G3: Event3-logit Estimate(T-2)
Within Sample
Area under ROC curve = 0.9019
G4: Event3-logit Estimate(T-2)
Hold-out Sample
Area under ROC curve = 0.8924
G5: Event3-logit Estimate(T-3)
Within Sample
Area under ROC curve = 0.9020
G6: Event3-logit Estimate(T-3)
Hold-out Sample
Area under ROC curve = 0.9668
H1: Event3-cloglog Estimate(T-1)
Within Sample
Area under ROC curve = 0.8840
H2: Event3-cloglog Estimate(T-1)
Hold-out Sample
Area under ROC curve = 0.9317
Sensitivity
1 - Specificity


H3: Event3-cloglog Estimate(T-2)
Within Sample
Area under ROC curve = 0.9031
H4: Event3-cloglog Estimate(T-2)
Hold-out Sample
Area under ROC curve = 0.9019
H5: Event3-cloglog Estimate(T-3)
Within Sample
Area under ROC curve = 0.9015
H6: Event3-cloglog Estimate(T-3)
Hold-out Sample
Area under ROC curve = 0.9214
I1: Event3-cox Estimate(T-1)
Within Sample
Area under ROC curve = 0.8783
I2: Event3-cox Estimate(T-1)
Hold-out Sample
Area under ROC curve = 0.9447
I3: Event3-cox Estimate(T-2)
Within Sample
Area under ROC curve = 0.8955
I4: Event3-cox Estimate(T-2)
Hold-out Sample
Area under ROC curve = 0.8653
I5: Event3-cox Estimate(T-3)
Within Sample
Area under ROC curve = 0.8964
I6: Event3-cox Estimate(T-3)
Hold-out Sample
Area under ROC curve = 0.9556
Sensitivity
1 - Specificity
0.00
0.25
0.50
0.75
1.00