The discursive construction of the mastery curriculum in mathematics

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Abstract

Recent educational reform in England has been informed by evidence of successful strategies being adopted in high-performing international educational jurisdictions, in particular Shanghai and Singapore. The influential National Centre for Excellence in the Teaching of Mathematics (NCETM) and Maths Hubs have encapsulated the word *mastery* in relation to mathematics teaching and learning based on observations in Shanghai. Whilst the current mathematics curriculum in England can be interpreted as a *mastery curriculum*, there have been numerous so called *mastery* approaches in the last forty years and they elude a single definition. Within the shifting landscape of education, mathematics teachers are still adjusting to the demands of the *mastery curriculum*, and its discursive framing in a variety of policy settings. This study seeks to provide insights into ways in which the idea of a *mastery curriculum* shapes ideological understanding of becoming a mathematics teacher. It explores the discursive construction of the *mastery curriculum* through the lens of Lacan's four discourses. In repeatedly mapping out classroom interactions to different permutations of discourse, we generate alternative possible understandings. Through depicting competing discourses and the sheer difficulty of being in the classroom, teachers respond to the various demands that they think are placed on them. It considers at a macro-level how society influences and controls notions of the *mastery curriculum*. It then investigates these influences in the day-to-day teaching of mathematics. In particular, it considers how student teachers make sense of their worlds as they gain qualification to teach 11-16 mathematics. I consider conventional psychological theories of learning, such as Vygotsky's Zone of Proximal Development, but also draw on Lacanian theory of the Subject towards producing a more contemporary spin on conceptions of psychology. The study is centred on the premise that motives in both learning and teaching are channelled by identification with particular discourses; the need to comply with new directives, educate or get educated, to achieve 'outstanding' status, etc. With this work being carried out from a teacher educator perspective, data collection is centred on discovering how both student teachers and the researcher himself identify with the multifaceted discourses that shape their practice, with particular reference to the *mastery curriculum* in mathematics.

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The thesis is dedicated to my dear mother Jadwiga Pawlik.

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Chapter 1 Overview

1.1 Introduction

The phenomenon of mathematics teaching is informed by beliefs and identifications with a variety of alternative motivations. An individual teacher may hold views on their own preferred way of teaching, inevitably influenced by the more mechanical processes that govern our lives. Each school and university has an agenda. Student teachers make choices, which are ambiguous and complex. Where the subject (student teacher) is understood relationally in terms of how he or she identifies with aspects of becoming a 'secondary school mathematics teacher', such that it becomes unclear where personal beliefs end and the imaginary demands of the world begin. What discourses have influenced these teachers' beliefs? For example, curriculum policies in England promote both *mastery*[1] as a pedagogical approach and as an objective for pupil learning (Morgan, 2017). However, *mastery* is a slippery term that has led to a range of conceptualisations (NAMA, 2015). More generally, we build images of how teachers should conduct themselves and develop particular understandings of normality (Brown and McNamara, 2011).

To explore such ideas, I was aware of the need for a theoretical framework that would allow me to give attention to the complexities of discursive encounters. Psychoanalysis theory provides an approach to disrupting the researcher's habitual thinking patterns within regulative scenarios and opening alternative discursive avenues. This approach is made possible through the ways in which Lacanian psychoanalysis, with its emphasis on language and identity, understands the 'individual as both a conscious, rational subject and as an unconscious subject whose desires and fantasies form a significant aspect of being' (Brown, Atkinson, and England, 2006:11). Here, student mathematics teachers are acting according to a fantasy of who they think they are or who they think they should be. Ideas of the individual being understood relationally, whose speech reveals a position in

[1] Throughout the study *mastery* in terms of the *mastery* curriculum or *mastery* teaching is in italics, to avoid confusion with the term in master discourse or master –slave dialectic

response to the perceived demands placed on them are applied to the processes of becoming a mathematics teacher. I am interested in how student teachers, through these fantasies, construct knowledge of teaching and learning of mathematics. Given that the *mastery curriculum* is a framework for the learning and teaching of mathematics, influenced by what is often termed the neo-liberal agenda for economic social mobility, this would provide me with an opportunity to research something complex and current.

This study seeks to provide insights into ways in which the idea of a *mastery curriculum* shapes ideological understanding[2]. Here, ideologies are seen as having varied shelf lives and relevance, providing specific conceptions of teaching and learning of mathematics. It considers at a macro-level how society influences and controls notions of the *mastery curriculum*. The research tracks a group of secondary mathematics pre-service teachers. It shows how they grapple with the demands of their specific course, set against regulative policy, that define teaching in England today. It then investigates these influences in the development of student teacher identity and day-to-day teaching of mathematics.

1.2 Aims of the study

There are three aspects of the research, firstly **to draw on psychoanalytical theory in providing an account of how student teachers construct their professional identities through the training process and in response to the *mastery curriculum***. Secondly, **to create a theoretical framework towards capturing the habitual thinking patterns that underpin the multifaceted discursive dimensions of teaching/learning encounters, towards disrupting them with view to opening more generative interpretations.** Finally, I realised that I needed to **disturb my own habitual thinking by looking at my own perspectives on teaching education courses, examining both teacher educators' and student teachers' conceptions of their mutual encounters and how these encounters progressively produce conceptions of self with respect to curriculum demands**.

[2] Ideology here is referenced to Althusser's (1971) institutional ideological states apparatuses (ISA). Individuals are called into being through prescribed registers and discourses.

The study is rooted in theory and practice. Most prominently, it is rooted in the everyday interactions between student teachers and pupils[3] as they address mathematics together. Practice extends to the interaction between university tutors, subject mentors, student teachers and pupils. The data collection spanned 20 months (see 1.7.1 for selection of participants). The stories of seven student teachers enrolled on a teacher-training course at Manchester Metropolitan University are considered. The data was drawn from the following: interviews with students, field notes made from classroom observations, lesson plans were considered, pre and post lesson discussions were conducted and reflective writing analysed.

The research unpicks and analyses how the discourse of the *mastery curriculum* unfolds within the multitude of alternative demands. However, the term *mastery* is a nuanced and complex. For example, *mastery* is used in relation to teaching, assessment, and curriculum (NAMA, 2015). This study does not seek to define *mastery* teaching or define what a *mastery curriculum* is and neither does it offer suggestions on how to improve mathematics teaching. It seeks to understand how the idea of *mastery* teaching is used to create a set of demands that shape student teachers understanding of becoming a mathematics teacher. Here *mastery* teaching is seen as an ideology or movement that sets the parameters of becoming a particular type of mathematics teacher. In this way, the study problematises student teacher development and unsettles some of the presumptions of how individuals interact with discourse.

1.2.1 Personal motivations

To explain further it is worth summarising my own development as a mathematics teacher and subsequently as a teacher educator. As such, I consider some of the discourses that have shaped my professional identity, this partly rationalises why and how the above aims materialised[4].

In the late nineties, I began my teaching career in England. This coincided with the beginning of New Labour's (1997-2010) transformation of education, which consisted of

[3] The term 'pupil' is used to denote students of a secondary school age (11-16) and the term 'student' refers to trainee teacher.

[4] Chapter 6 considers in more detail my story of becoming a teacher and subsequently a teacher educator.

frequent introductions of new initiatives to 'improve' the quality of teaching and learning. At the heart of this movement was the ambitious programme known as 'The National Strategies' (DfE 2011), which prescribed national curriculums in England for both students and teachers. Without realising it at the time, I was conforming to being a particular type of teacher, the National Strategies providing points of reference to my imaginings of the socially defined role of a teacher. On reflection, I can see how I complied with its demands placed on me. For example, ensuring each mathematics lesson incorporated a 'starter' and 'plenary'[5]. As such, I was called into being a particular teacher. My language was that of the other, the discourse of the National Numeracy Strategy. This resonances to Althusser's (1971) idea of interpellation, called into being a particular teacher. It naturalised the way I understood the role of a mathematics teacher. Even though I was aware of the limitations of some of these points of reference, I still carried on using them. To be understood and to understand, to be accepted, to fit in. I was working within the system, reproducing the system.

In 2010, the opportunity arose for me to work at Manchester Metropolitan University as a teacher educator. Reflecting on my earlier years as a teacher, I became increasingly self-aware of the previous demands that were placed on me (only to be replaced by a new set of demands, which are discussed in chapter six). I came to the realisation that my understanding of being a mathematics teacher is constant flux. That is, representations of mathematics teaching are effects of discourses produced in a particular time and place (Foucault, 1972).

One of my initial challenges as a teacher educator was striking a balance between the theory and the practice. I was often 'surprised' and 'disheartened' to see student teachers previously noted for their innovation and creativity relapsing into transmissive teaching. This resonates with Tabachnick and Zeichner's (1981) findings that many notions and concepts developed during the teacher education programme were 'washed out during field experience'. In the early part of my doctoral studies, I investigated the relationship between theory and practice. In particular how a Finnish inspired 'University Schools' (Haniak-Cockerham, 2019) model of teacher training could bridge the gap between theory

[5] In 1997, the National Numeracy Pilot began and became the National Numeracy Strategy (NNS) in 1999. The strategy introduced amongst many other strategies, a three-part lesson structure (DfE 2011)

and practice. However, it was quickly becoming apparent that policy at a macro and micro level played an important part of becoming a teacher; of what is allowed and what is not allowed. That is, pedagogical strategies are a function of national and local administrative constraints and curriculum guidance that organise teaching. At a micro level, student teachers are obliged quickly to conform to the routines and procedures of life in a school.

With the introduction of the *mastery* movement in 2016, I recognised similarities to the discourse of National Numeracy Strategy. That is, the *mastery curriculum* calls teachers into being a particular type of teacher in a similar manner that the National Numeracy Strategy privileged certain pedagogical structures over others. Student teachers placed in schools incorporating aspects of the *mastery curriculum* are training to become mathematics teachers within the context of the *mastery curriculum*. That is, student teachers enter into relationships with not only policy discourse but they also interact with subject mentors, pupils and other teacher. In this way, secondary mathematics teachers are dynamic entities responsive to social-cultural conditions. What this means is that the understanding of becoming a mathematics teacher is relative to the prevalent environmental conditions. Specifically, pedagogical strategies are a function of policy constraints always determined in time and space.

Mastery sold as improving standards is another way of normalising practice. That is, the ideology of *mastery* teaching becomes the master of us all, and we are obliged to suspend our critical faculties and comply with its discourse (Williams, 2019). Within the neo-liberal climate of competition, it is as important as ever to consider implications of such discourse. That is, critically examine and interrogate how discourse unfolds within the assortment of alternative demands of becoming a teacher.

1.3 Thesis Outline

This study takes the premise that the motives underlying teacher practice in schools are channelled by identification with a range of social discourses, such as; personal aspirations as to what it is to be a teacher, expectations of one's students, the need to comply with new policy directives, to achieve 'outstanding' status within inspectorial assessments, etc. The study is conducted from a teacher educator perspective as assumed by the author, where data collection is centred on discovering how pre-service teachers identify with the

multifaceted discourses that shape their practice, with particular reference to what has come to be known as the *mastery curriculum* in mathematics. Specifically, it explores the discursive construction of the *mastery curriculum* using Lacan's notion of the master signifier, where this analytical tool provides an approach to disrupting habitual thinking patterns within regulative scenarios and opening alternative discursive avenues.

1.3.1 Part One: Introduction

Part one problematises student teacher identification with various discourses. It considers how the representation of school mathematics is shaped by the discourses that have prevailed in particular times and places. As such, it sets out the ever-changing landscape, which contextualises the research and its aims. Part one offers examples of teaching episodes, demonstrating how learning relates to specific cultural and curriculum parameters. Methodological arguments are considered, before I reflect on some of the literature surrounding mathematics education and the discourses that shape mathematics, specifically the *mastery curriculum*. Part one introduces some of the ideas that characterise Jacques Lacan's work to be used as analytical tools within this thesis.

Chapter two outlines some of the literature and theoretical perspectives that underpin the analysis of data in chapters four and five. It provides an account of mathematics education reform in England in the last forty years. Successive governments have reformed educational policies with a view to improving standards. I consider how in response to the competitive neoliberal ethos, *teaching for mastery* has emerged as a movement. The idea of *mastery* provides a framework for mathematics teaching and learning, which functions as an ideology that can provide a point of reference or identification for teachers giving a sense of collective purpose. In the background to contemporary discussions of *mastery* are concerns about the degree to which we understand individual humans as part of a collective.

In the second part of chapter two, my attention turns to some ideas of cultural-historical theory of the teaching and learning of mathematics. Curriculum reform is accompanied by different sets of expectations about how mathematics education is understood. That is, mathematical meaning and classroom mathematical practices are socially constructed norms; becoming a mathematics teacher occurs through participating in social activity. I

commence with , a discussion on how early radical constructivist perspectives, linked to the individualistic developmental psychology of Piaget, display incompatibilities with later social constructivist views referenced to Vygotsky. This discussion is centred on notions of subjectivity that provide alternative filters to traditional concepts rooted in more individualist psychology. The discussion then moves on to the roles of signs in mathematical understanding.

Chapter three introduces psychoanalytical theory as a theoretical framework for understanding the movement of discursive encounters. I introduce some of the ideas that characterise Jacques Lacan's work to be used as analytical tools. Two particular themes are considered. Firstly, I outline Lacan's mirror stage theory and the construction of the human subject as an approach to exploring how teachers conceptualise their actions. I consider how generally human beings build an understanding of themselves and specifically how student teacher identities are constructed according to a fantasy of what it is to be a mathematics teacher. Attention is given to Lacan's theory of the subject and its notions of desire. Secondly, I discuss Althusser's concept of 'interpellation'. The particular focus is on how individuals are called into being through prescribed registers and discourse, specifically the demands placed on individual teachers in response to the ideology of the *mastery curriculum*. The chapter concludes in how we can use Lacanian theory to unpick Althusser's (1971, 2014) theory of ideological state apparatuses.

1.3.2 Part Two: Analysis of data, student teacher conceptions of 'teaching for mastery'

Structural and pedagogical discourses shape school mathematics to facilitate learning in particular ways. Part two focuses on how student teachers negotiate these various discourses and in particular how they conceptualise the *mastery curriculum*, but also how the *mastery curriculum* conceptualises them. That is, psychoanalytical theory, suggests that 'it is not discourse that contains the subject but the subject that, in some sense contains discourse' (Alcorn, 1994:20). In problematising my understanding of aspects of discursive accounts, psychoanalytical theory opens up my analysis beyond the one-sided power dynamics into something more expansive and complex. Where 'discourse here is something belonging to, worked upon, or contained by the subject' (ibid:20). Data relating

to particular moments are intended to provide points of reference, to help capture how the student teachers identify with the various discourse they encounter.

Chapter four considers the various demands placed on student teachers: From the needs of pupils, meeting the teaching standards, navigating curriculum policy, to multiple social demands. Student teachers need to conform to the demands of becoming a teacher. Their progress as student teachers is oriented against conceptions of what it is to be teacher. Here, student mathematics teachers are acting according to a fantasy of who they think they are or who they think they should be. I am interested in how student teachers, through these fantasies, construct knowledge of teaching and learning of mathematics. Discussion is offered in connection to Lacan's four discourses; that of master (governance), university (education), hysteric (protesting), and analyst (renewal). Lacanian discourse analysis offers unique possibilities for understanding how a given discourse affects the subjects that receive or produce it. In particular, it provides insights into the formulations between knowledge, master signifier, divided subject and otherness[6]. In repeatedly mapping out classroom interactions to different permutations of discourse, we generate different possible understandings. With examples of classroom practice, I investigate how policy documents, such as the *mastery curriculum* and teaching standards shape the development of student teacher practice.

Chapter five investigates some effects of the metaphors of teaching and learning associated with a Vygotskian perspective on the *mastery curriculum*. I discuss how both learning to become a mathematics teacher and the emergence of mathematical meaning itself occurs through participating in social activity. Here, individuals are defined by the relations to other people and social-cultural artefacts rather than being individual entities. As an outcome of the interactions between student teachers -pupils, subject mentors and policy discourse- individuals come to occupy positions in the social world. Lacan's notion of the human subject provides the analytical filter for how individuals interact with their

[6] Otherness (*Object a* in Lacanian terminology) refers the object of desire, that which is supplementary to the subject and as such fuels our fantasies and desires to make up for the feeling of incompleteness (Lacan, 2007a).

social environment. Lacan's psychoanalytical position summarises the individual's understanding of who she is, in their response to the symbolic network (Brown, 2011). This symbolic network, society's unwritten rules, directs and controls our acts (Žižek, 2006). Participating in the activity of teaching, student teachers are evolving through a process of subjectification (Roth, 2012a). That is, student teachers use language to fit in and to be understood, in what Lacan (2000) calls the symbolic order. In this way the 'subject comes into being' (Pais, 2015:378).

I explore data collected from student teachers' engagement with *mastery curriculum* practices to provide some exemplification for these theories. Drawing upon Lacanian discourse analysis I explore different ways of thinking about social interactions in preference to theories of individual cognition. Student teacher pedagogical development and mathematical understanding are depicted as mediated experiences between *mastery* policies, the use of textbooks, pedagogical practice, teacher beliefs, tools and materials. This co-creation is explored through Lacanian discourse analysis, with a discussion about how subjects construct their identities through the training process and in response to the *mastery curriculum*.

1.3.3 Part three: Discussion and implications

I am conscious that my own personal perspectives have evolved during my career as a teacher, teacher educator and in the writing of this study. In part three, which comprises chapter six, I conclude by giving an account of my story of becoming a teacher, teacher educator and now researcher, and how these multiple persona have evolved during the research process. I reflect on my influences and considers the various demands that pull me in different directions. In the second part of chapter six, I revisit and summarise how student teachers construct their identities through the training process and in response to the discourse of the *mastery curriculum*.

1.4 Background to the research

Mathematics education is not ideologically neutral; it is influenced by culture, history, society and politics (e.g., Ernest, Sriraman and Ernest, 2016, Pais, 2015). Historically, it has been dominated by discourses around the development of better stratagems to teach and

learn mathematics according to the understandings, motivations and fashions of the day. Successive governments bring about curriculum reform to modify teacher practice with a view to raising standards in line with the given regime. In 2016, Schools Minister Nick Gibb described recent changes in mathematical learning as a *renaissance.*

> *"We are seeing a renaissance in maths teaching in this country, with good ideas from around the world helping to enliven our classrooms."*

'*Renaissance'* in the press release delivered by Nick Gibb is fabricated around a notion of progress and enlightenment; '*with good ideas around the world helping to enliven our classroom*'. Within this, teaching in England evokes a necessary relationship to good ideas from around the world. The emancipatory narrative being difficult to resist, shapes the future of school mathematics. The idea of successful mathematics teaching is seemingly being sold as a means to drive up standards. The words of Nick Gibb resonate with Althusser's (1971, 2014) concept of 'interpellation[7]', where the teacher recognises himself or herself in some supposed ideological calling.

International testing has broadened the so-called neoliberal agenda. Whilst Ball (2012) warns that the term neoliberalism is used 'widely and so loosely that it is in danger of becoming meaningless' (2012:3). I think it is relevant as a term as it sees competition as the spur to life and its development, leading to comparison between countries, providing a metric of how competent pupils are in areas such as mathematics, literacy and science.

Assessments of pupils in Programme for International Student Assessment (PISA[8]) function as barometers of socioeconomic class. After the round of rankings, published in 2013, there were warnings from ministers in England that results were "stagnating" - and reforms were promised to match international rivals (BBC, 2016). In 2016 England ranked 27th in the PISA test down from ninth, trailing behind Singapore and China. Interestingly,

[7] Interpellation is a term I return to in chapter 3.5. It refers to how ideology constructs (interpellates) the subject (Althusser, 1971, 2014)

[8] PISA is a worldwide study by the Organisation for Economic Co-operation and Development (OECD) in member and non-member nations of 15-year-old school pupils' achievement in reading, Maths and Science. Beginning in 2000 and repeated every three years, it allows countries to measure their performance in an international context.

England has succeeded in moving from 18th to 10th position in the TIMSS ranking in 2015 (Mullis, Martin and Hooper, 2016). Does this reflect a choice between opposing ideologies, with England sacrificing its earlier proficiency with problem-solving approaches in order to meet newly understood TIMSS objectives (Brown, Hodson, and Smith, 2013)? One way to increase performance is to replicate curricula from high-performing international educational jurisdictions. PISA even claims that low performance in its tests has an economic impact on countries (OECD, 2010). It is no surprise then that, as in many countries, recent educational reform in England has been informed by evidence in high-performing international educational jurisdictions, in particular Shanghai. In 2016, the Department for Education in England made a press release, 'South Asian method of teaching maths to be rolled out in schools' (DfE, 2016). Gibb (2016) stating

> *"The significant expansion of the south Asian maths mastery approach can only add to the positive momentum, with thousands more young people having access to specialist teachers and quality textbooks." Schools Minister Nick Gibb (12 July 2016)*

By outlining the method of teaching 'to be rolled out in schools', the task of secondary mathematics 'could be understood not in terms of the inherent properties of mathematics, but in terms of the role this school subject plays within political economy' (Pais, 2015:378). There is a risk that in using international comparison tools that mathematics education becomes subservient to those demands, and not necessarily meeting local specific needs. As such, *mastery* policy is influenced by high performing educational jurisdictions in the hope that students will gain a higher level of performance in international comparison testing, such as PISA. The 'South Asian maths mastery approach' functions as an ideology that can provide a point of reference or identification for teachers giving a sense of collective purpose, and with a toolbox of resources (Brown, 2011). That is, the vocabulary and language of the *mastery curriculum* can provide the orientation through which one recognises themselves as mathematics teachers. In a similar way, the teaching standards function as an ideology that can provide a point of reference for being an outstanding or good teacher and so on. Learning or teaching effectively in terms of the *mastery curriculum* only demonstrates subscription to that ideology. It only determines successful mathematics teaching if one buys into that ideology.

These messages provide a backdrop to individual beginning teachers making sense of their professional identity as mathematics teachers. The influential National Centre for Excellence in the Teaching of Mathematics (NCETM, 2015, 2016) and Maths Hubs (2020) have encapsulated the word '*mastery*' in relation to mathematics teaching and learning based on its observations in Shanghai. Some schools have integrated Asian *mastery* teaching within their curriculum, with the expectation that all pupils will learn and the role of the teacher is to design lessons and use practices that ensure that will happen (Boylan, Wolstenholme, Maxwell, Jay, Stevens and Demack, 2016). The NCETM (2016) describes *mastery* of mathematics as a 'deep, long-term, secure and adaptable understanding of the subject'. Amongst others, there is an organisation, Mathematics Mastery (2020), linked to the Ark Academy chain of schools in England that has a similar ideological position. Mathematics Mastery Director, Ian Davies (2015) encourages 'intelligent practice' to enable pupils to develop conceptual understanding. This approach is similar to the one characterised by the NCETM, *mastery* teaching is, 'underpinned by methodical curriculum design and supported by carefully crafted lessons' and 'practice and consolidation play a central role' (NCETM, 2016), but this carefully prescribed method of teaching could deny pupils agency. It is the teacher who defines the pace and challenge of the lesson. Such passivity, in which students have no responsibility for their own learning, is the opposite of a growth mind set (Blair, 2015). What is clear is that there have been numerous *mastery* approaches in the last 40 years and they elude a single definition (NAMA, 2015, Pawlik, 2016).

Within the shifting landscape of education, teachers are still adjusting to the demands of the *mastery curriculum* and its discursive framing. That is, teachers come to occupy positions in the social world. Through a multitude of filters, individuals recognise themselves as teachers, responding to what they think is expected of them. Radford refers to a 'unique individual who, through her engagement in social activities, continuously positions herself through other individuals in the cultural-historical world as an unrepeatable entity always in flux' (2018:22). Even though each individual is unique, they still play to the same rules, common cultural expectations, and the social organisation of how teachers should conduct themselves. Despite the emancipatory narrative of the *mastery curriculum*, the reality of teaching is often a far cry from the fantasy of a rational

and idealised teacher. For example, as student teachers transfer their attention from perhaps learning about finding the gradient of a curve, to confronting real thirteen-year olds with an anxiety towards adding fractions. This research seeks to provide insights into how teachers construct their identities through the training process and in response to the *mastery curriculum*. I consider classrooms experiences and student teacher reflections to provide an account to how student teachers develop an evolving sense of self, but a self that is forever responding to the perceived demands of the training process. My interest is to go beyond traditional interpretations of teaching and learning as the reproduction of endorsed procedures and examine some of the linguistic and cultural filters that shape secondary school mathematics.

1.5 Setting the scene: A students teachers early experiences

To situate my research in some examples of practice, I now offer an account of the experiences of Daniel, a secondary mathematics student teacher, currently enrolled in year two of a three–year degree. In the extract below, Daniel has recently commenced his first main teaching practice. I begin by looking at a piece of reflective writing that he has produced three weeks into his placement, later I discuss an observed lesson. I seek to identify some of the social-cultural factors that influence secondary school mathematics pedagogical approaches. It begins to illustrate how conceptions of mathematics and the *mastery curriculum* manifest in school practice.

1.5.1 A student teacher reflection

Daniel reflects on how he is making sense of his own teaching. Through these reflections, we can begin to build a picture of how he understands himself, the factors guiding his actions, his motivations and fears.

> *'Through my own personal experiences and observations, it is apparent that questioning- when used effectively- can be a largely influential aspect of any teaching episode. This is through observing lessons with multiple instances of questioning being used to a certain degree of effectiveness; causing noticeable positive change in direction of the thought process in the minds of the students. An example of this is when I oversaw a teacher question the students:*

"What about one? Is one a prime number?" and proceeded to choose students at random to give their answers/opinions until the correct answer was found. By asking this question the students developed a definition for a prime number in their own words. This is, by Mason's (2010) definition of such, an example of a close-fronted, but also a close-ended, question as there is a specified example, and there is only one correct answer. However, I believe this to be an effective question during this time in the lesson as, in my opinion, this definition is more likely to remain in the student's schema as it is not something they are just told to remember but are opposingly (sic) creating themselves and the thought process behind this should help retain the information. However, this question could cause fatigue in students who already know the answer to the question (meaning they lack the misconception being dealt with) which, although it is preferred for them to not have misconceptions, could actually decrease their 'thirst for knowledge' as they are going over something they are already familiar with.'

Extract from a student teachers reflection

Early in his teaching practice, Daniel observes his subject mentor asking pupils questions such as, "What about one? Is one a prime number?" Daniel is seemingly attentive to the way in which this activates a response in the pupils where the precise definition of a prime number comes into question and orients the pedagogical encounter. In this way, the questions asked provide a pedagogical discourse through which pupils consider their own schema as the encounter proceeds. These experiences provide a framework for how Daniel might conceptualise teaching. Yet no interpretation is final. Such understanding is always in a state of flux, as his understanding will be conditioned by future experiences. Later in his reflection, he seems to rethink how effective this type of question really is. What if the pupils already know the answer? He is concerned that pupils might lose interest. This reflection centres on this dilemma and his attempts to resolve it. Daniel's ideas are not fully established but are continuously evolving. In an emerging understanding of what he needs to achieve as a teacher, he is keen to keep the encounter open, to allow space for the pupils to introduce their own sense making. In writing his reflections, Daniel is putting down markers, reference points, highlighting conceptions of his professional role in relation to teaching and learning according to his particular priorities and perceived expectations. That is, Daniel finds himself being pulled in many directions as he recalls alternative advice that he has received from the various people

involved in his training course whilst at the same time trying to build his own ways of making sense of situations and responding in positive ways. Here formulations of the individual are seen as being a consequence of social structures, shaped through a multitude of discourses.

1.5.2 A teaching episode

I now offer a description of a lesson. Daniel is on his first major teaching placement where a *mastery* style of teaching is incorporated by the mathematics department. He is now five weeks into his placement and has been teaching this particular class for two weeks. In a previous lesson, students became familiar with the image of *linear* and *non-linear* graphs. They labelled two graphs, one that was *linear* and one that was *non-linear*. In this lesson, students are completing tables of values and then plotting points to draw graphs. The lesson is 'delivered' to a higher attaining group of pupils aged 11 and 12. The lesson is divided into eight sections. For the purpose of my discussions, I focus on the general organisation of the classroom, the start of the lesson and then I discuss stages three and four, which '*refreshes students' knowledge on coordinates'* and '*students applying their knowledge of sequences to complete tables of values*'. In preparation for this lesson, the subject mentor offered some advice:

> Subject Mentor: You need to encourage them to use mathematical language, linear and non-linear, and that is following the mastery approach.

At the outset of teaching this lesson, Daniel had a sense of what might be recognised as a teacher following a *mastery curriculum*. Using mathematical language, such as *linear* and *non-linear*, is providing a reference point as to what is expected from him and from the students. His identifications with becoming a mathematics teacher are being influenced by the expectations of his subject mentor. Accordingly, the three lesson objectives are centred on the language of graphs:

> *To be able to identify straight line and curved graphs from the equation of the graph*
>
> *To be able to identify similarities and differences between graphs*
>
> *To be able to accurately use correct terminology to describe similarities and differences between graphs.*

The learning objectives serve to enact the curriculum content. The actions of the teacher and students are recognised through these prescribed filters. That is, the statements define the mathematics; create boundaries of what is included and excluded in the lesson. In his attempts to connect with the *mastery curriculum*, key mathematical language commands a central role in the learning objectives and the structuring of his lesson. Throughout the lesson, pupils are encouraged to talk about the mathematics unless Daniel, the student teacher, indicates 'independent work', with the intention that students work in silence. The commonly used metaphor *'*delivered*',* presupposes many assumptions about how teaching and learning are understood, and especially of how teachers should behave, for example, that of knowledge being transmitted to passive learners (Ellis, Fox and Street, 2007). The classroom itself is organised into a particular culturally defined structure. Thirty pupils sit at tables arranged in pairs.

