

Please cite the Published Version

Woollacott, LC, Tripathy, AK and Potgieter, JH (2021) The effect of operating conditions on density stratification in a batch jig II: the influence on stratification kinetics. Minerals Engineering, 164. 106846 ISSN 0892-6875

DOI: https://doi.org/10.1016/j.mineng.2021.106846

Publisher: Elsevier

Version: Accepted Version

Downloaded from: https://e-space.mmu.ac.uk/628317/

Usage rights: Creative Commons: Attribution-Noncommercial-No Derivative Works 4.0

Additional Information: Author accepted manuscript published in Minerals Engineering by Elsevier.

Enquiries:

If you have questions about this document, contact openresearch@mmu.ac.uk. Please include the URL of the record in e-space. If you believe that your, or a third party's rights have been compromised through this document please see our Take Down policy (available from https://www.mmu.ac.uk/library/using-the-library/policies-and-guidelines)

The Effect of Operating Conditions on Density Stratification in a Batch Jig Part 2: The Influence on Stratification Kinetics

L. C. Woollacott^{a*}, A.K. Tripathy^a, J.H.Potgieter^a

^a School of Chemical and Metallurgical Engineering, University of the Witwatersrand, Johannesburg, South Africa, 2050

* Corresponding author: lorenzo.woollacott@wits.ac.za

ABSTRACT

This paper presents the second of a two part study on density stratification in a laboratory batch jig. The first study, reported elsewhere, investigated the effect of operating conditions on the stratification profile achieved at equilibrium. The study reported in this paper investigated their effect on the kinetics of density stratification under 18 different sets of conditions—5 for binary systems with different particle densities; and 13 for a ternary system with different configurations of the jig cycle. Although the study was academic in nature (it involved simple systems of 8mm glass beads) it has provided insights into stratification can be expected in systems with density differences as low as about 150 kg/m³ provided sufficient jigging time is allowed. In addition, the study findings suggest that poor jigging performance with ores that are difficult to beneficiate may be the result of inadequate attention to the kinetics of stratification and that longer residence times or appropriate circuit modifications may achieve unexpected and significant improvements in both the recovery and grade of the values being beneficiated.

Keywords: Jigging kinetics, stratification, equilibrium stratification, batch jig

1. Introduction

Despite being one of the oldest mineral processing techniques available for separating particles of different densities, jigging is still inadequately understood at a fundamental level. The dynamic nature of the stratification processes and the multiple operating conditions that influence those dynamics make it difficult to untangle and understand the relationships between the operating variables of a jig and jigging performance. The difficulty is exacerbated further by the variety of types of jigging equipment in use—different types generate different relationships between operating variables and jig performance. The study reported in this paper focuses only on over-the-screen jigging where the upper or lower layer of a stratified bed is split off and removed as a concentrate.

The previous paper in this series of two (Woollacott et al., 2021a) presented a study on how operating conditions influenced the stratification achieved in a laboratory batch jig once it had reached an equilibrium state. Focusing only on the operating variables available to jig operators for controlling jigging performance, the paper found that the stratification profiles achieved at equilibrium were essentially the same for all operating conditions provided these remained within a broad range of optimum values. This implied that the stratification that a jig is capable of achieving, as indicated by the equilibrium stratification profile, is determined primarily by the *properties* of the particles being treated, and not by how the operating variables might be manipulated within the broad zone of optimum values. Significantly, it also implied that the influence of operating variables on jigging performance must derive from their influence on stratification kinetics rather than from their influence on stratification at equilibrium.

A further implication of these observations is that it makes sense to disentangle kinetic from equilibrium effects when thinking about the optimization of jig performance or when troubleshooting performance that is below par. In this regard, the equilibrium stratification profile should be thought of as indicating the quality of stratification that *can* be achieved *inherently* by a jig, as determined by the properties of the particles being treated. On the other hand, the kinetics of stratification should be thought of as providing information about the rate at which that equilibrium condition can be achieved and how operating conditions as well as particle properties affect that rate. As such, kinetic studies should pay particular attention to the approach to equilibrium rather than the common practice of focusing just on how stratification progresses—see examples of this in Mayer (1964), Karantzavelos and Frangiscos (1984) and in the review by Mehrotra and Mishra (1997). A focus on how stratification progresses with time combines two sets of factors—those that influence what is achievable at equilibrium and those that influence the rate of approach to equilibrium. Put another way, a conventional kinetic study entanglesand confuses-two important questions. What is achievable? And how best to achieve what is achievable? To the authors' knowledge, no previous study on stratification kinetics in jigs has attempted to disentangle these two questions. The previous paper (Woollacott et al., 2021a) addressed the first of these questions. This paper addresses the second.

2. Experimental Design

2.1 The Equipment Used

The test equipment and particles used in this study were the same as in the equilibrium study. The batch jig consisted of a cylindrical jig chamber, 200mm in diameter, made up of annular rings, each 15mm high, that were clamped together on top of a support screen. A pulsing unit located below the jig chamber consisted of a hutch chamber fitted with bellows, a water supply and a drain. The lower part of the bellows was driven up and down by a pneumatically powered piston. Adjustments made through a PLC controlled the movement of the piston and consequently the amplitude of water displacement in the jig. The PLC also controlled the run time for a jigging test; the time settings that controlled the shape of the jig cycle; and the movement of the bellows. The air supply pressure to the piston was controlled through a pressure regulator.

Nominally 8mm, spherical glass beads with four different densities were used in the study: boro-silicate beads with a density of 2226 kg/m³ and a matte appearance, (referred to as 'boro' beads); and soda-lime beads with three different densities and colours—i.e. green beads with a density of 2463 kg/m³; red beads with a density of 2554 kg/m³; and blue beads with a density of 2567 kg/m³. The majority of the tests were conducted on a ternary system consisting of boro, green and red beads. The differences in the appearance of the beads enabled them to be manually sorted into their respective density classes.

