Spatial skills and counting sequence knowledge: Investigating reciprocal longitudinal relations in early years

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A B S T R A C T

Despite the robust findings indicating the positive role of spatial skills in a variety of mathematics skills (e.g., calculation), no empirical evidence has been provided concerning the longitudinal reciprocal relations between spatial skills and oral counting – perhaps the earliest form of symbolic representation. The purpose of the present study was to investigate the reciprocal longitudinal relations between spatial skills and three types of counting sequence knowledge (counting range, counting forward, and counting backward), which were measured at three time points between ages 3 to 4 years in a group of 109 young Hong Kong children. A series of family background and domain-general variables (i.e., family SES, vocabulary knowledge and behavioral regulation) were measured as covariates. Results from the cross-lagged panel analyses showed a reciprocal relationship between the children's counting sequence knowledge and their spatial skills, but this relationship only occurred between Time 2 (i.e., fall semester in the first year of kindergarten) and Time 3 (i.e., spring semester in the first year of kindergarten) and not between Time 1 (i.e., end of the nursery year) and Time 2. Further analyses showed that this pattern of results was task dependent. Reciprocal relations were found only in the counting range task and the counting backward task; no reciprocal relationship was found between spatial skills and the counting forward task. The findings reveal the developmental reciprocal link between spatial skills and early mathematics competency (i.e., the production of counting lists) and further highlight that this link varies according to children's conceptual levels of counting knowledge. Implications for classroom practices and teaching strategies are discussed.

1. Introduction

The abilities to process numbers and spatial information has been recognized as core components in the early year education of Hong Kong. In the Kindergarten Education Curriculum Guide (2017) by Hong Kong Education Bureau, it states "to cater to children's development and thinking ability, teachers should assist children to understand basic mathematical language step by step and foster their sensitivity to numbers and space" (pp. 41). One potentially useful way to foster the sensitivity to numbers and space would be to link them together in early year education (Siegler & Ramani, 2009; Vasilyeva, Laski, Veraksa & Bukhalenkova, 2020). Therefore, it is an educationally important topic to look at whether the abilities to process numbers and spatial information are reciprocally related and feed into each other over development. In literature, some influential theoretical frameworks of numerical cognition have proposed this possibility (Dehaene, 2011; Walsh, 2003), and supporting evidence has been obtained in terms of similar behavioral patterns and shared neutral areas in these two domains; yet direct empirical evidence of the developmental reciprocal relation between number skills and spatial skills is still lacking in the field. Using a longitudinal design in children's early preschool years, the present study aimed to fill this educationally and theoretically important research gap by examining the reciprocal longitudinal relations between spatial skills and three types of counting sequence knowledge (counting range, counting forward, and counting backward). The study will present a unique contribution to the theoretical understanding of the developmental onset and patterns of interplays between symbolic number representation and spatial skills.

1.1. Theoretical frameworks

Our research questions are driven by two theoretical frameworks in the field of numerical cognition. First, critical to investigate the present research question is understanding of the magnitude representation of numbers (Dehaene, 2011). The mental representation of non symbolic numbers emerges early over development before the acquisition of language and follows a Weber's ratio. With the acquisition of language, the widely accepted idea is that symbolic representation of numbers, by the means of being mapped upon the non symbolic representation of numbers, gradually develops from basic levels (e.g., counting) to more sophisticated levels (e.g., algebraic calculation) (Lipton & Spelke, 2005). In the ontogenetic sense, counting skills represents the very first symbolic representation of numbers to lay the cognitive foundation of advanced mathematics skills. In this regard, testing counting skills in young children in our research question of the reciprocal relation between spatial skills and number skills are developmentally important.

Second, A Theory Of Magnitude (ATOM) by Walsh (2003) gives rise to the hypothesis of the present study. ATOM, derived from Gallistel and Gelman's (2000) idea, has proposed that a common magnitude system is shared by both of the mental representations of numbers and space. Supporting evidence for ATOM comes from that the same behavioral patterns and the overlapping neural basis (i.e., IPS) are observed in both number representation and spatial representation. Specifically, over development, it has been observed that both numbers and area representation reach the same level of accuracy at a particular age. Six-month-old infants detect a change in numerosity or area when the difference between the two stimuli in each dimension reached the ratio of 1:2, but they failed when this change reaches a ratio of 2:3 (Brannon, Lutz & Cordes, 2006; Xu & Spelke, 2000). Based upon ATOM, it is quite tentative to make a prediction that numbers and spatial skills feed off each other over development, i.e., a reciprocal relation between the two is expected. However, direct evidence is still lacking in this respect.

Overall, against the frameworks, we aimed to provide direct empirical evidence to fill the important research gaps in the literature concerning the developmental reciprocal relations between counting sequence knowledge and spatial skills in early years. This will greatly contribute to our theoretical understanding of the interplay between numbers and spatial representation from a development perspective indicated by ATOM.

1.2. Spatial skills and mathematics competency

Spatial skills are a set of skills with mental representation and manipulation of objects in the external world (Linn & Petersen, 1985). Spatial skills are widely recognized as being concurrently and longitudinally related to a wide variety of mathematics outcomes, such as arithmetic competence (Georges, Cornu & Schiltz, 2019; Yang, Huo & Zhang, 2021; Zhang et al., 2014, 2017) and general mathematics achievement (Gilligan, Flouri & Farran, 2017, Gilligan, Hodgkiss, Thomas, & Farran, 2018; Rittle-Johnson, Zippert & Boice, 2019; Tam, Wong & Chan, 2019; B. N. Verdine et al., 2014; B. N. Verdine, Irwin, Golinkoff & Hirsh-Pasek, 2014).

Recent research has paid increasing attention to the link between spatial skills and mathematics abilities in young children (Assel, Landry, Swank, Smith & Steelman, 2003; Cheng & Mix, 2014; Geer, Quinn & Ganley, 2019; Huo, Zhang & Law, 2021; Mix, Levine, Cheng, Stockton, & Bower, 2021; Rittle-Johnson et al., 2019; B. N. Verdine et al., 2014; Yang, Zhang, Huo & Zhang, 2020; Zhang, 2016; Zhang & Lin, 2015, 2017, 2018). Specifically, the relation between spatial skills and mathematics competency emerge in the very early years. For example, Lauer an Lourenco (2016) found that infants' spatial change-detection performance measured between 6 and 13 months of age predicted applied mathematics problem solving measured at 4 years of age after controlling for general cognitive abilities and spatial memory.

1.3. Counting sequence knowledge in early years

Counting skills are emphasized educationally and theoretically for young children's mathematics outcomes (Gallistel & Gelman, 1992; Hong Kong Education Bureau, 2017). As a fundamental component of counting skills, counting sequence knowledge refers to the oral production of the counting list, which makes concrete counting practice possible.

In early years, researchers have argued that counting sequence knowledge can be interpreted to represent not only the storage of number words in the verbal long-term memory but also to the conceptual understanding of counting words. For example, Ho and Fuson (1998) found that children's counting sequence knowledge was related to the children's cardinal understanding of number words. Johansson (2005) found that counting forward and counting backward were correlated with arithmetic performance in a group of 4 to 8-year-olds. Koponen, Salmi, Eklund, and Aro (2006) found that counting sequence knowledge in 7-year-olds predicted calculation fluency 2 years later. These studies and others (Koponen, Aunola & Nurmi, 2019, 2007; Zhang et al., 2014, 2017, 2020) collectively suggest that the process of acquiring counting sequence knowledge does not just involve the rote memorization of counting lists but is an indicator of early mathematics competency.