On the interactive whiteboard there is a title '5 a day' and under the title there are five 'starter questions'. The daily rituals and practices are defined to depict how pupils act and what they say. The pupils enter the classroom, sitting down quietly, taking out their exercise books, copying the five starter questions and begin to write their answers. Daniel is being socialised into acting a particular way by school routines and structures, and is inducted into the local cultural practices of his school, in a similar manner the pupils are also socialised into particular patterns of behaviour. In this sense, the conceptions of mathematics are shaped by norms of classroom practice rather than by abstract nature of the mathematics itself. The label '5 a day', has connotations of being good for you with national campaigns in the UK to encourage consumption of at least five portions of fruit and vegetables each day. The starter questions illustrate an approach that serves as a pedagogical form, possibly predicated on the *mastery* principle of developing fluency and regularly assessing pupils (NCETM, 2014). In this way, Daniel who is following departmental policy is successfully participating in a socially constructed schema designed to improve the quality of mathematics teaching. However, another way at looking at the starter activity is the suppression and socialisation of pupils (Biesta, 2014) to behave in a particular way. Either way the collective schema provides teachers with a framework against which they can orient themselves.

After the starter activity, the students engage with stage two, in which they make corrections to the previous lesson's classroom assessment booklet (CAB).

During stage three of the lesson, Daniel uses the interactive whiteboard to project a coordinate grid, with the coordinate (3, 2) highlighted. The Cartesian coordinates serve to depict the location of a point relative to a fixed reference point. The process of pointing, highlighting brings to attention the object, the sign, a social process of producing meaning. This knowledge about Cartesian coordinates offers students to understand mathematics in a specific way.

Daniel: I want you very quickly in pairs to talk about the coordinate on the graph.

Pupils collectively discussing and confirming the coordinate is (3, 2). Daniel moves around the room, observing and listening to the interactions of the pupils.

Daniel: The reason why I chose this coordinate is because some of you are getting them the wrong way around. I heard someone say a nice phrase.

Daniel: Steven, what did you say?

Pupil 'Steven': Along the corridor and up the stairs.

Daniel: Very good.

Daniel: Ok, I want you to copy this table into your books and fill in the missing values. I want you to work in pairs to complete the table.

$y = x^2$

x	*0*	*1*	*2*	*3*	*4*
y					

After two minutes, Daniel asks the pupils to stop working.

Daniel: Pens down, look this way. Ciara what are the answers in order.

Pupil 'Ciara': I times zero by zero and that is equal to zero.

Daniel: I just want the answers.

Pupil 'Ciara': Zero, one, four, sixteen.

Daniel: Very good, did everyone else get that.

Pupil 'Maria': No, I got zero, two, four, I times them by two.

Daniel: Yes, you need to be careful.

The coordinate grid brings the mathematical object into existence. There are different ways of understanding teachers and their subjective position that locates them. In this stage of the lesson, Daniel initiates discussion between pupils to agree on the positioning of the coordinate. That is, discussion is favoured as the mathematical activity to promote thinking. Although Maria's current interpretation does not yet resonate with the expected mathematical expectation, her understanding of x^2 is being shaped through social activities, continuously being positioned through other individuals in the cultural-historical world. Here understanding and meaning is developed through forms of collective learning. The teacher's response, 'Yes, you need to be careful', is reinforcing the meaning of the sign x^2. As we saw, the logic of interpreting a Cartesian representation of the location of a point and subsequently understanding the function of x^2 became progressively apparent through the social interactions of peers and the teacher intervention. Thus, the teacher was able to create in Vygotskian terms a Zone of Proximal Development (Vygotsky, 1978), something I come back to in chapter four. That is, through the social interaction between peers and the teacher, pupil understanding became more and more refined.

If we look in more detail at the passage above, we can see that Ciara offers a detailed explanation to how she calculated her answers, 'I times zero by zero and that is equal to zero'. Ciara appears to be 'performing' *mastery* expectations of answering in full sentences but it could also be indicative of the discourse of general mathematics teaching pedagogy. This resonates with Vygotsky's (1978) work and the role of social-cultural activity in the development of the individual. By participating in the activity of schooling, certain practices are normalised (Foucault, 1972). However, Daniel responds by saying, 'I just want the answers'. This is in contradiction to what Daniel was describing in his earlier reflection where '*thought process(es)... should help retain the information'*. Here the reality of teaching is in stark contrast from the rational and idealised teacher as they might be described in the teaching standards. Perhaps Daniel feels like he is under pressure to keep to his lesson timings and ensure the content of the lesson is *delivered*. For instance, comments like 'very quickly in pairs' or having a stopwatch on the board is indicative of certain perceived demands that are placed on him. In this case, it could be the fear of not progressing through the lesson content, or the fear that he alluded to in his earlier

reflection- *could cause fatigue in students.* He could be anxious that pupils could become disinterested.

In this lesson, pupils talking about the mathematics is a particular pedagogical form that has become a marker to represent the teaching of mathematics. However, Daniel's subject mentor feedback adds another layer of discourse that structures his conception of teaching.

> Subject mentor: Mastery is about spending time talking about the maths. I liked the way you got them to talk to their partner. At times there was a little bit of off-task behaviour. You just need to iron out any talking between tasks. For example, when you said 'does everyone agree?' you might want to choose a pupil, for example, do you agree, why? Don't forget that mastery is all about the discussion. So ask more why did you do that?

In my observation and post lesson discussions with Daniel and the subject mentor, 'questioning', 'mathematical vocabulary' and 'discussion' emerged as key words or concepts in Daniel's alignment with the *mastery curriculum*. The subject mentor's actions and language are beginning to reveal the demands to which he is responding. Are his activities based on rationality and beliefs or are his actions referenced to the performance of particular pedagogical structures that have come to represent school mathematics resultant to the demands of the *mastery curriculum*. This study seeks to consider alternative interpretations of how mathematics teaching is conceptualised.

1.5.3 Discussion

These brief examples illustrate how Daniel, a student teacher, negotiates his position at the intersections of multifaceted discourses that influence the choices that he has to make. In accepting a particular view on teaching, or in taking sides on a particular issue, it could be said that teachers are accepting a particular theoretical position. Pedagogical strategies are based on our epistemological outlook of how learning takes places. Teachers' theories or viewpoints are often based on experience, intuition, a multitude of discourses and perhaps fantasies. The examples above show how interactions with the university, subject mentor, pupils and Daniel's own reflections provide triggers that challenge or develop conceptualisations of teaching to new levels. For example, the

process of writing his reflection requires Daniel to make explicit his ideas that may remain implicit at the level of speech or classroom interaction.

The reality of classroom practice challenges Daniel's conceptualisations of 'asking pupils questions'. For example, he wants to incorporate discussion but also feels he needs to maintain the pace of the lesson. He is negotiating a path through the perceived demands of teaching a *mastery curriculum* but also meeting alternative demands that he thinks are placed on him (e.g., managing behaviour, maintaining pace of the lesson, progressing through the curriculum, etc.). This perhaps echoes Roth and Radford's (2011) notion that through a process of reflexivity individuals are continuously reconstructing their self-identity. In conforming to regulative structures, Daniel is becoming a mathematics teacher within the context of *mastery* teaching or the *mastery curriculum*. That is, the idea of *mastery* teaching is used to create a set of imaginary demands that shape Daniel's understanding of becoming a mathematics teacher. In complying with departmental policy, there is a risk of teaching becoming a technical job, where policy marginalises judgement and creativity.

Ideologies influence our behaviour (e.g., Eagleton, 2007, Ernest, 1991, 1998). However, what is not clear cut is how ideologies are formed and what influences and discourses shape our beliefs. Brown (2016) suggests that our beliefs are often more politically embedded than often depicted in mathematics education. Classroom behaviours and the choices we make are often based on a mode of activity referenced to a fantasy of what it is to become a teacher, fitting in with what we think is required of us. Thus, professional identity is developed through the discursive practices that regulate what is said and written (Cherryholmes, 1988). Navigating a series of demands placed by competing discourses, we ascribe to becoming a teacher. However, with so many diverse demands there is never any saying it all, just a series of interpretations and misinterpretations through interactions with our environment. As such, a social environment exists where teachers grapple between what they think is expected of them and their own personal liberty and aspirations to becoming a teacher. From a psychoanalytic perspective, pleasures may be derived through playing these expectations and aspirations off against each other. Of particular interest to this study is to how individuals experience these social

structures, such as the *mastery curriculum*, how they position themselves and respond to the discourse, as individuals participating in collective activity[9].

1.6 Why do this research?

The teaching of mathematics is often influenced by the dominant procedures prevalent in certain places and particular times. For example, the Cockcroft report (1982), the introduction of the National Curriculum (1988), the National Numeracy Strategy (1999), the Maths Hubs (2014), the Program for international Student Assessment (PISA), reform of national testing and more recently a Mastery Curriculum. These ideologies shape teachers' understanding of how they talk about their work, and they influence the way in which pupils achieve recognition and are thus cultivated or discarded. A central focus of this thesis will be upon the regulative discourses, such as the *mastery curriculum* or the teaching standards and how such discursive constructions are instrumental in forming teacher identities and subjectivities. However, within these regulative discourses, student teachers still have a voice of their own, through which they can resist some of the structured frameworks and express their own personal aspirations to become a teacher on their own terms. That is, they can assert their own subjectivity.

The demands of educational priorities such as league tables and Ofsted grading, pull teachers and mathematics departments in certain directions, specifically the need to prepare pupils for summative assessment, for example, GCSE qualifications, a framework for defining mathematics in particular way at the age of 16. At what cost? School mathematics is held in place by the regulations of examinations and demands of accountability. However, educational policy is not exact science. A diverse number of agencies regulated by a range of alternative and at times conflicting agendas with varying degrees of influence shape teachers actions and ideologies. With government policy, school mathematics and university ideology sharing the same space, these often conflicting agencies shape the identifications of beginning teachers. In building conceptions of what it is to be a teacher, decisions and choices are channelled by identification with particular discourses. We might imagine that we should teach in mixed

[9] I pursue ideas of collective activity in chapter five. Influenced by Leont'ev and Vygotsky. Roth and Radford (2011) describe subjects as subjects of collective activity.

attainment groups or maybe we should set according to attainment in our mathematics classrooms but these are just illusions of an imagined reality. Our reality is symbolised by previous attempts to make sense of the world, to represent it, for example, symbols, books, language and so on. It is through these filters that we experience reality. The *mastery curriculum* is a piece of the multitude of filters that process what it means to be a teacher.

The discourse of the *mastery curriculum* might offer the seduction and fantasy of mixed attainment classes, whereas assessment in mathematics offers a conflicting discourse, where students sit exams based on their 'perceived' ability in a set of assessment instruments being applied. Do we teach mathematics through enquiry or direct instruction? These are all real issues that teachers have to make choices. Within such a changing and contradictory landscape, I want to investigate how teachers grapple with the fantasies and desires. The predicament comes from the contrast between demands of day-to-day teaching and meeting the targets of data predictions and developing pupils that really understand what they are doing in a meaningful way. In the very serious business of education what are the drivers that inform their choices. What impact does society have on trainee teachers' professional identity? Yet within this changing landscape, mathematics teaching and learning still offer surprises. Individual teachers are on an endless quest to form an imaginary picture of themselves in relation to the world around them and the others who inhabit it. Rather than a biologically self-contained identity, that is waiting to be discovered and that can be objectively described, identity depends on the student teachers relations with others and is governed by fantasy, and modes of identification (e.g., Grosz, 1990). This study problematises the conception of the subject and how individuals orientate themselves through cultural and historical filters. By doing this research, I offer an alternative story to the development of identity and practices of teaching and learning.

1.7 An experiential study: ethical considerations

The study follows the development of a group of pre-service teachers; I examined how the student teachers were initiated into the notion of *mastery* at successive stages of their

training. The study was carried out at Manchester Metropolitan University and in various inner-city schools in the Northwest of England.

Student teachers were chosen on the premise that their placement school was incorporating aspects of the *mastery curriculum*. Lessons were observed, structured and semi-structured interviews were conducted (appendix 4), lesson plans analysed (see appendix 6 for an example of a lesson plan) and pieces of reflective writing scrutinised (see appendix 5 for guidance on reflective writing). The interviews were designed to assess how student teachers conceptualise the *mastery curriculum*, but also how the *mastery curriculum* conceptualises them and the teacher educators with whom they work. These data relating to particular moments are intended to provide points of reference; to help me capture my own developing identity and how the student teachers construct knowledge of the *mastery curriculum* in mathematics. Student teachers were encouraged to successively revise their developmental story across the alternative discursive spaces of school and university. My own actions were integral to the situation being described and thus my narrative became an essential part of the research. I am located within the research but also attempted to move outside the context to become an observer. Although never objective in what I notice, I tried capturing my development through a journal of my critical reflections and analysis. I also monitored and analysed the writings of student teachers at various stages of their development and sought to examine my own evolving contributions to this process.

1.7.1 Recruitment of participants.

The initial sampling frame contained student teachers enrolled on a teacher-training course at Manchester Metropolitan University. Two routes into teaching were considered: *BSc Secondary Mathematics with Qualified Teacher Status* and *Secondary Mathematics Postgraduate Certificate in Education*. I am a tutor on both these courses. My role as a tutor involves leading sessions on mathematics pedagogy and observing student teachers on placement.

Placements are organised around students' term time postcode and proximity to schools, as well as taking into account their own personal experiences. Once students had been placed, schools following a *mastery curriculum* were identified. All of the student teachers

placed in these schools were invited to volunteer to take part in this study. Fifteen students attended a 20-minute meeting where information about the study was shared. It was made clear that participation was voluntary and any data used would remain anonymous. Participant information and a consent form was distributed (appendices 2 and 3). Students had one week to consider the information before returning any consent forms. Ten out of the fifteen identified students returned their consent forms, agreeing to participate in the study. Out of the ten students that agreed to participate seven students were subsequently identified as having suitable teaching timetables following a *mastery curriculum*. Data collection took place from February 2018 to May 2019 (see appendix 1 for research plan).

1.7.2 Ethical considerations

- I was aware of the ethical issues regarding the process not being too arduous in terms of time commitments for the participants. I carefully considered the construction of questions for the semi-structured interview, minimising the amount of time required. I was mindful to not undertake excessive data collection for this study or collect data that was beyond the scope of the study and therefore could not be used. Students were invited to participate in an interview at a time that was convenient to them. Interviews lasted a maximum of twenty minutes on three occasions during the academic year.

- Both non-participants and participants were reassured that there was no advantage or dis-advantage of participating in the study. Participants' opinions were valued but remained anonymous and independent of course outcomes. I made it very clear to participants that involvement in the study was at all times voluntary. I also stressed to the participants that they could withdraw at any point during the process without explanation and there would be no adverse consequences as a result.

- During lesson observations, I reassured students to teach as they normally would for any other lesson. I did not want them to do anything different. I did not assess the participants on how they delivered a *mastery curriculum*. The participants had the same amount of lesson observations as any other student on their courses.

- Placement schools were informed about the study and it was made clear that the focus was student teacher development and their conceptualisation of the *mastery curriculum*. Names of schools and any data used would remain anonymous.

- There was a risk that students could feel exposed when sharing their ontology and epistemology. I worked hard to create a non-threatening, non-judgemental atmosphere. I engaged in active listening, trying to get a hold of the fine nuances, allowing participants to finish what they were saying. The discussions were not about confirming a correct version of the *mastery curriculum*, but much more about how we work and what we think along the way. By having an emphasis on these aspects, I hoped to minimise any feelings of potential distress.

- In asking the participants to share their developing ideas on the *mastery curriculum*. Their writing was scrutinised, teaching observed and semi-structured interviews took place. I am aware of the imbalance in power relations between students and myself as their tutor as well as the researcher. This increases the risk that students will join the project to 'please me'. There may also be concerns that there will be repercussions if they say anything negative and/or they may tell me things that they think they want me to hear. To minimise risk students were informed that data analysis would take place after they completed their placement (see appendix 1 research plan). Additionally, participants were reassured that there was no obligation to take part - it was purely voluntary and anything that was said would not impact on their studies or on future relationships with me as a tutor at Manchester Metropolitan University.

- Published work arising from the study will be anonymous and I will not discuss student responses with anyone else.

Chapter 2 Literature Review

2.1 Introduction

This chapter outlines some of the literature and theoretical perspectives that underpin the analysis of data in chapters four and five. There are two parts to this chapter; in the first part, I discuss the emergence of the term *mastery* within secondary school mathematics. In the second part, I pay attention to ideas of cultural-historical theory of the teaching and learning and mathematics.

To contextualise the emergence of the *mastery* movement I outline a brief account of mathematics education reform in England in the last 40 years. With education policy and standards becoming a national issue (e.g., Brown and McNamara, 2011), successive governments have reformed various policies with a view to improve education standards. Since the introduction of the National Curriculum in 1988, there have been a sequence of five failed versions to achieve high positions in the international league tables (Brown, M., 2014). International testing has broadened the competitive neoliberal agenda, providing comparison between countries, leading towards a more corporate model of 'successful' education, where structural priorities and accountability take precedence over teacher autonomy. Thus, unsurprisingly, the recent curriculum reform agenda in England has been referenced to high-performing international educational jurisdictions, in particular Shanghai. I consider how specifically *mastery* teaching, reflecting educations systems found in Southeast Asian countries has evolved as a key theme in mathematics education. In the background to contemporary discussions of *mastery* are concerns about the degree to which we understand humans as a collective. Here individual student teachers are understood as collectively participating in the wider social activity of teaching.

In the second part of this chapter, my attention turns to a more cultural-historical reading of the teaching and learning of mathematics. The reforming of the curriculum is accompanied by a new set of expectations on how mathematics education is understood. This chapter takes the premise that mathematical meaning and classroom mathematical practices are socially constructed norms. That is, becoming a mathematics teacher occurs

through participating in social activity. I begin with the discussions that took place in connection with the emergence of constructivism over twenty years ago and its later turn to the social. Radical constructivist perspectives emerged in line with Piaget's individualist development psychology, whilst social constructivism later developed Vygotsky's more socially oriented psychology. This discussion is centred around contemporary notions of subjectivity that provide alternative filters to traditional concepts of psychology. That is, traditionally, mathematical meaning is the 'real and objective description of the intrinsic properties of objects' (Radford, 2006a:39). However, this study takes the premise that the meaning of mathematical objects is accessed through the stories told about them. Individuals develop mathematical meaning through shared experiences and language. Additionally, student teachers, pupils and the researcher also identify themselves through social cultural filters, such as the *mastery curriculum*. Through these identifications, student teachers develop their practice as teachers. This sets the scene for the following chapter where I discuss in more detail Lacanian subjectivity and how the individual is continually seeking to complete the picture they have of themselves in relation to the perceived demands of the teaching standards, policy discourse and professionalism.

The purpose of this review is to show how mathematical educational research has positioned me in relation to the research questions guiding this study. The questions asked and my interpretations of data are both subjective and political. Why am I looking at particular issues in a particular way? This chapter is much more than a literature review; my discussions illuminate my own preferences, situating my current position in relation to the research, that is, the subjective stances that I assume.

2.2 The emergence of the mastery curriculum

The *mastery curriculum* presents particular images of mathematics that can provide filters through which one recognises himself or herself as a mathematics teacher. It prescribes roles for teachers and students. The actions of student teachers can be recognised and assessed in how they conform to the enactment of this version of mathematics. In this section, I reflect on how *mastery* has evolved as a key theme in mathematics education and the demands it places on teachers, but first I give some background in the evolution of recent policy discourse.

2.2.1. A short review of mathematics policies in the United Kingdom

The phenomenon of mathematics teaching is affected by curriculum policies, beliefs and identifications with a variety of alternative motivations. Over the last 40 years, regulative systems of central government have increasingly determined teacher practices and conceptions of mathematics. The fundamental reason for this continuous change has been a drive to 'improve' educational standards and in particular, since the 1990s, mathematics performance has been compared internationally (e.g., Brown and Clarke, 2013, Harris, Keys and Fernandes, 1997). In a climate of continuous transformation, the effects of neo-liberalism affect conventions of teaching mathematics. Where pedagogical strategies and administrative procedures reshape and package mathematics into objects that are more easily tested or monitored. Teachers, students and teacher educators face challenge as both experts and novices grapple with the perceived demands of change.

2.2.3 New Directions: The National Curriculum

The National Curriculum in England provides statutory guidance for "programmes of study" in mathematics (DfE, 2014). As such, teachers' behaviours and actions often comprise interpretations of curriculum and policy guidance. *The Cockcroft Report*, published in in 1982 (Cockcroft, 1982), which emerged six years before the first National Curriculum, was a turning point in UK mathematics education and continues to influence the teaching of mathematics (Brown, M., 2014). In 1977, responding to perceived concerns from employers and higher education providers, the Labour government commissioned a report on the teaching of mathematics, five years later, *The Cockcroft Report* was published. The resulting report emphasised that in order to 'apply mathematics it was necessary not only to have mastered procedures but also to have a connected understanding of mathematical ideas and practice in solving problems' (Brown, M., 2014:2). In particular, the report led to a rise in practical and problem-solving work. It lay the foundations for then Conservative party education minister, Kenneth Baker to introduce the Education Reform Act 1988. Subsequently, the first statutory mathematics curriculum was introduced in 1988. Its purpose was not only to determine subject content but also it promoted certain classroom management and teaching strategies. The subsequent introduction of school leagues tables in 1992 by John Major's Conservative

government fuelled the government and the public thirst for monitoring performance and accountability (Brown and McNamara, 2011). Since then, successive governments have used the league tables as levers to direct school systems down one particular route or another (e.g., Pais, 2015, Llewellyn, 2016). Take for example, the standards measure of five A*-C GCSE[10], it did not take schools long to recognise that there were different ways of reaching this target. Eventually the government realised that many students were achieving this threshold without achieving 'passes' in mathematics and English, so accordingly in 2006 the government adjusted this headline measure to include mathematics and English.

Since the Cockcroft report, there have been five versions of the mathematics curriculum, with the current curriculum implemented in September 2014. Alongside the National Curriculum, successive governments have endorsed various strategies in supporting its implementation (Llewellyn, 2016). New Labour's (1997-2010) version of education was based upon an unprecedented 'depth, breath and pace of change' (Coffield, 2006:2). In part, responding to observations of high performing international educational jurisdictions and about the dynamism of government, seen to be doing something (Ball, 2008), the government published a revised National Curriculum in 2000. Curriculum changes were implemented in conjunction with the published *Key Stage 3 National Strategy, Framework for teaching mathematics* (DfEE, 2001). Specifically, the strategy focused on both individual and national progress. It comprised a detailed framework that prescribed the specifics of what mathematics should be taught and how. For example, it described and explained types of teaching methods and offered many practical ideas for classroom activities, with an emphasis on 'mental calculation'. Many such factors guided school mathematics. The prescribed level of detail removed much of the agency from teachers. Being a mathematics teacher had come to be defined on how well you implemented the framework for teaching mathematics. In 2007, the Labour government developed a new secondary National Curriculum with a focus on removing content, while adding emphasis on problem-solving and process skills. As such, the 2007 National Curriculum was realigned to recommendations of the Cockcroft report in 1982. However,

[10] In the UK, the General Certificate of Secondary Education (GCSE) is an academic qualification, generally taken in a number of subjects by pupils in secondary education.

these curriculum changes coincided with changes to the General Certificate of Secondary Education (GCSE) in the UK. Ironically, these changes in GCSE phased out the coursework element in 2009, which removed the incentive for practical work and problem solving.

2.2.4 Mastering the curriculum

Following the change of government in 2010, Conservative Education Secretary, Michael Gove started planning for a new National Curriculum. As such, the current *mastery* movement can be traced back to 2010 with the publication of the Schools White Paper 'The Importance of Teaching' (DfE, 2010). A policy paper outlining the government's proposals to reform the education system in England. Its main concern was 'how we're doing compared with our international competitors' (3). At that time, England fell from eighth in the world in mathematics in the 2000 PISA survey to 24th in 2006. The White Paper stressed that the only way to move up the rankings 'is by learning the lessons of other countries' success' (3). The document also highlighted the need for a new approach to the National Curriculum, a 'model of knowledge which every child should expect to master in core subjects at every stage' (10). Unsurprisingly, in 2013, the UK government, in response to the 2010 White Paper, announced plans to overhaul the National Curriculum. Observations of high performing educational jurisdictions, in particular East Asian countries informed the content and principles of the 2014 mathematics curriculum (NCETM, 2014). Although the word *mastery* does not appear in the 2014 National Curriculum documents, the influential, National Centre for Excellence in the Teaching of Mathematics (NCETM) has encapsulated the word *mastery* in relation to professional development and the teaching and learning of mathematics (NCETM, 2016, 2018, 2019a). Masked by the appearance of independent actors, government funded agencies, such as the NCETM; promote specific classroom practices with an emphasis on East Asian styled *teaching for mastery* (Boylan and Adams, 2019). This is compound by government rhetoric, for example, the words of Schools Minister, Nick Gibb *(12 July 2016)*, highlights the direction of education policy.

> *''We are seeing a renaissance in maths teaching in this country, with good ideas from around the world helping to enliven our classrooms….[the] maths mastery approach can only add to the positive momentum"*

More recently, the Department for Education (DfE) has funded the Mathematics Teacher Exchange (MTE). The aim of the exchange is to learn from East Asian practices (Boylan, Wolstenholme, Demack, Maxwell, Jay, Adams, and Reaney, 2019). In conjunction with MTE, the DfE is also subsidising the development and dissemination of master-aligned textbooks. Nick Gibb visited Shanghai in March 2016 to see mathematics teaching in practice, and this was followed by the press release '*South Asian method of teaching maths to be rolled out in schools'* (July 2016). Subsequently a range of interconnected policy initiatives that promote East Asian practices was formulated under the banner 'Teaching for mastery' (TfM) (Boylan, Maxwell, Wolstenholme, Jay and Demack, 2018). The *teaching for mastery* programme currently implemented by the NCETM, is covertly transforming government policy into practice. Boylan et al. describe the role of the NCETM as 'developing, refining and deepening school and teacher's understanding of Shanghai teaching for mastery' (2016:7). Through the national network of Maths Hubs the NCETM are providing professional development and offering support in the implementation of a *teaching for mastery* curriculum.

The NCETM have organised mathematics into six themes: structure of the number system, operating on number, multiplicative reasoning, sequences and graphs, statistics and probability, and geometry. Each theme is supported by professional development material 'to develop subject and pedagogical knowledge' (NCETM, 2019b). Interestingly the absence of algebra is as a key theme; Carol Knights from the NCETM argues in her podcast (ibid) that high performing jurisdictions see algebra as generalised number, as such, Knights reasons algebra is embedded in the number themes. It is evident that the *teaching for mastery* strategy advocates particular teaching structures and methods. In particular, another key component is the use of variation theory that includes multiple representations of what a concept is, and what it is not (e.g., NCETM, 2019c, Kullberg, Runesson and Marton, 2017, Watson, 2017). Additionally the NCETM are prompting certain principles and features that have been adopted from education systems of Southeast Asia. These include:

- *Teachers reinforce an expectation that all pupils are capable of achieving high standards in mathematics.*

- *The large majority of pupils progress through the curriculum content at the same pace. Differentiation is achieved by emphasising deep knowledge and through individual support and intervention.*

- *Teaching is underpinned by methodical curriculum design and supported by carefully crafted lessons and resources to foster deep conceptual and procedural knowledge.*

- *Practice and consolidation play a central role. Carefully designed variation within this builds fluency and understanding of underlying mathematical concepts in tandem.*

- *Teachers use precise questioning in class to test conceptual and procedural knowledge, and assess pupils regularly to identify those requiring intervention so that all pupils keep up.* (NCETM, 2014:1)

However, on twitter in 2015, Mike Ollerton, an influential member of the Association of Teachers of Mathematics argued that many of the features of *mastery* teaching have been around for 40 plus years. Irrespective of policy commitment, various official and unofficial discourses conceptualise *mastery* learning. That is, the current *mastery* rhetoric in England is a product of social cultural mediations; a conglomeration of approaches that is packaged by contemporary educational policies. For example, Daniel whom I introduced in chapter 1.5.1 is influenced by his subject mentor advising him to encourage pupils to '*use mathematical language…..and that is following the mastery approach*'. Here, Daniel is developing to become a mathematics teacher within the context of the mastery curriculum. Many schools are adopting a *mastery curriculum* influenced by East Asian approaches. However, they often appear to be reconceptualisation's of earlier successful educational practices. For example, the term *mastery* can be traced to the work of Benjamin Bloom, such as, requiring that pupils achieve a level of mastery in prerequisite knowledge before moving forward to learn subsequent information (Bloom, 1968). Skemp's (1976) work on relational and instrumental understanding retains currency in the ongoing debate on the *mastery curriculum*. Singaporean approaches draw on Bruner's forms of representation; concrete-pictorial-abstract (Bruner, 1966). Nevertheless, the NCETM and Maths Hubs are providing a framework and act as vehicles or provide tools to

implement particular pedagogical approaches and government policy. Alongside the NCETM the government founded Education Endowment Foundation (EEF, 2017) produced a report on improving mathematics in key stages two and three. The report endorses many of the components of *teaching for mastery*. Even though there is a drive to centralise the content of professional development through the work of the NCETM and the Maths Hubs there are other providers and business systems which have set up versions of the *mastery curriculum*, for example, White Rose Maths and Mathematics Mastery. As such, the *mastery curriculum* is packaged as a new product, with curriculum materials, professional development, and a range of expertise and so on.