2.2 Variables Affecting Stratification Kinetics in Jigs

The previous paper (Woollacott et al., 2021a) reviewed the variables that influence stratification in a jig and identified 15 as being influential or potentially influential. These included 4 particle properties; jigging time; and eleven operating variables that control aspects of the behaviour of the jig bed.

The 4 particle properties that influence stratification—the density, size, and shape of the particles treated and their relative proportions in the bed—were not investigated in the first study as they are not operating variables which jig operators can manipulate in practice. However, these properties are likely to have a profound effect on stratification kinetics, an influence which has not been adequately investigated or reported in the literature. For example, the larger the difference in density among particles species in the bed, the faster the particles would be expected to stratify and vice versa.

Three operating variables control basic aspects of the conditions in the jig bed: namely, the depth of the bed; the hutch water flowrate; and the extent to which the bed remains in a flooded condition during jigging. In the previous study, the jig bed was always flooded during jigging and the hutch water flowrate was found to have little influence on stratification at equilibrium. It was found that bed depth had essentially no influence on the equilibrium stratification profile provided it remained within a range of optimum values—namely 80 to 100mm in the jigging conditions tested. Accordingly, in the study

reported in this paper, the bed depth was maintained at 90mm, the hutch water flowrate was zero, and the bed was always flooded during jigging.

Typically 4 operating variables determine the shape of the jig cycle, i.e. whether sinusoidal, saw-tooth, or some other shape. Generally, these are time settings that control the pulsion and suction phases of the jig cycle. In pneumatic jigs, for example, the valve introducing air into the air chambers below the support screen is opened for a period T1 and then is closed. This state is held for a period T2 after which the exhaust valve is opened for a period T3 and then closed. After a hold time of T4, the cycle begins again. Many mechanically driven jigs, such as the bellows-with-drive-piston arrangement used in this study, mimic the time settings of a pneumatic jig. The piston rises for the pulsion period T1 compressing the bellows and forcing water into the jig chamber. It is held at its maximum extent for a pulsion hold-time period T2 after which it is moved downward to its lowest position over a period T3, the suction time. It is then held in that position for the suction hold-time period T4 before the cycle begins again.

The dimensions of the jig cycle—i.e. its frequency and amplitude—are controlled by several operational variables. Frequency (cycles per unit time) is the inverse of the cycle time (time for one cycle) which is the sum of the time settings T1 to T4. The previous study fixed the frequency at 45 cycles per minute after finding that neither T3 nor T4 had any significant influence on stratification. Given this and the necessity to reserve the time settings T1 and T2 for controlling the shape of the jig cycle, T3 was set at 0.25 seconds, and T4 was adjusted so that the overall cycle time was 133 seconds (equivalent to a frequency of 45 cycles per minute).

In pneumatic jigs the control of the amplitude of the jig cycle, i.e. the maximum water displacement, is complex (Rong and Lyman, 1991). In the bellows-with-piston arrangement used in this study, the amplitude was controlled primarily by the length of the stroke of the piston driving the bellows and, secondly, by the pressure of the air supply to the pneumatic system driving the piston. In the study, the length of the piston stroke was expressed as a percentage of its maximum travel, i.e. as %Stroke. It was found that if the pressure of the air supply was kept above 58 psi, control of the cycle amplitude could be exercised by means of %Stroke alone.

In summary, the primary operational variables that were considered relevant to stratification kinetics in the test jig used were T1, T2, %Stroke and particle properties. Because the focus of the first study was density stratification, the only particle property included in the study was particle density.

2.3 Test Procedure

Kinetic tests were conducted on both a ternary mixture and binary mixtures of the glass beads. The ternary tests simulated a context where the separation of a nominally binary mixture of light (boro) and heavy (red) particles was confounded by incompletely liberated particles with an intermediate density, i.e. the green particles. In all these tests, the composition of the particle sample was the same as that used in the equilibrium study described in the previous paper—26 vol% boro; 45% green; and 29% red.

A series of 13 tests were conducted on the ternary sample to investigate the effect of T1, T2 and %Stroke on stratification kinetics. The series was constructed around the base case set of operating conditions summarized in Table 1.

Table 1: Operating conditions for the base case jigging condition

 (The cycle time and air supply pressure were respectively 1.33 sec and 4.4 bar)

Variable	T1	T2	T3	%Stroke	Bed depth
Value	0.22 s	0.28 s	0.25 s	38%	90 mm

In the remaining 12 tests, all operating variables were held constant except for the one being investigated. The advantage of this design is that the effect of each variable could be investigated and evaluated separately. An experimental design that would enable interactive effects to be investigated

was not attempted because, at the outset of the study, no suitable response variable could be identified to describe quantitatively the quality of the stratification profile achieved.

To investigate the effect of differences in particle density on stratification kinetics, tests were conducted on binary mixtures selected from the four different density beads described earlier. Of the six possible combinations of density components, only five were tested as the density difference between the red and blue beads (13 kg/m^3) was too small for a meaningful investigation.

Each kinetic test involved a series of batch tests conducted under the same set of operating conditions but with different jigging times. The procedure for a single batch test was as follows. Once the operating variables had been selected, the jig chamber was flooded with water, the particle charge was added to the chamber, and jigging proceeded for the time selected. At the end of the test, the chamber was drained, the rings unclamped, and the stratified particle bed was split into layers by forcing a thin slide plate between the annular rings. The particles removed from each layer were then dried using a hair dryer, were sorted into the different density components and weighed, and the volumetric concentration of each component in the layer was calculated.