1.4. Reciprocal relations between spatial skills and counting sequence knowledge

Although spatial skills have been found to play positive roles in many types of mathematics competency, its relationship to counting sequence knowledge in young children remains unclear. Nuerk et al. (2015) indicated this possibility to point out there are various explicit and implicit ways for children to make an association between space and numbers before they receive school instruction in the left-to-right oriented number line. Specifically, in terms of the relation between counting sequence knowledge and spatial skills, multiple possible routes suggested by Nuerk et al. may exist. Specifically, first, counting sequence knowledge may be acquired through repeated counting practices both at home and at school (Fuson, Richards & Briars, 1982). In a typical counting practice scenario, children count objects by pointing to them one by one from left to right, which is synchronous with the oral production of counting numbers. After repeated practice, counting sequence knowledge itself may eventually be linked to the spatial direction embedded in the counting practice. Second, numbers and space may be linked as a result of monitoring adult reading, writing and counting or spontaneous pretend reading. Until recently, however, very few studies have attempted to provide evidence concerning the potential longitudinal reciprocal relations between spatial skills and counting sequence knowledge.

The unidirectional relation between spatial skills and counting sequence knowledge has received some empirical support. Barnes et al. (2011) found that visual-spatial ability at age 5 was related concurrently to counting sequence knowledge. Very recently, Cornu, Schiltz, Martin and Hornung (2018) found that visuo-spatial skills were the only significant predictors of concurrent verbal number skills in a group of 5- to 6-year-olds. In their study, verbal number skills were latent variables on free counting, backward counting, number naming and finger rapid automatized naming (RAN) tasks. In a longitudinal study, Zhang et al. (2014) found that counting sequence knowledge was a mediator in the relation between early spatial skills and arithmetic performance. One of their findings relevant to the present study was that spatial skills assessed at the end of kindergarten (around 7 years old) predicted counting sequence knowledge six months later in first grade. Some studies with an intervention design have also suggested the positive role of spatial skills in learning counting sequence knowledge (Lowrie, Logan & Ramful, 2017). For example, Cornu, Schiltz, Pazouki and Martin (2019)) trained 4- to 7-year-olds in visuospatial skills and found that the training had a positive effect on counting sequence abilities.

At the same time, the role of counting sequence knowledge in spatial skills remains largely unknown. Some exceptions are two recent studies focusing on whether general mathematics abilities can improve spatial skills. For example, Mix et al. (2016) reported that spatial skills were concurrently predicted by mathematics abilities in kindergarten through Grade 6 although the specific predictors varied across grades. In their kindergarten sample of 6-year-old children, place value, word problem solving and calculation concurrently predicted spatial skills. Very recently, Geer et al. (2019), in a longitudinal cross-lagged analysis, reported that primary school students' scores in a general mathematic test showed positive effects on a spatial task assessing spatial perception and mental rotation and on another task assessing spatial visualization. It should be noted that the relation in their study was unidirectional, with no reciprocal relations found during the same time window.

To conclude, some empirical evidence has pointed to the positive relation between spatial skills and counting sequence knowledge in children as young as 4 to 7 years of age. Regarding the other directional relation – between counting sequence knowledge and spatial skills – existing research has been conducted on older students' advanced mathematics abilities (Mix et al., 2016) or performance in general mathematics tests (Geer et al., 2019). Regarding this directional relation, no studies have been conducted on young children's most fundamental mathematics competency, such as counting sequence knowledge; however, assumptions concerning the effect of counting sequence knowledge on spatial skills can potentially be inferred from the findings of prior research. Moreover, no empirical evidence so far presents the two directional relations within the same time window, i.e., the reciprocal relation between spatial skills and counting sequence knowledge.

1.5. Levels of counting sequence knowledge

Another question arises pertaining to whether different types of counting sequence knowledge are reciprocally related to spatial skills to the same extent. Counting range knowledge, counting forward knowledge and counting backward knowledge require different levels of conceptual knowledge of counting. According to Fuson et al. (1982), children experience five stages in developing the three levels of counting sequence competency, and progress from one stage to the next requires children's conceptual reconstruction of the number naming sequence. In the first step, children remember the counting list as a whole; they always start from 1 when they count. When children understand that each number word in the counting sequence is separable, they are able to count forward. When children understand the bidirectionality of the counting sequence, they are able to count backwards.

The majority of previous studies have failed to take into account different conceptual levels of counting sequence knowledge. Most have used a composite score or a latent variable score consisting of different levels of counting abilities to represent counting sequence knowledge. This is probably problematic because children's counting range, counting forward, and counting backward are at different ability levels at a particular developmental point. In other words, they may demonstrate different strengths of association with cognitive or mathematics variables. Indeed, findings from previous research indicate the importance to distinguish these differences in investigating the correlates of counting sequence knowledge. For example, Nguyen et al. (2016) tested children's counting range, counting forward, and counting backward in the preschool stage and their general mathematics achievement in fifth grade. Their results revealed that in predicting later mathematics achievement, the composite score of counting forward and counting backward was a stronger predictor compared to counting range. Liu et al. (2016) also found that in Hong Kong kindergarten children, their counting range, counting forward and counting backward were differentially related to linguistic and cognitive precursors. Interpretations of these results are in relation to the conceptual understanding of counting sequence knowledge in these three tasks. The second aim of the present study, therefore, was to investigate the reciprocal relations between spatial skills and different levels of counting sequence knowledge in early year.

1.6. The present study

Although there is mounting evidence that mathematics competency and spatial skills are closely associated, no attempt has been made to investigate the reciprocal longitudinal relation between counting sequence knowledge and spatial skills. To fill this important gap, the current study investigated the reciprocal longitudinal relations between counting sequence knowledge and spatial skills in young children, and whether these reciprocal longitudinal relations varied across counting range, counting forward and counting backward. Counting sequence knowledge and spatial skills were measured at three time points using a 1-year longitudinal design, which allowed an investigation into the reciprocal relations between the two in the early years. The present study also controlled for family contextual factors (i.e., family SES) and domain-general cognitive skills (e.g., vocabulary knowledge and behavioral regulation) because these variables have long been found to predict mathematics competency (Zhang, Hu, Ren & Fan, 2017). Two research questions guided the present study:

RQ1. Are children's spatial skills and counting sequence knowledge, the earliest and most rudimentary symbolic number representation, reciprocally and longitudinally related during the very early years, and if so,

RQ2. Does this reciprocal relation vary depending on children's conceptual levels of counting sequence knowledge?

In light of prior research, we expected that spatial skills and counting sequence knowledge would show reciprocal relations in early years. Further, given counting development stages by Fuson et al. (1982) and prior evidence documenting differential relations of different counting tasks to cognitive abilities (e.g., Liu et al., 2016), we expected that the reciprocal relations between spatial skills and counting sequence knowledge vary in relation to their counting tasks. The study was the first of its kind to systemically examine the reciprocal longitudinal relationships between counting sequence knowledge and spatial skills in young children, and was aimed at contributing to an understanding of the development of children's spatial representation of symbolic numbers (Dehaene, Bossini & Giraux, 1993; Siegler & Opfer, 2003) and of ATOM.