2.2.5 Discourse of the mastery curriculum

Subscribing to the ideology of a particular definition of a *mastery curriculum* does not necessarily mean that student teachers view mathematics education in the same way. Neither does it mean that policy documents have a fixed meaning. Neill (2013) argues that it is impossible to approach a text or policy document, without already distorting that text or discourse. Collective practice such as *teaching for mastery*, shape the individual practice of teachers. As Foucault asserts, that 'discourses construct rather than describe meaning' (Llewellyn, 2016:301). From this poststructuralist perspective, the *mastery curriculum* is defined by the way it is enacted upon in the classroom rather than having an inherent meaning. Furthermore, Carabine (2001) argues that discourses have the power to develop a particular form of the 'normal' that in turn this becomes taken for granted, an acceptance of this is the ways things are (Foucault, 1978). Walshaw (2007) argues that discourse also structure forms of social organisation, creating cohesion, in particular, she argues that, 'Education is one such social institution. It provides a perfect demonstration of how easy it is to be seduced by its emancipatory narrative' (2007:130). This perhaps echoes Althusser's (1971) concept of ideology, where ideology calls the subject into being. Discussion on how student teachers experience and negotiate the multitude of discourses are central in the remaining chapters. In particular, chapter three considers Lacanian discourse analysis and draws on the ideas of Althusser in creating a theoretical framework to analyse the data.

2.3 A short review of mathematics education research

My attention turns to some ideas of social-cultural theory of the teaching and learning of mathematics. The reforming of the curriculum is accompanied by a new set of expectations of how mathematics education is understood. This chapter begins from the premise that mathematical meaning and classroom mathematical practices comprise a socially derived norm. That is, I look to understand mathematical thinking, teaching and learning as a cultural-historical phenomenon. The discourse of the *mastery curriculum* presents teaching and learning as a particular package of mathematics and becoming a mathematics teacher occurs through participating in (this) social activity. Here, the relations to other people and social-cultural artefacts define individuals rather than being wholly separate entities. The purpose of the next section is to give some insight into my ensuing theoretical perspectives. Looking at teaching and learning through social cultural filters allows for alternative understandings' of the human subject, and how this subject identifies with caricatures outside of himself. I begin by briefly reviewing the rise of social theory.

2.3.1 Individual and the social

Historically questions about teaching and learning in mathematics are often seen as being centred on the operation of individual cognitions confronting mathematical phenomena (as suggested by Cobb and Yackel (1996) who began to assert a more social constructivist perspective). Even though there has been some attempt to regard the student as a member of a social group there is still a trend to favour the individual and simplify the role of the social. That is, contemporary concepts of the mind, thought, and consciousness are seen as something individual and makes learning a private and subjective enterprise (e.g., Roth and Radford, 2011, Radford, Miranda and Lacroix, 2018). However, Radford (2018) argues that knowledge is something cultural and historical. That is, teachers, the curriculum, textbooks all contribute in the students' learning.

My purpose is not to go into any detail about constructivism but to give some insight to my ensuing theoretical perspectives. Briefly discussing the problems I have found with

constructivist approaches before moving onto theoretical orientations related to cultural-historical attitudes to teaching.

Constructivist perspectives on cognition and learning were developed out of Piaget's (1952) work on child development (e.g., Confrey and Kazak, 2006, Steffe and Kieran, 1994). Its main thrust was that the individual produces knowledge internally. In England, Skemp (1961) 'championed Piaget's notions of schema, assimilation, accommodation, equilibration, and reflection as ways to conceptualise students' mathematical thinking as having an internal coherence' (Thompson, 2014:2). That is, knowledge is not passively received but is actively constructed by individuals (e.g., Cobb and Yackel, 1996, Cobb and Bowers, 1999, von Glasersfeld, 1990, Thompson, 2014). It generally appealed to mathematics educators as it offered an alternative to the constraints imposed by the narrow conceptions of transmissive teaching. However, the lineage of educational research following Piaget had given rise to a number of different conceptions of constructivism, for example von Glasersfeld's (1990, 1995) radical constructivism[11].

Despite the dominance of constructivism in mathematics education, its theoretical premises had its limitations. One of them, of particular interest to this study is its emphasis on individual cognition, ignoring or at the very least down playing the social (e.g., Radford et al., 2018, Bibby, 2008). For example, Cole criticises the individualistic approach for simplifying consciousness, thinking, and psychological processes. His main argument was that, 'the human individual's activity is a system in the system of social relations' (2009:vii). That is, the individual exists through social relations. Leont'ev (2009b) suggests that development between the individual and the sociocultural can occur in and through relations with others in the pursuit of collectively motivated *activity*. From this point of view, the mind is conditioned by the past, through culture and upbringing. As such, experiences are translated into stories that individual's tell to psychoanalytically construct themselves using reference points from the past that may orientate possibilities for the future.

2.3.2 Social turn

[11] A theory of knowing that provides a pragmatic approach to questions about reality, truth, language and human understanding (von Glasersfeld, 1995).

Lerman (2000) analysed a sample of papers from *Educational Studies in Mathematics* and found that during the 1990's there had been a 'social turn in mathematics education', he argued that there was a move in focus from the cognitive to the social. That is, there was a shift from theories of individual cognition to theories that privileged cognition produced by social activity. By identifying the 'social turn in mathematics education', not only did he 'describe a phenomenon but also helped shaped the phenomenon by naming it' (Wagner, 2015:5). Lerman introduced the social turn in this way:

> *'I have called these developments the social turn in mathematics education research. This is not to imply that other theories, mathematical, Piagetian, radical constructivist, or philosophical have ignored social factors […]. The social turn is intended to signal something different; namely, the emergence into the mathematics education research community of theories that see meaning, thinking, and reasoning as products of social activity'*. (Lerman, 2000:23)

Lerman's naming of the social turn raised the importance of sociological theories that privileged social activity in developing thinking, reasoning and knowledge. Such work 'grounds itself in classroom practices and relationships within wider social institutional processes and influences' (de Freitas and Walshaw, 2016:6).

Inglis and Foster (2018), in their study of *Five decades of mathematics education research*, found the proportion of words from sociocultural theory has increased significantly since the early 1980's across mathematical journals[12]. However, Mousley (2015:154-155) describes social constructivism as being a compromise between individual cognition and social activity, that is, it can be used to explain 'how the notion of individual cognition could remain viable in the context of social group interaction'. As such, even though there is an attempt to regard the student as a member of a social group, much mathematics education research still emphasises the individual. As Bibby (2008) suggests '[the social] is construed as an aggregation of individuals each of whom is fundamentally more important than the group' (39). Indeed Lerman (1996) raised the issue of integrating the social and the individual, 'the notions of the social construction of knowledge into a radical constructivist view of learning is, at the very least, problematic' (133). That is, a major

[12] The full text of all articles published in Ed*ucational Studies in Mathematics* and the *Journal for Research in Mathematics Education* since their foundation

problem for radical constructivism is a sufficient explanation of intersubjectivity (Cobb, Wood and Yackel, 1991).

How could constructivism, with its focus on the individual cognition explain intersubjectivity? Leont'ev (2009b) suggestion of collectively motivated activity was in part a response. Leont'ev considered thinking to be, 'mediated reflection on the world in accordance with the form or mode of the activity of individuals' (Radford, 2006b:5). Here Radford is making the point that thinking is a social practice. Mediated reflection echoes Vygotsky's (1978) ideas on the role of artefacts and cultural tools in bringing about changes in higher mental functions. For example, government policy, professional development, as well as artefacts, such as textbooks, pictures, and so on, all provide filters on the relationship with how classroom mathematics is formed. That is, we think through the language of cultural tools.

2.3.3 The role of signs in mathematical thinking

Returning to the question about teaching and learning mathematics, I consider interpretations of Vygotskian ideas of social interaction (see chapter five for more detail), because his work is hugely influentially in mathematical education and his emphasis on the role of social interaction in human development. Vygotsky (1978) with his group of students including Leont'ev (1981) and Luria (1979) researched *development of higher mental functions* from the standpoint that signs and symbols are embedded within socio-cultural practices, such as artefacts and language. Vygotsky's work has had a significant influence on how we might understand children's development through their interactions with other people and the social environment. That is, when we consider the relationship between *mastery* policies, the use of textbooks, pedagogical practice, teacher beliefs, tools and materials, we need to consider the relationship within wider social institutional processes (de Freitas and Walshaw, 2016).

Of particular interest to this study is Vygotsky's ideas on the way individuals repeatedly position themselves through engagement in social interactions and discourses in the cultural-historical world (Radford, 2018). The use of signs, which are located within cultural practices, such as language and artefacts, are seen as integral to the development of identity and individuals' positioning in the social world. Mediated reflection through

artefacts is in line with Vygotsky's (1978) thinking, as Cole, a leading exponent of contemporary cultural-history activity theory, argues, 'the central thesis of the Russian cultural –historical school is that the structure and development of human psychological processes emerge through culturally mediated, historically developing, practical activity' (Cole, 1996 in Brown, 2011:116). If we accept this line of thinking then artefacts are more than just aids to thinking, they have the power to establish and organise thinking (Radford, 2006b). For example, when Emily- a student teacher to whom I shall come back to in chapter five, uses bar models to represent simultaneous equations, this can be seen as mediation through artefacts. By reinterpreting the problem using a bar model the activity could be described as being, 'semiotically mediated by the pupils' subjective interpretations and feelings' Radford (2006b). That is, these artefacts shape thinking. Radford (2018) considers individuals in the classroom as signs too. He argues that similar to signs, individuals occupy positions in the social world and address an *Other*[13]. It is this response to the Lacanian Other that I am particularly interested in this study. Here, teachers and students, as individual subjects, are seen as dynamic entities, in constant flux, responsive to ever changing social demands.

2.4 Summary

2.4.1 *Teaching for mastery*: A social cultural activity

A key aim of this study is to provide insights into ways student teachers' conceptions of school mathematics might be shaped through regulative discourse, such as the *mastery curriculum*. Discussions above centred on how mathematical meaning, thinking and reasoning can be seen as products of social activity. In a similar manner, becoming a mathematics teacher is also a product of social activity. Accordingly, student teachers' mediated relationships with the *mastery* policy, pupils, university tutor, subject mentor etc. inform the individual human subject of what it is to be a teacher. That is, how they understand themselves and how others understand them is based on the participation of

[13] A Lacanian term I discuss in detail in chapter three. Put simply Lacan equates the big Other with language and the law, and hence the big Other is inscribed in the symbolic order. It is what makes social organisation possible.

a common shared agenda (Symbolic network[14]). Both learning to become a mathematics teacher and the production of mathematical meaning occurs through participating in social activity. As Foucault (1972) argues that knowledge is an issue of social, historical and political conditions, always determined in time and space.

As an outcome of the interactions between student teachers - pupils, subject mentors and policy discourse - individuals come to occupy positions in the social world. This highlights the notion that individuals are constantly adjusting and making sense of the way they see their world. In this way, the reality of teaching mathematics is understood as a social activity, where student teachers interpret the *mastery curriculum*, teaching standards, subject mentor advice and so on and enact them in their own particular ways. As such, the 'truth' of the *mastery curriculum* is never final and certain; it is interpretations of discourse, rooted in dynamic social interactions. Teachers demonstrate their understanding of these social structures through their school practice, and at the same time student teacher practice is assessed through these filters. This circularity between understandings and explanations is an example of the *hermeneutic circle*, where hermeneutics is the theory of interpretation (e.g., Brown, 2011, Brown and Heggs, 2005, Ricoeur, 1981). That is, the *mastery curriculum* may seek to impact on collective teaching and learning but this impact will always be function of how these collective practices are currently understood and how those understandings are processed as narratives of what we are doing (Brown and McNamara, 2011). In this way both the NCETM and Maths Hubs seek to represent the reality of the teaching and learning of mathematics and impact on collective understandings and social practices of individual teachers.

2.4.2 *Teaching for mastery*: The master of us all

It might seem that teachers have agency within the neo-liberal market driven forces. However, paradoxically, teacher autonomy is reduced in the process of *statification* (Boylan and Adams, 2019). That is, on the one hand, discourses of a free market and competition may seem to offer notions of choice and the autonomous individual. On the other hand, discourses of accountability and improvement do not leave much space for

[14] The Symbolic is a Lacanian term that I introduce in chapter three. de Freitas and Walshaw describe it as 'the domain of laws, words, letters and numbers that structure our institutions and cultures (2016:70)

teachers to explore and be more expressive (Fielding and Moss, 2011). As Clarke argues this, 'can be read as the subordination of this same self to the 'other', who determines and dispenses knowledge in the form of mandated curriculum, and who monitors its achievement through test and targets' (2012:48). As such, *mastery*, sold as good practice and as a means to improve standards has an impact on the conceptualisations of mathematics education. It can be seen as a means of directing and controlling the actions of teachers and learners.

Government rhetoric is difficult to refuse, the NCETM occupies a position of power, it represents systematic knowledge that addresses schools and teachers to enact and reproduce the knowledge system (Clarke, 2012). The NCETM could be described as disguising an authoritarian discourse with rationality. This systematic knowledge, the *mastery curriculum*, operates on the subject's desires to fit in, be successful, subordination to the Other. Independent providers such as Mathematics Mastery might appear to offer spaces for professional development and teacher autonomy, but they too could be described as disguising the dominance of government discourse. As Brown, Rowley and Smith (2014:285) put it, such 'policy documents define the parameters of teacher practice to the extent that participation in teaching and teacher education becomes a form of bureaucratic compliance monitored by an inspection regime that insists upon this taking place'.

Particular values and ideals of the *mastery curriculum* are presented as an absolute truth, or in Lacan's terms, as a master discourse, (for example, the supposed need to reduce the attainment gap and raise attainment). The emancipatory narrative being difficult to resist, shapes the future of school mathematics. That is, the vocabulary and language of the *mastery curriculum* can provide the orientation through which one recognises themselves as mathematics teachers. Within this politically charged sociocultural reality, how might we conceptualise teacher agency?

Chapter 3 Lacan and psychoanalysis: The subject of discourse

3.1 Introduction.

In the previous chapter, discussions were centred on how mathematical thinking, teaching and learning are effects of the social-cultural environment. This perhaps echoes Wittgenstein's (1983) notion that words do not have meanings in themselves; the meaning of a word depends on how it is *used* in a sentence. Individuals develop mathematical meaning through shared experiences and language. In a similar fashion, student teachers, pupils and the researcher also identify themselves through social-cultural filters, such as the *mastery curriculum*, teaching standards and stories about what it is to be a secondary mathematics teacher. That is, is by participation in social activity student teachers are called into being a particular teacher by the process of subjectification (Roth and Radford, 2011).

Subjectification here refers to the process of reflexivity, whereby the individual, participating in activity is continuously reconstructing their self-identity. Individual teachers may have personal aspirations but if they want to be employed as a secondary mathematics teacher, they have to conform to regulative structures. In this way, student teachers are members of a social cultural group, in so much that individual development is related to social cultural development (Leont'ev, 2009a). As Cole (2009:vii) puts it, 'the human individual's activity is a system in the system of social relations. It does not exist without these relations'. Student teacher activity either takes place collectively (with other people) or in a situation where the student teacher interacts with cultural objects, for example, the classroom, the curriculum, or the teacher standards. For example, when Daniel - a student teacher whom I introduced in chapter one - says 'I am training to become a mathematics teacher', he is identifying with particular configurations of what he thinks is expected of him. Such a label is understood both by Daniel and by members of his culture.

Becoming a 'mathematics teacher' comes with many expectations and demands but particular teachers interpret various discourse in their own individual way and enact 'teaching' in a particular way, their subjectivity (e.g., Brown, 2008b, Brown, 2011). This

study is interested in how student teachers respond to the multiple discourses that shape their practice. It is firmly rooted in the everyday interactions between student teachers and pupils as they address mathematics together. I place attention on the sense student teachers make of their immersion in social structures. How do they experience and negotiate the various discourses and demands that are placed on them? To explore such ideas, I was aware of the need for a theoretical framework that would allow me to give attention to the complexities of discursive encounters.

Psychoanalytic theory provides an approach to disrupting habitual thinking patterns within regulative scenarios and opening alternative discursive avenues. For example, this is made possible through the ways in which the Lacanian subject is understood as being, conscious and rational but at the same time, unconscious desires and fantasies filter the way an individual's sense of self is construed (de Freitas and Walshaw, 2016). It opens up ways of understanding how student teachers partake in the social practice of teaching and how they negotiate the multiple discourses. Here the student teachers' sense of self is understood as being related to a never-ending attempt to how they understand the perceived demands that are put upon them.

3.2 Psychoanalysis; the Lacanian subject.

This study takes the premise that the motives underlying teacher practice are produced and regulated through identification with a range of social discourses, such as; the need to comply with new policy directives, personal aspirations as to what it is to be a teacher, expectations of one's students, etc. In this way, individual teachers are constituted according to individual conceptions of self. However, this self is divided between what they are doing and what they say they are doing. Brown, Rowley and Smith (2014:285) describe it as a 'division located differently for different people, and the type of division determines who you are, who we are and how power and displeasure/pleasure function to secure alignment or nonalignment with particular discursive formulations'.

This notion of the subject suggests that there are specific subject functions that manipulate the discourse and give distinct shape to how discourse interacts with the self (e.g., desire). Additionally there are different discourse functions that operate on the subject (e.g., ideology). In this way, conscious subjectivity is an unstable entity dependent

on a self, that is regulated and disrupted by unconscious processes (Grosz, 1990). Lacan sees the human subject as being caught up in language that describes himself or herself, but the language never quite fits. As Lacan asserts 'when I say use of language I do not mean we use it - it is language that uses us' (2007b:66). In this way, the Lacanian subject is a divided subject, never complete. A subject divided by the 'I' that thinks and the 'I' that does the thinking. As such, Lacanian theory provides us with a model to look at how student teachers understand their development, fitting in with what they think is required of them but never actually meeting those diverse demands. As is becoming evident, in using terms such as subject and discourse, there are risks of oversimplification. Lacan's concept of the subject emerges in the mirror stage by seeing it as 'the subject of the unconscious' (Murray, 2016:174).

3.3 The making of self; the mirror stage.

The Lacanian subject is defined according to the wider discursive network. A good place to start my discussions about Lacan is to consider 'who I am?' and what is meant by the idea of myself? I may think I know who I am, as Descartes' famous dictum, 'I think, therefore I am' equates thinking with being. A conception of consciousness and reason, the human being rational, unified and conflict-free. The view that there is a fundamental core to who we are 'has been central to the Enlightenment's basic understanding of humanity' (Elliott, 2002:9). Whereas, psychoanalytical theory, suggests that the self (the ego), is an illusory sense of self. That illusory self then becomes the basis of subjectivity, interpretations, or rather misinterpretations. Reality becomes a reflection of the original illusion.

I can reflect on the nature of my own identity, consider the attributes of my personality or emotional investments, describe myself as someone who has two children, is married to Sally, enjoys mountain biking, teaches mathematics education, the son of Polish immigrants but this is just an illusory sense of self. This does not mean it is not important to me but when I say, 'I', you are not describing the infinite depth of who you are but you are uttering sounds which represent whatever the 'I' has identified with. Our sense of self is not handed down to us by the external world (Elliott, 2002). The Lacanian subject focusses on this sort of self-identification. In Freud's later work, the ego was understood

as a relational identity produced by the subject's identifications with the world around him (Brown, 2011). Building on this idea, Lacan 'promoted a shift from bio-scientific to narrative emphases in interpreting Freud's work' (Brown, 2011:108).

According to Lacan, the socialisation of the subject can be traced back to the early stages of life, which Lacan calls the 'mirror stage'. Whether understood as a literal or figurative concept, the essential point is that we see who we are in others. That is, the construction of self-identity, the self as it would like to be, is an illusory self. As Lacan puts it.

> *We have only to understand the mirror stage as an identification, in the full sense that analysis gives to the term: namely, the transformation that takes place in the subject when he assumes an image - whose predestination to this phase - effect is sufficiently indicated by the use, in analytic theory, of the ancient term imago* (Lacan, 2007a:2).

The implications of this egotistic construction are complex because it is during this stage that the child begins to construct its own image in relation to the other objects or people in the mirror. This ranges from family interactions, early childhood, and schooling. It is through looking at the Other[15], that the child constructs a self-identity. The interactions and reactions of the Other helps create the self-identity (ego) of the child. Yet that self is an illusory self, built around 'distortions and traps of the imaginary order' (Elliott, 2002:21). The child, in seeing herself in the mirror is identifying with an image that is both oneself and not-oneself. That is, the image the child sees in the mirror is an alienated one. However, it brings a unity that she had not previously conceived (Brown, 2011). It is a critical period in the construction of the ego or self-identity. Lacan describes this as the 'Imaginary', an *identification* with objects of the external world. Nevertheless, there is always a gap between the individual's sense of self-identity (ego) and the demands of particular ideologies placed on them. This self, this image is an illusory self. As Lacan says

> *I am led, therefore, to regard the function of the mirror-stage as a particular case of the function of the imago, which is to establish a relation between the organism and its reality - or, as they say, between the Innenwelt and the Umwelt….. The mirror stage is a drama whose internal thrust is precipitated from insufficiency to anticipation - and which manufactures for the subject, caught up in the lure of spatial identification, the succession of phantasies that*

[15] The recognition of the Other in the mirror is representative for the entry of the subject into the Symbolic. 'What the Symbolic allows (and what it disallows) is derived from the 'laws' of the larger social order or, in Lacanian terminology, the 'Law of the Father' and the 'Big Other'' (de Freitas and Walshaw, 2016:70-71).

> *extends from a fragmented body - image to a form of its totality that I shall call orthopaedic - and, lastly, to the assumption of the armour of an alienating identity, which will mark with its rigid structure the subject's entire mental development* (Lacan, 2007a:4).

Lacan's account of the mirror phase is the stage at which the child becomes able to conceptualise itself as complete marked by particular limits. In recognising himself or herself in the mirror, the subject is beginning to recognise the external reality that shapes their existence. Lacan describes it as an 'alienating identity' because the reflecting image is shaping the subject through imposed social filters. That is, the subject is formed in the realm of the *Symbolic*, in the domain of language and representation. Thus, the implications of the 'I' being created upon the reflections of the Other begin to misrepresent the human's sense of self. The ego or self is never complete, there is always a gap between the psyche and the perceived demands in relation to world around him. This version of the self is temporal, never fixed, always redefining itself with the perceived demands of the Other. This gap between, whom I think I am and what is expected from me, between reality and fantasy, brings in to play desire. For Lacan, desire motivates our conceptions of who we are in life (Brown, Dore and Hanley, 2019).

3.4 The Imaginary, the Symbolic and the Real.

As a student teacher, I may fully participate in the act of teaching, subscribe to the *delivery* of the *mastery curriculum* but somehow I can never complete the picture. Always something exceeds the demands that are placed on me, a gap between the Imaginary and the perceived reality. The thought of perfection or new opportunities without any defined limits fuels the subject's desire. Brown et al. argue that desire 'often mistakes it object, or lacks a well-defined object' (2019:21). We can perhaps gain a better understanding of these fantasises by looking at the developing practice of a student teacher. That is, a student teacher might have a fantasy of what is expected of being a 'teacher' and works hard to construct her sense of self according to those expectations. However, she can only know what successful teaching looks like through the Symbolic. For example, her subject mentor might encourage dialogue between the student teacher and the pupils and peer discussion. In her attempts to meet the subject mentors demands the student teacher misses the mark, there is a gap between her performance and the demands of the Other.

This might be recognised through subject mentor feedback or interactions with the pupils. In Lacanian thinking the subject is conflicted between the Imaginary and Symbolic registers (de Freitas and Walshaw, 2016). As de Freitas and Walshaw go on to argue, that in this way, it is not possible to break down subject identifications to the identities that the individual constructs of himself or herself but the self is contingent to the shifting relationship of subjection and agency. The narratives that I analyse in this study may reveal some of the desires and fantasies that form a significant aspect of becoming a mathematics teacher. Here the productions of the self are not depicted as entirely rational, nor are they represented as completely social. Rather, the self is constructed according to fantasy of what it is to be a mathematics teacher. However, Lacan argues these fantasies are mistaken, characteristic of an order the he terms the Imaginary.

Thus, Lacan paints a picture of the subject being caught up between the fantasy of his or her self and the fantasy of the perceived demands that are placed on them. However, neither fantasy achieves to offer a complete picture. Because of this, somethings are left unaccounted for, yet they remain present in the unconscious and might disturb how we might account the world around us. In tackling these issues, Lacan refers to the three psychic registers or orders; the Imaginary, the Symbolic and the Real[16]. These orders work interpendently, as de Freitas and Walshaw, point out; they work 'together to inform the subject's experience and sense of perception' (2016:70). As discussed above, the Imaginary order comprises of self-identification. For example, when I assume to understand something or put meaning to text or what is being said, I see an image of myself. A fantasy of what is expected from me. For Lacan these imaginings are always illusions, but necessary as a process to attempt to make meaning. As Neill (2013:337) argues, 'to attempt to read without imagining an identification is to try to read without meaning'.

The Symbolic refers to social organisation, the domain of language, words, laws and numbers that structure our world. It is what makes social organisation possible, to be understood and to understand. What the Symbolic 'allows' is derived from the 'laws' of social order, the Other (de Freitas and Walshaw, 2016). For example, in education, the

[16] In the study the Lacanian terms the Imaginary, the Symbolic and the Real are denoted in uppercase. This convention is not followed consistently by Lacan, or by other writers.

Other places specific demands on how we act in the classroom, it might include curriculum documents and school policies. However, the Symbolic does not include meaning. By way of example, I return to Daniel's lesson on Cartesian coordinates (Chapter 1.5.2).

Daniel: 'I want you very quickly in pairs to talk about the coordinate on the graph'.

The Symbolic requires that the discussion between pupils is meaningful and that particular cultural rules are followed. In this case, the pupils talk to their partner, they stay focused on the discussion topic, and they listen to each other and so on. In a similar fashion, the *mastery curriculum* requires that particular mathematics be taught in a particular way. As the discourse of the *mastery curriculum* unfolds, there are moments when meaning is pinned down. Moments of clarity that define what it might mean to be a successful teacher. Lacan refers to these moments as points de capiton, after the buttons that are used to secure the stuffing in cushions. Nevertheless, such moments are temporal, new discourse develops new meaning. To assume meaning implies the convergence of the Symbolic and the realm of the Imaginary.

As is becoming evident, reality for Lacan is a problematic concept. The closest Lacan gets to reality is in the register of the Real. Brown, et al. describe the Real as 'the space in which the Imaginary and Symbolic are enacted' (2019:24). Discourse is always framing and structuring how we see and engage with the world. Thus, there is no escaping the Symbolic and similarly, through the process of identification, there is no escaping the Imaginary. Both the Symbolic and the Imaginary work interdependently to inform the subject's experiences. In this way, Neill (2013:339) points out, 'the reality of experience is always an experience mediated and distorted through imaginary prisms and symbolic frames'. However, since the Real comprises the space that hosts the Symbolic and the Imaginary it is elusive and defies symbolisation. There is always that which escapes language and that cannot be accounted for or described. Even though the Real cannot be represented, it still leaves its mark on the subject, repeatedly 'waiting in the wings' to disturb the reality (Lacan, 2007a) as constructed within the symbolic network.

As an example, the discourse of the *mastery curriculum* combines the Imaginary, the Symbolic and the Real in Borromean knot of interdependency. Student teachers negotiate their understanding of prescribed pedagogical strategies. At the imaginary level, we have

the signifiers, 'mastery curriculum', 'questioning', 'discussion', and so on. Each signifier is attributed meaning. However, each signifier has a different meaning attached for different individuals, at particular times and circumstances. Let us take 'questioning', for example, any role this has in the Symbolic is dependent on the individual's imaginary perspective. At the Symbolic level, 'asking pupils questions' refers to cultural social organisation, certain rules that are the norm. To navigate between the Symbolic and the Imaginary the student teacher invests in a particular position made available (Bibby, 2009). De Freitas and Walshaw (2006) point out that in Lacanian theory, an individual's investment within a particular discourse over an alternative discourse is based on the notion of desire and through notions of obligation. Student teachers and in general the subject desires to close the gap between the fantasy and the reality it seeks to capture. The gap between whom I think I am and what is expected from me. Yet the understanding of 'asking pupils questions', the enactment of the symbolic rules in the classroom does not cover it all. Other factors influence what we do; emotions, resistance of the pupils, relationships, anxiety and unconscious desires, etc. For example, a student teacher with an anxiety of managing classroom behaviour may try and control learning rather than listening to what the pupils are actually saying.

3.5 Althusser and Ideology

The next section considers how student teachers might generally construct their identities through the training process and in response to the *ideology* of the *mastery curriculum*. As an idea, the Symbolic echoes Althusser's concept of 'interpellation' (de Freitas and Walshaw, 2016). Butler (1997) suggests the subject recognises himself or herself in some supposed ideological calling. Where ideology constructs (interpellates) the subject. Althusser (1971, 2014) who was one of Foucault's teachers is a key figure in the development of contemporary understanding of ideology. Both Althusser and Lacan reject the promise of a complete subject, an aspiration to make things better as implied by Habermas and Foucault, (Žižek, 1989). Althusser pursues a 'study of the imaginary constitution of the subject' as an 'effect of the structure of ideology' (Balibar, 2014:xvi). In this way, Althusser recognises the subject as created through a mis-recognition of

ideological discourse. That is, the human subject's relation to social and political demands is an imaginary construct of what is expected from them.

Althusser (1971) regards education as one of the institutional ideological state apparatuses (ISA). For example, *mastery* teaching as a pedagogical tool is seen as a means to drive up standards. In this way, it looks like, 'this is the way teaching has to be'. As Brown suggests, 'in subjecting oneself to the ritual of institutionalised mathematics one is inadvertently materialising one's belief in it and this belief creates a successful link between ideological state apparatus and interpellation' (2020:47). Interpellation here can be understood as the subject feeling valued, fitting in within the establishment of the imaginary domain. Fundamentally, individuals are called into being through prescribed registers and discourses. The *mastery curriculum* places specific demands on individual teachers, to teach in a particular way. As such, the *mastery curriculum* is resourced with a kitbag of ideological state apparatuses (professional development, information, resources and so on). In establishing the self in relation to such discourses, the student teacher is interpellated as a particular subject. However, Brown argues this sort of interpellation can be 'delusional through its failure to embrace the whole picture' (Brown et al., 2006:33). These ideas reverberate with Lacan's view that fantasies are deluded, characteristic of the Imaginary.