3. Results

The stratification kinetics for the base case condition are shown in Figures 1a to 1c. Each figure shows how the volumetric concentration of each of the three components changed with time in each of the 6 layers split from the bed. Figure 2 shows the same data from the perspective of the kinetics in each of the layers. The kinetic curves for the other 12 different operating conditions tested are similar in nature to those shown in Figures 1 and 2 and so are not presented here.



Figure 1. Stratification kinetics of each of the three components in each layer (base case)

It can be seen that the experimental data establishes quite clearly the trend of each kinetic curve plotted. The small deviations from the trends are the result of each point on a curve being derived from a separate experiment and the associated inevitable experimental error. (For example, errors could arise from slight differences in the way layers were split from the bed, and from segregation effects during initial mixing of the sample or when it was poured into the jig chamber before a test.)

Several features of the kinetic curves deserve comment. The light (boro) particles stratify much more quickly than the other two components. They reached equilibrium within 120 to 150 seconds compared to between 250 and 700 seconds for the other two components. This aligns with the expectation that the rate of stratification will depend on the difference in density of the stratifying components; the density difference between the light and intermediate particles was 237 kg/m³ and between the intermediate and heavy particles was 91 kg/m³.



Figure 2. Stratification kinetics in each layer in the jig bed (base case)

A second feature to note is the shape of the kinetic curves. In the bottom and top layers (layers 1 and 6 respectively), the kinetics of the light and heavy components look more or less exponential in form. In the middle two layers (layers 3 and 4), there is a tendency for the kinetic curves to change direction soon after jigging begins. This tendency is more marked for the intermediate density (green) particles and is most pronounced in Layer 2 (best seen in Figure 2b). The effect can be explained as being the result of the stratifying system consisting of three components. In a binary system, it would be expected that the kinetic curve for one component would mirror that of the other component. However, when there are more than two components stratifying, the concentration of any one component in a layer will be the combined result of the drift of that component in one direction and the drift of other components in perhaps different directions, with the drift velocities being different for each component. This can be seen clearly for the intermediate (green) component in layer 2 in Figure 2b. Here the light (boro) particles rise rapidly out of the layer with the heavy particles (red) building up more slowly in that layer. The initial part of the kinetic curve reflects the rapid depletion of light particles from the layer combined with a buildup of both of the other two components. Once the layer is depleted of the light particles, the kinetic curves of the remaining two components more or less mirror one another. The mirroring is not exact, however, because stratification is causing the intermediate particles to drift higher into the bed at a rate somewhat faster than the buildup of the heavy particles near the bottom of the bed.

A third feature to note in Figures 1 and 2 is that the rate of stratification of any given component is not the same in each layer. This is most clearly seen in Figures 1a and b. More will be said about this feature later in the paper.

Meaningful analysis and interpretation of the test results requires the raw kinetic data to be manipulated into a more convenient form as explained in the next section.

4. The Rate of Approach to Equilibrium

4.1 An 'Approach-to-Equilibrium' Metric

Figures 1 and 2 are interesting in that they show in detail how the different components migrated over time and how their concentrations changed with time in each of the six layers. However, the figures portrayed are complex, overly-detailed and non-generic; for example, they would be somewhat different if the bed had been sliced at different heights, or if the initial composition of the bed had been different. A more generic and holistic method for describing and understanding the kinetics of stratification is therefore needed. The method adopted in this study involved two metrics: a 'profile-metric' and an 'approach-to-equilibrium metric'.

The profile-metric provides a measure of the extent to which a stratification pattern in a jig bed has shifted from the pattern for an unstratified bed. Rong and Lyman (1992, 1993), in their research into jigging dynamics, used this idea and devised a profile-metric which they termed a 'concentration profile index' or CPI, Figure 3a. This was derived from a plot of the concentration of a component (mass%) in a layer plotted against the height of the center of the layer from the bottom of the bed. This plot deviated from the concentration pattern in the unstratified bed which is described by a vertical line indicating the feed concentration. The deviation could be measured as the 'integral area' (Rong and Lyman, 1993, p167) between the plot and the vertical line as suggested in Figure 3a.



Figure 3. Stratification profiles, the CPI and the Profile-metric (Fig. 3a is from Rong and Lyman (1993). Figs 3b and 3c are for the base case.)

The profile-metric utilized in this study was conceptually similar but derived from a plot of the component recovery (volume %) to the lower layer when the bed was split at a relative height h. In an unstratified bed, the recovery profile would be a straight line from 0% recovery at the bottom of the bed (h=0) to 100% recovery at the top of the bed (h=1). As illustrated in Figure 3b, the area between this line and the recovery profile in a stratified or partially stratified bed can be used as a profile-metric to describe the extent of stratification in a bed as a whole. Figures 3b and 3c illustrate how this metric progresses from the initial value of zero when jigging begins (t=0), to a maximum value at equilibrium ($t\to\infty$).

The 'approach-to-equilibrium metric', ATE, provides a measure of the extent to which the profilemetric at a given time is different from the profile metric when the system is at equilibrium. It is defined as the ratio of the profile-metric after a jigging time t to the profile-metric at equilibrium but expressed as a percentage, Equation 1. The profile metric for intermediate density components is not as simple as those for the heavy and light components. This is evident in Figure 4 which illustrates how the recovery profile for the intermediate density (green) component changes as stratification progresses.



Figure 4. Stratification kinetics of the intermediate density (green) component (base case)

What is evident from this figure is that the stratification kinetics of the intermediate density component is different in the upper part of the bed than in the lower part of the bed. In the upper part, the green component is in effect 'separating' from the light component and does so quickly because of the relatively high density difference between the two components. In the lower part of the bed, the green component is effectively separating from the heavy component and the kinetics are much slower because the density difference is much smaller.

