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Multi-commodity Optimisation of Peer-to-Peer Energy Trading Resources in Smart Grid

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***Abstract*—Utility maximisation is a major priority of energy prosumers participating in peer-to-peer energy trading and sharing (P2P-ETS). However, as more distributed energy resources integrate with the distribution network, the impact of link communication becomes significant and should therefore be considered. This paper presents a multi-commodity formulation that allows dual-optimisation of energy and communication resources in P2P-ETS. While the proposed technique minimises energy generation cost and communication delay on one hand, it also maximises the global utility of prosumers with fair resource allocation on the other hand. We evaluate the algorithm in a variety of realistic conditions including time-varying communication network with delay and lossy links. The results show that convergence is achieved in a fewer number of time-steps than previously proposed algorithms. It is further observed that the entities with a higher willingness to trade energy acquire more utility satisfaction than others.**

***Index Terms*—Distributed algorithm, social welfare, P2P-ETS, multi-commodity networks, economic dispatch, packet loss, peer-to-peer energy trading, DDG.**

NOMENCLATURE

α_τ Step size at time τ.
$\bar{k}_{i,j}$ Message signal delay on link (i, j).
$\lambda_{i,j}$ Incremental cost per unit energy generated and transffered using link (i, j).
$\mathcal{L}$ Lagrangian multiplier.
$\mathcal{M}$ Set of Virtual microgrids.
$\mathcal{W}$ Social welfare.
ω Weight associated with the fairness parameter of prosumer.
σ Fairness parameter associated with a prosumer.
D Total energy demand in the network.
d_i Energy demand of each prosumer in the network.
E Set of network links (i, j) connecting the prosumers.
$f_{i,j}$ Signal loss probability on the link (i, j).
$g(\tau)$ (Sub)gradient to Lagrange dual problem $\lambda(\tau)$.
$G(t)$ Time-varying network graph.
$g_i(q_i)$ Conjugate convex function.
K Messages from different peers in the network.
$l_{i,j}$ Lower bounds of energy flow capacity on link i, j.
n_c Number of energy consumers.
n_p Number of energy generators.
$u_{i,j}$ Upper bounds of energy flow capacity on link i, j.
V Interconnected nodes representing the prosumers.
$w(\cdot)$ Dual function.
x_i Power generation of prosumer n_p.
x_i^{max} Upper bound of power generation of prosumer n_p.
x_i^{min} Lower bound of power generation of prosumer n_p.
$C_i(.)$ Energy generation cost function of prosumer n_p.
$U(x_i)$ Utility of commodity x_i.

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I. INTRODUCTION

SINCE the past decade, advances in technology have been driving the rise of distributed energy resources (DERs) at the community level [1]–[4]. These DERs create a chain of independent energy producers and consumers that coexist with different energy generating capacities and demands [1]. The existence of these prosumers could result in power grid instability and unreliability if their energy supply and/or demand requirements are not properly coordinated. A common approach to energy coordination and control is utilising distributed control algorithm to eliminate the single point of failure in centralised control systems [5], [6].

Distributed algorithms have been proposed in the literature for energy coordination [7] and in peer-to-peer (P2P) energy trading [5], [8]. In these algorithms, each prosumer keeps a local approximate value of its energy profile and in most cases, communicates this estimated value directly to its connected neighbour. The energy profiles of all the prosumers converge to an optimal value over a communication network [9]. In a previous study [8], and a survey conducted in [10], the authors assumed a perfect communication link between these energy prosumers, and thus ignored the typical communication issues in real networks.

In practical P2P networks with digital capabilities, end-to-end transaction may be affected by several communication-related factors including topology, jitters, latency, reliability and attenuation due to weather, physical environment or contingencies on the link and the number of P2P prosumers in the network. Other factors include link capacity, number of nodes, and message size [11], [12]. This has been exemplified using a distributed consensus algorithm [13] that fails to converge in the presence of prolonged communication delays. Some recent works considered the impact of the imperfect communication links on optimal dispatch of energy among DERs [9], [14]–[17]. The performance metrics considered include communication delay [14], [15], time-varying topologies [16], time-varying directed network with delays [17], and unreliable communication links subject to packet drops [9]. While these studies considered, one or two network constraints of imperfect communication links, a typical economic dispatch problem (EDP) should encompass more, to incorporate the diverse generation mix of DERs in the power grid.

In a distributed EDP involving several DERs, the underlying communication network has a huge effect on the ability of the prosumers to reach a consensus on the optimality of their energy demand and generation cost. Thus, maximising the economic benefits of P2P energy trading and sharing (P2P-ETS) while respecting sustainability and environmental obligations is crucial to incentivise prosumers' participation in P2P-ETS market. Here, P2P-ETS is a collective term to indicate P2P energy interaction which could include energy trading, energy sharing, energy exchange, etc. This paper aims to solve the EDP among distributed DERs over realistic imperfect communication links. Furthermore, the utility derived based on the optimal distributed EDP is assessed. In this case, we consider fairness in the allocation of network resources to ensure balanced energy network.

This work employs the multi-commodity network flow (MCF) technique [18], [19], for optimising the distributed flow of resources in a network. This is because MCF optimisation offers the opportunity to consider the communication links whilst solving the optimisation tasks for energy trading. For instance, MCF optimisation provides insight into both the communication and the energy transfer between prosumers, which can be modelled simultaneously. The suitability of MCF for dynamic energy management has recently been assessed by [20] and applied to the smart grid in [21]–[23]. Their results show faster convergence of the algorithm and robustness to delay and packet loss in delay-sensitive networks like smart energy systems.

The main contributions of this paper are summarised as:

- presentation of MCF approach that allows dual optimisation of energy and communication resources in P2P-ETS where prosumers work in consensus to meet aggregate demand and maximise their utilities;
- although EDP was previously investigated in [9], [13], [15], [17] without considering imperfect communication links, the proposed algorithm offers faster convergence under the imperfect communication links characterised by delay, signal loss, and asynchronous communication. These imperfections usually result in stringent impacts on the optimal utility of energy prosumers due to stale energy prices;
- in addition, we evaluate the optimisation of utility satisfactions perceived by the prosumers when considering such imperfect communication links in the smart grid with interest on fair allocation of network resources in terms of supply and demands.

In the remaining sections, the literature review is presented in Section II. The problem formulation, MCF optimisation, and utility maximisation among P2P energy traders are presented in Section III. The simulation and results are discussed in Section IV. Section V concludes and identifies potential future work.

II. Literature Review

The performance limitation posed by centralised control approaches for energy dispatch among DERs connected at the edges of the power distribution network has birthed the increasing proposals on distributed algorithms [9], [13], [15], [17], [24] for energy control and P2P energy trading.

A P2P energy trading scheme is proposed in [25] using a leader-follower Stackelberg game for the power system to reduce its electricity demand during peak hours. For additional control to reduce the curtailment of renewable generation, [26] proposed a local energy market for distribution systems integrating P2P energy trading with locational marginal pricing. To increase user participation in P2P energy trading, a game-theoretic design is proposed in [27] which shows potential in attracting users to participate in the energy trading for more carbon and cost reduction, through a proposal of a bilateral contract in [28].