Althusser, like Lacan, maintains that cultural forms of ideology are constructed on an 'imaginary relation to their real conditions of existence' (Althusser, 2014:181). However, Althusser differs from Lacan in his discussion of subjectivity. The idea that interpellation brings the subject into being, suggests that the interpellated subject does not assume a prior conscious standpoint because the 'subject emerges through ideological interpellation' (Brown et al., 2006:70). Althusser downplays the fragmented nature of repressed desire, and thus displaces the role unconscious forces have on everyday actions. As Althusser expresses it

> *'*every subject endowed with consciousness and believing in the ideas that it inspires in her or freely accepts should 'act in accordance with her ideas' and therefore inscribe her own ideas as free subject in the acts of her material practice' (2014:185).

The assumption here is of the teacher as a rational agent, self-conscious and able to make clear judgement of ideological practices. For example, some teachers might subscribe to the government rhetoric that the *mastery curriculum* will foster a radical shift in mathematics teaching and improve England's performance in the PISA rankings. Whereas at the same time, other teachers might be sceptical about the motives of that ambition. Yet both groups comply with the master discourses and find that their practice is defined by the *mastery curriculum*. As such, the successful implementation of *mastery* policy is not necessarily an improvement in standards but by convincing teachers, teacher educators and the public that this version of teaching and learning mathematics is the correct one. Standards from this point of view have not changed but the parameters through which successful mathematical teaching and learning is understood have.

As discussed earlier the practice of student teachers involves imaginary identification of others and the self. The vocabulary and language of the *mastery curriculum* can provide the orientation through which one recognises themselves as mathematics teachers but as Pecheux (1982:111) states the meaning of a word or expression 'does not exist in itself, but is determined by the ideological positions brought into play in the socio-historical process'. That is ideology establishes individuals as social subjects, that are brought together by the 'dominant relations of production in a society' (Eagleton, 2007:18). For example, I return to Daniel, he reflects on the purpose of questioning, *'Through my own personal experiences and observations, it is apparent that questioning - when used effectively - can be a largely influential aspect of any teaching episode*.' These words make sense in the discursive environment in which they are produced. That is, student teachers are interpellated as speaking subjects (as subjects of their discourse) (Alcorn, 1994). The expression *questioning in mathematics* does not have a meaning of its own but its' meaning is established in the layers of discourse and relationships to other words or signifiers. If this assumption is correct, then a teacher incorporating 'precise questioning' (NCETM, 2014) into their lessons is not someone who responds to the phenomena of teaching with a critical analysis of what works best for his students but is someone who is called into being as a subject in response to the ideology of the *mastery curriculum*.

3.6 Ideology of the mastery curriculum

As discussed above I consider the *mastery curriculum* functioning as an ideology that provides a specific conception of teaching and learning mathematics. It defines the key parameters to successful mathematics teaching. It promotes certain pedagogical approaches (e.g., visual representations and knowledge of mathematical facts). Further, it points to particular styles of social organisation, (e.g., interactive teaching, pupils sit facing the teacher). Žižek (2012) suggests, that ideology organises our desires and explains our motivations in wanting to acquire something. In this case, the commodification of mathematics sets markers and structures school mathematics. For example, *mastery* teaching and learning requires the fluent recall of multiplication facts, the emphasis of learning 'key' facts becomes part of the way school mathematics is understood.

Many of the ideas that *mastery* teaching promotes seem arguably well-founded and this study does not consider whether pedagogical strategies are effective or not but it is concerned with the 'mastery' of the teaching and learning relationship. That is, the discourse of *mastery* teaching becomes the ideology of student teacher development and requires student teachers to suspend their beliefs and meet the demands of the Other. In relation to my research, I use Lacanian psychoanalysis in order to understand the positioning of the subject in relation to the ideology of the *mastery curriculum*. However, Althusser's theory of ideological state apparatuses is unable to account for individual agency and the complex interplay between the fragmented, desiring subject. As such, I use Lacanian theory of the subject and its notions of desire underscored by the Imaginary, the Symbolic and the Real orders to unpick how student teachers respond to the ideology of the *mastery* policy. I consider, what makes a teacher desire to teach mathematics using a *mastery* approach? Pais argues that 'a subject desires an object not due to its particular characteristics but because of the place such an object occupies within their libidinal economy' (Pais, 2015:380). In other words, the desire for teaching the *mastery curriculum* is not in its applications but the *desire of the Other* (Lacan, 2006), the symbolic network that signifies the master curriculum.

If we remind ourselves of the language of Nick Gibb[17], '*We are seeing a renaissance in maths teaching in this country, with good ideas from around the world helping to enliven*

[17] Schools minister Nick Gibb addresses delegates at the Advisory Committee on Mathematics Education (ACME) conference, 26 July 2016.

our classrooms' (2016). It is the idea of the *mastery curriculum* that is being sold, not the *mastery curriculum* itself. Lacan's contention that '*the unconscious is politics'* (Lacan, 1967, in Žižek, 2012), suggests that our inner most thoughts, our desire, 'is not only unconscious but schematized by politics' (Pais, 2015:378). That is, policy operates on how student teachers act and what they say. As Lacan stressed, 'the self is an Other', when the subject speaks, 'an Other speaks for her' (Fink, 1995:1). Yet, the human subject interacts differently with the discourses of the *mastery curriculum*.

As has become evident the Lacanian subject is a complicated idea to grasp. To summarise, using Lacanian theory I consider student teacher development in response to an ever-shifting symbolic network. This symbolic network using the domain of language, words, laws structures and organises our world. The symbolic comprises the discourses that we inhabit. As such, the *mastery curriculum* is one of the discourses student mathematics teachers might encounter. Here student teachers are subject to the regulative discourse of the *mastery curriculum* that shapes their actions and informs what classroom mathematics might look like. In later chapters I provide an account of how these discourses are enacted by student teachers; how they construct their identities through the training process and in response to the *mastery curriculum*. My aim is to listen to students talking about the *mastery curriculum* and look at the research data to see what it might reveal, what constitutes mathematics in the student teachers mind. Seeking to understand the 'truth of desire' (Lacan, 2006) of student teachers might help me better understand how teachers construct their identities through the training process and in response to the *mastery curriculum*.

Chapter 4 Discourse of the mastery curriculum

There is no Universe of discourse.
—Lacan, Seminar XIV November 16, 1966

4.1 Introduction

Teaching mathematics manifests itself differently in individuals and in different social settings. Students teachers navigate through various demands placed on them. From the needs of pupils, meeting the teaching standards, navigating curriculum policy, to multiple social demands. However, student teachers need to conform to the demands of the Other; their progress as student teachers is oriented against conceptions of what it is to be a teacher. Here, student mathematics teachers are acting according to a fantasy of who they think they are or who they think they should be. I am interested in how student teachers, through these fantasies, construct knowledge of teaching and learning of mathematics.

In this chapter, I use Lacan's notion of four crucial discourses, in which I begin to construct understanding of student teacher encounters with any discourse. With examples of classroom practice, Lacan's schema of discourse helps us guide the way for a clearer understanding of how discourse of policy documents, such as the *mastery curriculum* work and why often they do not work, at least not as intended. In particular, it provides insights into the formulations between knowledge, master[18] signifier, divided subject and the *Object petit a*[19]. In repeatedly mapping out classroom interactions to different permutations of discourse, we generate different possible understandings. As such, Lacanian theory is used to capture the habitual thinking patterns that underpin the multifaceted discursive dimensions of teaching, towards disrupting them with view to opening more generative interpretations.

[18] Mastery curriculum is an example of a master signifier, where the coincidence of names is at least partly coincidental.
[19] *Object a* (utre) in Lacanian terminology refers the object of desire, that which is supplementary to the subject and as such fuels our fantasies and desires to make up for the feeling of incompleteness (Lacan, 2007a).

4.2 Discourse: Real or Imaginary?

Understanding teaching as a social construct does not mean that it is not real to teachers, students, educators and so on. However, representations of education are effects and objects of discourses produced in a particular time or place that regulate what is said and written (Cheek and Gough, 2005). We might imagine that there is an absolute truth, a single correct answer but that would mean assuming that knowledge is finite. However, as Foucault (1972:117) writes, a discursive practice is, 'a body of anonymous, historical rules, always determined in the time and space that have defined a given period, and for a given social, economic, geographical, or linguistic area, the conditions of operation of the enunciative function'. In this way 'knowledge is neither finite nor permanent nor universal' (Neill, 2013:342). That is, our knowledge and understanding of the world is constituted by the discourses we live through. For Lacan, the individual's understanding of self is in response to ever-shifting discourses, encapsulated by the Symbolic network. In problematising my understanding of aspects of discursive accounts, psychoanalytical theory opens up my analysis beyond the one-sided power dynamics into something more expansive and complex.

It is impossible to approach discourse, for example text, without already distorting that discourse or text (Neill, 2013). This struck a chord with me when I revisited an early narrative journal of mine:

> *I ask student teachers to share their emerging thoughts on John Mason's conjecture, 'an adult asks a learner a question when the adult, while in the presence of the learner, experiences a shift in the focus of their own attention. The question is intended to reproduce that shift of focus in the learner' (Mason, 2010:1). The conjecture itself is cognitively challenging and students discuss how they understand the statement.*
>
> *After a period of silence, Nick suggests 'Is he saying that the adult has seen a way that the student can understand something, a revelation and they are asking that question to shift that learners focus on to that revelation?' and then Alistair adds, 'you are trying to get the pupils to come to their own realisation'. Impulsively, students begin to discuss their developing ideas, one shouts out 'I think that it's about, getting kids to think'. The students are beginning to articulate and construct their understanding of questioning as described by John Mason. A 'revelation' or 'realisation', is that how we can define learning? I think this is a pivotal point in the session; the students (well some of them) are*

beginning to generate new ideas about questioning and understanding of mathematics. Listening to this, I also come to a realisation, every time I read Mason's conjecture, I imagine, I understand what Mason is saying, but every time my interpretation or understanding is slightly different. The words do not have independent meaning; the reader approaches the text, whether it is I as the tutor or the student teachers. I realise it is impossible to approach the text without already distorting that text.

Such moments of clarity operate on what we might describe as the Imaginary. Mason, through the commentary of the student teachers, and myself becomes the kind of intellectual we imagine. Text only has meaning when it interacts with a 'subject'. Meaning 'is produced as language is driven or operated by subject-functions such as desire, temporality, the Imaginary' (Alcorn, 1994:23). If we consider Mason's conjecture as the relationship between discourse and subject, the text is manipulated, resisted and transformed by the subject. Lacan emphasises, there is 'no Universe of discourse' (1966:11). This could be understood, as there is no singular definition of anything, there is never any saying it all. There are only gaps between the elements of experience, that is there is always something missing. In the process of re-telling, a new version of understanding is created.

It is useful to consider the general discourse of the text to be held in place by a dominant term, a master signifier. Neill (2013) describes the master signifier as a term, which allows the other terms of a discourse to operate together and create meaning. So the conjecture 'an adult asks a learner a question when the adult, while in the presence of the learner, experiences a shift in the focus of their own attention. The question is intended to reproduce that shift of focus in the learner', makes sense through a multitude of discourses. Terms such as, 'adult', 'learner', 'question', function at the Symbolic level (the domain of language that structure our world) and interact with other social discourses, (e.g., social organisation of the classroom) which operate together to create some kind of meaning.

How we address the master signifier of the text depends on our positioning concerning the other factors of the discourse being played out. We are naturally inclined to position Mason as the author behind the text but Neill suggests looking for how the text constructs the subject. The subject of the text could then be seen as the actual author of the text, or

the one who reads the text and fills it with meaning. That is, we can begin to generate competing possible understandings. Lacan (2007b) argues that a subject can take different positions in respect to its reception of discourse. In his theory, the subject can take four different positions; the discourse of the master, the university, the hysteric and the analyst.

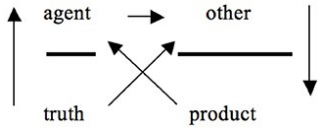
4.3 Lacan's Four Discourses

Lacan's discourse theory is relevant because he asserts that 'all determinations of the subject, and therefore of thought, depends on discourse' (2007a:152). Finks adds to this by claiming that Lacan's theories of discourse 'allow us to understand the functioning of different discourses in a unique way' (1995:129). It provides insights into the relationship between truth, knowledge, subjectivity and the Other. The aim is to demonstrate how mathematics and *mastery* teaching can be seen in different ways and how they are enacted upon by student teachers.

If we were to assume an actual master signifier, the true *mastery curriculum*, then we would be assuming that we have stepped outside of the discourse, looking in on it. We might imagine that we understand that there is a true version of the *mastery curriculum*, a particular way of teaching mathematics, but as Neill (2013:342) suggests, 'if knowledge is neither finite nor permanent nor universal, then no master signifier could exist'. That is to say, while the knowledge of the *mastery curriculum* is never finite, there are layers of interpretation. The meaning of text always resides with the reader. The way the reader interprets text is always subjective, always embedded with extra layers of meanings from the position the subject occupies and the desires based on their situation. Brown, Rowley and Smith (2014:285) portray Lacanian psychoanalytical theory as, 'a subject divided between what she is doing and what she says she is doing'. The subject, for Lacan is always divided in the various ways in which we fail to identify ourselves. If we recall earlier discussions of the subject (chapter 3), this helps us grasp an understanding of the divided subject. For example, when the subject says 'I', you are not describing the infinite depth of who you are but you are uttering sounds which represent whatever the 'I' has identified with. That is, there is a gap between 'the 'I' of the 'I think' and the 'I' of the 'I am'' (Neill,

2011:15). We never really know ourselves; rather we are a composition of identifications in the Symbolic and Imaginary. For further discussion, see chapter 3.

A subject can take different 'positions' in respect to its reception of discourse. Any discourse comprises the master (represents a position of power), the university (represents systems of knowledge), the hysteric (represents the subject who asks questions of the master) and the analyst (represents resistance to oppressive power structures). The different subject positions that the subject takes in relation to the discourse can help analyse different forms of social relation, which can in turn provide clues for understanding how student teachers respond to and process discourse. We have four structural positions: the agent, the truth, the Other and product.



The positions within the schema are important. The left-hand positions are occupied by productive factors in the discourse, and the right-hand positions are occupied by receptive factors (Clarke, 2012). The top position on both sides represent the overt or conscious factors, the bottom position the covert or unconscious factors. More specifically the top left corner, the agent of the discourse could be the author, speaker, institution, a position of power or it could be understood as an ideology. The agent addresses someone or something (an Other). Each agent or act is supported by a certain truth, the factor that underpins and supports the agent. At the other end of the discourse is a product, which cannot be accounted for by the agent. In other words, the discourse is never contained in the transmission of agent to the Other. Such an idea is complex because it describes the excess or residue of the discourse. As Neill (2013:342-343) explains, 'a discourse is encountered but a meaning emerges beyond what could be strictly be said to have been in the discourse'. This suggests that discourse is contained by the subject. Meaning or understanding, positions itself in a particular relation to a subject's conceptualisation of the truth, their subjectivity.

Lacan's framework of the four discourses uses mathemes to represent the factors in discourse; master signifier (S_1), knowledge (S_2), divided subject ($) and *Object petit (a)*. By populating the structural positions with different permutations of the four factors of

discourse, always in the same sequence, we produce four different models of discourse. Each discourse reflects different forms of social relations in society. A brief explanation of the four elements are:

S_{1-} The master signifier. This is the dominant signifier. Bracher (1994:112) describes: 'master signifiers are simply accepted as having value or validity that goes without saying'. In education, there are many unsaid demands that are accepted without question. For example, discourses of standards, accountability and accuracy. Like the authoritarian parent or teacher, who must be obeyed because of who they are.

S_2- Knowledge is needed in establishing the subject, creating an identity. This is what is ordered by or set in motion by the master signifier. For example, this is what I am supposed to be doing when I teach mathematics. It could represent the curriculum, teaching standards, *mastery* teaching and so on.

$- The divided subject. The subject for Lacan (in Neill, 2013) is always divided, in that sense that it is always incomplete. Bracher (1994) describes one manifestation as the gap between thinking and being. The 'I' that I think is an illusory sense of identity and never coincides completely with the 'I' that does the thinking. That illusory self then becomes the basis for my being and action. Reality becomes a reflection of the original illusion.

a- Object petit a. The Object a refers to the marker in the Symbolic realm of the relationship between subject and object. Lacan (in Bracher, 1994) says we all begin life as the Object a. That is, the gaze and desire of our parents determine our being. The position we occupy in relation to our parents determines the fundamental parameters within which we operate. It can be seen as the object of desire, that which is supplementary to the subject and as such fuels our fantasies and desires to make up for the feeling of incompleteness.

In populating the structural positions with the four factors of discourse, we can begin to build a picture of how the multifaceted discursive dimensions of teaching work through student teacher practice.

4.3.1 Discourse of the University

$$\frac{\mathrm{S}_2\ (systematic\ knowledge;mastery\ curriculum,teaching\ standards)}{S_1\ (master\ signifiers:\ qualifications,accountability,policy)} \rightarrow \frac{a\ (student\ teachers)}{\$\ (divided\ subject;students,teachers)}$$

In the discourse of the university, 'systematic knowledge', S_2 occupies the position of agency, which addresses the *Object a,* the object of desire. In other words, expert knowledge addresses the Other to enact and reproduce the knowledge system (Clarke, 2012). Here systematic knowledge is underpinned by the 'truth' of the master signifier. For example, in order for the *mastery curriculum* to take the position of agency there has to belief that this is the 'best way of teaching' or 'this is going to help my students pass'. As such, it comprises a disguised master using rationality to defend a position rather than mere whim. This in itself makes the university discourse powerful, as the appearance of objective, neutral knowledge is underpinned by the 'truth' of the master discourse. This systematic knowledge operates on the subject's desires to be successful, embody the characteristics of *mastery* learning, to fit it. In doing so, the systematic knowledge projects an ideal, complete teacher. As such, it can govern how we operate, behave, teach and so on.

4.3.2 Discourse of the Master

$$\frac{S_1\ (master\ signifiers:\ qualifications,accountability,policy)}{\$(divided\ subject;students,teachers)} \rightarrow \frac{\boldsymbol{S_2}\ (systematic\ knowledge;\ mastery\ curriculum)}{a\ (student\ teachers\ object\ of\ desire)}$$

The discourse of the master is associated with dominance. The master signifier is in the place of agency. Particular values and ideals are presented as an absolute truth, which in Lacan's formulation, can only occur if it is underpinned by subjective endorsement. We can see the discourse of the master operating in the realm of education, for example at a macro-level, the need to be qualified, accountability, and at a micro-level, school policy, the *mastery curriculum*.

The apprenticeship model of learning to teach is also characterised by the structure of the master discourse, with the imposition of the basic concepts of teaching. The domineering tutor or subject mentor, who must be obeyed because of who they are, not because it is underpinned by valid knowledge (Clarke, 2012).

While belief remains in place the master discourse remains in position to control and construct ideology. That is, the receiver of the master discourse enacts the function of knowledge S_2 (Bracher, 1994). The *mastery curriculum* or teaching standards as master discourses prescribe particular conceptions of teaching that predicate particular pedagogical approaches. By participating and enacting the master discourse, student teachers become recognisable as good teachers, making good progress. In the process of enacting this function of knowledge, the subject is never entirely satisfied and excesses are produced *Object petit a*. It is this *a*, for which there is no place in the system of knowledge S_2, the realisation the there is a price to pay to be involved in the system, 'that carries the power of resistance and revolution' (Bracher, 1994:121). While the subject endorses the master signifier, it remains in place to produce unquestioned authoritative knowledge. It is only when we place the subject in the position of agency that the master signifiers are brought into question.

4.3.3 Discourse of the Hysteric

$$\frac{\$\,(Divided\ subject;students,teacher}{a(Student\ teachers\ object\ of\ desire)} \rightarrow \frac{\boldsymbol{S_1}\,(Master\ signifiers\colon qualifications,accountability,policy)}{S_2(Systematic\ knowledge;mastery\ curriulum\ )}$$

In the discourse of the hysteric, the divided subject is the position of agent who disrupts the authority of the master discourse. The subject driven by lack of certainty, questions the master signifier. As Zupančič (2006:165) argues, 'the truth of her or his basic complaint about the master is usually that the master is not enough'. The subject driven by uncertainty tries to reduce the gap between the ideology and reality. In addressing the master signifier, the student teacher no longer accepts the *mastery curriculum* as an absolute truth. 'Why are we teaching in this way?', 'Is there another way?'. Despite these questions, they remain in unity with the master signifier. As Clarke notes the divided subject 'is still underpinned by an unacknowledged and repressed other, *a,* the tendency in this discourse is for the subject to seek a new master' (Clarke, 2012:56). In the search of meaning and security, the subject responds by providing a new master. As the schema represents, in the search of meaning and security, it covertly produces a system of knowledge S_2 (Bracher, 1994). While the discourse of the Hysteric disrupts the authority of the master, the Analyst's discourse places the subject's desire (a) in the place of agency, underpinned by the truth of their unconscious knowledge (S_2).

4.3.4 Discourse of the Analyst

$$\frac{a(Student\ teachers\ object\ of\ desire}{S_2(Systematic\ knowledge;mastery\ curriculum} \rightarrow \frac{\$(Divided\ subject{:}students,teacher)}{S_1\ (Master\ signifiers;qualifications,policy)}$$

In the discourse of the analyst, the *Objet petit a, is* the agent of the discourse. The receiver of the discourse is the divided subject $. That is, the subject's desire, *a,* asks the subject to 'critically consider how the world is presented to them and the ways they situate themselves within the world' (Thomas, 2014:53). In this way, the subject recognises that discourse is not fully within his or her control but somewhat involves a process of identifying the master discourses, thinking about it repeatedly and hence reducing its intensity by gaining insight about its workings (Bailly, 2009). Through this process, the subject produces a re-worked master signifier, thus making the process circular. However, there is a fundamental difference in this new discourse of the master. It has not been imposed from the outside but has been produced by the subject. As such, the discourse of analyst produces a master signifier that is less authoritarian, oppressive, and rigid and is more fluid and adaptable.

For example, in education, a student teacher could be in conflict with alternative systems of knowledge and cause disturbance to the subject. This means, educational policy, such as the *mastery curriculum*, can be challenged, re-worked. In doing so the subject produces new understandings, new meanings (Neill, 2013).

4.4 Using Lacan's discourse schema

Lacan's model of the four discourses allows us to describe different forms of social relations between the sender of a message and the subject of discourse. For example, it can offer the means for a clearer understanding of how discourses of the training process and the *mastery curriculum* construct student teacher identities or on the other hand, induce a state of anxiety. It can help us understand why the training process works and why it sometimes does not, at least not in the intended manner. Student teachers are encouraged to reflect upon and evaluate their teaching, which exposes their reasons for pursuing a particular pedagogical approach.

4.5 Mastery teaching as a discourse

I examine how a group of student teachers are initiated into the notion of *mastery*, at successive stages of their training. *Teaching for mastery,* can be viewed as an idealisation of teaching, that is to say it can provide a point of reference or identification for teachers giving a sense of purpose. It is easy to consider student teachers as rational individuals. A conscious individual whose practice is carefully planned and initiated. However, within this process, there are intrinsic tensions that student teachers experience; negotiating the teachings standards, emotions, relationships, past experiences, subjectivity leading to a series of conscious actions and unconscious processes. In this chapter, I draw on data from the training of three student teachers over two school placements. They have been placed in schools as part of the 'University Schools Model' (Haniak-Cockerham, 2019). The training model places six student teachers in a school, where they teach collaboratively in two groups of three, as well as taking an individual class. The unique attribute of the model, is that of the university tutor, who supports the students one day a week in school. This allows for frequent discussions about pedagogy, approaches to teaching and provides opportunity to regularly observe lessons and give feedback.

I consider student teachers development from their first placement school, time spent at university and then on their second placement, after which they are finally awarded qualified teacher status. In their first placement, they are initiated into how teachers act and behave, and what is expected of them, at the latter stages of this first placement they are more attentive to their subject pedagogy and the language they use in the classroom. After completing a three-month school placement the students spend nine months in university developing subject knowledge and consider different approaches to teaching such as realistic mathematics education and cognitive conflict (e.g., Streefland, 1991). The final part of their course consists of a four-month placement at school, where the student teachers negotiate the demands of, *teaching for mastery*, school policy and the teaching standards.

4.6 Stories from the classroom

As discussed in chapter 1.7 student teachers were chosen on the premise that their placement school was incorporating aspects of the *mastery curriculum*. Research was carried out over two successive teaching placements with student mathematics teachers enrolled on: *BSc Secondary Mathematics with Qualified Teacher Status* and *Secondary Mathematics Postgraduate Certificate in Education* programme. As part of the programme requirements, students were asked to carry out weekly reflections on their experience, learning, practice and professional development (appendix 5). Commencing October 2018 weekly reflections by the student teachers were inspected and organised into themes. Analysing their writing reveals part of their emerging understanding of what they think is required of them. It provides reference points to how individual student teachers understand and capture personal experiences. The three stories were selected to indicate the variety of participation in the research and was indicative of the general themes noticed.

4.6.1 Natalie's Story

Natalie's story begins in her first school placement; she is in the fifth week of her placement reflecting on an introductory lesson on positive and negative numbers.

> *When delivering the lesson of adding and subtracting negative numbers, I started with ordering a variety of positive and negative numbers which proved to be more manageable than I thought. I used the temperature idea to help aid the discussion and most students were confident with this and were able to independently answer questions. I then moved onto the cauldron analogy, with hot coals and ice cubes. I found this easy to explain, however it was difficult to get the students to use this idea as they had a very fixed knowledge of the 'rules' in their mind, e.g. 'two minuses make a plus', however I encouraged the use of the cauldron to aid answers.*

Natalie is motivated by developing students' conceptual understanding of negative numbers. The context of the witches' cauldron has previously been introduced during university sessions in order to develop meaning within the abstract context of negative numbers. Placing, 'conceptual understanding' in the position of truth allows us to understand the possibilities of the story. That is, unless we accept the status of 'developing

a deeper understanding of mathematics' the story cannot start to make sense. With the master signifier, 'conceptual understanding', in the position of truth a certain perspective on teaching is endorsed, i.e., the use of context or 'real' life situations. This knowledge or perspective on teaching, that 'students need to develop understanding', underpins what Natalie is doing, giving her a sense of identity and direction. To succeed in adding and subtracting negative numbers, students need to understand what a negative number is. In terms of discourse, this is fact.

Whilst the master signifier is in a position of truth, the authoritarian discourse of the master is rationalised (discourse of the university). It is only when Natalie challenges the system of knowledge; that the authority of the master signifier is coming into question (discourse of the hysteric). Natalie reflects, '*it was difficult to get the students to use this idea as they had a very fixed knowledge of the 'rules' in their mind'.* Here, Natalie acknowledges the difficulties in learning, and how previous experiences shape the students thinking. However, despite this disruption, Natalie remains loyal to the master signifier and carries on with the use of the cauldron but through this process, it produces the possibility of new knowledge as we can see in the next example. We fast-forward to Natalie's second placement, which takes place 15 months later. Here, Natalie is beginning her second teaching placement and is again teaching an introductory lesson on negative numbers (see appendix 6). Similar to the previous lesson, Natalie is using the analogy of the cauldron to develop conceptual understanding of negative numbers.

> *This week we introduced the topic of negative numbers to our low ability year 9 group; a small class of 7 students, with low but varied ability. We used the cauldron concept to aid us in this teaching which produced very interesting results, particularly in terms of class discussion and clear conflicts between student's thoughts.*
>
> *A pivotal part of the lesson was when we moved onto adding cold cubes to the cauldron, which demonstrated what happened when we add negative numbers – this is an area I have seen many students struggle with, all the way up to year 11. When discussion decreasing the temperature in the cauldron by 2, one began to take the hot colds out but was quickly reminded he couldn't touch these hot coals. This then clearly caused some conflict, which students showed through sighs of frustration.*
>
> *At this point, from previous teaching I have come to notice that it is important for the teacher to respond to these signs of annoyance and conflict with thought*

> *and in a way, that encourages further thought into the problem, rather than in a negative way, which may shut them down. I feel like although this will take years of practice, we were able to respond in a neutral way at first, while still encouraging students to continue in their patterns of thought. This conflict led onto exploration of the idea of hot and cold coals 'cancelling each other out'/using the annihilation method which is where we were hoping to get into in this lesson. Students discussed how the hot coals and cold cubes cancel out and our temperature is what we have left. The process the students went through to reach this point was one filled with disagreement, discussion, reflection and re-evaluation*

Natalie's reflection seems to confirm the value of reflective practice. She has changed some aspects of interaction with students based on her previous experiences and observations. Natalie, in addressing the master discourse, in this case the school curriculum or even her own experiences of being taught, no longer accepts things as they are or seem to be? How else could it be? In her first reflection, Natalie stated it was '*easy to explain'* but there was something missing, the students were fixed in their thinking. In her second reflection, she acknowledges that there is more to just 'telling them', rather it is a process of '*disagreement, discussion, reflection and re-evaluation*'. She identifies the struggles students may have as learners, for example, the problematic nature of adding two negative numbers. Natalie's use of context and conflict are indicative of her teaching strategies to developing a more meaningful understanding of the addition of negative numbers. That is, Natalie is motivated by an underlying interest in seeing negative numbers as objects not just operations. What holds the assumptions of 'using context' and 'cognitive conflict' in place is its endorsement by Natalie; this is going to help my students learn. Natalie's own desires are important in the process of her development as a teacher. As a student teacher, she is still acquiring the skills and strategies, which allow her to respond to the different ways her students think. Identifications through such discourses shape the individuals thought, mode of practice and even their professional identity. There are several permutations of Lacan's framework that can be used to analyse the discourse.