4.2 The Approach-to-equilibrium Metric and Stratification Kinetics

Figure 5 shows plots of the approach-to-equilibrium metric, *ATE*, against jigging time for the light (boro) and heavy (red) components under base case operating conditions. It presents a less cluttered, more generic picture of the stratification kinetics as a whole than the detailed plots in Figures 1 to 3.



Figure 5. Stratification kinetics for the light and heavy components (base case)

In Figure 5a, the stratification kinetics, expressed in terms of the *ATE* index, have a general exponential form. This is demonstrated in Figure 5b where the kinetics of the light and heavy components individually are described in terms of Equation 2 and involve a time constant Θ , an offset time t_o and the jigging time t.

$$ATE/100 = 1 - \exp\left[-\left(\frac{t+t_0}{\theta}\right)\right]$$

or
$$-\ln\left(1 - ATE/100\right) = \left(\frac{t_0}{\theta}\right) + \left(\frac{t}{\theta}\right)$$
(2)

The physical significance of the time constants in these expressions is well known (see Table 2). It represents the time taken for whatever time-based process being described to reach 63.2% of its value at equilibrium. However, in the context of stratification kinetics, the significance of the offset time t_0 is less clear. It has to do with the initial effects in the bed noted earlier (such as the reversal in the direction of change of component concentrations). In addition, it is affected by the fact that a component may stratify quickly with respect to one component and rather more slowly with respect to another (as has been described in the previous section for the intermediate density component). The complexity of these effects will undoubtedly increase as the number of components in the system increases. Consequently, the idea of an offset time in stratification is not conceptually very helpful. Indeed, the offset times measured for the 13 tests that were conducted do not show any discernable pattern as the operating conditions in the jig change (see our earlier paper Woollacott et al. 2021b).

Table 2: Significance of the Stratification Time Constant, Θ

Percentage approach to Equilibrium	63.2	86.5	95.0	98.2
Time taken to reach that percentage	θ	20	3 0	40

A more conceptually meaningful approach to modelling *ATE* and stratification kinetics is to account for the fact that a proportion of the particles may stratify quickly and a proportion may stratify more slowly. For example, in the context of this study, the lighter (boro) component stratifies relatively quickly because the density difference between it and both of the other components is relatively large. On the other hand, the heavy (red) component stratifies relatively quickly with respect to the light component, but relatively slowly with respect to the intermediate density (green) component. On this basis, the stratification kinetics of the ternary systems tested can be modelled by means of Equation 3, where Equation 3a applies to the light (boro) component and Equation 3b to the heavy (red) component.

$$ATE (fast)/100 = 1 - \exp[-\left(\frac{t}{\theta_{fast}}\right)]$$
(3a)

$$ATE (slow)/100 = 1 - p_{slow} \exp\left[-\left(\frac{t}{\theta_{slow}}\right)\right] - (1 - p_{slow}) \exp\left[-\left(\frac{t}{\theta_{fast}}\right)\right]$$
(3b)

Here, θ_{fast} and θ_{slow} are respectively the time constants of the 'fast' and 'slow' processes, and p_{slow} is the relative proportion of the particles that are associated with the 'slow' process. In the case of a binary system, where the kinetics of one component mirror those of the other component, Equation 3a can be used to model either component.

One consequence of a model that involves three or more particle components is that the physical interpretation of the time constant is compromised. The time taken to achieve 62.3%, 86.5% etc. of the equilibrium stratification profile is no longer always a simple multiple of the time constant as suggested in Table 2. Rather, the time taken to achieve these specific levels of approach to equilibrium must be calculated by solving Equation 3b for whatever value of *t* is of interest. So for example, the base case condition—with values of Θ_{fast} and Θ_{slow} of 36 and 176 seconds respectively (with $p_{slow} = 0.475$ as discussed shortly)—will have an effective time constant for the heavy (red) component, Θ_{red} , of 86 seconds. In addition, the time taken to reach 95% of the equilibrium condition is not $3x\Theta_{red}$ (=3x86 = 258) nor is it $3x\Theta_{slow}$ (=3x176 = 528 seconds). Solving Equation 3b for *t* when ATE = 95% gives a value of $\Theta_{red,95\%} = 397$ seconds. (These complications do not apply to the light (boro) component, however, as its stratification kinetics is described by Equation 3a, so that $\Theta_{boro} = \Theta_{fast} = 36$ seconds.)

4.3 The test results expressed in terms of the ATE metric

Figure 6 presents the result of fitting Equation 3a to the binary test data and shows how stratification kinetics are influenced by $\Delta \rho$ —the degree to which the densities of the stratifying particles are different. The fits are good although there is some scatter in the data when the density difference is small and stratification kinetics are relatively slow.



Figure 6. Effect of density difference on stratification kinetics

Figures 5a and 7 show the fits of Equations 3a and 3b to the *ATE* data for the 13 tests conducted on ternary systems. As the value of p_{slow} should be the same for all tests (because the composition of the jig bed was the same for all tests), its value was estimated as the average of values obtained from the individual tests. The value obtained, 0.475, is not unreasonable. (For example, if one assumes that the volume of green or red particles that quickly stratify matches the volume of boro particles—i.e. 26% of the particle volume in the bed—then the proportion of green and red particles that stratify slowly would be 48% = 100-26*2. However, the basis for such an assumption and calculation is rather speculative.) With a value of 0.475 for p_{slow} , the fits in Figures 5a and 7 are good. The values of the relevant time constants for the different tests are presented in Table 3.



Figure 7. Stratification kinetics under different operating conditions (tests K2 to K13). (The kinetics for the base condition are shown in Figure 4a.)