2. Method

2.1. Participants and procedure

The present study was part of a longitudinal study focusing on Hong Kong kindergarteners' numeracy development. One hundred and nine children (mean age = 38.01 months; *SD* = 2.68 months; 64 boys and 45 girls) were recruited from 10 nursery classes in 6 not-for-profit kindergartens in Hong Kong. Among the participants, 67.6% of fathers and 59.5% of mothers reported to have at least a Bachelor's degree. Further, 63.4% families reported that their monthly income was HK$40,000 or above; they could be considered to be middle to high family socioeconomic status in Hong Kong when the study was conducted in 2014 (Zhang, 2016).

In Hong Kong, when children reach 3 years old, all of them are entitled to receive kindergarten education. All the kindergartens in Hong Kong are privately run (Lau & Rao, 2018). There are government guidelines on curriculums for all of the kindergartens in Hong Kong, i.e., Kindergarten Education Curriculum Guide (2017). According to this, Hong Kong kindergartens provide formal instructions on basic number skills, such as counting, sequencing and ordering from the first year of kindergarten education (i.e., when children are 3 years old). Children typically are admitted to the kindergartens by location or by parents' preferences. Teachers in Hong Kong kindergartens are required to hold Qualified Kindergarten Teacher [QKT] qualification or its equivalent since 2003. Further, in these six participating kindergartens, Cantonese is the main instruction language, which is also the language used during the daily interaction between teachers and children. Among the participating kindergartens, the number of classes per grade approximately ranged from 1 to 3. The size of each class was approximately ranged from 8 to 30 children. The teacher children ratio approximately ranged from 1:7 to 1:11. In Hong Kong, the kindergarten education ends when children reach around 6 years old to start their primary school education.

Meanwhile, all of the participating kindergartens provide daycare services. In Hong Kong, there are also daycare classes for 2- to 3-year-old toddlers in some, but not all, of the kindergartens. Not every child is expected to attend daycare classes before 3 years old in Hong Kong. Parents have their options to take care of their children at home by themselves, and ask for help from grandparents or hire helpers for childcare. Some parents may have their children to attend playgroups. In Hong Kong, only approximately 30% of the toddlers attend the daycare classes before the kindergarten education (Zhang, 2016). So, the current sample might limit the generalizability of our findings. Further, currently, there are no government curriculum guidelines on teaching among toddlers in Hong Kong.

Data collection involved three time points: the spring semester of nursery classes (June and July, Time 1 [T1]; $n = 109$), the fall semester of the first year of kindergarten (November and December, Time 2 [T2]; $n = 84$), and the spring semester of the first year of kindergarten (May and June, Time 3 [T3]; $n = 80$). Counting sequence knowledge and spatial skills were tested at T1, T2, and T3. Vocabulary knowledge and behavioral regulation were tested at T1 and used as control variables. Students' family SES information was collected at T1 and used as a control variable.

Before the study, the parents of all the participating children were informed about the purpose of the study and gave their written consent. The experimenters were undergraduate students who studied in the major of education or psychology. Before the study, all the experimenters were trained to ensure that they were familiar with the implementation of the measures. All of the participants were tested individually by 10 to 15 experimenters at each assessment wave. For one participant, it typically took 20 min to complete all the tasks. In each of the participating kindergartens, the test took place over two or three days within a week. The children were rewarded with small gifts such as pencils and erasers at the end of each assessment wave.

Longitudinal attrition was due mainly to children transferring to other kindergartens. Attrition analyses were conducted to see if students who left the study were different from those who did not. The analyses showed that students who left the study at T2 had higher family SES and lower levels of behavioral regulation (both $ps < 0.05$) than those who stayed in the study. Similarly, the students who left at T3 had higher family SES ($p < .05$). This might limit the generalization of our results to some extent.

2.2. Measures

2.2.1. Counting sequence knowledge

This was assessed using a verbal counting task which includes three sub-tasks, a counting range task, a counting forward task and a counting backward task. This verbal counting task was designed to assess young children's counting sequence knowledge (Koponen, Salmi, Eklund & Aro, 2013; Zhang et al., 2014). It has been recently adopted with Hong Kong kindergarteners in previous studies (e.g., Liu, Lin, & Zhang, 2016). This task is a pure verbal counting task, which means that no physical items were shown to facilitate counting. The Cronbach's alphas for this task across T1, T2 and T3 were 0.666, 0.831 and 0.861, respectively.

In the counting range task, the children were asked to verbally count from 1 to a pre-determined number. To avoid a ceiling effect, different counting ranges were used at different time points. The required ranges for T1, T2 and T3 were 1–50, 1–80, and 1–100, respectively. Two scoring methods were used to measure the children's performance. First, the highest number that the children could reach was recorded as their counting range. Further, one point was awarded when the children were successful in the range of 1–10, two points in the range of 1–50, three points in the range of 1–80, and four points in the range of 1–100.

In the counting forward task (4 items), the children were asked to start from a pre-determined number and count on. The four items were 1–5, 6–13, 18–25, and 39–48. The items were scored using a 3-point scale, with 2 = no errors, 1 = one small error (e.g., the child stopped counting one number too early), and 0 = two or more errors.

In the counting backward task (4 items), the children were asked to count down from a specific number. The four items were 12–7, 23–18, 33–17, and 23–19. The items were scored using a 3-point scale, with 2 = no errors, 1 = one small error (e.g., the child stopped counting one number too early), and 0 = two or more errors.

2.2.2. Spatial skills

Spatial skills are not a unitary construct; they include at least three types of spatial skills according to Linn and Petersen (1985), namely, spatial perception, mental rotation and spatial visualization. The spatial perception task typically does not involve manipulations or transformations, thus individual differences in spatial perception can be reliably detected in young children. By contrast, mental rotation and spatial visualization tasks have been found to be typically difficult for children under 5 years old (Frick, Mohring & Newcombe, 2014; Noda, 2014). Considering that the mean age of the participants of the present study were around 3 years old at T1 (e.g., 9.5% of the reported age were 36 months or below), spatial perception was considered as a proxy of spatial skills, and the task assessing spatial perception was considered as age-appropriate and adopted. The task was the Visual-Spatial Relationship subtest from the Test of Visual-Perceptual Skills Revised (Gardner, 1996). This task has been successfully adopted with Hong Kong kindergarteners in prior research to assess their spatial skills (Zhang & Lin, 2017)

For each item of the task, there were five drawings, among which one was differently oriented. The children were asked to identify the drawing that was distinct from the other four in terms of the orientation. There were 16 test items in total. One point was awarded to each correctly answered item. The Cronbach's alpha for this task across T1, T2 and T3 was 0.657, 0.814 and 0.831, respectively.

2.2.3. Family socioeconomic status (SES)

In order to assess the family socioeconomic status of the participants, information about parents' educational level and family monthly income was gathered using parent questionnaires. Parental education was measured on a 5-point scale: (1) junior high school; (2) high school to associate degree; (3) Bachelor's degree; (4) Master's degree; (5) Doctoral degree. The participants' family monthly household income was assessed using a 5-point scale: (1) less than HK$10,000; (2) HK$10,000 to HK$29,999; (3) HK$30,000 to HK$59,999; (4) HK$60,000 to HK$99,999; (5) HK$100,000 and over.

2.2.4. Vocabulary knowledge

This was assessed using 30 items from the Hong Kong Cantonese Receptive Vocabulary Test, which was developed by Cheung, Lee, and Lee (1997). The test is modeled on the Peabody Picture Vocabulary Test for 2- to 6-year-old Cantonese-speaking children in Hong Kong. All words were familiar to the children.