When *mastery* learning occupies the position of truth (discourse of the university), it holds meaning in place, systematic knowledge becomes fact. This is to say, certain pedagogical strategies are expected without question, a perspective on teaching. This knowledge, this way of understanding pedagogy says that, '*disagreement, discussion, reflection and re-*

evaluation' are important if not essential in the process of learning. In terms of discourse, this is fact, non-negotiable. It presents the illusion of neutral knowledge and addresses Natalie, and thus she tries to enact and reproduce the knowledge system. This knowledge, this way of seeing addresses the idealised subject in the form of the characteristics of the 'good' teacher. In doing so, systematic knowledge projects an image of complete teacher. As always, there are different interpretations of the discourse.

If we rotate the structural positions and place the object of desire (*a*) in the location of agent, then the value of social structure is brought into question (discourse of the analyst). Natalie addresses the discourse as an incomplete subject, seeking to understand the knowledge structure. Through her *previous teaching* experiences, she seeks to rework, rethink her pedagogical practice. Seeking to understand her practice, she produces new meanings; this is the way a teacher should respond to student conflict. However, while trying to occupy a critical position, Natalie is still underpinned by the knowledge of language, previous discourses.

4.6.2 Ali's Story

Ali is seven weeks into his first school placement, where mathematics teaching is organised using a *mastery* approach. In this context, school mathematics is conglomeration of approaches that are packaged under the umbrella term 'Mastery'. For example, both Maths No Problem (2020) and White Rose Maths (2020) influence the mathematics curriculum framework. Currently in Ali's placement school only pupils aged 11-12 are following the *mastery* approach, with plans to roll out *mastery* teaching to successive year groups each academic year. The lesson structure is in five parts; diagnostic question, anchor task, journaling (sometimes replaced by guided learning), group task and independent work (appendix 7). The *mastery* discourse functions as an idealisation of teaching, that is, the vocabulary and language of the *mastery* approach can provide the orientation through which Ali recognises himself as a mathematics teacher. Although this *mastery* discourse offers a way of understanding what it is to be a mathematics teacher, the Imaginary domain, it lies some distance from the social processes that Ali experiences. Ali's story consists of a series of conscious actions, unconscious processes, disruptions and unplanned events. Whilst, in solidarity with the master discourse, there is the wish to

overcome anxiety due to behaviour of the students and create a meaningful and purposeful classroom.

> *With this class, I have found it can get difficult to manage the classroom when some students are not challenged and stretched. The lesson was based on percentage increase and decrease without a calculator and I included an anchor question which was on the GCSE exam last year as I felt it was accessible for all pupils. The question was "If I increase an amount by 20% and then decrease it by 20%, do I end up at my original amount?" My judgement was correct as I have got to build a good rapport with this class and know the level of understanding most pupils have. Students worked well independently to complete the task and used think-pair-share effectively.*

Ali is identifying his students as particular types of learners, that is, without appropriate challenge they are difficult to manage but also, he wants to ensure the work is accessible to the students. Here, the discourse of the students, their responses, provide an orientation of how successful his teaching was. Through his reflections, Ali is identifying his students' learning habits and his perceived teaching weaknesses. This analysis informs him on an alternative teaching strategy and he uses GSCE exam questions as an anchor task. Is Ali making his decisions based on pedagogical reasons or as a behaviour management strategy? As the discourse unfolds, there are moments of clarity and understanding. Lacan (in Parker and Pavon-Cuellar, 2014) claims these key points, *points de capiton,* function as anchors of representation. Fleeting moments 'this is the way things are' (ibid, 2014:41). These temporal *points de capiton*, come together to mean something. In this way, we can see how Ali has moments of clarity, *'My judgement was correct as I have got to build a good rapport with this class and know the level of understanding most pupils have.'* However, meaning develops in the realm of the Imaginary. Through these moments of clarity, Ali establishes an idea or identity of himself. Here, successful teaching is seen as building, '*a good rapport with this class'*. However, these moments are brief; the discourse is already moving on, other *points de capiton* are emerging. As Ali goes on to write:

> *However, there was one slight problem, which I could have picked up whilst planning or even adapted to better during the lesson. After this task, I planned for students to journal their methods in working out the answer and other discoveries they had including definitions of an extension task with examples of percentage increase/decrease like depreciation, inflation, interest etc. Due to*

> *the open-ended nature of the anchor question, most students began journaling and had already wrote paragraphs on their findings before I introduced the 'Journal' task.*

The *mastery* approach can be seen as a master signifier, with which Ali is identifying. Mathematics is taught in this way. To place the *mastery* approach in the place of agency it has to be endorsed, or supported by a certain truth, it is simply accepted as having a value or validity. This is significant because it plays a role is structuring Ali's sense of identity and direction. Thus, the *mastery* approach, serves to organise the way mathematics is taught and understood. In this case, the five parts to the lesson are accepted, as this is the way things are and must be. To accept the *mastery* approach as the master signifier, you have to believe in the value of teaching and learning mathematics. That is, mathematics as a subject and qualification offers opportunities to succeed in our world and culture. In using GCSE exam questions, Ali could be seen to be using the function of knowledge as a means to deliver the *mastery* approach. What we are doing is important; it is going to help you pass your exams.

If on the other hand, knowledge (discourse of the university) is placed in the position of agency then our actions are determined by the system of knowledge or belief. For example, the positioning of mathematics within the education system. When the education system works, it appears not to be a system, it is naturalised and undisputable. What after all, could you do without education, mathematics? Ali and his students are complicit in this and desire only in ways that function to enact, reproduce the system. For example, in gaining a pass grade in mathematics. The master signifier underpins the discourse of the university, which in this case could be the *mastery* approach.

Another reading of Ali's actions can be made by placing $, the divided subject, in the position of dominance. Ali driven by uncertainty, questions the dominance of the master. Does it have to be this way? Is there another way of doing this? Through reflection, Ali is questioning the role of the 'anchor' task, how does it subscribe to his vision of teaching? He feels the open-ended nature of the task is interfering with his desire for stability and security. He recognises there is a gap between what he is doing and his articulation of teaching; in particular the engagement of his pupils. In the previous reflection, Ali was pleased with the use of a formal examination question as an anchor task, 'students worked

well independently to complete the task'. The use of the exam question adheres to the nature of incorporating an anchor task, but in this case, Ali feels he is still in control. Through his writing, you begin to see what is driving Ali, whilst trying to act according to the master discourse (*mastery* learning); he is developing 'relationship maintenance' strategies as a subtle way of getting pupils to act in line with the required behaviour and thus the proper conduct of mathematics.

4.6.3 Emily's Story

Before Emily's main second teaching placement she spent two weeks at university looking at alternative pedagogical strategies. For example, students attend lectures titled, 'making sense of algebra'. These lectures attend to using contexts that are designed to motivate, engage and develop conceptual understanding of pupils (e.g., Hough, 2012). Subsequently when Emily starts her second main teaching placement in a secondary school she is 'impressed' with the school's approach to teaching mathematics (which is quite a contrast to her didactical experiences on her first placement). The school has recently decided to incorporate a *mastery* approach and as such, a considerable amount of time is spent within the department talking about different pedagogical strategies. Whilst many of the students are challenging, Emily's initial reflection at this school is full of enthusiasm:

> *A mastery style of teaching mathematics is promoted in the departments, with a priority at KS3. After spending some time on this whilst at university, I am truly impressed by the teaching style. Whilst this is more in depth and requires more time, my ideas and teaching have changed significantly. I find myself picking up on very small elements of language and proof that I would not have noticed before. I can see the benefits of teaching students the 'why' and 'how' some abstract concepts of mathematics is useful and can be applied. I have stripped back my own knowledge of maths to then reteach in another way.*

Emily's reflective writing is presented as a discourse in which she is forming herself as a particular type of teacher. She is constructing her identity (or being interpellated) in response to the ideology of the *mastery curriculum*. Emily has an image of the teacher she wants to be. Spending time both at university and with her mathematics department has organised her desires and might explain her motivations to become a particular type of teacher. That is, Emily is making a link between sessions at university and her school's approach to mathematics teaching. The discourse of *mastery* teaching is strengthened as

it is endorsed by Emily's identification with the idea of an ideal teacher. 'So, this is what is expected of me', a teacher that asks '*why'* and '*how'*. In her writing, she comes across as a unified subject, one that has no resources for resisting ideology (Smith, 1988). However, turning back to Lacan we can develop a more complex account of both ideology and the subject of discourse.

In Lacan's framework, a particular agency is 'only a temporary subject effect resulting from a temporary subject position, and in addition, subject structure is not stable' (Alcorn, 1994:30). In the discourse of the university, placing 'succeeding in mathematics' in the position of truth, allows us to understand the possibility of the *mastery* teaching as the agency of the discourse. This systematic knowledge, this way of 'understanding mathematics teaching', addresses the subject. In doing so, *mastery* teaching offers an idealised vision of the complete teacher. Emily strives to teach in this way, she is even attempting to 'strip' back her own knowledge of mathematics so that she can teach this way. Here the discourse of the university is having a 'totalising and tyrannical effect' (Bracher, 1994:115), where *mastery* sold as good practice disguises the authoritarian master signifier with rationality. This means no provision is made for individual agency. Individuals are to act and think in ways that enact and reproduce the system. In doing so, S_2, systematic knowledge, functions to enact or reproduce *mastery* teaching.

Emily states '*my ideas and teaching have changed significantly'.* Meaning is produced by language, which is 'driven or operated by subject-functions such as desire, temporality, repression, the imaginary' (Alcorn, 1994:24). Lacan (2007b) proposes that many different discourse functions such as ideology or knowledge operate upon the subject. Desire might be expressed in relation to the type of teacher Emily strives to be, '*teaching students the 'why' and 'how''* of mathematics. That is, the Lacanian subject is connected to the realms of the Imaginary, the Real and the Symbolic. These unique subject functions 'produce the subject's particularity of discourse- a singular style of discourse that characterises the subject' (Alcorn, 1994:37).

As Emily learns new knowledge, she is motivated to change and modify her actions and even her identity as a teacher. Smith (1988) notes that the Lacanian subject can never be equivalent to a particular composition of knowledge but is operated by many layers of internal organisation. All these layers form a system, but the many parts of system are

never fully configured and this means that the subject can never purely be one thing, but be the divided subject $. In the case of Emily, the discourse component of 'mastery style of teaching' is the agency operating on her but it is not a simple reflection of the discourse system but through a synthesis of remembered discourse (in part a history of discourse). This echoes with Alcorn's theory of subjectivity where 'the subject operates discourse' (1994:20). Emily is not a mere reflection of the discourse but through her subjectivity, she manipulates and transforms the discourse. In this way, there is the possibility in the production of original discourse, new knowledge.

As the teaching placement progresses Emily is more concerned in building relationships, (in particular with her year 10 class) issues of classroom management are a constant concern. In the Lacanian framework, a rotation of the mathemes offers possibilities of new understandings. The next extract highlights some of her concerns and anxieties.

> *I feel more in control of the students-particularly the targeted students in year 10 that caused me issues. I spent a lunchtime detention with them talking things through and getting to know them. This has definitely helped and improved my relationship with them. I hope that moving forward this will continue and I especially look forward to parents evening next week meeting their parents.*

At this moment in her teaching, the emphasis is on building relationships with her students. Emily's teaching of mathematics is taking second place to relationship maintenance. She is positioning herself and her students within a particular power discourse. Emily is finding that student desires take priority over master demands, even if they are anti-productive. These are producing real tensions and the power relations manifest themselves in Emily giving a lunchtime detention and looking '*forward to parents evening'*. However, at the same time she acknowledges that she needs to talk and listen to the students. She is forming herself as a particular kind of teacher in which herself and students acquire specific identities.

If we place Emily, the divided subject in the position of the agent (discourse of the hysteric) this disrupts the authority of the master discourse. As Bracher notes the 'hysterical structure is in force whenever a discourse is dominated by the speaker's symptom' (1994:122). That is, her concerns and anxiety about the behaviour of her pupils manifests as a failure of the subject, $. There is a gap between what she thinks is expected from her as a teacher and her awareness of the performance. The wish for security and stability is

helping Emily develop identities for herself and students. In the search for meaning and security, the subject responds by providing a new master signifier, S_1, in the form of a secure meaning that will overcome anxiety and give a sense of control, stability and respectable identity. It is thus, as represented in the schema, the production of new master signifiers, covertly produces a system of knowledge S_2. In other words, the hysterical discourse challenges Emily's position and identity as a teacher, in asserting control and imaginings of improved relationships with students, she is producing a new system of knowledge, this is what it means to be a teacher, this is what the society expects from me.

4.7 Discourse, Resistance and Subjectivity.

All we can perceive, experience, think about, is the surface layer of reality. That is, the Lacanian subject is always a subject of the Imaginary and Symbolic orders. As Roseboro (2008:72) notes the '(speaking being) is temporal, connected in complex ways to the realm of the Imaginary, the Real and the Symbolic'. When we make sense of something, I like to think of the analogy, 'tip of the iceberg'. The surface appearance of reality is connected to our history of discourse. As Lacan insists 'that the early history of the subject stamps upon the subject certain characteristic patterns that remain stable throughout later historical progression' (Lacan, 1988, cited in Alcorn, 1994:32-33). This could explain why the subject has particular ways of thinking (ideals and values). Even the divided subject that is in a sense incomplete, non-self-identical, is still organised according to patterns contained by the subject.

Alcorn observes that a subject's identity pattern is not easy to change, 'it seems to have vast resources for ideological and psychoanalytical resistance' (1994:33). However, Butler (1997:97) reflects on the possibilities of 'subjective resistance to given forms of social reality'. To consider the complexity of the relations of resistance and the subject, Alcorn considers two types of resistance. In the first instance, the subject can resist 'bad' ideology or in the second instance, the subject can resist knowing that ideology is 'bad'. Whilst the two resistances are related, they are distinctly different. In the first case, Alcorn describes resistance to 'bad' ideology as *political resistance*. Alcorn's point can perhaps be illustrated in the discourse of assessment. For example, assessment driven requirements

promote particular conceptions of teaching which standardise school mathematical practices and steer learners according to an arbitrary assessment criteria. There is a risk that pupils are served generally, rather than according to their specific needs. However, Alcorn argues that through political resistance, the subject (teacher) motivated by 'knowledge and self-consciousness, is able to intervene in the production of ideological effects' (ibid:33). In teaching, this could be seen as 'not teaching to the test', resisting the restrictions imposed by assessment.

Perhaps we can explore this idea of resistance through Natalie's reflections by considering the discourse of the analyst; this resonates with Alcorn's description of political resistance. By placing the object of desire (*a*) in the location of agent, Natalie is questioning the dominance of the master discourse. Her previous teaching experiences serve to offer her some resistance to the master discourse. The words she uses 'conflict', 'disagreement', 'discussion', 'reflection' and, 're-evaluation' are both her reflections on the processes the pupils went through and possibly indicative of her own emotions. In re-working and disrupting the master discourse, the analytic discourse, 'makes it possible to produce a master signifier that is less oppressive' (Bracher, 1994:124). Generally, in teaching this could be seen to offer new possibilities, a means of re-working or re-thinking the purposes of education.

Alcorn's (1994) other form of resistance, which he describes as *analytic resistance*, the subject conforms to ideology knowing it is 'detrimental'. Here, 'the subject does not use knowledge to effect a freedom from suffering' (34). Resistance in this case is repressing something the subject knows or thinks they know, so that can conform, 'get on with life'. In this way, the resisting subject does not see what is in front of it. This resistance to knowledge relies on the repression of the divided subject. This type of resistance could be evident in the discourse of the master, where the dominant and univocal master signifiers, S_1, demand compliance to certain structural or operational forces. For example, compliance to the assessment of pupils on a regular basis. These ideas raise difficult questions about knowledge and subjectivity and begs the question can knowledge change subjectivity?

We can analyse Emily's remarks by considering subjectivity, knowledge and modes of resistance. For example, to what extent is Emily, resisting the knowledge of the *mastery*

curriculum by repression and fear, her need to control the students? On the other hand, she is also motivated by knowledge, and is able to resist previous beliefs by acknowledging the need to '*(strip) back my own knowledge of maths to then reteach in another way'.* This suggests that as Emily learns new 'knowledge', she is motivated to change her actions, values and even identity. However, Alcorn maintains that a Freudian perspective would suggest that, 'subjects can never use knowledge in a disinterested way because knowledge is always intertwined with the structure of subjectivity' (ibid:35). That is, the subject always produces knowledge. The intimate link between knowledge and human interest was the central theme of a classic text by the major social theorist Jürgen Habermas (1972). Assimilation of knowledge is essentially connected to subject structure. In essence, it would mean a Capitalist is always predestined to be Capitalist, a Marxist forever condemned to be Marxist. How can a being be anything other than what one is positioned as being? This would be a disappointing extrapolation from the theory. Whilst some forms of knowledge, seem more independent of subjectivity than others do. For example, performing algebraic manipulation, multiplication tables and so on. These forms of knowledge seems less problematic to transfer and less prone to subjectivity. However, other forms of knowledge have a stronger relationship to subjectivity, for example, ethical, political and so on. Alcorn (1994:36) argues that 'while it is difficult, it is not impossible to achieve knowledge in these fields'. That is, through political resistance (discourse of the analyst) but also being attentive to the features of analytic resistance; it is possible to develop knowledge implicated in the structures of subjectivity. In this way, student teachers would acquire more agency in their practice.

The analytic discourse disrupts the demands of the master signifiers by thinking about it repeatedly and hence lessening its intensity by gaining insight about its workings. In this case, resistance to the master is motivated by knowledge and self-consciousness. Change in the subject is possible as the ego, '"processes" discourse and "learns" to respond differently to the insistence of the unconscious' (Alcorn, 1994:40). That is, through the recognition that reality is an illusion, the subject gains insight of who they are not, with new possibilities for the self.

A reflective and critical stance towards teaching encourages student teachers to analyse their practice and hence lessen the intensity of master signifiers. Taking a critical stance

provides an interrogatory position, within which the subject can (as far as possible), unsettle the dominance of initial identifications and is open to new possibilities. As discussed earlier, Ali in reproducing the master signifier (mastery approach) reflects on the behaviour of his pupils, in effect disrupting the dominance of the master discourse, through this process the master discourse is reworked and ultimately a new version of the master signifier is developed. Throughout this process, *Object a*, the object of desire is underpinned by subjective knowledge structures.

These accounts involve imaginary identifications of reality and as such, the discourse of *mastery* teaching is an effect of subjectivity. As Alcorn points out, 'the subject can do nothing other than largely internalise - and thus in some manner *be* - some manifestation of the discourse system' (ibid:40). However, through conflict and resistance, the subject is not a simple reflection of the discourse system but contributes to the production of original discourse, new knowledge. That is, the subject of discourse is operated on by many internal agencies such as desire, repression, that are unique to the subject. Through these narratives, it is evident student teachers are forming and reforming their identities. Similar discourses have different effects on the teachers involved.

4.8 Systematic Knowledge: Mastery curriculum

We gain significant insights into *mastery curriculum* policy discourses when they are read through the discourse of the university, particularly because systematic knowledge occupies the position of agency. Firstly, the *mastery curriculum* can be understood as being dominated by government rhetoric to outline systematic knowledge that addresses schools and teachers to enact and reproduce the prescribed knowledge system, accountable to high stake international testing. Secondly, it disguises the master signifier, using rationality to defend a position. For example, teachers seem to be offered a 'choice'. They endorse the *mastery curriculum* and choose to teach in a particular way. In doing so, the systematic knowledge projects an ideal, complete teacher. However, such attempts to colonise, through seduction and coercion (Bracher, 2006) inevitably fails and produce a compromised individual.

Returning to Ali's story, he uses the five-part lesson structure, which is endorsed by the mathematics department as a proxy for the *mastery curriculum* (see appendix 7). The

importance of using this approach becomes understandable when the master signifier-*master* curriculum is drawn upon. Placing *mastery curriculum* in the position of truth allows us to understand the possibilities of the story. That is, the *mastery curriculum* underpins the pedagogical approaches that Ali is using and this presents rationalised knowledge, a perspective on teaching. Ali is striving to be an outstanding teacher by embodying the characteristics of the *mastery curriculum*. In doing so, the *mastery curriculum* presents an idealised vision of the complete the teacher. This knowledge, this way of structuring a lesson is non-negotiable. In terms of discourse, this is fact. Teachers operating within this discourse all embody the five-part lesson, 'this is the way it is and the way it has to be'.

In order for the discourse of the university to function, that is, for systematic knowledge to occupy the place of agency it requires the recipient of the discourse to be receptive to established knowledge. As Bracher states, it requires the subject to empty 'themselves of any knowledge that might interfere with the knowledge in the discourse becoming an amorphous, non-articulated substance, *a,* to be articulated by discourse' (1994:109). The product of this discourse is the divided subject, $: the disaffected teacher. We can see this in several accounts about. For example, Natalie's continuous attempts to use the witch's cauldron to conceptualisation negative number, ends in frustration for both her and the pupils. Amir's difficulties with managing the classroom. Emily's concerns with building relationships. It is necessary for each of these student teachers to put the master signifier in the dock to interrogate the efficacy of its influence.

4.9 Summary-A theoretical framework

> *There is no whole. Nothing is whole.*
>
> *—Lacan, Scilicet 2/3 (1970*:93*)*

Frosh (2014:20) theorises on the subjects' often-fragmented position and describes the human subject as,

> *'never a whole is always riven with partial drives, social discourses that frame available modes of experience, ways of being that are contradictory and reflect the shifting allegiances of power as they play across the body and mind'.*

As such, it is quite common for the subject of discourse to take conflicting positions. There is movement between the divided subject and the desire to be complete. This might begin to explain how the subject can take different structural positions within Lacanian discourse analysis.

In repeatedly mapping out classroom interactions to different permutations of discourse, we generate different possible understandings. As Fink (1995:129) claims 'it allows us to understand the functioning of different discourses in a unique way'. In particular, it provides insights into the formulations between knowledge, master signifier, divided subject and otherness. It combines in one model, psychic structures, motivation, with semiotics and discourse (Bracher, 1994). In considering the various positions of the master signifier, we produce different understanding of how the subject engages with discourse. I suggested above that the master signifier (e.g., *mastery curriculum*), could take the position of truth, which endorses systematic knowledge, which addresses schools and teachers to enact and reproduce the prescribed knowledge system. However, the framework allows us to rotate the factors. How in each rotation does this affect the subject?

In each of the rotations, we need to consider how we theorise that which is the otherness, the *Object petit a* to the discourse. The *Object a*, is that part of the subject that is 'simultaneously left out of and produced by the identity established for the subject in the S_1-S_2 articulation' (ibid:114). Put simply the *Object a*, is the object of desire. The function of *Object a* can be filled by various things, for example, in teaching it could be achieving outstanding status, pay rise, promotion and so on. However, this is just a stopgap, insofar, because whatever the subject 'seeks can never be found because it was never something to be found' (Neill, 2013:343). Thus, if we consider *Object petit a* in the position of truth (hysteric), how does that affect our understanding of the divided subject being the agent of the discourse. Significantly, if *a* is the indefinable, that which is constantly unattainable, out of reach, uncovering it seems an impossible task. As Lacan stated 'There's no such thing as a universe of discourse' (Seminar XIV November 16, 1966). Yet the process of trying determine *a*, that which is indescribable, that which we can never find, leads to what cannot be said. As such, through the permutations of the framework there are multiple understandings.

Lacan's schema of discourse can help us guide the way for a clearer understanding of how discourse of policy documents, such as the *mastery curriculum* work and why often they do not work, at least not as intended.

Chapter 5 Social relations

5.1 Introduction

For many years, first as a teacher and later as a teacher educator I engaged with the work of the Russian developmental psychologist, Lev Vygotsky, in particular his idea of the *Zone of Proximal Development* (ZPD). This might be because it feels in part intuitively right. It provides a model and metaphor offering a sound rationalisation for the relationship between a students' (novice) learning and their cognitive development. That is, there exists a space between what the learner can do by themselves and what they can do with the assistance of a more capable other. This supposed connection provides an attractive concept for the teacher-student relationship. The expert providing a scaffold to support learning and then gradually removing the scaffold. However, the alluring nature of the Zone of Proximal Development masks the reality and difficulties encountered in learning. Why would a pupil resist the transfer of knowledge? Bibby (2008) argues:

> *'The seductive imagery conjured by Vygotsky's metaphor of the 'zone of proximal development' leaves hanging the nature of the zone and obscures the space it occupies, it allows us to ignore the difficulties and resistances which the learner will encounter and develop'* (Bibby, 2008:38).

Bibby warns of the *seductive imagery* of Vygotskian ideas, underlining the intrinsic tensions between the notions of 'the individual' and 'the social'. Much of what occurs in classrooms is beyond conscious reach. That is, the reality of teaching is very different from the rational process of conventional psychological theories of learning such as Vygotsky's Zone of Proximal Development. Rather than depicting individuals as biologically self-contained entities, this chapter takes the premise that individuals are relational beings implicated in symbolic networks. As Grosz argues, the Lacanian ego 'depends on the subject's relations with others' and 'is governed by fantasy, and modes of identification, and introjection' (1990: 31). In other words, the Lacanian ego is a relational entity, a result of fantasy. To make sense of learning encounters and to make sense of how policy discourse is enacted; this chapter considers different forms of social relations between policy and the subject of discourse.

Much educational research has been oriented on notions of individual cognition independent of emotion (e.g., Cobb and Yackel, 1996, Steffe, 1983). That is, social relations and individualistic learning tend to be different domains of study. Traditionally, mathematical meaning is the 'real and objective description of the intrinsic properties of objects' (Radford, 2006a:39). Thus, conceptions of meaning are not subject to negotiation. For example, the meaning of the sign-drawing of a circle reveals the real and objective description of a circle. Here meaning is an objective point of reference. However, Radford argues if meaning rests on the subjective intentions of an individual how then can real and objective knowledge be guaranteed? Following Radford's argument, this chapter takes the view that mathematical meaning and classroom mathematical practices are socially constructed norms. Mathematical objects develop meaning through negotiation. That is, they do not have meaning in themselves but subjects develop meaning through shared experiences built through shared language. As Alcorn argues, 'meaning is produced as language is driven or operated by subject-functions such as desire, temporality, the Imaginary' (1994:23). Here, Alcorn is referring to Lacan's belief that speech or Language is not neutral. The meaning of speech and language in this sense, always resides on the side of the receiver. That is, speech is something contained by the subject, their subjectivity.

Discussions in this chapter are based on how both learning to become a mathematics teacher and mathematical meaning occurs through participating in social activity. As an outcome of the interactions between student teachers - pupils, subject mentors and policy discourse - individuals come to occupy positions in the social world. As such, I use Lacan's notion of the human subject being produced through symbolic frameworks. In other words, both individual student teachers and pupils are seen more as actors in a symbolic framework, responding to perceived demands.

In the first part of the chapter, I discuss Vygotsky's work and the role of socio-cultural activity in the development of the individual. Next, I give a brief account of Lacan and how his theories privilege the notion of the subject and their subjectivity through social relations rather than individual cognitive entities. Lacanian subjectivity in relation to Vygotskian ideas provides a theoretical framework that disrupts habitual thinking patterns, with a view to opening more generative interpretations. I draw on data collected from student teachers' delivery of the *mastery curriculum* to provide some

exemplification for these theories. Mathematical development is explored and problematised. Drawing upon Lacanian discourse analysis I explore different ways of thinking about more familiar Vygotskian theories about social interactions in preference to theories of individual cognition. Here pupils' mathematical development and understanding are depicted as mediated experiences between *mastery* policies, the use of textbooks, pedagogical practice, teacher beliefs, tools and materials. This co-creation is explored through Lacanian discourse analysis, with questions being asked as to how subjects construct their identities through the training process and in response to the *mastery curriculum*.

5.2 The individual and the social subject

Vygotsky's work has been hugely influential in mathematics education (e.g., Chaiklin, 2003, Cole, 1996, Confrey, 1991, Leont'ev, 1981, Lerman, 1998, 2000, Radford, 2006a, Roth and Radford, 2010, 2011, Watson and Dawes, 2017) and his social-cultural theories have played a big part in the practice of teaching and learning for many years with many alternative nuanced interpretations that defy easy synopsis. Much has been written about Vygotsky in mathematical educational research and the purpose of this chapter is to problematise Vygotsky's ideas as they are perhaps understood by western cultures. For example, his ideas have encouraged attention in student collaborative work, in which students can 'discuss mathematical ideas and construct understanding' (Watson and Dawes, 2017:43). He has also contributed to interest in student dialogue and questioning of pupils. In a similar fashion the *mastery curriculum* encourages pedagogical approaches, specifically increasing whole-class interactive teaching, more teacher-pupil interaction, including increased questioning throughout the lesson and so on (Boylan et al., 2016). However, even though there has been some attempt to regard the student as a member of a social group there is still a trend in mathematics education research to favour the individual and simplify the role of the social (e.g., Bibby, 2008, Roth and Radford, 2011, Cobb and Yackel, 1996). As such, mathematics is often seen as being centred on the operation of individual cognitions confronting mathematical phenomena.