Data for ternary systems (obtained using Equations 3a and 3b)								
Operating variables*	Θ_{fast} (seconds)	O _{slow} (seconds)	θ _{red} ** (seconds)	θ _{red,95%} ** (seconds)	<i>p</i> _{slow} *** average for all tests (individual test)	Test		
Stroke=34%	43	234	107	527	0.475 (0.475)	K6		
Stroke=36%	40	184	93	417	0.475 (0.504)	K10		
Stroke=38%	36	176	86	397	0.475 (0.453)	K1 (base)		
Stroke=41%	28	125	64	282	0.475 (0.485)	K8		
Stroke=45%	28	86	51	194	0.475 (0.523)	K7		
Stroke=48%	25	96	46	218	0.475 (0.409)	K12		
T2=24 cS	43	209	98	472	0.475 (0.411)	K4		
T2=28 cS	36	176	98	397	0.475 (0.453)	K1 (base)		
T2=32 cS	28	133	51	300	0.475 (0.390)	K5		
T2=35 cS	26	130	61	294	0.475 (0.435)	K9		
T2=38 cS	23	95	54	213	0.475 (0.487)	K13		
T1=18 cS	43	165	90	373	0.475 (0.601)	K2		
T1=22 cS	36	176	86	397	0.475 (0.453)	K1 (base)		
T1=26 cS	30	144	79	326	0.475 (0.551)	K3		
T1=30 cS	25	113	55	255	0.475 (0.446)	K11		
Data for binary systems (obtained using Equation 3a) Density difference								
	(kg/m^3)		θ (seconds)	Component mix (50 - 50 vol%)				
91			189	green – red				
104			185	green – blue				
237			25.0	boro – green				
328			16.4	boro – red				
341			16.7	boro – blue				

Table 3: Kinetic parameters for stratification under different operating conditions

* The operating variables for each test were those for the base case (Table 1) except for the variable indicated in this table.

** Calculated by solving Equation 3b for $t = \Theta_{red}$ when ATE = 63.2%, or $t = \Theta_{red,95\%}$ when ATE = 95%

*** The proportion $p_{slow} = 0.475$ was determined as the average from the individual tests indicated in the table in brackets. The time constants Θ_{fast} and Θ_{slow} were determined using $p_{slow} = 0.475$.

5. The Effect of Jigging Conditions on Stratification Kinetics

5.1 The effect of density difference

Figure 6c (and Table 3) show how stratification kinetics are influenced by the difference in density between the stratifying species. The effect is dramatic. In a two component particle system, a density difference of 341 kg/m³ results in a stratification time constant that is more than 10 times smaller than for a system where the density difference is only 91 kg/m³. The figure also suggests that reasonable stratification kinetics—i.e. with time constants around 25 seconds—can be achieved with density differences as low as 237 kg/m³ provided the systems are binary. However, for density differences smaller than this, the rate of density stratification becomes significantly slower and appears to deteriorate rapidly for density differences below about 150 kg/m³.

As Figure 6c indicates, the relationship between the *ATE* metric and density difference asymptotes in two directions. As the density difference approaches zero, the *ATE* asymptotes to infinity as expected— no density stratification occurs when there is no difference in density among particles. As the density difference increases, stratification kinetics become much faster and the *ATE* metric appears to

asymptote to some small, non-zero value. A rough model of the influence of density difference on the stratification time constant is indicated in the figure.

There is little in the literature that can usefully be compared with the findings of this study. Vetter (1987) measured the stratification kinetics in binary systems of plastic cubes with density differences of 1100, 1200 and 1300 kg/m³. Stratification kinetics improved in that order. Rao et al. (2016) conducted a kinetic study on the jigging of iron ore fines. Interpreting their data from our perspective of approach-to-equilibrium suggests that the stratification time constant in their context would have been between 10 and 11 seconds in a system with a density difference in excess of 600 kg/m³. This data point has been included in Figure 6c.

5.2 The effect of operating conditions

To show how the key operating conditions—%Stroke, T1 and T2—affect the stratification kinetics of the light (boro) and heavy (red) components in the ternary systems, the trends shown in Figures 5c and 7 have been combined in Figure 8. For clarity, only the model fits are shown in the diagram. (Note that the time scales on the plots for the heavy component are considerably longer than those for the light component.)



Figure 8. The effect of operating conditions on stratification kinetics expressed in terms of *ATE*, the Approach-to-Equilibrium metric (In each plot, the kinetics for the base case (K1) is indicated by the solid line.)

The figure (and Table 3) show the general trend that an increase in the value of any one of the three operating variables improves stratification kinetics—i.e. the associated stratification time constants decrease and equilibrium is reached more quickly. However, there are some anomalies in the data. The test with 45 %Stroke (K7 in Figures 7a and b) had faster kinetics than the test with 48 %Stroke. The test with T1 = 0.18s has an odd shape for the heavy component (K2 in Figure 7d); initially it had slower kinetics than the test with T1 = 0.22s (K1) but then approached equilibrium faster. There are also two instances where the measured kinetics for different conditions are very similar: these are the 36 and 38%Stroke tests in Figure 7b (K10 and K1 respectively); and the 0.32 and 0.35s tests for T2 in Figure 7f (K5 and K9 respectively). No specific reason for these apparent anomalies is offered beyond the general observation that they could be the result of experimental errors in the kinetic data, errors in the estimation of the equilibrium stratification profile, or errors arising from the way the profile metrics have been calculated. Despite these anomalies, however, the improvement in the kinetics as any of the three operating increase is clearly established by the test results. This is demonstrated in Figure 9.



Figure 9. The effect of operating conditions on stratification kinetics in terms of stratification time constants

(* The 'optimum range' is the range of %Stroke that achieves the optimum stratification profile at equilibrium as established by Woollacott et al., 2021b.)