A Mixed Integer Linear Programming based predictive design and a dispatch optimisation algorithm was proposed in [29], while, [24] utilised a two-level incremental cost consensus distributed algorithm to solve EDP in smart grid. With the evolving digitisation of power network, communication systems have become an integral component of the smart grid. This poses the problems of stale energy prices due to the commonly known problems of time delay and packet losses from imperfect communication links. Thus, the influence of time delays on the distributed algorithms was investigated in [13] and [15]. The authors in [13] investigated the influence of time delays over different types of information exchanged among the DER units, and found that the consensus algorithm either converged to an incorrect value or failed to converge altogether. Further, [17] proposed a distributed algorithm based on push-sum and gradient method to solve the EDP among connected DER units over fixed and time-varying network delays. On the other hand [9] proposed a robustified extension of [17] using the same method but solving the coordination problem over packet-dropping communication links. Other efforts in reducing the communication delay in DERs are found in [30], [31].

In maximising the social welfare of generators and consumers, [32] propose an incremental welfare distributed consensus algorithm, which was further extended in [33] to incorporate transmission loss and directed communication topologies. In [34], a social welfare maximisation problem using open control law to minimise generator and load adjustment rates was addressed. In contrast, this study presents the use of MCF optimisation technique in solving EDP in smart grid considering the disjoint electrical and communication variables. The results are then analysed for maximising prosumer social welfare in a P2P energy trading network.

III. Problem Formulation

Consider an energy distribution network, for example, the IEEE 5-bus system as shown in Fig. 1-A, to which distributed energy generators are connected. In Fig. 1, the physical network represents the physical connectivity of the energy generators depicted as (G) in Fig. 1-A, to a distribution network. The virtual network denotes a nodal representation of the physical nodes to reflect communication among them. Fig. 1 also illustrates the relationships between the assets (physical energy network and virtual communication network) and multi-commodity modelling (economic dispatch problem, resource constraints, and social welfare of prosumers) presented in this study. The energy trading assets are depicted in Fig. 1-A while the energy multi-commodity resources model that harnesses the asset representations and the constraints as shown in Fig. 1-B.

To formalise the relationships, let the connectivity of the prosumers be represented using a strongly connected graph network, that models the pairwise relations between nodes and links. The nodes are called the vertices and links connecting the vertices are edges. In this model, the strongly connected energy network is defined by graph $G = (V, E)$ of $V = \{1, 2, \cdots, N\}$ interconnected nodes, $E \subseteq V \times V$ set of bidirectional links of any (n_p, n_c) interconnected prosumers and N is the number of prosumers in the network. Note that $\mathcal{P} \triangleq \{1, \cdots, n_p\}$ represents the set of energy generators with index $i \in \mathcal{P}$ and $\mathcal{C} \triangleq \{1, \cdots, n_c\}$ is the set of energy consumers with index $j \in \mathcal{C}$. No energy prosumer has the combined characteristics of generator and consumer simultaneously at a given trading period, t, and thus $\mathcal{P} \cap \mathcal{C} = \varnothing$. It follows also that the set of all prosumers is $V = \mathcal{P} \cup \mathcal{C}$ and the total number of energy prosumers is $N \triangleq n_c + n_p$. The goal of each prosumer is to optimise its energy output to maximise its benefit and to collectively meet the total energy demand in the network in a distributed way.

A. Communication Network

The power network is overlayed by a communication network that conveys energy trading messages as shown in Fig. 1-A. Let the communication network be represented as a time-varying graph $G(t) = (V, E(t))$ with $E(t)$ links, where each set of links changes over time, based on the state of the communication link at a time, t. A directed link from prosumer n_p to prosumer n_c is denoted by $(i, j) \in E(t)$. Each directed link $(i, j) \in E(t)$ is characterised by its upper bounds of energy trading messages through the links, $u_{i,j}$, delay, $\bar{k}_{ij}$, and signal loss probability, $f_{i,j}$ on links connecting n_p to n_c.

B. Energy Generation and Demand

For tractable solutions, we assume that the prosumers are virtually clustered using communication systems into M number of virtual microgrids (VMG) [35]. In this case, we are interested in minimising energy generation costs and maximising social benefits within the VMG for prosumers. This problem can be approached by minimising the total aggregate energy cost and assuming small clusters of energy generators. Through clustering, energy demand can be matched with a supplier within an m^{th} VMG in a local energy P2P trading fashion [36]. Let $\mathcal{M} = \{1, 2, \cdots, M\}$ be the set of virtual microgrids such that $m \in \mathcal{M}$. Thus, during the t^{th} trading interval, there exists $E_m(t)$ set of links in the m^{th} VMG. Each VMG is thus characterised by $n_p \in \mathcal{P}$ energy generators and each producing $x_i(t), i \in n_p$ units of energy. Other motivations for clustering the prosumers are to reduce non-commodity charges, for optimal node density and reducing energy trading cost [35], [36]. The generation cost minimisation problem during $t \in \mathcal{I}_{ij}$ trading interval is formulated as

$$\min_{\{x_i\}} \sum_{t \in \mathcal{I}_{ij}} \sum_{i \in n_p} C_i x_i(t), \ (i, j) \in E_m(t), m \in \mathcal{M} \quad \text{(1a)}$$

$$\text{subject to :} \sum_{t \in \mathcal{I}_{ij}} \sum_{i \in n_p} x_i(t) \leq D_m(t), \ (i, j) \in E_m(t), m \in \mathcal{M} \quad \text{(1b)}$$

$$x_i \geq 0, \ \forall i = 1, \cdots, n_p \quad \text{(1c)}$$

where $C_i(\cdot)$ is the cost function of prosumer $i \in n_p$ for generating x_i units of energy. It is assumed that the cost function follows a convex function model for tractability. The model in (1b) implies that the total energy generated within a cluster m must satisfy the total energy demanded for energy conservation to hold. Henceforth, we shall focus on a single market period of one hour in the P2P energy market similar to [37] as the single-period problem can be extended to a multi-period problem with temporally coupled constraints. The solution for the single market period demonstrates the performance of the proposed method in a more explicit manner [37]. In that case, we shall be dropping the notation t in (1).

C. Energy as a Multi-commodity Flow Problem

The most basic MCF problem can be represented as

$$\min_{\{x_{ij,k}\}} \sum_{k \in K} \sum_{(i,j) \in E_m} C_{ij,k}(x_{ij,k}), \ m \in \mathcal{M} \quad (2)$$

where $x_{ij,k}$ is the flow of commodity k on the link between the nodes (n_p, n_c), whilst the objective function $C_{ij,k}(x_{ij,k})$ is the cost of flow in the links, which is convex monotonically increasing function [20]. The decision variables in this model are energy flows $x_{ij,k}$ which must follow flow conversation criterion for the power network to be balanced, i.e., the flow entering the node must be equal to the flow leaving the node. In addition, energy flows through the links are limited by lower and upper bounds, which translates to the maximum energy that can flow through the link at a time. For consistency, throughout the rest of this paper, the term commodity represents message flows from the different prosumers in the network which is a communication parameter.