Vygotsky (1978) highlighted the importance of learning through action or activity. The central thesis of his work places social interactions at the forefront of his theories (e.g.,

Cole, 1996, Confrey, 1991). Confrey argues that for Vygotsky, 'activity is inherently social, and it is through the engagement of activity, in the company of parents, peers, teachers and others that intellectual development transpires' (1991:28). However, the lineage of educational research following Vygotsky had given rise to a number of different conceptions of learning through activity (e.g., Davydov, Leont'ev and Holzkamp). For example, Leont'ev argued that activity consists of those processes 'that realise a person's actual life in the objective world by which he is surrounded, his social being in all the richness and variety of its forms. In other words, these processes are his activity' (Leont'ev, 2009a:2). That is, individuals participating in activity are not making discoveries and constructing knowledge on their own; they are 'subjects of collective activity' (Roth and Radford, 2011:10). Leont'ev's (2009b) approach suggests that development between the individual and the sociocultural can occur in and through social interactions in the pursuit of collectively motivated *activity*. As such, the mind is conditioned by the past, your culture, your upbringing. For example, when Natalie, - a student teacher to whom I return to later - asks students to compete the following statement $\frac{1}{2} = \frac{2}{?}$, she has articulated the concept of equivalent fractions in a form that is recognisable to others. Such a depiction of fractions is possible in her culture, and is understood by other teachers and students. Therefore, in asking students to complete the fractional statement, individuals are being conditioned in response to a social framework, they are responding to what they think is expected of them. Fractions develop meaning in relation to other fractions or integers and so on. They do not have meaning themselves but they are accessed through narratives about them. The sign fraction is a cultural artefact that shapes possibilities for thought and actions, and its meaning is temporal, constructed in relation to experiences and socially defined filters (de Freitas and Walshaw, 2016).

It is through such motivated activity, that the psyche is a product of cultural and historical reflection (Roth and Radford, 2011). As discussed in chapter 3.3, the 'I', our sense of self is relative and relational, bound by sociocultural factors. When you say, 'I', it is the ego speaking. Hence the individual and the social can function as the 'I' and the 'Ego'. When we say 'I', our sense of self is an illusory sense of identity that is conditioned by our past, symbolic, and material reality (Elliott, 2002, Roth and Radford, 2011). In a similar fashion, through the participation in socio-cultural activity student teachers become 'cultural

beings through an unending process of subjectification' (Roth and Radford, 2011:10). Subjectification here refers to the process of reflexivity, whereby the subject, participating in cultural-historical experiences is continuously reconstructing their self-identity. For example, when Daniel -a student teacher whom I introduced in chapter one- reflects on his observations and experiences:

> *'I believe this to be an effective question (close-fronted, close ended) during this time in the lesson as, in my opinion, this definition is more likely to remain in the student's schema…. However, this question could cause fatigue in students who already know the answer to the question'.*

Daniel describes a subjective experience that is still expressed in a form that is understood by others. He is not merely describing what he has seen but uses specific language evolved in time to describe his experience. That is, the language used is constrained within his cultural-historical environment, to be understood and to understand. Each expression is a process of identification, and through these identifications, individuals develop their subjectivity. As Bakhtin states, 'it is not so much that the expression adapts itself to our internal world but that our internal world adapts itself to possibilities of our expression, to its possible ways and orientations' (Bakhtin, 1977 in Roth and Radford, 2011:11). Returning to Daniel, his understanding of *effective questions* may be described as a cultural tool, but it is only when he is able to actively deploy such *effective questions* that Daniel has agency in the process. As such, Daniel might understand his actions, thinking and speech as independent processes but his capacity to act is a product of collaborative cultural practices.

5.3 Internal and external processes

The process of thinking is problematic to describe, often we use metaphors to try to explain invisible thinking processes. Are there any real connections between how we describe thinking, 'made a discovery', 'pictured it', 'came to a dead end', 'worked it out', 'visualised' and the mental processes that we experience. As von Glasersfeld argues, 'among the most intriguing human activities that can never be directly observed is thinking or reflecting. At times one can infer thoughts or reflections…but the actual process of thinking remains invisible' (1995:77). Vygotsky (in Wood, 1988) suggests there are more than metaphorical relationships between the language used to describe mental processes

and that used to talk about activities in the physical world. Vygotsky explored the theory that our experiences in the social plane are gradually internalised, and so when we describe a mental process, there is a real sense that it has derived from our experiences and physical activities. When we ask pupils to describe their thinking they will often use metaphors based on their experiences so far. In Chapter 4 of *Mind in Society*, the concept of 'internalisation' was introduced, summarised by Vygotsky 'every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level: first between people and then inside the child' (Vygotsky, 1978:57). That is, through activity and communication the child learns to internalise the values and structures of their culture. As Vygotsky put it: 'social relations, real relations of people, stand behind all the higher functions and their relations' (1997/1931:106). Only subsequently, upon internalisation of social interactions does learning occur. As Hedegaard (2001) explains:

> '*By internalisation, Vygotsky did not mean copying but transforming the external interaction to a new form of interaction that guides the child's actions. Internalisation does not directly mirror the external social relations; it is a transformed reflection.'* (Hedegaard, 2001:16-17).

Thus, internalisation is not a simple transfer of knowledge from social activity to an internal plane but a transformation, which occurs, in the social environment and between individuals before being processed by the individual. That is, pupil's mathematical knowledge can be understood as a 'process of individualising the social' (de Freitas and Walshaw, 2016:18). This idea has been formulated by the metaphor of the Zone of Proximal Development, the notion there exists a space between what the learner can do by themselves and what they can do with the assistance of a more capable other. We develop meaning of mathematical objects and for that matter any objects by negotiation and relation to other objects. Objects do not have any meaning until we create an account for them, how they relate to other objects. For example, when a pupil first sees a square, it only gains meaning when he recognises it has four sides, it is different to triangle, it is called a square and so forth. Through linguistic and relational mediations, we begin to grasp what the social acceptance of what a square is. Rather than being passive, the pupil actively constructs knowledge of a square through these interactions. Looking at it through a Vygotskian lens this mediated object will be subjective, dependent on existing knowledge, and the social interactions with an expert other. When the pupil is familiar

with the properties of a square he can engage in classifying other shapes, for example, is a square a rectangle? He is drawing on shared meaning to engage as a member of a group. As Walshaw describes it, 'developing shared understandings is an ongoing responsibility of the teacher who must ensure interaction with as well as engagement and commitment from the student' (Walshaw, 2017:293). *Teaching for Mastery* as defined by the NCETM (2017) highlights the function of a teacher to *guide* pupils *to* actively construct knowledge and deepen understanding of key mathematical ideas.

> *'Lessons are designed to have a high-level of teacher-student and student-student interaction where all students in the class are thinking about, working on and discussing the same mathematical content. Challenge and the opportunity to deepen understanding of the key mathematical ideas is provided for all'* (NCETM, 2017).

The high-level of teacher-student and student-student interactions can be understood in terms of Vygotsky's Zone of Proximal Development at two levels. Firstly, at the whole-class level, the teachers are interacting with the students, allowing ideas to emerge from the collective activity of students. Secondly, student-student interaction allows 'understanding' to occur through students' engagement with each other and the designated task. Collectively, therefore, the Zone of Proximal Development allows 'key' mathematical ideas to develop, and individually, each student internalises a version that is supposedly constitutive of the shared knowledge after the intervention.

5.4 Mediation

Vygotsky (1978) with his group of students including Leont'ev (1981), Luria (1979) and Luria's student, Cole (1996), researched the development of higher mental functions through tool mediation. Mediation is grounded in the idea that the mind is co-constructed through 'culturally mediated, historically developing, practical activity' (Cole, 1996:104). This implies that external factors such as language, artefacts, text books, worksheets, mathematical symbols, policy and so on influence individual development. For example when Emily, whom I discuss later in this chapter, uses boxes and circles to represent variables in an algebraic equation it provides pupils with a way to express their algebraic generality. Emily is producing algebraic text, although not yet formalised, it provides pupils with a way understanding mathematical objects in a progressive manner. In a similar

fashion student teachers development is affected by continuous intervention of subject mentors, university tutors, policy documents and so on. For example, in chapter 1.5.2, I discuss how Daniel negotiates his position as a student teacher, where his subject mentor draws attention to 'the use mathematical language'. From a Vygotskian perspective, the constant intervention by the subject mentor is mediating Daniel's understanding of becoming a teacher. Over time the subject mentors role as external mediating agent will be reduced as Daniel initiates the process internally. That is, the processes first occur between individuals before they occur within the individual.

5.5 Lacan, language and the Symbolic

So far, in this chapter, I have discussed Vygotsky's work and the role of socio-cultural activity in the development of the individual. I now focus my attention on Lacan's notion of the human subject being produced through symbolic frameworks. Lacanian conceptual tools allow for the analysis of the interactions between the subject and the social. That is, they provide a theoretical framework that disrupts habitual thinking patterns. I problematise the individual in relation to Vygotsky's theories and consider how the subject mediates the Symbolic. For example, subject mentors through curriculum documentation, such as the *mastery curriculum*, guide individual student teacher development. That is, conceptions of mathematical pedagogical knowledge are mediated through the various discourses. Student teachers' production of mathematics is referenced by the demands of what they think is expected of them. Listening to student teachers talk about mathematics teaching and observing the interactions of student teachers and pupils can reveal how they mediate the curriculum, subject mentor advice, the teaching standards, and their subjectivity.

In chapter three, I discussed Lacan's psychoanalytical position as the individual's understanding of who she is, in their response to the symbolic network (Brown, 2011). The assumption follows that our speech and gestures are controlled by some indescribable agency. Lacan describes this agency of apparent control as the big Other, the multitude of symbolic networks that shape who we are. Our desire to fit in, to be socially accepted, defines how we act, dress, and speak and so on. This starts with the parent's fantasies of the child, and their desire for their child to do well at school, connect

with family members, fit in. Brown (2011) explains; 'the human subject is not an entity itself but a relational entity built through social interaction in the discursive environment where the relations have many subtle or concealed features' (115). To make further sense of the Lacanian subject it is useful to consider how it differs from the poststructuralist subject. A poststructuralist perspective assumes the subject is constantly taking position with respect to discourse, i.e. the subject is a consequence of discourse (e.g., Foucault, 1977). However, in contrast Fink (1995) describes the Lacanian Subject as a split or divided subject, the individual that can never be fully explained or fully self-aware within the language that is available. Brown and McNamara describe the subject as being 'forever on a quest to complete the picture she has of herself in relation to the world around her and the others who also inhabit it' (2011:20). Accordingly, Alcorn pays particular attention 'to the organization of discourse within the subject that produces the subject's uniqueness' (1994:37). Which suggests, a unique personal history of discourse produces unique discourse matter within the subject. As Lacan (1988) states 'language is completely burdened with our history' (285). In this model of discourse, the subject has an image of the world and their place in it. Each country, region, culture, and so on, has particular language and sites of inflection. Individuals move about the world according to a particular itinerary with perceived demands placed on them. This symbolic network, society's unwritten rules, directs and controls our acts (Žižek, 2006). However, as Alcorn mentioned each subject encounters this big Other in unique ways and because of this unique subjectivity, each subject processes social interaction differently.

Like Vygotsky, Lacanian theories privilege social relations rather than individual cognitive entities. As Brown indicates, 'both would claim that humans feed off the linguistic apparatus that surrounds them' (2011:116). Vygotsky argued that through the inherently social relations, 'We become ourselves in others' (cited by Roth, 2012b:465). In a similar but also contrasting fashion, Lacan rejected a self-contained biological ego that can be objectively described. He conceptualised the ego as dependent 'on the subject's relations with others' and 'is governed by fantasy, and modes of identification, and introjection' (Grosz, 1990:31). In this sense, both agreed on the subject being a consequence of social relations. However, at that point, their similarities diverge; they differ in their understanding of how individuals interact with social discourses. Let me explain further.

Participating in the activity of teaching, student teachers are evolving through a process of subjectification (Roth, 2012a). That is, student teachers use language to fit in and to be understood, in what Lacan (2000) calls the Symbolic order. In this way the 'subject comes into being' (Pais, 2015). In other words, student teachers (subjects) are transformed by their own actions in the classroom that are themselves a function of the Symbolic order. Student teachers may pay attention to the words (signifiers) that subject mentors say but Lacan would argue there is always a disconnect between concepts (signified) and words (signifiers), between what we say and what is interpreted (Roseboro, 2008:32). According to Campbell (2004) in the Lacanian model of knowledge, 'there is no possibility of a neutral representation of reality precisely because to describe an object involves representing it in language' (2004:35). Speech in this sense is a complex relationship between signifiers and responses. A signifier is the form that the sign takes, while the signified represents the concept. If, for example, we ask a pupil to draw a square, the word 'square' is a signifier. It is the form of a word or its sound image. If we see the word square in the mathematics classroom, it becomes a signified or a concept that it is consonant with and understandable to others. The idea that is represented by that word. Depending on the learner's experiences so far, he might have an image of a shape with four equal sides, right angles and so on. We interpret the signifier (word) and signified (concept) together as a sign. If the context of the situation changes, our grasp of the sign changes. If, for example, we hear someone say "the ball hit me square in the forehead", this representation has nothing to do with a shape. Lacan believes that the meaning of a sign is not fixed until a sentence is completed.

There is always a gap between what we say and how it is interpreted, between the signifier and the signified. For Lacan, these gaps and disconnections represent the movement of the unconscious into the Symbolic order (Roseboro, 2008:33). As Lacan stressed the 'the self is an Other' (Fink, 1995:1). At its most basic level when the subject speaks, 'an Other speaks for her' (Pais, 2015:378). According to Brown (2020), any attempt to identify with specific discourse or ideology is located by the individual's desire to please, to respond to the perceived demands of the Other. Freud described that Other place as the unconscious, and Lacan states that 'the unconscious is the Other's discourse' (Lacan in Fink, 1995:4). Of interest to this chapter is how did that Other discourse develop 'inside of us'? I focus on

how student teachers interact in Other discourses, such as the *mastery curriculum*. Analysing the speech of teachers and learners gives insight into their unconscious desires, the perceived demands of the Other. The interaction between teacher and learner is never neutral. To put a concept into words (signifier) is to put it into context, to give it meaning, and to attempt to convey that meaning. With such a layered and complex conceptualisation of subjectivity, I try to discuss subjectivity in terms of what can be discovered about it both individually as a researcher and by analysing the language and writing of both pupils and student teachers. Let me explain further by sharing a short extract of a lesson observation and a reflection from my journal. I observed Ali, seven weeks into his first main teaching placement. The mathematics department are following a *mastery curriculum* and Ali is teaching a lesson on the multiplication and division of decimal numbers:

> *Using the lattice method to multiply 13 x 42. It is good that you are linking back to previous learning, although by using this method you are distracting pupils from noticing the structure of mathematics.*
>
> *It might be worth asking question such as:*
>
> *If 13 x 42 =546*
>
> *What is 1.3 x 42? Why?*
>
> *What is 1.3 x 4.2? Why?*

Extract from my observation notes 20 November 2018

After the lesson observation, I noted in my journal.

> *I notice something, I then write about it, but when I put it in to words it is just partial story of what I think I imagined. In the process of writing, something is lost. In considering what I notice, I realise it is a subjective task. The meanings I produce are based on my particular configurations of how I make sense of teachers, mathematics and education. I imagine I understand this discourse. That it has some true meaning, an underlying truth that is to be uncovered. Here lies the crux of the problem, my imagination; interpretation is just one way of viewing the discourse. Observing the lesson last year would create a different version of events. Another analyst would see it differently. Even if the teaching episode was videotaped and played back to the teacher, they would in all likelihood see it differently from what they intended in the first place.*

(Journal entry November 2018)

In analysing my writing, it is useful to consider the specifics of the activity and I will attempt to specify the Other, to which my feedback is intended. My words constitute my understanding of becoming a mathematics teacher. However, I do not find myself in the text, for I am not there, it is the discourse of the Other. In the above example, I make sense of an observed lesson through the network of symbolic structures; the teaching standards, curriculum and departmental policies, or in Lacanian terminology the Big Other of mathematics education, representing the reality of teaching and learning of mathematics. For example, when I write*, 'It is good that you are linking back to previous learning*', my use of language is making use of the official discourse of the teaching standards. In being obliged to use such terms I am responding to what I perceive is expected from me. My own performance as an *observer* or *teacher educator* is itself open to scrutiny. Such comments point to an apparent demand to build on previous learning, take account of prior learning. I start in a positive tone, praising the student teacher with words such as '*I like*', but the intonation changes with the comment, '*although by using this method you are distracting pupils from noticing the structure of mathematics'*. Such comments point to an apparent demand of what is expected from a teacher. Why is it important to notice the structure? What do I mean by structure? Mathematics? Let us consider the word mathematics, a google search provides a succinct definition 'the abstract science of number, quantity, and space'. However, to make sense of this you would need to know the definition of number, which in turn would reference arithmetic, which in a cycle would take you back to a 'branch of mathematics' (Oxford University Press, 2019). As such, words are defined through other words, in this sense it is closed system, a symbolic network. Or perhaps by combining two words we produce new meaning effects that extend the network. To assume meaning of a word implies the convergence of the Symbolic into the Imaginary realm (Neil, 2013). When we encounter discourse whether it is text or speech we might imagine understand each other and we are carried away by this imagining. However, if meaning rests on our subjective intentions, then, how can the objectivity of meaning be guaranteed (Radford, 2006a)? Radford argues that ideas and meaning of mathematical objects are 'conceptual forms of historically, socially, and culturally embodied reflective, mediated activity' (ibid:42). That is, the text does not belong to the author or the reader, but constitutes the reality for both. I might imagine Ali understands

what I am writing but the words do not exist independently of the subject's encounter with those words. Ali might identify with particular words or phrases in the text, such moments of identification operate on the Imaginary. That is, the discourse is processed and contained in a unique way, with unique demands place on the subject.

5.6 The Zone of Proximal Development

I now turn my attention back to Vygotsky's Zone of Proximal Development before drawing on Lacanian theories of subjectivity to explore different ways of thinking about social relations. With the publication in 1978 of *Mind in Society,* the English reading audience was introduced to the metaphor of the Zone of Proximal Development (ZPD). Vygotsky defines it as:

> '*the distance between the actual developmental level determined by individual problem solving and the level of development as determined through problem solving under guidance or in collaboration with more capable peers'* (Vygotsky, 1978:86)

A simplified interpretation of Vygotsky's metaphor conveys an unproblematic interaction between two individuals, one that is a more competent person and a less competent person. The idea that there exists a space between what the learner can do by themselves and what they can do with the assistance of a more capable other. The teacher and student engage in an activity where the learner, with the assistance of more capable other constructs knowledge. Described by Roth and Radford (2010:299) as 'an interaction within the individual consciousness and what happens in collective consciousness'. The expert providing a scaffold to support learning and then gradually removing the scaffold. This concept provides student teachers with an appealing metaphor to rationalise their role in the classroom, providing a point of reference or identification for teachers giving a sense of purpose. However, it places the teacher on a pedestal, and master of all knowledge, a position of idealised omnipotence. The teacher's authority must not be challenged, a place where the teacher's difficulties can be concealed, masked. By taking control of discussions, the teacher places the responsibility on the learner to jump from the position of not knowing to one where they can do with the assistance of a more capable other. It can mask their insecurities. The questions teachers ask are based on what they consider important in the acquisition of knowledge. The way the questions are asked aligns with

the preferred social dynamics of pedagogical encounters. Knowing and more importantly not knowing has implications for our confidence, the fear of exposure of not knowing can create anxieties. Pupils will often try to hide their lack of understanding by muddling through questions saying 'I don't care about this' or, 'what is the point?'. Alternatively, saying what they think the teacher wants to hear. Teachers are complicit in this interaction, often not wanting to expose the students' lack of understanding especially as this could have implications for their own esteem as experts that are passing on knowledge.

The transfer of knowledge by an expert to a novice gives the impression of a rational benevolent classroom. Why would a pupil resist the transfer of knowledge? In doing this, the metaphor allows the teacher to ignore any differences between learner and teacher suggesting that the learner's difficulties are trivial and readily subdued to the teacher's benevolent intentions (Bibby, 2008). How pupils interpret the teacher's instructions and how the teacher interprets pupils' responses are imaginings of a truth. I think the teacher is saying this… I think the pupils understands this as …Well there is a lot to imagine. It is not surprising that learning is a difficult subjective task. Take for example, a classroom observation where the passing on of knowledge is interpreted in a different way to the original intentions of the student teacher (expert in this situation). In the following, I reflect on Natalie's lesson. Natalie is a student teacher ten weeks into her first main teaching placement where a *mastery* style of teaching has been developed by the mathematics department. She has planned a 'feedback lesson' based on a test that was completed the previous week. Natalie has marked the tests and each pupil received individual feedback given through Strengths (S1-S7) and Targets (T1-T7). Each strength and target is based on a competency of algebraic manipulation, for example, S1 'I can collect like terms' or T1 'I need to practice collecting like terms'. At the start of the lesson, Natalie discusses some of the more common 'target' questions.

> Natalie writes on the board:
>
> $3q + 5q^2 - 7q^2 - 2q$ 'Like' terms were highlighted in the same colour. Mini whiteboards are on the pupils' desk.
>
> Natalie: 'Show me on your mini whiteboards the answer', pointing to the algebraic expression on the board.

Most pupils are scribbling on their boards. Some writing and then rubbing out. After 30 seconds, Natalie asks the pupils to show their mini whiteboards. Some pupils hold their boards high, some tentatively, Natalie acknowledges all answers.

The class, split into two camps, about half opting for $5q + 2q^2$ and the other half writing, $q - 2q^2$. One pupil had written $q - (-2q^2)$.

Natalie: What have you written on your mini white-board? Pointing to $q - (-2q^2)$.

Pupil: q minus brackets minus two q squared.

Natalie writes on the whiteboard at the front of the classroom $q - (-2q^2)$.

Natalie: I don't think you need that extra sign.

Natalie proceeds to rub out the negative sign in the brackets.

Natalie: Three q minus two q is q and five q squared minus seven q squared is two q squared.

Natalie then writes on the board $q - 2q^2$

Natalie: Is that ok?

Some of the pupils respond with nods. The teacher takes this sign (or absence of signs) as confirmation to carry on with the lesson.

In this classroom, the daily rituals and practices are defined to guide how pupils act and what they say. For example, pupils are participating in using the mini-whiteboards in socially developed ways. They know how to interact with the mini-whiteboards, scribbling their answers in a timely fashion. The explanation provided by Natalie (social activity), is part of wider social framework. How to act in the classroom, the pedagogical activities are representative of the expectations of the curriculum. For example, the use of mini-whiteboards are indicative of contemporary mathematics pedagogy and thus reveal the forms through which mathematics can be expressed, recognised and validated. Natalie has the potential to promote or hinder conceptual thinking in the pupils. That is, the mathematics in this classroom comes into existence in relation to the mediations between Natalie and the pupils, both in the past and present (Walshaw, 2017:295). The effectiveness of the activity is dependent on the strength of the connections between previously acquired knowledge with the goal focus of the activity. She uses specific algebraic notation to make those connections. Vygotsky argued that 'societal functions…become functions of the personality' and 'development proceeds not toward

socialization but toward individualization of societal functions' (Vygotsky, 1929/2005, cited in Roth, 2012b:453). In this example, this could be understood as a process of the pupil(s) internally trying to generate meaning of collecting like terms but also the socialisation of the individual, i.e. to behave or act in a particular way, the two processes are mutually constitutive.

Natalie asks the pupils to show what they have written on their whiteboards. Pupils are required to use a shared language - in this case writing algebraic expressions - to be included in the social exchange. Pupils may or may not make sense of collecting like terms. Natalie is using strategies, such as pointing, asking, writing, to organise individuals' thinking. For Lacan, 'dialogue seems to function as the alienating experience, the *stade du miroir* phase of a child's development' (Emerson, 1983:256). That is, there is a gap between the speech of the teacher and place pupils occupy. For the teacher, it is unknown how the pupil will develop meaning of the mathematics they are teaching, while, for the pupils the unknown is the mathematics. The pupil, in attempting to make productive sense of the mathematics is struggling to make connections with Natalie's intentions. Natalie ignores the 'extra negative sign'. It is disruptive to the master discourse. Knowledge acts as the ultimate object of desire. However, the by-product is the divided subject, the disengaged pupil. In trying to meet the demands of the teacher, in agreeing to Natalie's demands, 'is that ok?', both Natalie and the pupil are missing the mark. That is by micro managing the teaching, through performative standards such as strengths and targets; teaching misses the point, the very thing it is trying to achieve.

Natalie is trying to rationalise the authoritarian discourse of the master. That is, it comprises a disguised master using rationality to defend a position rather than mere whim. Abstract thinking is at the core of mathematical thinking but this is also the main issue for learners. The reason the pupil does not understand the mathematics is not down to inherent incompetence but it is a breakdown of communication between the expert and novice. The message of the expert as received by the novice lacks jointly constructed meaning. However, the pupil in their desire to please, to respond to the perceived demands of the Other can give the impression of a rational benevolent transfer of knowledge.

The strengths and targets that Natalie is setting are based on the discourse domain of what pupils at age 12 should be able to do. The curriculum discourse in this case, the algebraic statements 'I can collect like terms', ultimately holds the rewards of secondary school mathematics. The fantasy of being successful in mathematics places the individual in acceptance of the mathematics curriculum. The teacher offering strengths reinforces the truth of the curriculum discourse. In other words, it can be seen as a coercion to submit to the 'master'. The targets are there to shape the students in such a way that they are successful in their exams. Žižek (in Brown and McNamara, 2011:6) argues that such rational structures that guide practice can provide a substitute for the deeper desires that we wish to satisfy. In the example above, the strengths and targets provide a framework for receiving a fast and easy solution in validating the achievement of both the pupils and the student teacher. Pupils are implicit in this. Žižek portrays a complex culture where there is desire for simple solutions. As such, the complexities of teaching, perceived demands provide a backdrop that can activate a desire for a framework to shape teachers' practice, allowing for a quick fix to get their achievement validated. In this case, the pupils demonstrating that they can do the mathematics as described by the curriculum and in doing so, the student teacher demonstrates that she is meeting the teaching standards.

5.7 Problematic nature of the interactions between individuals

The passing of knowledge from a more experienced other implies that the transfer of knowledge is one way. This perhaps explains the popularity of the term 'delivery', where teachers deliver a lesson. The metaphor of 'delivery' perhaps centres on notions of transmissive teaching, where the role of the pupils is that of a passive learner. Pupils learn from a more experienced other (often seen as the teacher). This interpretation challenges the importance the learners play in the acquisition of knowledge. It ignores the role of the teacher learning about the pupils knowledge, in the important time pressed profession of teaching there is no time to listen to what pupils really want to say, rather the teacher desires to hear a version of themselves, of what they are thinking. Anything different is disruptive to learning, to the authority of the experienced other. Obviously, this is a troubling interpretation of Vygotsky's Zone of Proximal Development. It goes against the grain of the *mastery curriculum* and the much acclaimed 'assessment for learning', where

the teacher at the very least takes into account of where the 'pupil is at'. The following example, serves to demonstrate the problematic nature of interaction between individuals, one that is a more competent person and a less competent person. Alice, a student teacher on her sixth week of her school placement is teaching a lesson on fractions. The classroom situation centres on how Alice presents a mathematical object, equivalent fractions. In doing so, it focuses on her conceptions of self, her beliefs in teaching mathematics and her engagement in the social structures of a classroom. She enacts how she understands the *mastery curriculum*. Alice poses the initial problem with the hope that pupils may notice the structure of the mathematics, to make connections to prior learning. However, there is a gap between her initial intentions and how the lesson progresses. It shows how tensions and difficulties arise when the purpose of the activity and pupil engagement with that activity are in conflict.

> Alice is standing at the front of the classroom; the 26 pupils sit in pairs behind desks. On each desk, there is a worksheet, mini-whiteboard and marker. Alice writes the following fractions on the board:
>
> $$\frac{1}{4} = \frac{2}{?}$$
> $$\frac{2}{5} = \frac{6}{?}$$
>
> Alice: Think about what should go in the missing spaces.

One of Alice's function as a teacher is to manage the social classroom to ensure pupils are participating and interacting within the social framework of instructing, questioning and listening. The procedures, practices and concepts that Alice is attempting to impart on learners are socially accepted cultural inventions. The pupils' mathematical development hinges on their mediation through the interactions of the teacher's words. In asking the pupils to notice an object within the culturally negotiated norms of a classroom, the notion of equivalent fractions is a shared social normal. Why should $\frac{1}{4} = \frac{2}{8}$, who says so? The teacher is trying to replicate cultural knowledge. In a Vygotskian interpretation, the pupils need to perceive the mathematical properties of fractions that define its possible uses in relation to their own thinking and in relation to the notion of an equals sign as equality rather than just the answer to a question. This interaction can be understood in terms of

the Zone of Proximal Development at the whole-class level, the teacher in asking the question, is highlighting the structure of mathematics. What do you notice? Mason (2010) conjectures that the process of asking questions and responding is intended to reproduce a shift of focus in the learner that the teacher has experienced. Mason suggests that when we ask closed questions, we are anticipating pupils notice what we, as experts want the learners to notice. The process of asking questions and structuring activities provides insights into what Alice is trying to achieve. Drawing the attention of the learner to the structure of the mathematics, Alice is helping to maintain control of learning; guiding the learner to understand mathematics in a particular way. The teacher is directing the learning. However, the process of transferring knowledge is problematic, there is a disconnect between the signs Alice is using and the meaning being created by the learners.

> Alice paused, but noticed that the pupils were confused and not sure, what was expected of them. After four seconds, she added:
>
> Alice: What have I multiplied one by to get two?
>
> Pupil answered not very confidently 'err two '.
>
> Alice: I've got four here so this must be?
>
> Alice pointing to the empty space in $\frac{1}{4} = \frac{2}{?}$.
>
> Pupil: Two?
>
> Alice: What have I multiplied one by to get two?
>
> Pupil: Two
>
> Alice: So, four multiplied by two is?
>
> Pupil: Eight
>
> Alice: Great. So eight is the missing number
>
> The pupil receives a positive credit for his contribution (part of the rewards system).