The figure shows the influence of each of the three key operating variables on stratification kinetics as indicated by the stratification time constants Θ_{fast} (or Θ_{boro}), Θ_{slow} , and the effective time constant for the heavy (red) component, Θ_{red} . The figure highlights the considerable difference between the rate of the fast and slow kinetic processes. For example, the average time constant for the slow process, Θ_{slow} , (equal to 145 seconds) is about 5 times greater than the average time constant for the fast process, Θ_{fast} , (equal to 32 seconds). As discussed earlier, the reason is that the density difference driving the fast process is much larger that the density difference driving the slow process.

Over the ranges measured, the time constants decrease linearly as the values of the three operating variables increase. The stratification kinetics improved by between 42 and 46% for the light (boro) component, and by between 39 and 58% for the heavy (red) component. Linear models of these relationships are indicated in the figure.

T1 and T2 appear to influence the stratification kinetics of the light (boro) component to about the same extent; the models of the effect of both variables are virtually the same (Figures 9a and b). However, T2 exerts a stronger influence on the kinetics of the heavy (red) component than T1 does. Over the ranges measured, it reduces Θ_{slow} by about 55% compared to 32% by T1. With regard to the combined time constant, Θ_{red} , the difference between the two is smaller—45% reduction for T2 and 39% reduction for T1.

The influence of %Stroke on stratification (Figure 9c) is complicated by the fact that stroke can affect the quality of stratification at equilibrium. The previous paper (Woollacott et al., 2021a) reported that, at equilibrium, optimum stratification occurred when the %Stroke was between 36 and 41%. Outside

this 'optimum range' the equilibrium stratification profile deteriorates. Within the optimum range, changing the stroke had no discernible effect on the stratification profile at equilibrium. However, it does affect the kinetics; increasing %Stroke across the optimum range improves the kinetics significantly. All three time constants— Θ_{fast} , Θ_{slow} and Θ_{red} —were reduced by about 31%. A further reduction of up to 28% can be achieved if the %Stroke is increased above the optimum range up to 48%, but at the cost of a deterioration in the equilibrium stratification profile that is associated with such an increase.

6. Discussion and implications for practice

It is now appropriate to pull together the findings from this paper and the previous one and discuss their implications for jigging practice.

6.1 The Approach-to-equilibrium Metric

This study has found that stratification kinetics in a jig can be described by an expression that is essentially exponential in form. Some researchers in the field have also developed expressions for stratification kinetics that are exponential in form, but they differ in detail. Mayer (1964), for example, suggested that the rate of stratification, S, could be described in terms of a rate constant, k, and a 'jiggability index', J, (Equation (4)). Karantzavelos and Frangiscos (1984) assumed the rate of change of the yield, Y, from a jig was exponential in form with two parameters, θ and β , (Equation (5)).

Model by Mayer:
$$S = J \exp(-kt)$$
 (4)

Model by Karantzavelos and Frangiscos:
$$Y = 1 - \exp(-t/\theta)^{\beta}$$
 (5)

Both these expressions require two experimentally determined parameters with t representing the jigging time. In principle, both can be used as a means of describing how operating conditions affect stratification kinetics; i.e. by establishing how the parameters in the expressions are influenced by those operating conditions. However, the differences between these models and those developed in this study are such that meaningful comparisons of their respective predictions are not possible. Apart from the differences in the exact detail of the three expressions, Equations (4) and (5) relate to continuous jigging and focus on a context where the bed is split at one given height; in other words, they provide a local view of stratification kinetics. Equations 2, 3a and 3b, on the other hand, relate to stratification in general and focus on the rate at which the stratification profile *as a whole* approaches the most well stratified profile achievable, namely the profile at equilibrium. As such, it provides a global view of stratification kinetics.

In addition to giving a global view of stratification kinetics, the *ATE* approach has several significant advantages when studying jigging dynamics: it decouples equilibrium and kinetic effects; it enables an individual assessment of the kinetics of the two key components in the bed—the light and heavy components; and it describes stratification kinetics in terms of physically-meaningful parameters—i.e. the times taken to achieve specified levels of approach to equilibrium, and the proportion of the particles associated with those times. In the context of the study, just three parameters were needed—one for the fast stratification process, θ_{fast} , and an additional two for the slow process, θ_{slow} and p_{slow} . What is more, these parameters provide a viable basis for modelling how jigging conditions influence the kinetics of stratification (see Figures 6c and 9).

6.2 The equilibrium stratification profile

Separation performance and the quality of stratification in a jig is determined by the stratification profile achieved at equilibrium. Critically, the performance that a given jig can achieve at equilibrium appears to be determined primarily by the *properties* of the feed to the jig and its mechanical configuration and not so much by how the operating variables of the jig are manipulated or 'fine-tuned'. The study found that as long as inter-particle movement within the bed was in a range that was vigorous enough but not too vigorous the operating conditions did not have any noticeable influence on the equilibrium

stratification profile. In the study, this optimum range was determined by %Stroke, i.e. between 34% and 41%, and by the depth of the bed, i.e. between 80 and 100mm. As can be seen this range is quite broad; the water displacement can change over a range of 30%, and bed depth over a range of 25% without affecting the quality of the equilibrium stratification profile. Obviously, the limits of these ranges need to be properly established for each specific jigging context if jigging performance is to be optimized in that context.

When the bed depth and water displacement are within these ranges—i.e. within the optimum operational zone—the other operating variables can vary considerably without affecting the quality of stratification at equilibrium. In the study, the pulsion hold-time (T2) could change over a range of 58%, and the pulsion time (T1) over a range of 67% without a noticeable effect on the equilibrium stratification profile. Similarly, variation of the other variables considered, such as the suction time (T3), the suction hold-time (T4), the cycle frequency, and the air supply pressure to the piston driving the water displacement in the jig, had no effect.