In each trial, the experimenter first read a word to the children. The children were then asked to select one picture among four options that could represent the meaning of the word they heard. Options included an item with a similar sound to that of the target, an item with a similar meaning to that of the target, an unrelated item and the target item. One point was awarded to each correctly answered item. The Cronbach's alpha for this task was 0.686.

2.2.5. Behavioral regulation

This was assessed using the Head-Toes-Knees-Shoulders (HTKS) task (McClelland et al., 2014), which is commonly used to capture the integration of executive functions in a game format. This has been used in other studies testing Hong Kong kindergarteners' behavioral regulation (Liu et al., 2018).

The children were instructed to play a game in which they did the opposite of the experimenters' instruction. In the first 10 trials, the experimenter instructed the children to touch their head (or toes); instead of following the instruction, the children were expected to do the opposite and touch their toes (or head). In the second 10 trials, the opposite options in the instruction were knees and shoulders. In the third 10 trials, the opposite options in the instruction mixed head, toes, knees and shoulders together (e.g., head-knees). The scoring standard was 0 (incorrect), 1 (self-correct), or 2 (correct) for each item. A self-correction was defined as any incorrect action that was self-corrected, ending with the correct action. The Cronbach's alpha for this task was 0.997.

2.3. Data analysis

Mplus 8.0 (Muthén & Muthén, 1998–2010) was adopted to conduct data analyses. Mplus has the advantage of handling missing data by using the Full Information Maximum Likelihood (FIML) procedure. In adopting the FIML approach, the sample size in all of the analyses was 109. The MLR estimator was used in Mplus because some variables were not normally distributed. All parameters were standardized estimation. The bootstrapping procedure in Mplus was used to calculate the 95% confidence interval of the parameter estimates. Family SES was calculated by a combination of z-scores of family income, mothers' education level and fathers' education level. Raw scores were calculated and used for analysis in other measures.

To investigate the reciprocal relations between counting abilities and spatial skills, a series of cross-lagged models was built to examine the autoregressive effects of variables across three measurement points and the cross-lagged relations between counting sequence knowledge and spatial skills. Variables at the same time point were set to be correlated. Further, four control variables – family SES, vocabulary knowledge, behavioral regulation and gender – were included in the cross-lagged models by being regressed on counting sequence knowledge and spatial skills at three time points. To address the first research question, a cross-lagged model was built, with counting sequence knowledge indexed by the composite score of counting range, counting forward, and counting backward. To address the second research question, four cross-lagged models, in which counting abilities were indexed by counting range accuracy scores, the highest number the children could reach in their counting range, counting forward accuracy scores, and counting backward accuracy scores, were tested, respectively.

Five indicators are generally considered to assess whether a model fits the data: the $\chi 2$-test, the Comparative Fit Index (CFI), the Tucker-Lewis Index (TLI), the Root Mean Square Error of Approximation (RMSEA), and the Standardized Root Mean Square Residual (SRMR). If the model fits the observed data, a nonsignificant $\chi 2$ value is expected. If the CFI and TLI are .95 or greater and the RMSEA and SRMR are .05 or less, the model fit is considered to be good (Hu & Bentler, 1999). If the CFI and TLI are .90 or greater and the RMSEA and SRMR are .08 or less, the model fit was considered to be adequate (Hu & Bentler, 1999).

Some preliminary analyses were conducted on each of the models mentioned above. When the model fit was not adequate, modifications based upon the modification indices in Mplus were considered and included to improve the model fit.

3. Results

3.1. Descriptive statistics

Table 1 presents the means, standard deviations, skewness, and kurtosis of the variables. The correlation between all variables can be found in Table 2. Counting range knowledge from T1 through T3 was significantly correlated with spatial skills at T2 and T3 (= .224 - .550) but not with spatial skills at T1 (r =.026 - .164). Counting forward knowledge from T1 through T3 was significantly correlated with spatial skills at T2 and T3 (r = .267 - .346). Counting forward knowledge at T1 was significantly correlated with spatial skills at T1 (r =.236). Counting backward knowledge from T2 through T3 was significantly correlated with spatial skills at T2 and T3(r = .268 - .369). Counting backward knowledge at T3 was significantly correlated with spatial skills at T1(r =.229).

3.2. The relation between counting sequence knowledge and spatial skills

The cross-lagged model examining the relation between counting sequence knowledge and spatial skills is illustrated in Fig. 1. The lagged effects between spatial skills at T1 and at T3 based on preliminary analyses were included. The model fit the data, with $\chi^2(3, N= 109) = 3.253$, $p = .354$, RMSEA = .028(90% CI = [.000, .166]), CFI =.998, TLI =.978, and SRMR = .014. In terms of reciprocal effects, a reciprocal relationship was observed between counting sequence skills and spatial skills from T2 to T3. That is, counting sequence knowledge at T2 significantly predicted later spatial skills at T3 (β = .232, SE = .099, p = .019; 95% CI = [.037, .426]), and spatial skills at T2 significantly predicted later counting sequence knowledge at T3 (β = .291, SE = .080, p < .01; 95% CI = [.133, .448]). This reciprocal pattern did not appear between T1 and T2.

Regarding the stability of counting ability and spatial skills, both were stable across the study period. That is, there were significant autoregressive effects of counting sequence skills from T1-T2 (β = .514, SE = .148, p < .01, 95% CI = [.225, .803]) and from T2-T3 (β = .499, SE = .083, p < .01, 95% CI = [.337, .661]), and of spatial skills from T1-T2 (β = .210, SE = .102, p= 0.040, 95% CI = [.010,.411]) and from T2-T3 (β = .198, SE = .098, p= .044, 95% CI = [.005, .390]).

Table 1
Mean, Standard Deviation, Skewness, Kurtosis

	N	Mean	*SD*	Skewness	Kurtosis
Gender	109	–	–	–	–
Family SES	69	.08	2.35	−0.43	−0.70
Vocabulary (T1)	109	15.55	4.38	−0.15	−0.06
Executive function (T1)	107	14.33	12.83	1.05	.53
Counting range_highest numbers (T1)	109	15.69	11.11	1.10	1.32
Counting range_highest numbers (T2)	84	21.15	17.86	1.84	3.74
Counting range_highest numbers (T3)	80	40.04	31.03	1.09	−0.19
Counting range_accuracy (T1)	109	0.80	0.47	−0.63	0.32
Counting range_accuracy (T2)	84	0.95	0.60	1.39	5.14
Counting range_accuracy (T3)	80	1.53	1.11	1.54	0.86
Counting forward (T1)	109	2.52	1.96	0.82	.68
Counting forward (T2)	84	2.89	1.88	0.88	.87
Counting forward (T3)	80	4.00	2.38	0.21	−0.84
Counting backward (T1)	109	0.07	0.47	6.99	52.17
Counting backward (T2)	84	0.30	1.07	5.31	33.54
Counting backward (T3)	80	0.99	2.05	2.40	5.18
Spatial skills (T1)	109	2.43	2.23	0.96	.37
Spatial skills (T2)	84	3.43	3.23	1.03	.22
Spatial skills (T3)	80	5.16	3.66	.52	−0.50