By validating contributions and asking pupils to deduct the missing numbers, Alice is trying to use students' ideas to shape instruction and to occasion particular mathematical understanding in the classroom. However, with the pupils not particularly forthcoming in their responses, Alice quickly adopts the role of 'explainer', while students respond to closed trivial questions. The agenda was procedural- pupils are presented with a method to imitate. There is little evidence of a mediated social understanding. There is a gap

between what Alice is saying and how the pupils connect with that account of fractions. On behalf of the pupils, there is a desire to please, meet the expectations of the teacher. The expectation is for the pupil to comply with the arbitrariness of the master discourse, where particular values and ideals are presented as an absolute truth. 'So, four multiplied by two is' the answer, there is very little rationalisation here. When the pupils says what Alice wants to hear, she offers praise, acts of confirmation. Pointing to, reminding, asking questions all serve to dictate what is being learnt and in what way, under the guidance of an expert. Given enough instruction, learners are often able to perform tasks such as, factorise quadratics, and add fractions and so on. However, when does learning become performance? A one-way process that is an instrumental reproduction, 'mirror of the expert'. How much instruction is adequate? Depending on prior experiences, children require different amounts of instruction. At what point is the teacher doing all the thinking for the novice. If learners do not understand an instruction given at one level, then further instructions are offered, the process of funnelling occurs (Mason, 2010). If the pupil does not provide the required answer, what does the teacher do? I need to be more precise, they still 'don't get it', be even more precise until they practically tell the pupil the answer. The teacher's questions are sequenced towards a predefined objective.

When a teacher hears a desirable response, they often use positive praise to indicate what the teacher wanted all along, thus preventing other pupils expressing their responses or difficulties. For a teacher working within the constraints of the curriculum, words have specific meaning that she needs to communicate with the pupils. If Alice is to maintain control of the learning and always be the 'expert' then developing mutual educational relationships is not going to be so easy to achieve. There will always be a one-sided dynamic, the expert passing on knowledge to the novice. This non-mutual, one directional learning will always be problematic, the pupil left unsatisfied.

5.8 Language and thought

The considerably recent invention of schooling and creation of the characters such as 'teachers' and 'pupils' create particular demands on adults, as they consciously try to transmit knowledge and culture. It also places demands on pupils as they absorb knowledge and culture, memorising and thinking in specifically constructed ways. Valuing

pupils' responses and contributions, helps generate social activities with the possibility of shared knowledge being constructed. For example, asking pupils to say or write what they notice supports learning as a social process, between people and then internally, within the pupil, a transformed reflection (Vygotsky, 1997/1931). That is, learning can be seen as the residue between the interaction of the teacher and the pupil. Of course, learning is never entirely successful and excesses are produced. This could possibly described as the discourse of the hysteric; where there is a gap between performance and articulation of that performance. To discuss this further, I offer an account of the experiences of Charlie. Charlie, a student teacher on her ninth week of her first placement is teaching a lesson on geometric sequences to a year ten class (ages 14-15). She is growing in confidence and routinely is able to engage and sustain pupil's interest and learning. Similar to Alice, Charlie poses an initial set of problems designed to produce a shift of focus in the learner.

> As pupils enter the room there are three sequences presented on the board;
>
> 2, 4, 8, 16, 32
>
> 3, 6, 9, 12
>
> 3, 9, 27, 81, 243
>
> Charlie: Think about what happens from one term to the next. Write a sentence about what you notice.

The pupils are set the task of noticing patterns according to a particular structure. The teacher is attentive, giving pupils space to consider and write down their thoughts. Through this social interaction, pupils are encouraged to notice particular structures. Paul, a pupil in the classroom offers a response to Charlie's enquiry.

> Charlie: What is happening in the first one? Hands up, hands up, don't worry about your answers.
>
> Paul: You multiply or divide.
>
> Charlie: What do you multiply by?
>
> Paul: Two.
>
> Charlie: Good. Okay, we call this a geometric progression, where each term is obtained by multiplying and dividing the previous term by a constant r.
>
> Paul: What do you mean by constant r?
>
> Charlie: When you are doubling, you multiply by 2.

> Charlie shows a new slide on the whiteboard, students have to decide whether each sequence is a geometric progression.
>
> Pupils write down what they notice. In response to the sequence 2, 4, 6, 8, 10, one pupil wrote; it is a geometric progression because it is times by two.
>
> Charlie, slightly flustered, reiterated, 'the term before needs to be multiplied by two, it's not the two times table'

By reframing the pupil's speech and introducing the term 'geometric progression' (GP), Charlie is socialising pupils in an attempt to move them from their intuitive reasoning into more sophisticated formally recognised rules of practice. In a Vygotskian interpretation of this interaction, learning is supported by means of language. Mathematical language 'acts as an instrument of psychological activity in a manner analogous to the role of a tool in labour' (Vygotsky, 1978:58). Unlike face-to face speech, where mutual understanding is the responsibility of both speaker and listener, writing requires learners to articulate what they mean in an intelligible and accessible way to the reader, the burden of responsibility is squarely placed on the writer. The pupil writes, 'It is a geometric progression because it is times by two', this does not conform to the concept and meaning the teacher is trying to convey. The pupil, driven by uncertainty, questions the authority of the master. Can it not be a geometric progression when it is increasing by a constant of two? Why not? The unknown fuels the subject and this produces knowledge, new possibilities.

Lacan distrusts the written word (Roseboro, 2008:35), because it does not allow for so much ambiguity as when a word is spoken. Transforming speech and thinking to written text assumes that the written words selected actually represent thinking and the reader shares a similar discursive understanding as the writer, that they correctly interpret the intended meaning. The pupil writes it is 'times by two', what do they actually mean? Do they mean it is has constant difference of two. These presumptions are problematic at best. Neill (2013) suggests reversing how we might understand the subject of text by looking at how the text constructs the subject who would be taken to be its author. This leads to the splitting of the author, into two, the writer and the reader who fills the words with meaning. In this teaching episode, both the teacher and pupils are learners. Charlie, the teacher does not know how her language, the writing of number sequences on the board will be mediated, for the pupil the unknown is what Charlie is describing as a geometric sequence. They are both responding to what they think is required of them.

Charlie wants her teaching to be effective; she might imagine what the characteristics of an 'outstanding' teacher are. She may assume that she, as the authoritative teacher, is better able to see things than the pupils are, but inevitably they see the mathematics in different ways. There is a mismatch between her understanding of being a good teacher and how the pupils perceive it. For student teachers 'closing the identity gap is what learning to teach is all about (Walshaw, 2008:124).

5.9 Shared meaning in the Zone of Proximal Development

If we accept, Vygotsky's premise that all learning is socially mediated 'then mathematical thinking begins with a taken-as-shared sense of the expectations and obligations of mathematical participation' (Walshaw, 2017:299). Mercer (2000) argues that teachers use linguistic strategies to develop shared meaning in curriculum-based goals. These linguistic strategies, informed by curriculum design, organise teaching. Curriculum design, such as *teaching for mastery* inform how students and teachers interact, develop norms of practice, advise the questions we ask, modes of assessment and so on. However, Roth and Radford (2010) challenge traditional conceptions of knowledge transmission, they argue that it does not necessarily mean a transfer of knowledge by an expert to a novice but propose that we acknowledge the differences in the 'interacting participants who become each other's teachers and students independent of their institutional position'(300). That is, learning is mediated, it is based on the communication of individuals. We might imagine that we understand each other, that what we hear, the text we read has a meaning and we might think that we comprehend what that meaning is. However, communication is a two-way process, belonging to both the speaker and listener. That is to say, each word has two sides. Meaning belongs to both the producer and receiver of discourse. For example, if we look at another teaching episode, where Alice observes and listens to what the pupils are saying, she notices the gap between what she as a more capable person is instructing and how the students are participating in the activity. Alice has been working on establishing routines and affirming her expectations; developing social norms, how to behave in the classroom and how to participation in mathematical discussion. The organisation of the classroom amplifies the view of what is expected and how learning might take place.

24 pupils sit at tables arranged in pairs. On each table is a, 'Do Now Task', the teacher also has the 'Do Now Task' displayed on the board.

The 'Do Now Task' has 3 columns ; the first column shows fraction of a time, e.g., $\frac{1}{12}$ of an hour, the second column has an analogue clock face showing a time, the third column displays a period of time, e.g., 30 minutes.

The teacher starts the task by pointing to the fraction, then to the clock and then to the period of time. The teacher's speech is very brief:

Alice: You need to match the fraction to the clock to the time. Draw lines matching the fractions, clocks and time.

Thus the mathematics itself is formatted into a particular culturally defined structure to depict how we as a society organise time, within an established school day. The imposed pedagogical structures define conventional ways of looking at things. Collectively the Zone of Proximal Development allows a classification system to develop, and individually, each pupil produces a grouping of fractions, clock faces and a time. There is an assumed shared meaning. In a Vygotskian interpretation of development, students are provided with cultural objects and activities to assist in grasping an achievement, which by themselves would not be able to realise. Alice draws attention to what she has in her mind. She points to the fraction, then to the clock and then to the period of time. In matching fractions and time, the pupils are participating in the activity of schooling but they are not fully aware of the motives of the activity. The activity is intended to reproduce a shift of focus in the learner (Mason, 2010), to make connections to prior learning and social experiences. A social interaction in a specific context. The expert conveying a message. However, the discourse of this message is far reaching, with unexpected consequences. This mediated object will be subjective, dependent on existing knowledge, and the social interactions with an expert other. How pupils or teachers interpret words of others or objects is built through shared social experience. There is a desire, imaginings by the teacher that the pupils can notice a connection between the fraction, the clock and the period of time. 'See what I see'.

It was quite noticeable that the teacher's requests were carried out immediately. The pupils were eager to begin and straight away started drawing lines that criss-crossed the page, joining corresponding fractions, to clocks and times. This being indicative of a familiar pattern that they have experienced before. At first glance, it seemed that all

pupils were to understand the task and were matching successfully. However, the superficial appearance of pupils successfully matching may govern most of the perspective of a teacher needing to check a class of 24 pupils. However, at closer inspection Alice notices that the majority of pupils had only matched three of the fractions ($\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$) with clocks and times. Observing two boys at the back, they spent the next 5 minutes drawing lines from $\frac{1}{3}$, $\frac{1}{12}$, $\frac{5}{6}$, to various clocks and then rubbing them out. They had more success matching the times to various clocks (20 minutes, 15 minutes etc...). Is this a bridge too far, the limited success of matching fractions to time would suggest there is a too big gap to navigate in the Zone of Proximal Development? Within the constraints of this task, the boys found it difficult to identify some of the fractions with times. It was quickly apparent that they had difficulty in completing the activity. They recognised common fractions such as $\frac{1}{4}$ and $\frac{1}{2}$ as I assume they have seen and heard them before, e.g., 'half past six'.

Alice noticed that the boys were struggling and asked one of them

Alice: Which fraction do you think matches with 12.50?

Andrew: I think it is $\frac{1}{12}$.

Alice hesitated in her reply and frowned and the pupil quickly interjected.

Andrew: No, I mean it is $\frac{5}{6}$.

Alice: Good, now have a go at the other questions.

Did Andrew realise the answer or was he guessing. Following the interactions with the teacher, Andrew is submitting to the master discourse, he 'successfully' matches the time with the fraction. From a psychoanalytical perspective, Alice holds the assumption of authority and Andrew wants to show his alliance by trying to 'guess' the answer. In giving the first answer of $\frac{1}{12}$, Alice's reaction, her hesitation gave Andrew the signal that this was incorrect. Recognising he has given an incorrect answer, he quickly tries another answer, $\frac{5}{6}$, this time it is correct. However, this does not imply there is a successful transfer of knowledge, not knowing the answer, he may have guessed at the other 'hard' fraction. He got lucky, and Alice plays the role of pleased teacher. This superficial appearance may be governed by what the teacher wants to hear. A desire to be understood and understand;

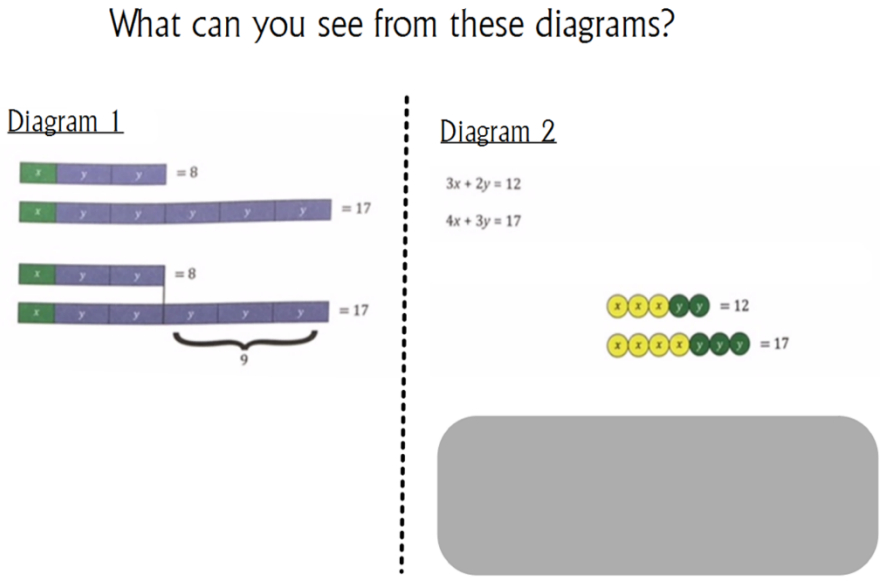
to be subversive to the acquisition of knowledge. Both pupils and teacher are complicit in this relationship. Andrew makes it known that he does not understand what is being asked of him and Alice tries to remedy the problem. Not only does Alice guide Andrew to the point of matching the fraction and time but Andrew also guides Alice. In this interaction Andrew exhibits social competence, in responding to signs that Alice gives he understands what is expected of him. By responding, Andrew becomes an active participant and 'opens up possibilities for intersubjectivity to appear' (Roth and Radford, 2010:303). The actual question, 'which fraction do you think matches with 12.50?' is problematic, rather than the demand for an answer. Andrew might have responded with 'I do not know?'

5.10 Developing mathematical meaning: Visual representations

Boylan et al. (2019) found schools influenced by the Mathematics Teacher Exchange (MTE) showed an increase in the uses of models to develop conceptual understanding. Additionally the National Centre for Excellence in the Teaching of Mathematics (NCETM) through their programme of professional development supported the use of models. Unsurprisingly, during this study, student teachers were observed using visual representations and artefacts in attempts to make mathematics meaningful. Student teachers put in a lot of effort into making and using visual representations (e.g., bar models, number lines, whole-part diagrams) and the use of physical materials (e.g., multilink cubes and Cuisenaire rods) in the classroom. These pedagogical forms inevitably provide filters on the relationship on how we come to understand mathematics teaching and mathematics itself. In a Vygotskian interpretation of children's development, models and representations assist in mediating and materialising thinking, providing the opportunity to realise and achieve that might otherwise remain out of reach (Walshaw, 2017). To discuss this further let us return to Emily who was introduced in chapter 4. Emily is nine weeks into her second teaching placement in a secondary school where a *mastery* style of teaching has been developed by the mathematics department. In this lesson, she is using visual representations to assist in developing mathematical understanding of simultaneous equations.

> After a brief introduction to the lesson, Emily shows two diagrams, and asks pupils

‘What can you see from these diagrams? You have two minutes to think about this. On your mini-whiteboards quickly write down what you notice’.



Sitting in silence, not sure what is expected of them, pupils start scribbling on their mini-whiteboards. Many pupils copy diagram 1. Emily circulates the classroom, observing pupils writing their answers. She stops at a Kim’s (pupil) desk and utters ‘interesting’, after two minutes Emily asks the pupils to put their pens down.

Emily: Kim, could you read out what you have written.

Kim: Err, okay. So, I added the top line with the second line and that made 25.

Emily: So what did you write, can you read it and I will write it on the board. What did you write?

Kim: x + 2y + x + 5y = 25.

Emily: How did you get 25? What did you do?

Kim: I just added the top line with the bottom line and that makes 25.

Emily: Yes, good. Did you notice anything else?

Kim shakes her head in a negative response.

Emily: What would happen if you take away the first line from the second line?

Kim: You’d get nine.

Emily: Okay, good. How much bigger is the second line?

Kim: Nine.

Emily: Okay, but by how many boxes is it bigger, how many extra y's are there? So how much bigger is it?

Kim: Oh, three boxes, which is three y's

Emily: Very good. Does everyone see that?

A central element of the concept of this activity is to find two unknown quantities in a pair of algebraic equations. In this lesson, Emily used the visual representation of equations to develop the concept of simultaneous equations. The boxes and circles representing the unknown quantities help to mediate mathematical understanding. That is, the boxes and circles lead to visualisations of simultaneous equations in a particular form; they make understanding easier. As Radford (2006b) would suggest, the diagrams are more than mere aids to thinking but rather constitute thinking. Emily directs her pupils' gaze towards the visual representations, 'what can you see from the diagram?' In highlighting and validating Kim's contributions and asking further questions, Kim is developing a particular mathematical understanding. Emily is strongly engaged with the Symbolic of the *mastery curriculum*, she is enacting what she thinks is expected from a mathematics teacher. The strategies she is using are what she believes a 'mastery lesson' might look like. Throughout this interaction the objective of the activity may be clear for Emily, but this is not necessarily clear for Kim or the other pupils. As Radford argues, 'If the objective were to be clear to them [the pupils], then there would be nothing left for them to learn' (2006b:13). Kim's interactions with the Emily's words make it possible for her to reflect and notice the difference between the two lines and offer an alternative response to the original question. Emily watching and listening to what Kim is doing and saying, responds accordingly, in this way interactions between the teacher and the pupils, through use of words and symbols has the potential to modify knowledge. Radford's description of thinking explains how the problem upon which the pupils are reflecting on is a historically formed cultural reality.

> *Thinking is as a reflection, that is, a dialectical movement between a historically and culturally constituted reality and an individual who refracts it (as well as modifies it) according to his /her own subjective interpretations and feelings* (Radford, 2006b:6).

That is, solving the problem does not lead to some abstract new knowledge but constitutes ways of perceiving mathematics. In analysing Emily's enactment of the discourses of *mastery* teaching, I consider the Other, which Emily is responding to. From observations in the classroom and her reflections (See Chapter 4.6.3) it is becoming evident that Emily is working hard to construct an identity that is not merely an illusion of the teacher she wants to be. That is, there was little conflict between the Symbolic and Emily's image of herself (Imaginary). Emily's enthusiasm for *mastery* teaching is perhaps exemplified in discussions after the lesson. I ask Emily on how she understands *mastery* teaching.

> *I think in our school mastery teaching is about challenging students and extending their knowledge rather than just giving them something harder to do. It's about deepening their knowledge so putting it into real life context into various different forms to see how they can apply what they have done. Using diagrams helps them see and understand, rather than just telling them how to it.*

In Emily's short history of training to teach she has formed images of what type of teacher she wants to become. Emily's understanding of the *mastery curriculum* and her desire to succeed as a teacher shapes her actions. In the nine weeks that she has been in this school, a view of mathematics teaching has been developed: using diagrams helps [pupils] them see and understand. Here, Emily's concept of the *mastery* teaching is based a belief that it 'is about challenging students and extending their knowledge' and this is supported by using diagrams to help them achieve mathematical understanding.

5.11 Asymmetrical relationships: the problem with the Zone of Proximal Development

The Zone of Proximal Development can be described as an asymmetrical relationship, where the teacher is more capable than another individual (Roth and Radford, 2010). As Bibby playfully suggests, 'this idea provides an attractive metaphorical image of teaching as a somewhat gentle, benevolent, rational process of drawing the less-knowing learner towards the more-knowing teacher' (2008:37). As suggested earlier in the chapter, this interpretation challenges the importance the learners play in the acquisition of

knowledge. If the teacher remains in control of learning and is always the 'more experienced other' then forming mutually reciprocal relationships is not going to be possible. Perhaps we can see the asymmetrical nature of teaching and learning when Natalie ignores the 'the extra minus sign', as if the pupils were resisting the transfer of knowledge. Highlighting a tension between the demands of the teacher, a proxy for the master discourse and the production of knowledge by the pupil. It seems the pupil's differences are irrelevant and distracting to the teacher's intentions. The extent to which student teachers direct learning was evident in many of the observed classroom interactions. For example, if we return to Alice's lesson on fractions. Alice plans a lesson on equivalent fractions around what she thinks, as a student teacher, is expected of her. She wants pupils to notice the structural equivalence when two fractions are positioned as equal to each other. However, with the pupils not forthcoming with their responses, Alice adopts the role of active explainer and pupils became passive learners (Swan, 2007). Alice is directing and controlling the transmission of knowledge underpinned by the asymmetrical relationship of an expert and novice. In this conception of teaching, there will always be a one-sided dynamic, the expert passing on knowledge to the novice. This non-mutual, one directional learning will always be problematic, the pupil left unsatisfied. A more symmetrical possibility is suppressed, where the teacher could directs questions on their better understanding the pupil's current understanding. The point here would be negotiate meaning rather than to suppose that a correct meaning is fixed or known in advance.

5.12 Discussions and Conclusions

In the transfer of knowledge, the asking of questions, the pointing out, something is lost. Pupils and teachers are required to use shared language to be included in social exchanges, as Brown suggests 'in this way the human subject identifies with something outside of himself. They see themselves in the social language, but the languages never quite fit. And through these identifications they craft their subjectivity' (Brown, 2011:105). In similar fashion to the nature of the Zone of Proximal Development, there is an asymmetrical nature between the illusory image student teachers have of themselves and the Symbolic. That is, there is a space between the Symbolic, the demands of the Other

and the reality of teaching and learning. This creates an opposition between the 'ego' and the 'I', which gives rise to a desire to close this gap. Through the participation in teaching, Alice develops her sense of self but she is never satisfied. Emmerson (1983) uses Lacan's model of child development to argue ' the child is released from his alienating image only through discovering himself as Subject, which occurs with language; but this language will inevitably come to him from the Other. Thus speech is based on the idea of lack, and dialogue, on the idea of difference' (256). Pupils are learning to be what the student teachers want them to be. In contrast to asymmetrical relationships, Roth and Radford propose a symmetric perspective where 'interacting participants become each other's teachers and students independent of their institutional positions' (2010:300). Perhaps we can see this when both Emily and Kim assume the position of teacher, and both assume the position of learner. Such relationships involve mutual trust and dialogue, a space that Bibby (2008) characterises as intersubjective. In this way, we can consider the 'teacher' as less experienced other and the 'pupil' the more experienced other. This might begin to explain how student teachers respond reflexively to generate through time and an evolving account of becoming a teacher. Emily, by listening to Kim's responses finds out if her instructions have been successful or not, and whether her subsequent actions bring about a desired response. That is, student teachers' understanding of who they are is produced through their responses to the symbolic network.

Chapter 6 Discussions and new openings

6.1 The forming of a teacher educator

This study has attended to how mathematics teaching is informed by beliefs and identifications through a variety of alternative motivations. A central theme throughout considers how mathematics teaching is conceptualised by the multifaceted discourses that pull student teachers, pupils and the teacher educator in different directions. Although the stories and narratives of this study never reveal the 'truth', they provide insights into the fantasies of who student teachers believe they should be or what they are trying to achieve. *Mastery* teaching places specific demands on student teachers to produce a particular version of mathematics. Lacanian discourse analysis offers unique possibilities in the understanding of how student teachers construct their identities through the training process and in response to the *mastery curriculum*. My own personal perspectives have evolved from being a pupil, a student, a teacher, a teacher educator and now a practitioner researcher. My research questions are relevant to a particular stage in my career. When I first started teaching in 1995, my main concerns were more about my own teaching and how this impacted on the education of young people, whilst my concerns have now shifted to broader aspects of education, it is still a function of the imaginary demands placed on me. The first part of this chapter considers my story of becoming a teacher, teacher educator and now a practitioner researcher. In writing a narrative of the self, I begin to reflect on the interactions between the individual and the collective. I consider how I have identified myself and in doing so what has been marginalised?

I started my teaching career some twenty-five years ago. After completing my Post Graduate Certificate in Education (PGCE) in 1995, I was fortunate to gain my first teaching post at St Joseph's college in Botswana. Free from regulative structures that defined teacher practice in many European countries, I was given an unusual amount of autonomy for someone who had just qualified. Straight from university and full of enthusiasm, I was able to experiment and develop my ideological practice. My experiences as a student and subsequently as a student teacher provided me with points of reference to what I assumed

was a successful teacher. For example, I remember re-writing the end of year exams, taking the task seriously, I looked at previous the year's papers. However, I felt they just tested procedural knowledge with no scope for demonstrating problem solving skills. I busily spent a week writing long-winded, complex questions that would address this lack of problem solving. The expression 'went down like a lead balloon' summarises the responses from both students and other teachers. Undeterred I realised that apart from the wordiness of my questions, pupils had not been presented with any aspects of problem solving, so I decided to incorporate investigations into my lessons. It was during this time I acquired 'space' to develop my practice but also to challenge my own assumptions.

In 1998, after two years in Botswana, I returned to England and gained a teaching position as a newly qualified teacher. Educational standards had become a high profile national issue, with a particular thirst for monitoring performance and accountability. A stark contrast from my relatively free reign in Botswana. The combination of curriculum and pedagogic prescription, the focus on levels of progression was very much at the heart of becoming a mathematics teacher. The National Strategies, a detailed framework that prescribed the specifics of what mathematics should be taught and how. Compliantly I bought into the system. I progressed quickly through the educational system, and after two years, I was 'promoted' to Key Stage 3 coordinator and subsequently to Second in the Mathematics Department.

Early in my career, I was fortunate enough to have two inspirational teachers/mentors, Joe Murray and Dave Bellis, they were both engaged in a student-centred approach to mathematics, with a particular emphasis on practical and problem solving work. At the time, they were often seen as being 'counter-culture'. To a certain extent, them resisting the prescriptions of the National Strategies, helped me disrupt the authority of the discourse. In Lacanian discourse theory, this might be termed as the discourse of the hysteric (see chapter 4.6). That is, I no longer accepted the National curriculum as an absolute truth. Influenced by my two mentors I often asked myself, 'Why are we teaching it this way? Can it be done otherwise?'. Despite these questions, I remained in solidarity with the master discourse, the National Strategies.

In 2004, I was accredited Advanced Skills Teacher (AST) status by Salford Local Education Authority (LEA). This provided me with the opportunity to spend one day a week supporting

other teachers in their professional development. Much of my time was spent working with teachers in what was termed as, 'failing schools'. These schools often defied the production of a clear definition of success. Regularly pupils resisted the best intentions of teachers, the sheer difficulty of being in the classroom challenged the increasingly narrow definition of education.

My AST role allowed me a certain amount of autonomy (or so I thought), and enabled me to pursue different approaches to teaching. I would often observe lessons and give advice on teaching strategies. On reflection, I was a proxy for the master discourse, disseminating National Strategies priorities, delivering a version of 'best' practice, which marginalised both my judgement and the judgement of teachers I was working with. For example, I remember working together with another teacher developing different approaches for a plenary to her lesson. Here, the task of teaching was developed in response to how we both understood what was expected from us.

This resonates with Radford's (2018:21) 'dynamic cultural symbolic superstructure', which naturalises the way individuals make sense of the world. On another occasion, I remember organising a network meeting to support teachers in their approach to 'Assessing Pupil Progress' (APP). We spent two hours scrutinising pupils' exercise books to agree on what a 'level 5 in mathematics' might look like. On reflection, I am surprised at the level of prescription that shaped mathematics education. This could be linked to the regulation of teachers and how Althusser (1971, 2014) regards education as one of the institutional ideological state apparatuses (ISA) 'through which the symbolic machine of ideology is 'internalised'' (Žižek, 1989:43). That is, the ideology of the National Strategies was shaping how I acted and talked about mathematics.

I was becoming increasingly frustrated with the level of regulation. I felt that my creativity and initial desire to develop innovative practice was being chipped away. Teaching was becoming a tick box of accountability. As the level of prescription and monitoring increased, I questioned my own teaching identity. Does it have to be this way? I remember specifically, the school I was working within devised a system of book scrutiny to monitor teachers' marking, ensuring every piece of work was marked with a green pen and each piece of marked work had, 'two stars and a wish'. I would spend several evenings a week conforming to these demands, feverishly trying to come up with worthwhile comments for

pupils to improve their work, even if they got everything correct. Meanwhile, educational league tables were becoming more prominent with increasing pressure for pupils to gain five GSCE's A- C including mathematics and English. Teachers and pupils were becoming increasingly under pressure to 'succeed' in mathematics, at whatever cost! A key criterion for a successful Ofsted lesson was that pupils demonstrated measurable progress (Ofsted, 2008). Choices needed to be made between ensuring exam success, promoting a love of mathematics, a conceptual understanding of mathematics and functionality of mathematics. There were different ways of prompting these different priorities. For example, in 2007, the DFE introduced functional skills in mathematics, to make mathematics more applicable in the workplace. National curriculum and assessment changes valued certain versions of mathematics. Educational policy was becoming more politicised, fabricating what it is to be successful at mathematics.*

At this point in my teaching career, there was a gap between the teacher I wanted to be and the perceived demands of educational prescription. I began feeling like an object of change rather than an agent of change. It was about this time I became interested in a movement offering alternative modes of teaching, in particular, during 2009, I became involved in an EU Comenius funded project, Learning and Education in and through Modelling and Applications (LEMA) (Wake, 2011). The project helped me to see beyond the discourse of the National Strategies and consider different approaches to pedagogy. As a group of teachers, local educational authority (LEA) consultants and teacher educators, we met on a number of occasions, including one residential meeting, at which we discussed classroom pedagogy. This allowed me to consider my own case study of professional development. I felt like the shackles were off, I could experiment with my own teaching. The whole process re-vitalised my teaching but it also introduced me to the world of research.

Not long after this, in September 2010, I started working as a teacher educator at Manchester Metropolitan University (MMU), where a new set of demands were placed on me. Reflecting on my early years at MMU, I developed a paradigm shift, in which my epistemological, ontological and methodological premises were altered. This development occurred during my transformation from a teacher of mathematics to a teacher educator, a shifting identity changing across time and place.