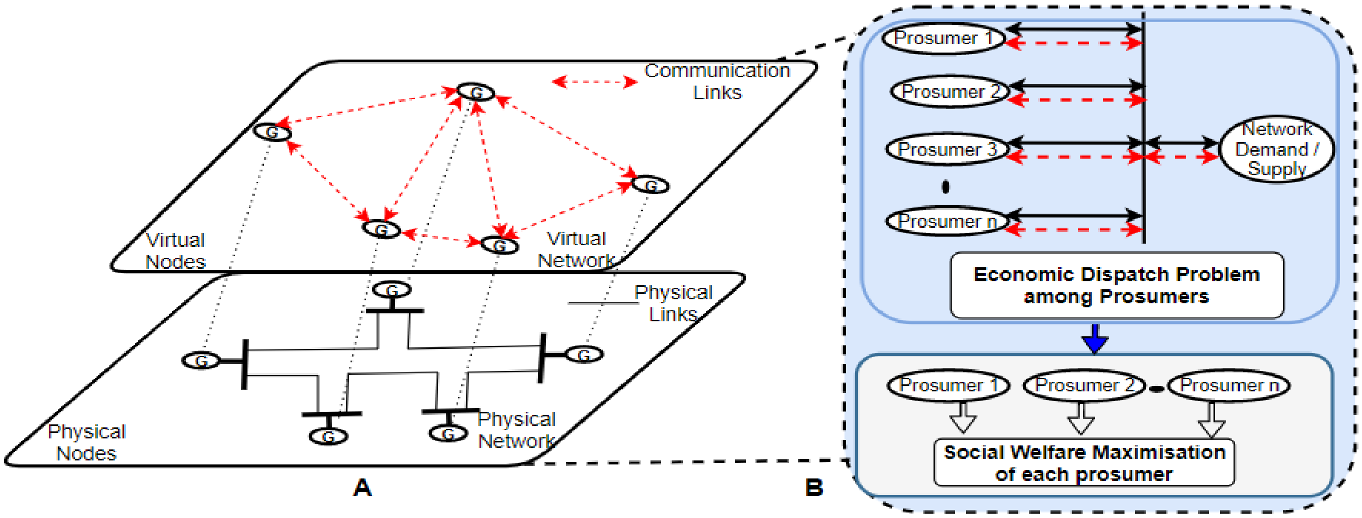

Fig. 1: A: IEEE 5-bus system of energy generators showing the physical and communication connectivities; B: Schematic of the multi-commodity modelling - economic dispatch and social welfare maximisation problems.

Without loss of generality, the EDP can be represented as an MCF optimisation of the form

$$\min_{\{x_{ij,k}\}} \sum_{k\in K}\sum_{(i,j)\in E_m} C_{ij,k}(x_{ij,k}),\ m\in\mathcal{M} \tag{3a}$$

$$\text{subject to}: \sum_{k\in K}\sum_{(i,j)\in E_m} x_{ij,k} \le D_m,\ m\in\mathcal{M} \tag{3b}$$

$$l_{ij,k} \le x_{ij,k} \le u_{ij,k}, \forall (i,j)\in E_m,\ m\in\mathcal{M} \tag{3c}$$

$$x_{ij,k} \ge 0, \quad \forall k\in K, \forall (i,j)\in E_m,\ m\in\mathcal{M} \tag{3d}$$

$$x_{ij,k}^{min} \le d_{ij} \le x_{ij,k}^{max}, \forall\, d_{ij}\in D,\ (i,j)\in E_m \tag{3e}$$

where d_{ij} is the demand at each bus, such that $\sum_{(i,j)\in E_m} d_{ij} = D$. Constraints (3b) is the conservation of energy flow constraint, (3c) is the upper and lower bounds of flows in the links, which must not exceed the capacity of the link and (3d) represents non-negativity constraints, i.e, a generating unit must generate energy x_i, satisfying the lower and upper bounds of their generation capacity (3e).

D. Dual Lagrange Problem for EDP

To solve the minimisation over variable $x_{ij,k}$ problem of (3a), this section first presents its dual problem, followed by the derivation of the distributed (sub)gradient algorithm. The Lagrangian function for relaxing the flow conservation constraints of problem (3a) is

$$\mathcal{L}(x,\lambda) = \sum_{k\in K}\sum_{(i,j)\in E_m} C_{ij,k}(x_{ij,k}) - \sum_{k\in K}\lambda_{ij,k}D + \sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}x_{ij,k} \tag{4a}$$

$$\text{subject to}: x_{ij,k}\ge 0, \forall k\in K,\ m\in\mathcal{M} \tag{4b}$$

$$l_{ij,k} \le x_{ij,k} \le u_{ij,k}, \forall (i,j)\in E_m,\ m\in\mathcal{M} \tag{4c}$$

$$x_{ij,k}^{min} \le d_{ij} \le x_{ij,k}^{max}, \forall d_{ij}\in D,\ (i,j)\in E_m \tag{4d}$$

where $\lambda_{ij} \ge 0$ represents the Lagrange multiplier, the incremental cost associated with the energy flow constraint. This is usually an optimal parameter that ensures the constraint conditions are not violated. Constraints (4b) - (4d), (4a) is summarised as

$$\mathcal{L}(x,\lambda) = \sum_{k\in K}\sum_{(i,j)\in E_m} C_{ij,k}(x_{ij,k}) - \sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}D + \sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}x_{ij,k} \tag{5a}$$

$$= \sum_{k\in K}\sum_{(i,j)\in E_m} C_{ij,k}(x_{ij,k}) - \sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}d_{ij,k} + \sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}x_{ij,k},\ m\in\mathcal{M} \tag{5b}$$

where $\sum_{(i,j)\in E_m} d_{ij,k} = D,\ k\in K$. Notice that the model discussed is peculiar to energy trading and may include energy sharing when producers do not charge peers. The model (5a) can further be summarised in terms of the energy flows, thus:

$$\mathcal{L}(x,\lambda) = \sum_{k\in K}\bar{C}_{ij,k}(x_{ij,k}) + \sum_{k\in K}\lambda_{ij,k}\bar{x}_{ij,k} - \sum_{k\in K}\lambda_{ij,k}\bar{d}_{ij,k},\quad m\in\mathcal{M} \tag{6}$$

where $\bar{C}_{ij,k}(\cdot) = \sum_{(i,j)\in E_m} C_{ij,k}(\cdot)$, $\bar{x}_{ij,k} = \sum_{(i,j)\in E_m} x_{ij,k}$ and $\bar{d}_{ij,k} = \sum_{(i,j)\in E_m} d_{ij,k}$. The argument that minimises the Lagrangian given in (6) by following a dual decomposition formulation and can be expressed as

$$x^{\star}_{ij,k} = \operatorname*{arg\,min}_{x_{ij,k}\in(4b),(4c),(4d)} \mathcal{L}(x,\lambda)\ s.t.\ \lambda_{ij,k}\ge 0,\ k\in K. \tag{7}$$

When $C_{ij,k}(\cdot)$ is strictly convex, the cost function can be investigated for the optimum (minimum) value.

The dual objective function $w(\cdot)$ can be shown to enable each energy generator in the network to participate in solving the distributed optimisation of the energy traded in the network. This is quite scalable and efficient and also could improve the trust level in the system. Besides, the energy trading information of each energy prosumer is private and

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thus the optimisation problem cannot be solved centrally because the central agent cannot access the private energy information. Thus, the dual objective function is expressed as:

$$\begin{aligned} w(\lambda_{ij,k}) &= \min_{x_{ij,k}\geq 0} \mathcal{L}(x, \lambda_{ij,k}) \\ &= \min_{x_{ij,k}\geq 0} \sum_{k\in K} \bar{C}_{ij,k}(x_{ij,k}) + \sum_{k\in K} \lambda_{ij,k}\bar{x}_{ij,k} - \sum_{k\in K} \lambda_{ij,k}\bar{d}_{ij,k} \\ &= \sum_{k\in K} \min_{x_{ij}\geq 0} \Big(\bar{C}_{ij,k}(x_{ij,k}) + \lambda_{i,j}\bar{x}_{ij,k} - \bar{\lambda}_{ij,k} d_{ij,k} \Big) \end{aligned} \tag{8}$$

Clearly, (8) shows a fully $k \in K$ distributed problems that each energy generator i participates in solving. Estimating the optimal dual solution in terms of the Lagrange of the dual function problem as

$$w^\star(\lambda^\star_{ij,k}) = \max_{\lambda_{ij,k}\geq 0} w(\lambda_{ij,k}) \tag{9}$$

the optimal pricing information, $\lambda^\star_{ij,k}$ is required to establish the best energy unit, $x^\star_{ij,k}$, transferred by the generator unit to the demand unit. This can be realised through an update of the pricing information in an iterative fashion which is presented in the next section (III-E).