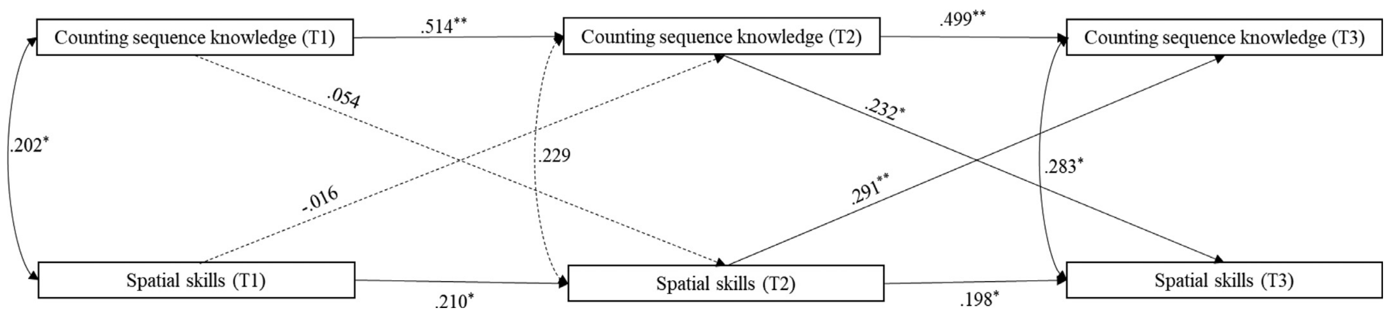

Fig. 1. The cross-lagged model of the relation between counting sequence knowledge (a composite score of three counting tasks) and spatial skills. *Note*. Gender, family SES, vocabulary knowledge and executive function were controlled but not presented for the simplicity of presentation. $^{*}P < 0.05$. $^{**}P < 0.01$, two-tailed.

Among the control variables, boys tended to perform better than girls on the task of counting sequence knowledge at T3($\beta = -.309$, $SE = .091$, $p < 0.01$; 95% CI = [−.489, −.130]). Vocabulary knowledge was a significant predictor of spatial skills at T1 ($\beta = .189$, $SE = .081$, $p = .019$; 95% CI = [.031, .346]), T2 ($\beta = .221$, $SE = .096$, $p = .021$; 95%CI = [.033, .409]) and T3 ($\beta = .278$, $SE = .114$, $p = .015$; 95% CI = [.054, .502]). Family SES ($\beta = .355$, $SE = .089$, $p < .01$; 95% CI = [.181, .529]) and behavioral regulation ($\beta = .333$, $SE = 0091$, $p < .01$; 95% CI = [.154, .511]) were significant predictors of counting sequence knowledge at T1.

3.3. The relation between counting range knowledge and spatial skills

In the counting range tasks, two indicators were used. The first indicator was the highest number that the children were able to reach when they counted from 1. The cross-lagged model examining the relation between the highest counting number and spatial skills is illustrated in Fig. 2a. The lagged effects between the highest number at T1 and spatial skills at T3 were included. The model fit the data adequately, with $\chi^2(3, N = 109) = 5.642$, $p = .130$, RMSEA = .090(90% CI = [.000, .203]), CFI =.980, TLI = .735, and SRMR = .022. Regarding reciprocal effects, a reciprocal relationship was observed between the children's highest counting number and spatial skills. Specifically, children's highest counting number at T1 significantly predicted later spatial skills at T3 ($\beta = .391$, $SE = .118$, $p < .001$, 95% CI = [.161,.621]), and spatial skills at T2 significantly predicted the children's highest counting range at T3 ($\beta = .437$, $SE = .101$, $p < .01$, 95% CI = [.238,.636]). Both the highest counting range and spatial skills were stable over time in the model. That is, there were significant autoregressive effects in the children's highest counting range from T1-T2 ($\beta = .391$, $SE = .166$, $p = .019$, 95% CI = [.066, .717]) and from T2-T3 ($\beta = .419$, $SE = .098$, $p < .01$, 95% CI = [.228,.610]), and in spatial skills from T1-T2 ($\beta = .217$, $SE = .103$, $p = .034$, 95% CI = [.016, .418]) and from T2-T3 ($\beta = .215$, $SE = .094$, $p = .022$, 95% CI = [.031, .398]).

Among the control variables, vocabulary knowledge was a significant predictor of spatial skills at T1 ($\beta = .191$, $SE = .082$, $p = .019$; 95% CI = [.031, .351]), T2 ($\beta = .222$, $SE = .100$, $p = .026$; 95%CI = [.026,.419]) and T3 ($\beta = .258$, $SE = .101$, $p = .011$; 95% CI = [.060, .455]). Family SES was a significant predictor of spatial skills at T3 ($\beta = -.162$, $SE = .080$, $p = .043$; 95% CI = [−.319, −.005]). Behavioral regulation was a significant predictor of highest counting range at T1 ($\beta = .301$, $SE = .089$, $p < .01$; 95% CI = [.126, .476]).

The other indicator was counting range accuracy scores. The cross-lagged model examining the relation between counting range accuracy and spatial skills is illustrated in Fig. 2b. The lagged effects between spatial skills at T1 and at T3 were included. The model fit was good, with $\chi^2(3, N = 109) = 1.779$, $p = .062$, RMSEA = .000(90% CI = [.000, .132]), CFI =1.00, TLI = 1.00, and SRMR = .013. From T2 to T3, a reciprocal relationship was observed between counting range accuracy and spatial skills. That is, counting range accuracy at T2 significantly predicted later spatial skills at T3 ($\beta = .279$, $SE = .107$, $p < .01$, 95% CI = [.069, .488]), and spatial skills at T2 significantly predicted children's counting range accuracy at T3 ($\beta = .506$, $SE = .099$, $p < .01$, 95% CI = [.312,.699]). Regarding the stability of counting range accuracy and spatial skills, there were significant autoregressive effects in counting range accuracy from T1-T2 ($\beta = .498$, $SE = .103$, $p < .01$, 95% CI = [.297, .699]) and from T2-T3 ($\beta = .292$, $SE = .093$, $p < .01$, 95% CI = [.109,