6.3 Stratification kinetics

The study confirmed that both particle properties and operating conditions influence stratification kinetics significantly. It found, as expected, that differences in particle densities influence stratification kinetics very strongly. Kinetics were fast when the density difference was 237 kg/m³ or more—with time constants around 20 seconds—but were very slow for density differences of 100 kg/m³ or less—time constants were 10 or more times greater.

With both slow and fast kinetic processes, the kinetics improved markedly and linearly as the values of the operating variables tested increased. Over the ranges tested, increasing T1 decreased the stratification time constants by 41% for fast kinetic processes and by 31% for slow kinetic processes. The influence of T2 was somewhat stronger—the time constants for fast kinetic processes decreased by 47% and by 55% for slow kinetic processes. Increasing the water displacement (i.e. %Stroke) within the range for optimum stratification at equilibrium also improved stratification kinetics but not quite to the same degree as varying T2 did; the time constants for fast kinetic processes decreased by 35% and by 47% for slow kinetic processes. Further improvements in kinetics were achieved when the extent of water displacement was increased beyond the upper limit of what was optimal for stratification at equilibrium. Overall, optimum stratification is achieved when the water displacement is near this upper limit and with high values of T2 and T1. (The optimum mix of settings for T1 and T2 was not investigated.)

The finding that the influence of T1 and T2 on the stratification kinetics of the light (boro) component¹ is fairly similar but is guite different for the heavy (red) component is interesting. There may be several reasons for this. The first is the theory that the initial bed expansion during the pulsion period T1 involves upward movement of the bed as a whole with little inter-particle movement before the lower layers begin to rise less rapidly so that "the bed loosens from the bottom, expanding by means of a loosening wave spreading upward through the bed" (Burt, 1984, p188). The consequence of this dynamic should be that stratification during bed expansion should be somewhat restricted compared to when the expanded bed begins to collapse and reconsolidate. By this theory, T1 should exert less influence on stratification than T2 does. However, although T1 encompasses the period when the bed rises 'en mass', it is not necessarily coincident with it; it may very well extend over into the 'settling period' when the loosening wave is active and when the expanded bed starts to reconsolidate. As such, the latter part of the T1 period may very well involve the same stratification processes that are associated with this 'settling period' and are characteristic of the dynamics during the pulsion hold-period T2. This could explain the finding that T1 and T2 exerts a similar influence on the stratification kinetics of the lighter component in the jig bed. However a different dynamic must also be at play because the influence of T1 and T2 on the heavy component is not the same. One possibility is that the heavy

¹ This contradicts the preliminary findings reported in Woollacott et al. (2021b) that T1 influenced the stratification kinetics of the light component less than T2 did. That report was based on time constants calculated by Equation 2 and overlooked the effect of the variable influence of the offset times, t_o , obtained with that model.

particles start to settle before the light particles do so that the light particles 'settle' over a longer settling period than the heavy particles do. Extending the length of the 'settling period' by extending the length of T2 would then be expected to result in a greater improvement in the kinetics of the light particles than is the case for the heavy particles.

6.4 Processing ores that are difficult to beneficiate in a jig

The feed to a jig is sometimes unexpectedly difficult to beneficiate well because liberation of the valuable mineral is incomplete. This leads to a distribution of particle densities in the jig and a reduction in the density difference between many particles species in the jig. Both are detrimental to the equilibrium stratification profile and to stratification kinetics. Options for optimizing jig performance in such circumstances must focus on optimizing stratification kinetics and lengthening the residence time during jigging. This is because the reduced quality of stratification achievable at equilibrium is predetermined by the properties of the material being processed; it can only be improved by measures that increase the degree of liberation in the feed.

The nature of the difficulty can be illustrated by the study presented in Figure 10. It compares the separation performance achieved with and without an intermediate-density component in the jig bed.



Figure 10: Grade-recovery curves for jigging systems after specific jigging times The figure shows grade-recovery curves for binary and ternary systems in which the light (boro) and heavy (red) components have a density difference of 328 kg/m^3 . In the ternary system, the density difference between the heavy and intermediate (green) components is 91 kg/m^3 .

The figure describes the performance achieved after specified jigging times for both a binary and a ternary system. The performance is presented in the form of grade recovery curves—i.e. the recovery of the heavy (red) component to the lower layer split from the jig bed vs the grade of that component in the lower layer. The points in each curve are data points from the study. The radial dotted lines are operational lines that show how the height at which the bed is split into two products (i.e. at the top of layer 1, 2 etc.) relates to the grade-recovery curves obtained.

In the figure, the data for the binary system demonstrates the excellent separation achievable when no intermediate-density component or unliberated particles are present and when a large density difference exists between mineral and gangue—328 kg/m³ in this case. Removing layers 1 to 3 as the heavy product from the jig (equivalent to a yield of 53%) recovers 99% of the heavy mineral at a grade of

86%. The figure also shows that the grade-recovery at equilibrium is achieved very quickly—i.e. within 100 seconds. The progress of the separation is indicated in the figure by the grade-recovery curves that would be obtained if jigging ceased after 30, 45, 60 or 100 seconds. A typical residence time in industrial jigging is 3 minutes (180 seconds) (Myburgh, 2010) and, as the figure shows, equilibrium is easily reached within that time so stratification kinetics do no constrain the separation and jigging performance is optimal.

The situation changes dramatically when the minerals are not completely liberated or a third component of intermediate density is present in the bed. This is illustrated in the figure by the grade-recovery curves for a ternary system in which the same light and heavy minerals (in a proportion similar to that in the binary system) constitute only 55% of the jig bed. The intermediate component has a density that is only slightly smaller than the heavy component—91 kg/m³ smaller. Again grade-recovery curves relating to different jigging times are shown in the figure.