E. Distributed Dual-Gradient (DDG) Algorithm for EDP

Problem (9) is solved using the (sub)gradient method. The (sub)gradient method is a generalisation of the gradient descent, using the iterations

$$\lambda_{ij,k}(\tau+1) = [\lambda_{ij,k}(\tau) - \alpha_\tau g(\tau)]^+ \,,\ k \in K, (i,j) \in E_m \tag{10}$$

where α_τ is the step size at time τ, and $g(\tau)$ is a (sub)gradient to $w(\lambda_{ij,k})$ at $\lambda_{ij,k}(\tau)$. Note that $[s]^+ = \max(s, 0)$.

Assumption A: *Since the cost function within the dual objective function is strictly convex, then the dual function $w(\lambda_{ij,k})$ is continuously differentiable* [38].

The (sub)gradient, $g(\tau)$ is realised by taking the first derivative of (8) and setting the result equal to zero as follows

$$g(\tau) = \frac{\partial w(\lambda_{ij,k})}{\partial \lambda_{ij,k}} = 0 \Rightarrow - \left(\sum_{k\in K} \bar{d}_{ij,k} - \sum_{k\in K} \bar{x}_{ij,k} \right) = 0. \tag{11}$$

Substituting (11) into (10), a (sub)gradient update of (9) along each dual variable is obtained and expressed as

$$\lambda^{(\tau+1)}_{ij,k} = \left[\lambda^{(\tau}_{ij,k}) - \alpha_\tau \Big(\sum_{k\in K} \bar{x}_{ij,k} - \sum_{k\in K} \bar{d}_{ij,k} \Big) \right]^+ , (i,j) \in E_m. \tag{12}$$

As can be seen in (12), when the demand is greater than the supply, the generators will increase the price of the excess demand energy units by α_τ. For example, when $\sum_{(i,j)\in E_m} d_{ij,k} > \sum_{(i,j)\in E_m} x_{ij,k}$, the second term in (12) will be greater than zero which leads to the $[s]^+ = \max(s, 0) > 0$. The dual variables are updated bi-directionally and synchronously at discrete time $\tau = \{0, 1, \cdots, \infty\}$, and only neighbours can communicate. For instance, each generating unit will wait a random time before transmitting the next update of its generated output. At every time step, there is an upper bound on the optimal value of the Lagrange function (4a), which is obtained by evaluating the dual objective function (9). Each link computes its (sub)gradient coordinate using the generator flow variable $x_{ij,k}$. To reduce excess overheads and delay that could result in assigning additional scalar variables to the estimate of each generator unit at each iteration as seen in [9], the information communicated among the generators is completely distributed and limited to the incremental cost $\lambda_{ij,k}$. The novelty in this update is that each generator ensures it uses the price as an indicator function to generate the required energy that satisfies the network demand. Each generator utilises the $\lambda_{ij,k}$ to update its generation output $x_{ij,k}$ at k^{th} flow. It is worth noting that the model (12) reduces to a consensus problem when all the incremental costs $\lambda_{ij,k}$ are identically equal to zero, (e.g. [39]).

F. Modeling the Communication Delay and Signal Loss

One of the ways to measure the robustness of an algorithm is its ability to converge in the presence of faults which could result from out of sequence delivery or loss of signalling messages. In a consensus network where all peers are minimising their objectives to achieve a collective goal, the higher the transmission delay in such a network, the longer it takes for the peers to reach the desired agreement. Communication delay is prevalent in distributed networks, we therefore observe the robustness of DDG when the communication network is subjected to high signalling/transmission delay. In a realistic scenario, there is always a communication delay, $\bar{k}_{i,j}(\tau)$, on the communication link (i, j) in sending a message from prosumer, n_p, to prosumer, n_c. Similarly, there exists an end-to-end time delay, $\tau + \bar{k}_{i,j}(\tau)$, to receive a response from prosumer, n_c, by prosumer, n_p [40]. The impact of high signalling delay would result in using an outdated link cost in the gradient iteration, which would generate algorithm oscillations without reaching an optimal solution. The gradient update of (12) becomes

$$\begin{aligned} &\lambda^{(\tau+\bar{k}_{i,j}(\tau)+1)}_{ij,k} = \\ &\left[\lambda^{(\tau+\bar{k}_{ij}(\tau))}_{ij,k} - \alpha_\tau \left(\sum_{(i,j)\in E_m} x_{ij,k} - \sum_{(i,j)\in E_m} d_{ij,k} \right) \right]^+ , \\ &\forall (i,j) \in E_m,\ m \in \mathcal{M}. \end{aligned} \tag{13}$$

It has been shown in [40] that introduction of communication delay would not affect the convergence speed of the algorithm but would result in convergence to a larger neighbourhood of the optimal value. However, the choice of step size determines the algorithm convergence. In this study, a constant step size is used which has been shown to converge to optimal value when the objective function is differentiable [41] [42].

Similarly, a probabilistic approach [9] is employed to model the signal loss on the communication link. A communication between prosumer n_p and n_c is said to be successful when the information sent by n_p is received by n_c without loss and in real-time. However, due to signal loss on the link $(i, j) \in E$, a failure vector $f_{i,j}(\tau)$ is introduced, where $f_i(\tau) = 1$ if the communication from prosumer i at iteration τ is received, 0 otherwise. Thus $w(\lambda_\tau)$ in (8) now becomes

$$\Bigg[\sum_{(i,j)\in E_m} f_i f_j \Bigg[\sum_{k\in K} \min_{x_{ij}\geq 0} \bar{C}_{ij,k}(x_{ij,k}) - \bar{C}_{ji,k}(x_{ji,k})$$
$$+\lambda_{ij,k}\Bigg[(x_{ij,k}-d_{ij,k})-(x_{ji,k}-d_{ji,k})\Bigg]\Bigg]\Bigg]^+$$
$$\forall (i,j)\in E_m,\ m\in\mathcal{M}. \tag{14}$$

G. Resource Allocation for P2P-ETS

The energy market of prosumers is further considered for fairness allocation of communication resources. The weighted general fairness utility model is given by:

$$U(x_i^*) = \omega_i \frac{x_i^{*\,1-\sigma}}{1-\sigma},\ i\in n_p,\ m\in\mathcal{M} \tag{15}$$

where σ is the fairness parameter and ω_i is the weight associated with the utility of prosumer i, x_i^* represents the optimal energy resources obtained from solving the EDP problem using MCF in the foregoing discussion. As in [36], it can be shown that the utility model follows weighted concave function of the energy resources expressed as

$$U(x_i^*) = \omega_i \ln(x_i^*), \forall i\in n_p,\ m\in\mathcal{M}. \tag{16}$$

Suppose that $x_i^* = 0, \forall i \in n_p$, then $\ln(x_i^*) = -\infty$. To overcome this problem, a constant $\theta_i \geq 1$ is introduced so that the utility becomes $U(x_i^*) = \omega_i \ln(x_i^* + \theta_i),\ i \in n_p$. Our interest is to maximise the resources allocated to a prosumer over a finite link capacity. This is approached by maximising the utility of each actor subject to a capacity constraint and considering the optimisation variable as the energy resources traded over the link. In that case, the optimisation problem becomes:

$$\begin{aligned}\max_{\{x_i^*\}_{i\in n_p}} & \sum_{i\in n_p} U_i(x_i^*)\\ \text{subject to}: & \sum_{i:\ell\in i} x_i^* \leq c_\ell \quad \forall \ell\in n_p\\ & x_i^*\geq 0,\ i\in E_m,\ m\in\mathcal{M}.\end{aligned} \tag{17}$$

where c_ℓ is the capacity of link ℓ. Fig. 2 demonstrates the utility function of six prosumers in a network with different weights, w_i (assigned weight in the bracket). The graph shows that the utility increases for varying increasing weights of the energy traders. Physically, the weights may be interpreted to willingness to trade energy with other peers. Producers with a higher willingness to trade energy achieve higher utility than other prosumers with little or no willingness.