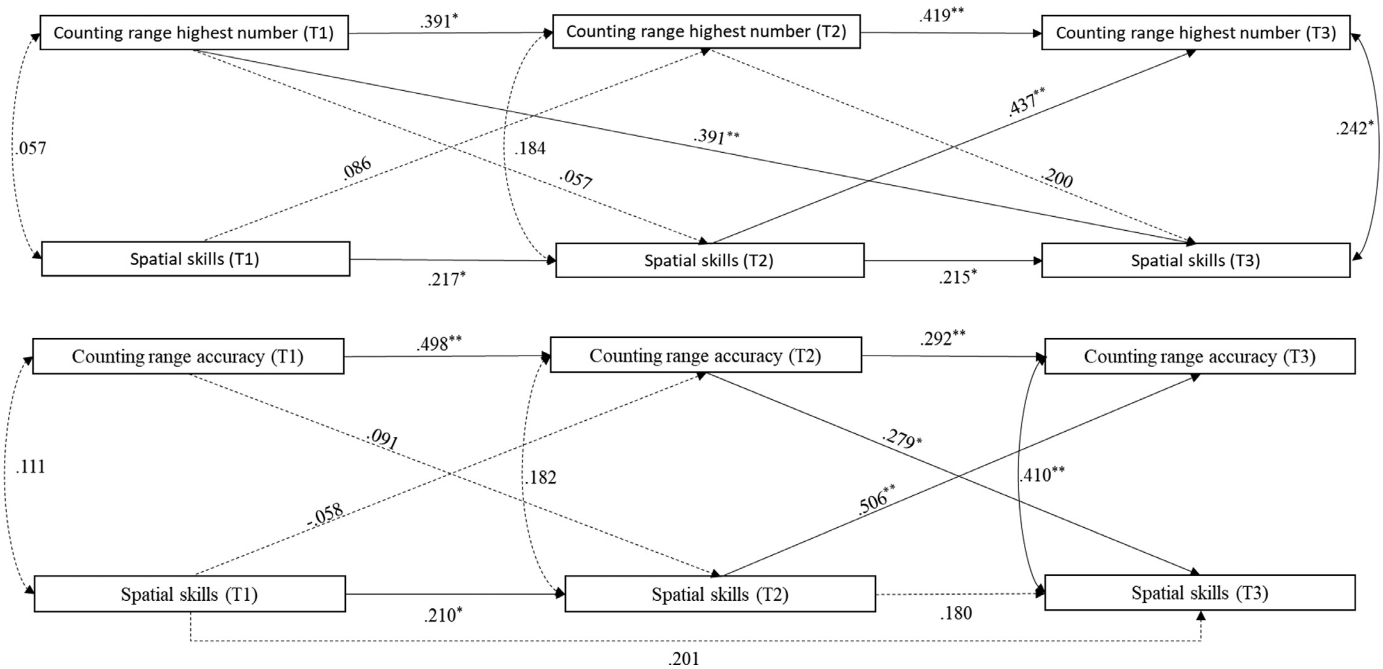

Fig. 2. The cross-lagged model of the relation between counting range knowledge in terms of highest number (top panel) and accuracy (bottom panel) and spatial skills. *Note*. Gender, family SES, vocabulary knowledge and executive function were controlled but not presented for the simplicity of presentation. $^{*}P < 0.05$. $^{**}P < 0.01$, two-tailed.

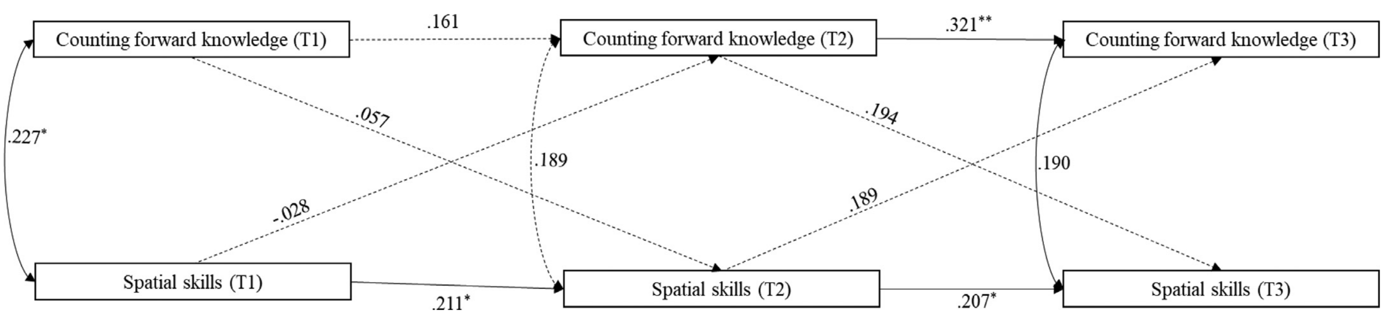

Fig. 3. The cross-lagged model of the relation between counting forward knowledge and spatial skills. *Note*. Gender, family SES, vocabulary knowledge and executive function were controlled but not presented for the simplicity of presentation. $^{*}P < 0.05$. $^{**}P < 0.01$, two-tailed.

.474]), and in spatial skills from T1-T2 (β = .210, *SE* = .104, *p*= .044, 95% CI = [.006, .415]) but not in spatial skills from T2-T3 (β = .180, *SE* = .107, *p*= .092, 95% CI = [−.030, .390]).

Among the control variables, vocabulary knowledge was a significant predictor of spatial skills at T1 (β =.189, *SE* = .081, *p*= .019; 95% CI = [.031, .347]), T2 (β = .204, *SE* = .097, *p*= .036; 95% CI = [.014, .395]) and T3 (β = .288, *SE* = .109, $p < .01$; 95% CI = [.074, .503]) as well as counting range accuracy at T1 (β = .244, *SE* = .089, $p < .01$; 95% CI = [.071, .418]). Family SES (β = .217, *SE* = .104, *p*= .037; 95% CI = [.013, .421]) and behavioral regulation (β = .186, *SE* = .077, *p*= .016; 95%CI = [.035, .337]) were significant predictors of counting range accuracy at T1.

3.4. The relation between counting forward knowledge and spatial skills

The cross-lagged model examining the relation between counting forward knowledge and spatial skills is illustrated in Fig. 3. The lagged effects between spatial skills at T1 and T3 were included. The model fit the data adequately, with $\chi^2(3, N= 109) = 3.657$, *p*= .301, RMSEA = .045(90% CI = [.000, .174]), CFI =.993, TLI = .909, and SRMR = .020. In contrast to the above models, counting forward knowledge at T2 was not a significant predictor of later spatial skills at T3 (β = .194, *SE* = .109, *p* = .076; 95% CI = [−.020, .408]), and spatial skills at T2 was only a marginally significant predictor of later counting forward knowledge (β = .189, *SE* = .097, *p* = .050; 95% CI = [.000, .379]). Similarly, counting forward knowledge at T1 was not a significant predictor of later spatial skills at T2 (β = .057, *SE* = 0.115, *p* = .621; 95% CI = [−.169, .283]), and spatial skills at T1 was not a significant predictor of later counting forward knowledge at T2 (β = −.028, *SE* = 0.124, *p* = .824; 95% CI = [−.271, .216]).

Autoregressive effects of spatial skills from T1-T2 (β = .211, *SE* = .102, *p*= .038, 95% CI = [.012,.410]) and from T2-T3, (β = .207, *SE* = .101, *p*= .040, 95% CI = [.009, .405]), and counting forward knowledge from T2 to T3 (β = .321, *SE* = .086, $p< .01$, 95% CI = [.153, .490]) were observed but not in counting forward knowledge from T1-T2 (β = .161, *SE* = .137, *p* = .241, 95% CI = [−.108, .430]).

Among the controlling variables, boys tended to perform better than girls on the counting forward knowledge at T3 (β = −.322, *SE* = .101, $p < .01$; 95% CI = [−.520, −.123]). Vocabulary knowledge was a significant predictor of spatial skills at T1 (β = .190, *SE* = .080, *p*= .018; 95% CI = [.033,.348]), T2 (β = .221, *SE* = .095, *p*= .021; 95% CI = [.034, .407]) and T3 (β = .267, *SE* = .118, *p*= .024; 95% CI = [.035, .498]) as well as counting forward knowledge at T2 (β = .221, *SE* = .104, *p*= .033; 95% CI = [.018, .424]). Family SES (β = .361, *SE* = .101, $p < .01$; 95% CI = [.162, .559]) and behavioral regulation (β = .344, SE = .096, p < .01; 95% CI = [.157, .531]) were significant predictors of counting forward knowledge at T1.