At equilibrium, the heavy product (layers 1 to 3 with a yield of 53%) now recovers only 93% of the heavy mineral at a grade of 51%. (The fact that the intermediate density component might contain locked heavy mineral is ignored for the purposes of this illustration.) The separation performance achieved is still reasonable but is considerably inferior to what can be achieved when no intermediate component is present. However, to achieve even this performance, which is the best possible for the ternary system, the residence time in the jig would need to be 12 minutes (720 seconds)! If the residence time in the jig was only the typical 3 minutes, only 85% of the heavy mineral would be recovered at a grade of 47%—a drop of 8% in recovery and 4% in grade from what could be achieved by the jig with a long residence time. If the residence time was increased from 3 to 5 minutes (300 seconds), the recovery and grade would improve somewhat—to 89% and 49% respectively—but would still be significantly less than what the jig could achieve given enough jigging time.

In order to improve performance in this context, the residence time in the jig must be extended. Lengthening the residence time in an existing jig can be achieved by reducing the feed rate to the jig but at the cost of a reduced throughput. Throughput would be compromised less if the jig were longer or if two stage jigging were to be employed, or a scavenger jig were to be added to the circuit. In either case, the residence time in the second jigging stage or scavenger would need to be appropriately long as highlighted by Figure 10. Adding a scavenger also has the advantage that liberation could be improved by a preceding comminution step.

6.5 Limitations of the study

The primary limitation of this study is that it was conducted in a small (200mm diameter) batch jig on essentially mono-sized, 8mm spherical particles. The effect of the size, shape and distribution of particles was not investigated. Nor were the effects of bed depth, bed composition, or hutch flowrate on kinetics investigated. Interactive effects between the influences of the various operating variables tested were also not investigated in the kinetic study. The analysis of the data generated focused on how the stratification profile in the bed as a whole changed with jigging time. The way stratification kinetics vary with height in the bed was not analyzed in this paper but will be investigated using a more fundamental approach similar to that used by Vetter (1987).

7. Conclusions

The experimental procedure and analysis used in this study parallels approaches used by others to investigate stratification kinetics. However, it has some novel features and has yielded some novel findings. Firstly, it has developed an approach which disentangles kinetic from equilibrium effects. It allows the complexities associated with stratification kinetics in a particle bed to be described and analyzed in terms of how the stratification profile as a whole changes with time. This can be modelled in terms of three physically meaningful parameters: two stratification time constants, θ_{fast} and θ_{slow} ; and p_{slow} , the proportion of particles that stratify slowly. The effect of operating conditions on stratification kinetics can then be modelled in terms of their effect on these parameters.

To the authors' knowledge, the study's investigation into the effect of particle density on stratification kinetics is more thorough than has been reported before. In addition, it has focused on the effect of relatively small density differences on stratification kinetics—a context that is particularly relevant to systems with significant proportions of locked particles. The detrimental impact of this on jigging performance has been illustrated in the paper using data from the study. Significantly, the illustration has shown that remedial measures in this context need to focus on improving the kinetics of stratification and on extending residence times in order to improve jigging performance. It also suggests circuit modifications that may also improve performance.

In conclusion, the study has focused on the disentangling of equilibrium and kinetic effects, and has found that the manipulation of the operating conditions—the bed depth, and the properties of the jig cycle—affects the kinetics of stratification but not what is inherently achievable in a jig as determined by the equilibrium stratification profile. What is achievable in a jig is determined primarily by the properties of the feed to the jig; it is predetermined and essentially is not amenable to manipulation in the jig.

References

Burt, R. O., 1984. Gravity Concentration Technology. Amsterdam, Elsevier.

- Karantzavelos, G. E., Frangiscos A. Z., 1984. Contribution to the modelling of the jigging process. In: Control '84. Mineral/Metallurgical Processing, New York, AIME. pp 97-105.
- Mayer, F. W., 1964. Fundamentals of a potential energy theory of the jigging process. In: Proceedings of the 7th International Mineral Processing Congress. N. Arbiter, Gordon and Breach, New York. pp. 75-86.
- Mehrotra, S. P. and Mishra, B. K., 1997. Mathematical modeling of particle stratification in jigs. Proceedings of National Seminar on Processing of Fines. Jamshedpur, CSIR Institute of Minerals and Materials Technology.
- Myburgh, H. A., 2010. The influence of control and mechanical conditions of certain parameters on jigging. JSAIMM. 110, 655-661.
- Rao, B. V., H. K. Velan, Kumar, K. S. M., Mulage, R., Dutta, A., 2016. Slicing the evolving batch jig bed. In: Conference proceedings. MIST.
- Rong, R. X., Lyman, G. J., 1991. The mechanical behaviour of water and gas phases in a pilot scale Baum Jig. Coal Preparation. 9, 85-106.
- Rong, R. X., Lyman, G. J., 1992. The effect of jigging time and air cycle on bed stratification in a pilot scale Baum jig. Fuel. 71, 115-123.
- Rong, R. X., Lyman, G. J., 1993. A new energy dissipation theory of jig bed stratification. Part 1: energy dissipation analysis in a pilot scale baum jig. Int. J. of Mineral Processing. 37, 165-188.
- Vetter, D. A., Brouckraer, C. J., Wright, D. W., 1987. A dispersion model of autogenous particle separations, with specific application to the batch jigging of particles. In: 20th International Symposium on the Application of Computers and Mathematics in the Mineral Industries, Johannesburg, SAIMM. pp. 127-139.
- Woollacott, L. C., Tripathy, A. K., and Potgieter, J.H., 2021a. The influence of operating conditions on density stratification in a batch jig. Part 1: The Influence on the Equilibrium Stratification Profile. (Paper submitted for publication.)
- Woollacott, L. C., Tripathy, A. K., and Potgieter, J.H., 2021b. The effect of operating conditions on stratification kinetics in a batch jig. Proceedings of the International Mineral Processing Conference. (Publication in process.)