H. Optimal Resource Allocation

Invariably, if the utility is proportional to willingness, higher energy flow will be experienced in the network, thus resources must be fairly and optimally allocated to each prosumer so as not to starve other prosumers in the network. Throughout this study, we consider fairness parameter $\sigma = 1$ as shown in (15).

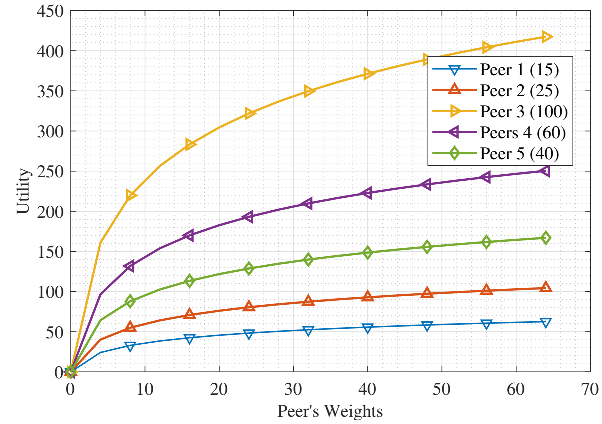

Fig. 2: Relationship between the utility function and varying weights of the peers.

Next, we consider realising the optimal resources that can be allocated to the link $i \in n_p,\ m \in \mathcal{M}$ considering capacity $c_\ell, \forall \ell \in E_m$. By taking the Lagrangian of (17), we express

$$\mathcal{F}(x,\eta) = \sum_{i\in n_p} U_i(x_i^*) - (\sum_{i\in n_p} x_i^*\eta_i - \sum_{i\in n_p}\eta_i c_\ell),\ \ell\in E_m \tag{18}$$

where $x = (x_1, x_2, \cdots, n_p)$, $\eta = (\eta_1, \eta_2, \cdots, n_p)$. By taking the first derivative of (18) with respect to x_i and setting the result equal to zero, the following is derived:

$$\frac{\partial \mathcal{F}(x_i^*,\eta_i)}{\delta x_i^*} = 0 \Rightarrow \sum_{i\in n_p}\frac{\omega_i}{x_i^*+\theta_i} - \sum_{i\in n_p}\eta_i = 0$$
$$\Rightarrow \sum_{i\in n_p} x_i^{**} = \frac{\sum_{i\in n_p}\omega_i - \sum_{i\in n_p}\eta_i\theta_i}{\sum_{i\in n_p}\eta_i}$$
$$\Rightarrow x_i^{**} = \frac{\omega_i - \eta_i\theta_i}{\eta_i} \quad \forall i\in n_p,\ m\in\mathcal{M}. \tag{19}$$

From (19), the optimal resource allocation $x_i^{**}\ \forall i \in n_p,\ m \in \mathcal{M}$ depends on the congestion price η_i and the number of actors on the link. For example, to reduce the resource flow on the link due to actor i, implies increasing the congestion price η_i. Similarly, to increase the resource flow due to prosumer i implies reducing the network congestion price η_i. In addition, from (19), increasing the congestion price will be useful in controlling congestion in the network as a lower amount of data will be sent by each actor over the link ℓ.

I. Social Welfare Maximisation

Using the foregoing utility function, this section introduces a social welfare maximisation objective to improve on the overall costs and maintain fairness for all generators and demands. Let $\mathcal{W}$ represent the total social welfare comprising of the energy generators $W_i(.), \forall i \in n_p$ and demand

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units$W_j(.)$, $m \in \mathcal{M}$, $\bar{p}$ is the price of electricity.

$$\max_{\{x_i^*, d_j\}} \mathcal{W} = \left[\sum_{i \in n_p} W_i(x_i^*, \bar{p}_i) + \sum_{j \in n_c} W_j(d_j, \bar{p}_j) \right] \quad (20a)$$

$$\text{subject to}: \sum_{i \in n_p} x_i^* = \sum_{j \in n_c} d_j \quad (20b)$$

$$x_i^{*\,min} \leq x_i^* \leq x_i^{*\,max}, \quad i \in n_p \quad (20c)$$

$$d_j^{min} \leq d_j \leq d_j^{max}, \quad j \in n_c. \quad (20d)$$

Constraints (20b) is the conservation of energy flow constraint while the operational constraints (20a) and (20d) represents the lower and upper bounds of energy generation and consumption respectively, which are further defined as follows:

1) **Generator welfare**: Let $\bar{p}_i x_i^*, \forall i \in n_p,\ m \in \mathcal{M}$ represent the revenue that generator i receives from selling x_i^* units of energy with selling price $\bar{p}_i$, then the social welfare of the generator i can be expressed as:

$$W_i(x_i^*, \bar{p}_i) = \bar{p}_i x_i^* - C_i(x_i^*) \quad (21)$$

where $C_i(x_i^*)$ is the cost incurred by i to generate x_i^* units of energy earlier shown in (1b). We model the cost as a convex quadratic function of the form:

$$C_i(x_i^*) = \frac{1}{2} a_i x_i^{*\,2} + b_i x_i^* + c_i \quad (22)$$

where $a_i \geq 0$, $b_i > 0$ and $c_i = 1, \forall i \in n_p,\ m \in \mathcal{M}$ are the cost parameters.

2) **Consumer welfare**: For the consumer side, social welfare is the difference between the utility it derives and the cost of procuring $x_j^*, j \in n_c$ units of energy.

$$W_j(d_j, \bar{p}_j) = U_j(d_j) - \bar{p}_j x_j^*,\ j \in n_c \quad (23)$$

where $U_j(d_j)$ is the utility function that defines the amount of satisfaction that consumer j receives from demanding d_j units of energy and $\bar{p}_j$ is the payment made for d_j. As shown in (16), the utility function of the consumer is continuously differentiable and non-decreasing.

Substituting (23) and (21) for the generator and consumer welfare in (20a), respectively, the total social welfare of the prosumer makes the optimisation problem to become

$$\max_{\{x_i^*, d_j\}} \mathcal{W} = \sum_{j \in n_c} U_j(d_j) - \sum_{i \in n_p} C_i(x_i^*) \quad (24a)$$

$$\text{subject to}: \sum_{i \in n_p} d_j \leq \sum_{j \in n_c} x_i^* \quad (24b)$$

$$x_i^{*\,min} \leq x_i^* \leq x_i^{*\,max}, \quad i \in n_p. \quad (24c)$$

Notice that in (24a), the power balance criteria earlier defined in (3b) enabled $(\sum_{i \in n_p} \bar{p}_i x_i^* - \sum_{j \in n_c} \bar{p}_j d_j)$ to be eliminated. Due to the concave properties of (24a), the model in (24) is a concave maximisation problem and can be solved using convex programming techniques. By inspection, the model terms in (24) are individually differentiable. Thus, we involve the use of DDG-algorithm as in the foregoing discussion in solving the welfare maximisation problem. This has also been similarly applied in the literature [36], [43]. To achieve this, we start by formulating the Lagrangian of problem (24) as follows