3.5. The relation between counting backward knowledge and spatial skills

The cross-lagged model examining the relation between counting backward knowledge and spatial skills is illustrated in Fig. 4. The model fit the data adequately, with χ^2(4, N= 109) = 5.970, p= .201, RMSEA = .067(90% CI = [.000, .171]), CFI =.992, TLI = .923, and SRMR = .021. In terms of the reciprocal effects, a reciprocal relationship was observed between counting backward knowledge and spatial skills from T2 to T3. That is, counting backward knowledge at T2 significantly predicted later spatial skills at T3 (β = .171, SE = .087, p = .049; 95% CI = [.000, .342]), and spatial skills at T2 significantly predicted later counting backward knowledge (β = .225, SE = .099, p = .023; 95% CI = [.031, .420]). This reciprocal pattern did not appear between T1 and T2.

Regarding the stability of counting backward knowledge and spatial skills, both were stable across the study period. That is to say, there were significant autoregressive effects of counting backward knowledge from T1-T2 (β = .767, SE = .146, p < .01, 95% CI = [.481, 1.054]) and from T2-T3 (β = .459, SE = .145, p < .01, 95%CI = [.175, .742]), and of spatial skills from T1-T2 (β = .223, SE = .103, p= .031, 95% CI = [.020,.425]) and from T2-T3 (β = .244, SE = .101, p= .016, 95% CI = [.046, .443]).

Among the control variables, boys tended to perform better than girls on the counting backward knowledge at T3 (β = −.265, SE = .097, p< .01; 95% CI = [−.454, −.075]). Vocabulary knowledge was a significant predictor of spatial skills at T1, T2 and T3 (β = .191, SE = .082, p= .020; 95% CI = [.030, .351]; β = .233, SE = .096, p= .015; 95% CI = [.045, .422]; β = .323, SE = .115, p < .01; 95% CI = [.097, .548], respectively). Family SES (β = .225, SE = .073, p < .01; 95% CI = [.081, .368]) and behavioral regulation (β = .165, SE = .078, p = .036; 95% CI = [.011, .319]) were significant predictors of counting backward knowledge at T1.

4. Discussion

The purpose of the present study was twofold: to investigate the reciprocal longitudinal relations between counting sequence knowledge and spatial skills in a group of 3- to 4-year-olds, and to determine the extent to which these relations varied according to the children's conceptual levels of counting knowledge. The present study presents new evidence showing that there were no relations between counting sequence knowledge and spatial skills from nursery to the first year of kindergarten, while from the first year of kindergarten, children's counting sequence knowledge and spatial skills were reciprocally related. The first hypothesis is therefore supported with developmental stages considered. Further, the same reciprocal longitudinal relations between counting sequence knowledge and spatial skills were observed when counting sequence knowledge was indexed by counting range knowledge and counting backward knowledge. No reciprocal relations were observed between counting forward and spatial skills in the 3- to 4-year old children. The second hypothesis is supported as the longitudinal reciprocal relation between counting sequence knowledge and spatial skills vary in relation to levels of counting sequence knowledge. Collaborating with prior findings (Mix, Levine, Cheng, Stockton, & Bower, 2021; Xu & Spelke, 2000), the findings of the present study provide novel empirical evidence in understanding A Theory of Magnitude (Walsh, 2003) from a developmental perspective. The findings further highlight the contribution of counting sequence knowledge to the development of spatial skills.

Table 2
Correlations between Variables.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Gender	–																		
Family SES	−.040	–																	
Vocabulary (T1)	.192*	.060	–																
EXEC (T1)	−.012	−.024	.216*	–															
CRAN_H (T1)	−.099	.171	.198*	.323**	–														
CRAN_H (T2)	−.074	.179	.144	.203	.436**	–													
CRAN_H (T3)	−.095	.153	.263*	.190	.387**	.559**	–												
CRAN_A (T1)	.037	.211	.300**	.226*	.580**	.486**	.391**	–											
CRAN_A (T2)	−.027	.110	.118	.204	.443**	.818**	.473**	.519**	–										
CRAN_A (T3)	−.107	.146	.158	.129	.344**	.514**	.937**	.290**	.421**	–									
CFORT (T1)	.091	.292*	.197*	.324**	.491**	.457**	.286*	.561**	.490**	.291**	–								
CFORT (T2)	.016	.098	.266*	.170	.404**	.537**	.534**	.481**	.542**	.470**	.251*	–							
CFORT (T3)	−.268*	.172	.237*	.178	.411**	.464**	.618**	.501**	.545**	.544**	.254*	.438**	–						
CBACT (T1)	−.108	.248*	−.056	.137	.488**	.490**	.069	.324**	.509**	.065	.404**	.300**	.256*	–					
CBACT (T2)	−.104	.215	.099	.218*	.508**	.577**	.282*	.376**	.566**	.278*	.507**	.549**	.295**	.791**	–				
CBACT (T3)	−.242*	.145	.119	.061	.443**	.638**	.494**	.433**	.601**	.485**	.320**	.494**	.546**	.472**	.530**	–			
SS (T1)	.003	−.054	.195*	.099	.104	.143	.090	.164	.026	.087	.236*	.054	−.002	.023	.209	.229*	–		
SS (T2)	.140	.336	.367**	.322**	.224*	.272*	.550**	.284**	.263*	.539**	.267*	.290**	.305**	−.004	.283**	.300**	.302**	–	
SS (T3)	.023	.008	.404**	.218	.528**	.435**	.525**	.394**	.368**	.525**	.270*	.335**	.346**	.104	.268*	.369**	.300**	.395**	–

EXEC, Executive function; CRAN_H, Counting range_highest number; CRAN_A, Counting range_accuracy; CFORT, Counting forward; CBACT, Counting backward; SS, Spatial skills.
* P < 0.05.
** P < 0.01, two-tailed.

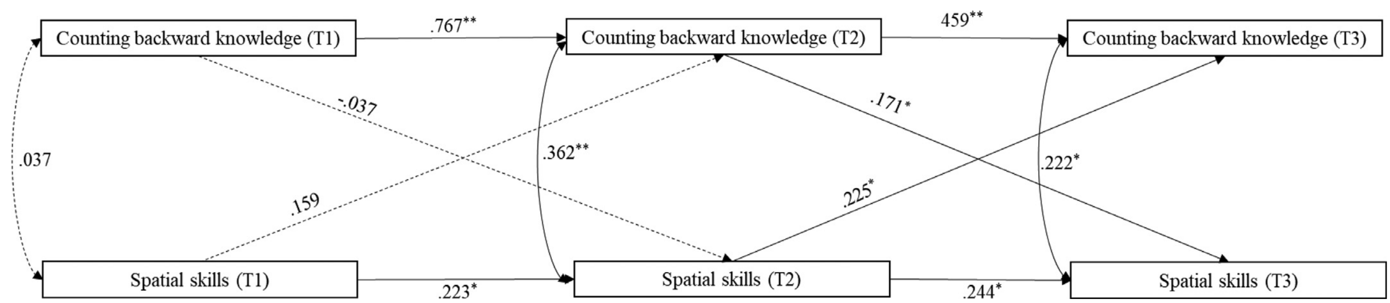

Fig. 4. The cross-lagged model of the relation between counting backward knowledge and spatial skills. *Note.* Gender, family SES, vocabulary knowledge and executive function were controlled but not presented for the simplicity of presentation. $^{*}P < 0.05$. $^{**}P < 0.01$, two-tailed.