$$\mathcal{J}(d_j, x_i^*, \rho_{i,j}) = \sum_{j \in n_c} U_j(d_j) - \sum_{i \in n_p} C_i(x_i^*) - \rho_{i,j} \Bigg(\sum_{j \in n_c} d_j - \sum_{i \in n_p} x_i^* \Bigg). \quad (25)$$

where $\rho_{i,j}$ represents the Lagrangian multiplier. In terms of generators and consumers, problem (25) can be decomposed and solved in a distributed fashion as follows

$$\mathcal{J}(d, \rho) = \sum_{j \in n_c} U_j(d_j) - \sum_{j \in n_c} d_j \rho_{i,j} \quad (26a)$$

$$\mathcal{J}(x, \rho) = \sum_{i \in n_p} \rho_{i,j} x_i^* - \sum_{i \in n_p} C_i(x_i^*) \quad (26b)$$

by taking the first derivatives of (26a) and (26b) with respect to d_j and x_i, and (25) with respect to $\rho_{i,j}$, respectively, and setting the result equal to zero. In that case, the optimal flow variables can be expressed as:

$$\frac{\partial \mathcal{J}(d, \rho)}{\partial d} = 0 \Rightarrow d_j^* = \frac{\omega_j - \rho_{i,j}}{\rho_{i,j}} \quad (27a)$$

$$\frac{\partial \mathcal{J}(x, \rho)}{\partial x} = 0 \Rightarrow x_i^* = \frac{\rho_{i,j} - b_i}{a_i} \quad (27b)$$

$$\frac{\partial \mathcal{J}}{\partial \rho} = 0 \Rightarrow \rho_i = - \sum_{j \in n_c} d_j + \sum_{i \in n_p} x_i^*. \quad (27c)$$

From (27a), the demand is inversely proportional to the price. In other words, consumers will assess the prices of energy to buy more at a low price or buy lesser energy units at higher energy prices. From the generator side in (27b), they are motivated to supply more, linearly, at higher prices and vice versa. From (27c), the update price function is expressed as

$$\rho(n+1) = \left[\rho(n) - \alpha_n \left(\sum_{i \in n_p} x_i^* - \sum_{j \in n_c} d_j \right) \right]^+. \quad (28)$$

In (28), the energy producers assign an additional α_n penalty to the network fees at n^{th} time-step if the total demand d_j exceeds the total supply x_i^* in the network. However, lower than the network fees will not be charged due to $[\cdot]^+$.

IV. Numerical Simulation and Result Analysis

To evaluate the performance of the developed distributed algorithm for EDP, simulations are performed using Java [19] [44]. An instance of 5 prosumers adopted from study [9], which uses an IEEE 5-bus system as in Fig.1 is considered for comparison purposes. The generation cost function is set to a value of ±20kWh of each generator's demand. For instance, each generator produces energy in the range of ±20kWh of what it consumes serving as flow bounds for each unit. This is because the objective is to optimise the generation output of each generator to satisfy the aggregate energy demand in the network. The 5 prosumers are connected by 16 links. A set of energy demands (kWh) of $d_1 = 40$, $d_2 = 30$, $d_3 = 100$, $d_4 = 40$, $d_5 = 90$ are considered. The step size, α, is set to a constant value of 1 for most of the cases considered. This is because, we have established in our previous study [22] that, with a constant step size of 1, the network achieves lower delay, and the algorithm converges faster.

A. DDG Algorithm: without communication delay

The ideal scenario, without communication delay, is the most basic case study that exists in the literature. This is used as a starting point for testing the robustness of the proposed algorithm. The stability of an algorithm requiring the algorithm to converge to a solution in a finite amount of time is a desirable property used to measure the algorithm performance and efficiency. The result is analysed based on

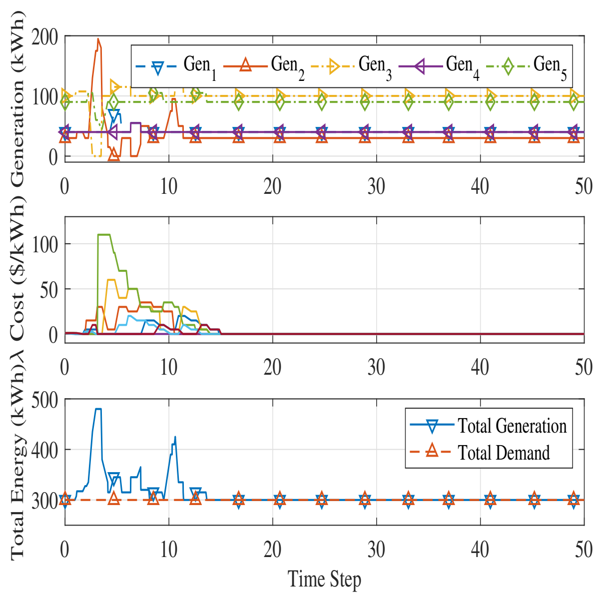

Fig. 3: Results for the ideal case showing the convergence of the generated energy and the incremental cost

the algorithm convergence time. For the ideal case without communication delay, the result is as shown in Fig. 3, where the upper plot shows the convergence of the incremental costs ($\lambda_{ij,k}$). The middle plot, also the generation plots, shows the evolution of the energy generated from each prosumer. The bottom plot shows the total generated energy and the total energy demanded by the prosumers. The optimal energy generated by each of the prosumers referred on the plots as Gen_1 to Gen_5) x_i, $i \in M$ is $x_1 = 40kWh, x_2 = 20kWh, x_3 = 115kWh, x_4 = 45kWh, x_5 = 80kWh$ with a total $\sum_{i=1}^{5} = 300kWh$. This shows that x_1 generated its own energy (note that the initial demand for generator 1 is $40kWh$, and the generated output is $40kWh$), while other prosumers generated below or above their energy demands to satisfy the total demanded energy in the network (an aggregate of $300kWh$). The three plots, including the incremental costs, $\lambda_{ij,k}$, settle at around 14^{th} time step, indicating about 4 communication steps $(O(n) - 2)$ for each prosumer before convergence.

Furthermore, from Fig. 3, it can be observed that at the 3rd time step, the total energy generated peaked (upper and bottom plot of Fig. 3). This results in an increase in the cost function (middle plot of Fig. 3). However, as the energy generation output descended overtime to meet the demand, the incremental cost equally descended to 0. The increased cost before convergence can be interpreted as meaning that additional storage space is required for the excess energy, this increases the cost of generation. Thus the optimisation algorithm works to minimise the cost by solving the EDP problem when generated energy meets demand.

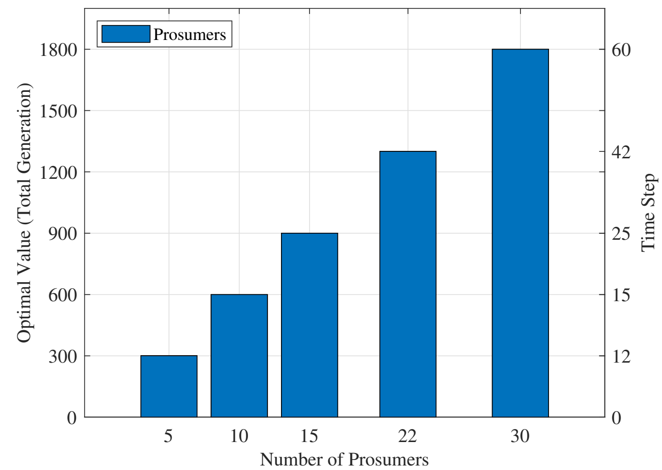

Fig. 4: Scalability results showing the convergence of the cost function for $5, 10, 15$ and 30 prosumers respectively

To explore the scalability of the algorithm, this research moves away from the norm of 4 prosumers [5] [8] and 5 prosumers [9]. Fig. 4 shows the convergence of the cost function (generators) for $5, 10, 15$, and 30 prosumers with total demands set to $300kWh$, $600kWh$, $900kWh$, and $1800kWh$ respectively. As expected, the networks with 5 prosumers converge faster than the network with $10, 15$, and 30 prosumers. For instance, a network consisting of 15 prosumers attains an optimal value of $900kWh$ at about the 25th time step as compared to 22 prosumers that attain $1300kWh$ optimal value at 42nd time step (Fig. 4). It is worth noting that during the simulation, the computation time and the number of iterations per time-step to reach the optimal value increases with an increasing number of prosumers.

In [24], a two-level incremental cost consensus (ICC) algorithm was proposed to solve the EDP in the smart grid. A comparison test of the convergence time of the proposed DDG algorithm to the ICC proposed in [24] is as shown in Fig 5. It can be observed that the DDG converged faster than the ICC. For instance, the total generation already matched the total demand at the $14th$ time step for DDG, whereas, the ICC converged at around $38th$ time step. The implication of this is that, while both consensus algorithms solved the EDP problem, DDG would be a better choice in a large scale network.

B. Evaluating the Impact of Communication Delay

A communication delay of 10 time steps is adapted from [17] with $\alpha = 1$, and the result is as shown in Fig. 6.

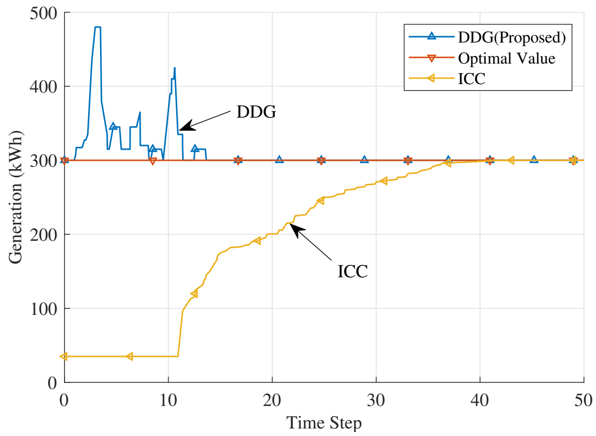

Fig. 5: Comparative analysis of the convergence properties for ICC [24] and proposed DDG

It can be observed that each of the variables ultimately

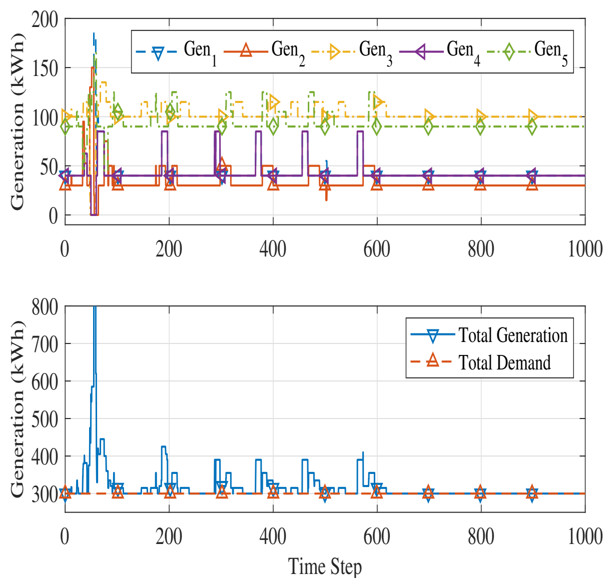

Fig. 6: Results for a network with communication delay showing fast convergence for the proposed DDG algorithm

converges to the optimal value as the ideal case, despite the signalling delays, i.e., the convergence occurred at the $610th$ time step. In comparison to the algorithm presented in [17], DDG attains its optimal solutions faster (refer to Table I for the convergence time analysis). It should also be noted that each agent in the algorithm presented in [17] holds a couple of variables that are updated and communicated at each iteration thus, adding to the communication delay. Whereas, in this study, the only communicated variable is the incremental cost signifying when to increase or reduce generation to meet the network demand. This significantly improves the communication delay leading to faster convergence. Further, Table I compares the contribution of the proposed algorithm to the literature by detailing the cases considered and the convergence time. Unlike the work that was completed in this

Description	Ref. [17]	Ref. [9]	This study
Signal delay	Yes	Yes	Yes
Communication signal loss	No	Yes	Yes
Signal delay & signal loss	No	No	Yes
Asynchronous communication	No	No	Yes
Result for message delay case			
Algorithm convergence time step	> 900	-	610
Result for the probability of signal loss case			
Algorithm convergence time step	-	> 45	18

TABLE I: Result comparison with related works.

paper and presented in Section IV-D, studies including [9] [17] did not consider the cases of both signal loss and signal delay simultaneously.

C. Impact of Communication Signal Loss

In this section, a case of an unreliable communication network with a probability of message signalling loss on the communication links is considered. Motivated by [9] and for comparison purposes, the probability of loss was set to 0.1. The results are as shown in Fig. 7. It can be observed that the

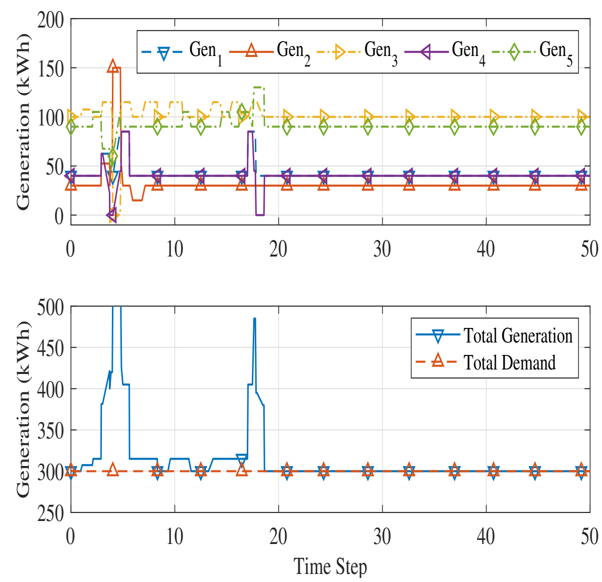

Fig. 7: Results for a network with a probability of signal loss showing fast convergence for the proposed DDG algorithm

signal loss probability has a negligible effect on the algorithm convergence which shows a better performance, in terms of convergence time to [9] (Table I). Comparing (Fig. 7) and (Fig. 6), it can be observed that the convergence for the delay is higher than that of a probability of loss. This could result from the fact that the loss is modeled as a probability function that could occur or otherwise, whereas, the delay is a constant value with high significance. For instance, when there are delays in the gradient updates, prosumers could advertise stale energy prices. Similarly, if the energy flow data is significantly delayed or lost or that bad data are detected, the energy trading information (e.g. price) could be significantly higher or lower than the prices advertised by the neighbours.

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In such a case, energy producers (or consumers) could resort to state estimation which elongates the decision and agreement periods.