4.1. Reciprocal relations between counting sequence knowledge and spatial skills

Although previous research has revealed the unidirectional relations between spatial skills and counting sequence knowledge (i.e., the effect of spatial skills on counting sequence knowledge), only one longitudinal study has investigated the reciprocal relations between mathematics performance and spatial skills (Geer et al., 2019). The findings of their study showed that spatial visualization/mental rotation scores in Year 1 positively predicted mathematics test performance in Year 2, and that mathematics test performance in Year 2 predicted significant improvement in spatial visualization/mental rotation skills as well as in spatial perception skills in Year 3. However, as the researchers acknowledged, this reciprocal relation was not observed over the same time period and may thus be considered to be only partly in accord with the concept of reciprocal relation. To our knowledge, the present study is the first to reveal the reciprocal longitudinal relations between mathematics competency and spatial skills over the same time period.

Furthermore, this finding also highlights the reciprocal longitudinal relations between spatial skills and mathematics performance in relation to the earliest symbolic number representation – counting sequence knowledge. Hartmann, Mast, & Fischer, 2015 eye-tracking data showed that when college students were asked to do the counting forward and counting backward tasks, their eyes moved from left to right when counting forward but from right to left when performing the counting backward task. This was not surprising because through education and repeated practices, adults' counting sequence knowledge itself involves spatial processing, although the oral execution of counting sequence does not involve concrete object counting behaviors. The present study extends these previous results to 3- to 4-year-olds with very little formal mathematics education, indicating that counting sequence knowledge and spatial skills are developmentally intertwined from the early years. Plausibly, in early years, the repeated instructed counting practices in schools combined with the reading/writing direction from left to right acquired readily in young children may lead to a directional mental representation of counting sequence knowledge (Göbel, Maier & Shaki, 2015; Shaki, Fischer & Göbel, 2012). The spatial perception task in the present study captured a basic spatial ability to discriminate spatial relations (Linn & Petersen, 1985; Zhang, 2016), which is assumed to be linked to the spatial relations of the counting words in the counting sequence knowledge.

Our longitudinal design allowed us to look at the reciprocal relations with a developmental lens. As shown in our results, the reciprocal relations between counting sequence knowledge and spatial skill emerged only in the first year of kindergarten. There are two possible explanations for this. First, as mentioned above, accumulated counting practices during development may play a role. During six more months' daily practice in the first year of kindergarten, counting sequence knowledge may gradually be linked to spatial skills in children to a greater extent. Second, in Hong Kong, formal mathematics instruction starts from the first year of kindergarten. In other words, the onset of the reciprocal longitudinal relations between counting sequence knowledge and spatial skills observed in the present study overlapped with the beginning of formal mathematics instruction. Formal instruction in counting emphasizes the direction of counting practices and increases the frequency of counting practices in schools (e.g., counting activities). Such schooling experiences in counting very likely enhance the spatial thinking of counting sequence knowledge in kindergarten children.

4.2. Levels of counting sequence knowledge matter

As our second research goal, the present study also revealed an interesting finding that the reciprocal relations between counting sequence knowledge and spatial skills were task dependent. The longitudinal reciprocal relation between counting sequence knowledge and spatial skills was present when counting sequence knowledge was indexed by counting range and counting backward. This piece of evidence suggests that it is crucially important to view different levels of counting sequence knowledge separately (Fuson et al., 1982) in investigating its relations to spatial skills.

Regarding the null result in the relationship between counting forward and spatial skills, counting only 5 numbers forward in a small range (e.g., 1 to 5) might involve some levels of automatic processing for 3- to 4-year-old young children. According to Fuson (1982), the acquisition and elaboration of counting sequence knowledge in some ranges may have reached a higher level while other ranges may still be at the acquisition level. In our tasks, counting in the 1–80/100 range and counting backward tasks may still have required children's cognitive recourses to process in order to keep track of the ordinal sequence of number words, whereas counting forward from 1 to 5 or 6–13 was relatively automatic. Thus, less attention may have been devoted to the ordinal relations of number words in the counting forward task. Nevertheless, this possibility is speculative and needs further investigation.

4.3. Educational implications

In the teaching of counting sequence knowledge at school and at home, parents and teachers could be recommended to include spatial elements in children's acquisition of counting sequence knowledge. For example, explicit use of directional words in formal teaching could be encouraged (e.g., "*please count from left to right*"). These activities are expected to help children to develop both spatial skills and counting sequence knowledge simultaneously as they are reciprocally influenced. Moreover, our finding indicates that the above suggestion could be implemented as early as the first year of kindergarten in the Hong Kong context.

Furthermore, the results clearly indicate that not all types of counting activities and spatial skills are reciprocally related. Although the underlying mechanisms are not clear, teachers could be reminded of this so as to discriminate between different counting sequence knowledge tasks for children to practice if they want to use the activities to improve children's spatial skills as well.

The findings of the present study indicate that intervention programs with a focus on counting abilities in the early years should include a training session on spatial skills (Dunbar, Ridha, Cankaya, Jiménez Lira & LeFevre, 2017), at least in the Hong Kong educational context. At the same time, to improve spatial skills in young children, a simple training task on counting sequence knowledge would probably be useful. Another important implication of the present findings is that children who are struggling in one domain should be closely monitored for difficulties in the other domain as well. Furthermore, since the reciprocal relations between the two are longitudinal in the early years, earlier difficulties in one domain may indicate later failure in the other domain. This monitoring may help the early screening of difficulties in counting sequence knowledge and spatial skills.

4.5. Limitation and conclusion

Several limitations of the study should be acknowledged. First, our results are very likely related to instruction and the curriculum in the Hong Kong context. Prior research has shown that Hong Kong 6-year-old children are able to count up to 50 in both forward and backward directions (Liu, Lin, & Zhang, 2016). The formal instruction on mathematics in Hong Kong kindergartens may serve as a stimulating factor for the directional mental representation of the counting sequence in 3-year-olds. Questions arise as to the extent to which the reciprocal relations between counting sequence knowledge and spatial skills are context dependent. For example, in other parts of China, the teacher student ratio is much higher than Hong Kong, and the average early childhood education quality is typically not comparable to that in Hong Kong (Li, Yang & Chen, 2016). In other words, differences in schools in China are a nonnegligible factors; not all children are formally instructed on mathematics since 3 years old. This also indicates that the implications on the intervention programs could potentially be constrained to the Hong Kong context to some extent. Further studies are needed to investigate this issue to better understand the reciprocal relations between counting sequence knowledge and spatial skills across educational contexts. The second limitation concerns the underlying mechanisms that may account for the observed reciprocal relationships between counting sequence knowledge and spatial skills. These mechanisms need to be addressed in further intervention studies.

The current study has elaborated upon previous findings on the reciprocal longitudinal relations between spatial skills and counting sequence knowledge in very young children. This is an important contribution to the literature because it is possibly the first evidence supporting the reciprocal relationships between counting sequence knowledge and spatial skills in the early stages of symbolic number representation and its variations according to children's conceptual understanding of the counting sequence. The study further contributes to a better understanding of development of children's spatial representation of symbolic numbers.

Authorship contribution

Yingyi Liu: Conceptualization, Formal analysis, Writing - original draft, Writing - review & editing. Xiao Zhang: Conceptualization, Data curation, Funding acquisition, Investigation, Methodology, Project administration, Resources, Supervision, Writing – review & editing

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