D. Impact of Communication Delay and Signal Loss

Motivated by the success of the convergence on the message with a probability of loss, a case where the communication network is both affected by signalling delay of 10 time steps and a probability of loss of 0.1 is considered. Studies [9] and [17] only considered cases of delay or packet loss and not both cases simultaneously. However, the combined impact on the network is remarkably different from the effect of each variable in isolation. As

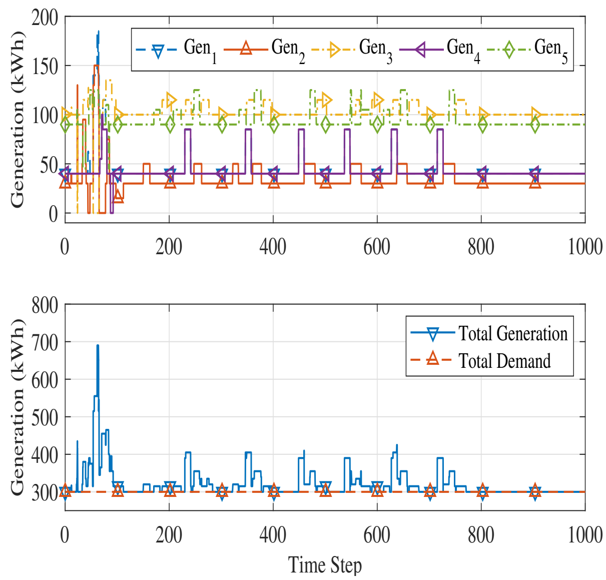

Fig. 8: Results for a network with both communication delay and signal loss showing fast convergence for the proposed DDG algorithm

shown in Fig. 8, the algorithm ultimately converges to the optimal value as the ideal case. In addition, of all the cases considered, the communication delay and signal loss resulted in the highest communication link cost, which is because the prosumers synchronously transmitted their update after the communication delay, thereby oversubscribing the communication links. This observation implies that the proposed DDG algorithm is robust against signalling delay and signal loss of the underlying communication link. However, a significant level of signalling loss and delay might result in algorithm oscillations without reaching convergence. One of the future work of this paper would be to extend the model by deriving the explicit relationship between delay, signal loss, and algorithm stability.

It is worth noting that the robustness of the algorithm resulted from the use of MCF optimisation, as it offers an opportunity to consider the communication links whilst solving the optimisation task. In addition, unreliable communication mostly results from link utilisation and congestion thus leading to signal drop and signal delay [22]. It can be noted, that by utilising the MCF optimisation, the research has already set a limit to the maximum allowed traffic based on the capacity of the link at the time, thus reducing the probability of maximum utilisation and congestion, and thus signal drop.

E. Numerical Example of Optimal Resource Allocation and Social Welfare

Given a linear network shown in Fig. 9, utilising the utility model and the optimisation problem of (17), with constraints as shown. The results shown in Fig. 10 demonstrate the

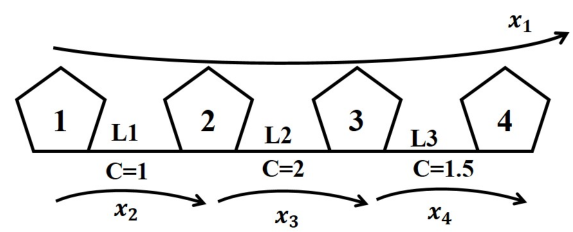

Fig. 9: An example of a linear network topology for resource allocation demonstration

optimal data flow rates under the σ-fairness condition, which indicates that Prosumer 3 has the highest utility function.

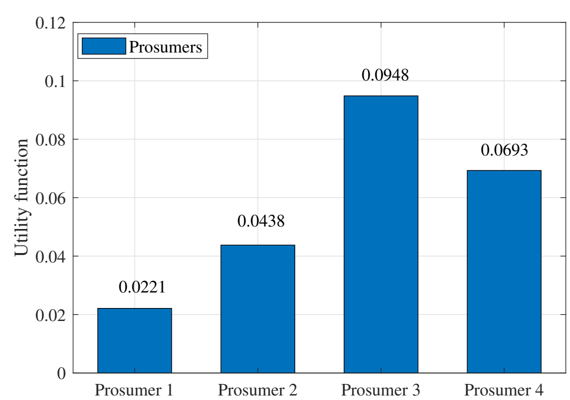

Fig. 10: Optimal data flow rates under the σ-fairness condition for the prosumers in the network

Solving for the optimal demand flow yields Fig. 11, which shows that Prosumers 2, 3, and 4 have more social welfare than Prosumer 1. In addition, Fig. 12 shows a typical run over 24 hour depicting the relationship with energy demands and supply in the network. The results reflect a reduction in the quantity of energy demanded when the supply is at the highest price, obeying the law of demand and supply. However, the ratio of producers to consumers in the network affects the price paid for the energy bought or sold as shown in Fig 12. Take for instance, at $40s$, with 20 producers and 8 consumers, the total supply is $2.2kwh$ and demand is $9kWh$. Whereas with 10 producers and 20 consumers, the total supply is $3.6kWh$, and demand is $7kWh$.

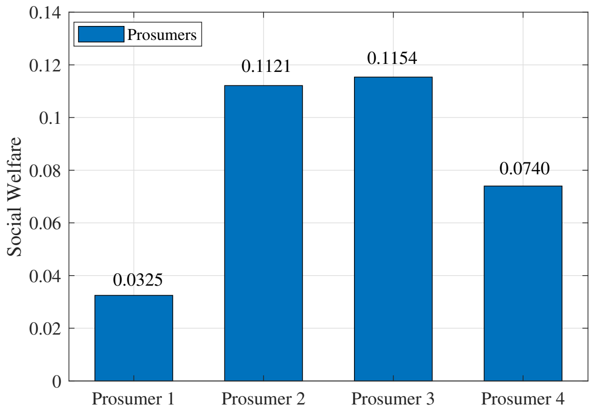

Fig. 11: Optimal social welfare under the σ-fairness condition for the prosumers in the network

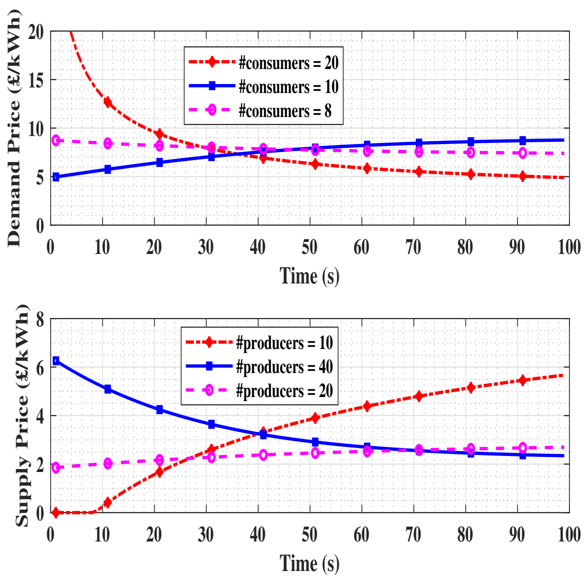

Fig. 12: Optimal social welfare for different numbers of producers and consumers in the network

V. CONCLUSION

This paper presented a distributed dual-gradient algorithm based on multi-commodity flow technique and a dual-(sub)gradient method for the distributed economic dispatch problem application. Specifically, we tested the proposed algorithm with an unreliable communication network by considering signal loss probability, message delay, and asynchronous communication of prosumers. The proposed techniques converges faster than previously proposed algorithms which is a desired feature especially in a large network connecting several distributed energy resources. The model is further extended to realise the global utility maximisation among market-based participants to improve overall costs and maintain the fairness of all generators and demands. Results showed a reduction in quantity demanded when supply is at the highest price, but the price paid is dependent on the ratio of producers to consumers in the network. For instance, the lower the number of producers, the higher the energy price, and the lower the energy demanded by the consumers. In the future, we shall investigate the flexibility of demands, time-variation, and other time-coupling constraints of prosumers on the proposed model.

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