Currently most of my 'directed' time is divided between two educational programmes; Secondary mathematics Postgraduate Certificate of Education (PGCE) and the undergraduate BSc Secondary mathematics with qualified teacher status (QTS). I am often pulled in different directions by different discourses, on the one hand ensuring that student teachers meet the teaching standards, and on the other promoting alternative pedagogical approaches such as Conflict Teaching and Realistic Mathematics Education (RME).

In my role, I spend a lot of my time in different schools observing student teachers and interacting with subject mentors, they provide me with points of reference. At times, these markers offer conflicting stories, for example, relational understanding versus instrumental learning. My use of these shared points of reference compromise my individual voice. It is not me speaking but the a history of my discourse, these reference points anchor meaning, these points de capiton offer fleeting moments of 'ah, so this is what it means to be a teacher educator', but these moments are fleeting, temporal, discourse never stands still and another point de capiton emerges to offer new meaning. Recognising that my understanding of mathematics education is defined by my past and activated by current priorities, might help me be more critical of both my past assumptions and new directives that might otherwise constrict my thinking into particular ways.

I think my story demonstrates how I have gradually become frustrated with the limitations of education that I am expected to work with. This seems to be a far cry from my own beliefs when I first started teaching in Botswana over 25 years ago. Britzman characterised formal education as an encounter with 'an avalanche of certainty' (2009:2), a meeting with prescribed knowledge, and pedagogical strategies, with tests and league tables measuring success and failure. To begin to makes sense of this, these reductions in education, I realised it would be helpful to think differently. I did not want to get drawn into a blaming culture, the blaming of assessment and curriculum for the limitations of education but to step aside and consider at some level how (student) teachers through their fantasies, construct knowledge of teaching and learning of mathematics.

Following a meeting with my now supervisor Tony Brown, I commenced my doctoral studies in September 2014. I remember Tony referring to psychoanalysis, 'even when we think we know ourselves, we do not'. I found this frustrating and I was irritated to hear

someone claim that I cannot understand myself. However, intrigued, I was nudged in the direction of Jacques Lacan. For example, I started to read the work of Tamara Bibby (2011) who used the work of Lacan to investigate the ways we learn about ourselves, our peers and how people and ideas interact. The work of Deborah Britzman (2009, 2011) and much of Tony Brown's work, in particular, 'Mathematics education and subjectivity', absorbed me. Callum Neill's (2013) work on Lacanian discourse analysis, drawing on accounts of South Africans' experiences of apartheid provided me with a practical and workable approach to using Lacan's four discourses. Meanwhile Mark Bracher and Marshall Alcorn's (1994) book on 'Lacanian Theory of Discourse' provided enough detail to join the dots. Eventually I started grappling with Lacan's work first hand, in particular reading 'The other side of psychoanalysis'. It took a long time to read Lacan, sometimes I would re-read a paragraph three or four times and then returning to it a month or so later and thinking 'ah that's what he means', only for this meaning to be temporal with new discourse throwing a different light on its meaning. I was often left confused but always wanting to know more. Much of my reading challenged my own perspectives on how I see the world around me. Nearly six years later, I feel I am beginning to understand what Tony meant 'even when we think we know ourselves, we do not'. That is, I am becoming comfortable with the uncomfortable.

6.2 Discussions and Conclusions

The insertion of my own personal history demonstrates how my perspectives have evolved over time. That is, my viewpoints have developed through a multitude of discourses and successive new demands. As Lacan would say my understanding of self is in response to the ever-shifting symbolic network.

As a teacher educator, I can never be sure of my influence on student teachers, nor can I be fully aware on how student teachers influence me. In a similar way, the *mastery curriculum* works on student teachers but also student teachers work on the *mastery curriculum*. When I say 'I' am a 'teacher', or 'I' am 'teaching a mastery curriculum', we are not describing some biological entity of what it is to be teacher, we are responding to what we imagine the socially defined role of a teacher to be.

6.2.1 Mastery curriculum and teacher education

I have drawn on psychoanalytical theory to provide an account of how student teachers construct their identities through the training process and in response to the *mastery curriculum*. Stories from the classroom illuminate some of the socially constructed norms, a useful indicator to show how things are. However, the stories are productive of a student teacher telling that story, and a practitioner researcher interpreting it. In the theoretical framework that I followed, both student teacher's and teacher educator's identities are understood as a function according to the particular 'normalised' proficiencies and practices, always determined in time and space (Foucault, 1972).

The *mastery curriculum* positions student teachers and pupils through a myriad of influences from national priorities to local contexts, from subject mentors advice to anxious fourteen year-olds. The stories and experiences from the classroom tells us of how the *mastery curriculum* might be enacted. The words and actions become part of a student teachers developing professional identity. As the discourse of the *mastery curriculum* unfolds, there are moments of clarity, '*this is the way things should be'*. These reference points in student teachers' development act as anchors to meaning. It structures the incessant discourse from meaning too much or from meaning nothing at all. However, these moments are temporal, as Lacan sees the human subject, 'caught in a never-ending attempt to capture an understanding of oneself in relation to the world in which one lives' (Brown, Dore and Hanley, 2019:6). From this perspective, the human subject is continually split between how they sees themselves, and a need to realign to the perceived demands of the Other. Perhaps this is illustrated if we return to Charlie's lesson on geometric progression (Chapter 5.8). In this example, Charlie is torn between how she imagines her as role as a teacher and the discourse of classroom interactions. Pupil responses disrupt the authority of the master discourse, and in this sense, Charlie is questioning her sense of self, how she identifies herself as the classroom teacher. Such interactions can be unsettling for both the student teacher and the pupil as they grapple with the uncertainty of what is expected of them. Thus, Charlie learns from her failed attempts to produce the results that she strives. Such narratives are indicative of the developmental process of becoming a teacher. A multifaceted collection of competing

discourses that pull student teachers in different directions. Particular attention was given to how students responded to the varied demands rather than checking some supposed 'truth' to their practice or in seeking its effectiveness.

Student teachers enter into relationships with not only policy discourse (e.g., teaching standards, *mastery curriculum*) but they also interact with other individuals (e.g., university tutor, subject mentor, pupils, etc…). In this way, understandings of becoming a secondary mathematics teacher are responsive to dynamic social-cultural conditions. In particular, pedagogical strategies are a function of national and local administrative constraints and curriculum guidance that organise teaching. However, student teachers did not converge to any one particular version of the *mastery curriculum* but rather their practice was developed through local priorities and filters. At a micro-level, this could be subject mentor feedback, departmental policy, and environmental constraints, all of which influence day-to-day teaching. For example, requirements for Ali to incorporate five-parts into his lessons, Daniel's subject mentor emphasis on discussion, or Emily's departmental policy on using 'visual representations' of mathematical objects. These are all examples of rules and conventions governing activity. In their attempts to reproduce a version of *mastery* teaching, 'discussion', 'representation' and 'anchor task', amongst other signs, become filters in the register that shape student teacher's practice. At the Symbolic level these key words make social organisation possible, to be understood and to understand. For example, Daniel asking pupils to discuss a mathematical object, expects a specific type of response. The Symbolic requires that certain cultural rules be adhered to (e.g., pupils listen to each other).

Some student teachers identified with the *mastery* approach more than others did. For example, Emily's early experiences of a 'mastery style of teaching' were progressive, to the point where she felt the need to, '*strip back my own knowledge of maths to then reteach in another way*'. In this way, Emily is being interpellated in response to the ideology of the *mastery curriculum*. As the discourse of the *mastery curriculum* unfolds, there are fleeting moment of clarity, '*this is the way things are'*. However, these moments are temporary; the discourse is already moving on.

There is often a gap between attempting to meet the ideology of *mastery* teaching and the reality of classroom interactions. Student teachers and in general the subject desires

to close the gap between the fantasy of *mastery* teaching and the reality in the classroom. Other demands and factors influence what we do. For example, as the teaching placement progresses, Emily's priorities have altered. Emily has become more concerned about managing behaviour and less concerned with developing pedagogical strategies. These concerns and anxieties about the behaviour of her pupils manifest in giving lunchtime detentions and arranging to meet pupils' parents, a far cry from her initial observations. The discourse challenges Emily's position and identity as a student teacher, in the process she produces a new system of knowledge, '*this is what it means to be a teacher*' (See Chapter 4.6). As Walshaw points out (2008:124), 'closing the identity gap is what learning to teach is all about'.

Student teachers are required to meet the demands that are placed on them. In this way, the activity of teaching and participating in the classroom are performative, not just neutral actions. In such a conceptualisation of teaching, student teachers are performing to someone, an Other. Similarly, pupils are responding to an Other. Here, the teacher's sense of self is developing in response to how they understand the task of teaching and becoming part of a teaching community. As Radford (2018) argues, the activity of teaching is normalised and framed by a *dynamic cultural symbolic superstructure* that naturalises the way individuals make sense of the world. This reverberates with Althusser's (2005:234) view on ideology, in that it, 'expresses a will, a hope or a nostalgia, rather than describing a reality'. In this way, *mastery* policy is coded into discourse, which works on individuals and society, normalising policy to give the appearance, that this is the way things actually are (Eagleton, 2007). In a similar manner, Lacan's psychoanalytical position encapsulates the individual as responding to symbolic networks that shape who we are.

The discourse of *mastery* teaching demands compliance to certain operational protocols in the appearance of making progress. In this way, the concept of *mastery* teaching resonates with the Hegelian master-slave dialectic. In short, 'the master's knowledge is produced as knowledge that is entirely autonomous with respect to mythical knowledge' (Lacan, 2007b:90). The discourse of the *mastery curriculum* presents a version of mathematics as though this is the only way to teach mathematics. As Williams (2019:2) argues 'the policy [Mastery mathematics] becomes the master of us all, and we are obliged to suspend our critical faculties and comply'. This process of assimilation is not

always welcome and there is always a cost to joining a community. For example, if we refer back to Emily's lesson on simultaneous equations (chapter 5.10). Models and visual representations are used. This insistence of particular pedagogical strategies may be well meant and justified but at the same time agency is removed from the teacher and this has the potential to repress creativity either consciously or unconsciously. In this way, teaching is only credited if it meets some external requirement of what it is to be a successful mathematics teacher.

The activities and narratives provide us with insights into the dynamics and tensions that underpin the processes of reflexivity, whereby the pupil and teacher, participating in activity are continuously reconstructing their self-identity. Additionally, the classroom rules and structures are more than just mere classroom management strategies, they regulate behaviour, and they remove the individual's experience from its pure subjective experience. They provide structures to how pupils and teacher develop their subjectivities. This is why it is a mistake to assume that the *mastery curriculum* merely promotes a deeper understanding of mathematics, it promotes a **particular** understanding of mathematics, particular pedagogical structures. However, naturally pupils fall short of these expectations, there is gap between what the teacher wants and the position the pupils take.

I have considered ways on what can be learned by looking at classroom practice in different ways, to consider mathematics education through different perspectives. This study has never sought out to define *mastery* teaching or to offer suggestions to improving mathematics teaching but to better understand the training process. I am more concerned with how both student teachers and the teacher educator make sense of the various discourses that pull individuals in different directions. My concerns for developing better stratagems to teach and learn mathematics is that they will always be according to a particular agenda. Many of the ideas of *mastery* teaching promote what seem to be like worthy goals. However, in valuing a particular version of mathematics, we exclude other versions. *Mastery* sold as good practice could be seen as just another way of telling teachers what to do. Where structural priorities surpass the autonomous teacher.

6.3 The contribution of this research to the field of knowledge

The study contributes to a growing body of research that takes a psychoanalytical perspective on (student) teacher development.

The main findings are:

- Teachers are continually split between how they see themselves and the perceived demands of discourse.
- The subject is responsive to ever-changing social conditions.
- Recognising that when you say 'I am a teacher', or 'I am teaching the mastery curriculum', it is not the self as a biological entity but the ego responding to perceived demands.
- The role of subjectivity in the construction of knowledge.
- How ideology calls teachers into being a particular teacher; normalising practice, pedagogical strategies and social interactions.
- Discourse can play out in different ways, with the unconscious always 'lurking' to disrupt the master discourse.
- Recognising how my own practice has developed through multitude of discourses and successive new demands.

The overarching theme for this study investigated the everyday interactions between student teachers, pupils and policy. It considered the different ways student teachers negotiate the different demands placed on them. My initial intention was to consider how student teachers construct their professional identities through the training process and in response to the *mastery curriculum*. However, as became apparent, various alternative demands and discourses pull teachers in different directions. As such, the study became more of a study of training to teach mathematics within the context of the mastery curriculum. The main reason for this refraction is that a great deal of what happens in the classroom seems beyond conscious reach; the reality of teaching is often far removed from the benevolent often well-founded policies such as the *mastery curriculum*. It is also often far removed from rational process of conventional psychological theories of learning such as Vygotsky's Zone of Proximal Development.

However much students teachers plan, follow policy, reflect on their practice, learning is unpredictable, pupils puzzle us. However much we attempt to pass on our knowledge, make a difference, pedagogical encounters are unstable. Teachers identify with various demands, thoughts of what is expected them. Nonetheless, every thought, every action is underscored by the unconscious. This is different for individual subjects influenced by a history of discourse, the individual's subjectivity. To make sense of these learning encounters and to make sense of the training process we need to think differently. We need to consider; unconscious struggles, fear, anxiety or resistances to knowledge that often disrupt the benevolent leaning process. We need to consider; different forms of social relations between policy and the subject of discourse.

This study posits itself at the interface of the complexities of becoming a teacher and psychoanalytical theory that disrupts habitual thinking patterns within regulative scenarios. Using Lacan's four discourses allows us to repeatedly map out classroom interactions to different permutations of discourse, generating different possible understandings. It provides insights into the relationship between truth, knowledge, subjectivity and the Other

In the neo-liberal climate of competition, it is as important as ever to consider the implications of discourse, look and interrogate the underlying issues. That is, critically examine and interrogate how discourse unfolds within the assortment of alternative demands of becoming a teacher.

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Appendix 1 Research plan

Research Plan			
Task Name	Start	End	Duration (days)
Complete and submit RD1	02/12/2016	01/03/2017	89
Application for ethical approval	29/11/2016	06/09/2017	281
Identify students and schools	15/09/2017	23/10/2018	403
Gain consent from students and schools	15/09/2017	23/10/2018	403
Collect data, Interview round 1	05/02/2018	04/05/2018	88
Collect data, Interview round 2	08/10/2018	10/12/2018	63
Collect data, lesson observations round 1	22/02/2018	07/05/2018	74
Collect data, lesson observations round 2	01/10/2018	01/12/2018	61
Collect data, lesson observations round 3	01/02/2019	01/05/2019	89
Organise and collate data	04/05/2018	01/07/2019	423
Chapter 1: Introduction, 1st draft	01/08/2018	01/02/2019	184
Chapter 1: Introduction, 2nd draft	01/08/2019	01/11/2019	92
Chapter 2: Literature review, 1st draft	01/12/2018	01/02/2019	62
Chapter 2: Literature review, 2nd draft	30/04/2019	30/08/2019	122
Chapter 2: Literature review, 3rd draft	01/09/2019	31/10/2019	60
Chapter 3: Lacan and psychoanalysis, 1st draft	01/04/2019	09/06/2019	69
Chapter 3: Lacan and psychoanalysis, 2nd draft	12/06/2019	08/07/2019	26
Chapter 4: Four discourses, 1st draft	01/12/2018	11/02/2019	72
Chapter 4: Four discourses, 2nd draft	01/04/2019	31/05/2019	60
Presentation of 'Four discourses' at the student conference	18/06/2019	19/06/2019	1
Chapter 5: Social Relations, 1st draft	03/06/2019	30/07/2019	57
Chapter 5: Social Relations, 2nd draft	01/08/2019	15/09/2019	45
Chapter 5: Social Relations, 3rd draft	01/10/2019	01/11/2019	31
Chapter 6: Discussion and conclusions,	01/12/2019	01/02/2020	62
Complete thesis: 1st draft	01/08/2019	31/12/2019	152
Final draft and corrections	09/01/2020	29/02/2020	51
Thesis submission	02/03/2020		
Thesis Viva	15/05/2020		

Appendix 2 Participant Information sheet

Participant information sheet - Students

Study Title

The discursive construction of the mastery curriculum in mathematics

You are being invited to take part in a research study. Before you decide it is important for you to understand why the project is being done and what it will involve. Please take time to read the following information carefully and discuss it with others if you wish. Please ask if there is anything that is not clear or if you would like more information. Take time to decide whether or not you wish to take part. Thank you for reading this.

What is the purpose of the study?

The study investigates how student teachers construct knowledge and understanding of policy and curriculum. In particular, it aims to provide an account of how student teachers construct knowledge of the "mastery curriculum" in mathematics. It also provides a theoretical framework towards capturing the habitual thinking patterns that influence decision-making.

Why have I been invited?

You have been invited to take part in this study, as you are a pre-service mathematics teacher, on placement in a school incorporating aspects of the mastery curriculum.

Do I have to take part?

It is up to you to decide. I assure you there will be no personal detriment either way. I will describe the study and go through the information sheet that I will give to you. You will

have time to consider the study before you decide. I will then ask you to sign a consent form to show you agree to take part. Even if you agree to take part, you are free to withdraw at any time, without giving a reason.

What will happen to me if I take part?

I would like permission to use your reflective writing at various stages of the course as data. I would also like permission to use your lesson observations as data.

In addition, I would like to invite you to participate in semi-structured small group interviews to explore your views and understanding of the mastery curriculum at successive stages of the course: start, middle and towards the end. The interviews will be audio-recorded and will last not more than 20 minutes.

What if there is a problem?

If you have a concern about any aspect of this study, you can contact me and I will do my best to answer your questions (Peter Pawlik p.pawlik@mmu.ac.uk). If you prefer you can contact my supervisor (Tony Brown a.m.brown@mmu.ac.uk). If you remain unhappy and wish to complain formally you can do this by following the University complaints procedure.

Will my taking part in the study be kept confidential?

All information collected about you during this research will remain strictly confidential. Any audio-recording and any quotes I use from the study in writing up the research will be completely anonymised. Electronic data will be stored on a password-protected computer accessible only by researcher. Once the interviews have been transcribed, the sound files will be deleted. No-one will be identified in any way, and any students mentioned will be given pseudonyms.

What will happen if I don't carry on with the study?

If you withdraw from the study, I will destroy all your identifiable data including recorded interviews, but I will need to use the anonymised data collected up to your withdrawal.

What will happen to the results of the research study?

The results will be written up as part of my Doctor of Education thesis. It will be used in academic papers and conference presentations as part of dissemination.

Who is organising or sponsoring the research?

The research is for my Doctor of Education study at Manchester Metropolitan University.

Further information and contact details:

If you have any more questions or would like any further information please do not hesitate to get in touch.

Appendix 3 Consent form

Name: Peter Pawlik
Course: Secondary Education
Department: Faculty of Education
Building: Brooks
Manchester Metropolitan University

Title of Project: The discursive construction of the 'mastery curriculum' in mathematics

Name of Researcher: Peter Pawlik

Participant Identification Code for this project:
Please initial box

1. I confirm that I have read and understood the information sheet for the above project and have had the opportunity to ask questions about the interview procedure. ☐
2. I understand that my participation is voluntary and that I am free to withdraw at any time without giving any reason to the named researcher. ☐
3. I understand that my responses will be sound recorded and used for analysis for this research project. ☐
4. I understand that my responses will remain anonymous. ☐
5. I agree to take part in the above research project. ☐
6. I understand that at my request a transcript of my interview can be made available to me. ☐
7. I agree to the use of the data in the study. ☐
8. I agree to the use of the data in research papers, articles or conference proceedings ☐

Name of Participant _____ Signature _____Date_____

Researcher_____ Signature _____Date_____

To be signed and dated in presence of the participant

Appendix 4 Semis-structured interviews

Semi-structured Interview schedule

The interviews will take place at successive stages of the course: From January 2018 to December 2018. Interviews will last a maximum of 20 minutes.

The interviews are semi-structured to allow a journey within the sessions to be followed, while also leaving room for discussions to develop along unanticipated directions. A proposed format for the interviews:

Introduce the interview session. Reassure participants that there is no obligation to take part - it is purely voluntary and anything that is said will not impact on their studies or in future relationships with me as a tutor at MMU. Published work arising from the study will be anonymous.

Explain from the perspective of a mathematics educator why I want to carry out this research. Emphasise that it is the process, the thinking behind how student teachers construct knowledge of the mastery curriculum. There are no right or wrong answers here, just their views that count.

Ensure audio recording equipment is tested and switched on**.**

Interview discussion points.

Could you describe the mastery curriculum? What does it mean to you?

What does the mastery curriculum look like in the classroom?

What elements of mastery teaching have you observed?

What do you think are the advantages of the mastery curriculum?

What are the challenges/ disadvantages?

Do you think pupils should be taught in mixed attainment or sets?

Whole class versus other activities

Describe the teaching approaches you have observed. How much time is spent on whole class activities? How much time is spent in individual and small group work?

Activities during whole class time

In your observations of whole class activities. How much time is spent on instruction and explanations? How much time is spent on 'Questioning and interaction, dialogue and discussion?' Do you agree with these proportions? If not what should they look like?

Individual versus small group

How much time is spent on pupils working individually compared to small groups (including pairs)?

How do you think you can help develop pupils' conceptual understanding? What about procedural fluency. Do you have any thoughts on which should be taught first? Procedural fluency or conceptual understanding?

Frequency of teaching mathematics topics

Do you think it is better to teach mathematics topics frequently but in less depth, visiting them often or spending more time on an area of learning?

Do you think all pupils can succeed in mathematics?

What do you think are barriers to implementing a mastery curriculum?

Appendix 5 Guidance on reflective writing

REFLECTION ON EXPERIENCE AND LEARNING

Rationale and Introduction

What do we mean by ‘Reflection on your Experience and Learning’ (REAL)?

The programme requires you to carry out weekly reflections on and evaluation of your experience, learning, practice and your professional development throughout the programme. This enables you to develop as a 'reflective practitioner', to chart and monitor your progress and to set targets for your further development.

Your weekly ‘REAL’ develops into a reflective log of your learning journey and helps you attempt to explain significant events in your professional development and to use these to inform your future practice.

Your weekly reflections should make clear how you understand particular events in your training, your experiences and your learning, including your own impact on pupil learning and progress.

Answering certain prompt questions will help you in this process:

- **What happened (BRIEF description) or what have you noticed?**
- **How do you feel about it?**
- **What have you learned?**

analyse: why it is important to you

analyse: what you think it means

analyse: why you have come to this particular explanation and not another; might there be an alternative explanation for the event or experience witnessed?

- **How has your training impacted on what you have learned? What are the implications of your learning for your future professional development?**

• How has (or might) your practice and its impact on young people be enhanced or changed as a result of your analysis?

Events written about in your REAL log as a whole **MUST** include reference to your:

- School-based training in **Placement Schools A and B** and in your **transitional settings** e.g. observation of others, mentor training sessions, whole-school training, other school study.
- **University** sessions and training.
- **Feedback on your practice** from, and discussion with, mentors, tutors and other teachers and how you have responded to this feedback and discussion.
- Evidence from the tracking and development of your **subject knowledge for teaching**.
- **Reading**, including school policies, published professional and academic articles and books, is an expected context of training and as such, should be used to inform experience and other training referred to in regular entries in the REAL as a whole.
- Your **teaching**, including its planning, preparation, students' assessed work and any collaborative work with experienced colleagues, and your lesson evaluations.

Look to critical incidents in school-based and university training, classrooms and schools that provide rich sources upon which to reflect; it will be important to consider what inspires you to reflect across a ***range*** of training and learning experiences. In particular, look to critical incidents in your own classroom too. In other words, reflect on examples from a range of contexts and how, for example, something you have tried in the classroom relates to a discussion you had at a central training session, or something you read. **Do not focus exclusively on training and experience in the classroom**. Rather, use as many training contexts as possible and the links between them to show how your action as a teacher is shaped and developed.

Your weekly REAL entries must be *analytical, explaining* events and should be as long as they need to be to achieve this (aim for 150 – 200 words each). They should not be too long and definitely not overly descriptive.

Reflections, should capture significant moments of your learning and should also be specifically related to the teaching standards and throughout consider your progress and impact on pupil learning. Although personal, they are used as a basis for professional discussion with mentors and tutors and must therefore observe professional expectations.

Reflection on Experience and Learning (REAL) during the Programme

At first, it is normal to note much of what happens to you during the programme in the REAL log. However, as the programme develops you will become more selective about the experiences you refer to, including only those learning experiences that are most significant to your professional development. Experience tells us writing two longer entries each week, rather than many smaller ones tends to be most effective. **You must, however, provide weekly REAL entries throughout your programme.**

Appendix 6 An example of a lesson plan

MANCHESTER METROPOLITAN UNIVERSITY

DEPARTMENT OF SECONDARY TEACHER EDUCATION

PGCE SECONDARY LESSON PLAN TEMPLATE			
Trainee Teacher Name: Miss N., Mr P., Mr D.		**Class Teacher Name:**	
Subject: Maths	**Class:**	**Lesson Duration:** 1 hour	**Date:** 06/02/2019
No. in class: **Boys:** 4 **Girls:** 3	**Ability/set:** Set 4	**Lesson No:** 1	

Unit of Work: Number	**Enquiry Question:** How do we use the 'cauldron' to help understand negative numbers
Learning Objective/s: • To be able to use negative numbers to work the witch's cauldron	**Learning Outcomes/Success Criteria:** 1. Students will have related sums such as -2 + -3 to hot and cold cubes and can calculate an overall temperature change
Resources and Organisation: Multilink cubes Number cards	**Inclusion strategies/Differentiation:** **G&T/MORE ABLE:** **SEND:** **PUPIL PREMIUM:** **EAL:** **VULNERABLE GROUPS:**
Extended Learning (Homework): **Date due:**	
Deployment of teaching assistants: Miss N and Mr P are classroom support. They might be needed to represent numbers on a numberline. Miss N	**Key Terminology and Concepts:** Positive Negative Minus (subtract etc) Add (plus etc)
Links to numeracy: Implicit	**Links to literacy (reading, writing, speaking and listening):** Students are required to use correct terminology. Prompted to only use positive/negative when describing and

	number, and minus/add when describing operations.

Plan of Teaching and Learning Activities

Real Time	Lesson Section	Outcome	Teacher led activity	Student led activity
0-10 minutes	**Starter –** link to future and prior learning	Reviewed angles on parallel lines	If struggling, draw parallel lines on board and get students to match diagrams to name e.g. corresponding to 'f' angle	Students work out missing angles and give reasons as to how they know. They have to use key words, e.g. corresponding
10-20 mins	**Learning Activity 1**	Check understanding of negative numbers		Students order the numbers from smallest to biggest unsupported and without number line Students then place the numbers on the number line and see if their order matches.
	Mini-plenary		If students find this tricky, spend extra time using a number line and placing positive and negative numbers on. Could do this as a class where Mr. P is 20, Miss. N is 0, Mr. D is -20. Give students a number and ask them to stand where they think their number is on the line.	
20-30 mins	**Learning Activity 2**	Using the Cauldron to add positive	Only add hot and cold cubes – demonstrate how the temperature of the cauldron changes when	

Real Time	Lesson Section	Outcome	Teacher led activity	Student led activity
		and negative numbers	you add cubes. Talk about hot 'cancelling' cold	Students are given multilink cubes to represent hot and cold – they can use the cubes to help with section A of the worksheet
30-35 mins	**Mini-plenary**			Students relate number sentences such as 3 + -2 to the cauldron and work out the temperature
35-45 mins	**Learning Activity 3**	Using the cauldron to subtract positive and negative numbers	Demonstrate taking hot and cold cubes and discuss effect on temperature	Students complete section B of the worksheet and can use cubes to help
45-55 mins	**Plenary**			Students relate number sentences such as 3 - -2 to the cauldron and work out the temperature
	Extension			Students can make their own recipe for the cauldron

For lesson observations please attach a copy of the seating plan and progress data to the lesson plan. The additional information provided should include the target grade/level and current working grade/level for each student and should indicate if they are SEND, Pupil Premium, a member of a vulnerable group, EAL or G & T.

Appendix 7 An example of a lesson structured into five parts: diagnostic, anchor

Solving Equations 2

LO: To be able to solve multi - step equations to find an unknown

DIAGNOSTIC

Solve the following equation for a:

$$8 = 2a - 4$$

A $a = 2$

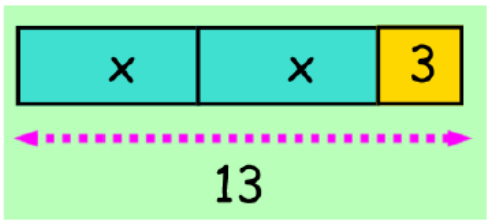
B $a = 6$

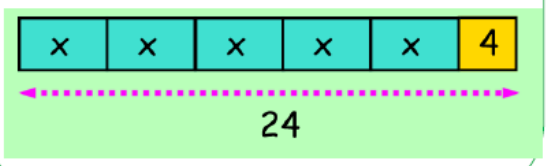
C $a = -6$

D $a = -2$

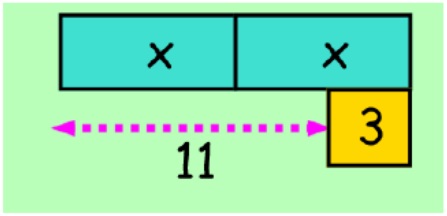
ANCHOR

What equations do these represent?
Can you solve them?





Challenge

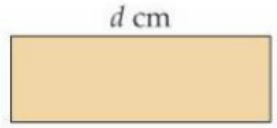


How would you show $\frac{x}{5} = 6$ using bars?

How would you solve these using a function machine?

GROUP

The area of this rectangle is 28 cm².
Find *d*.



Can you write an algebraic equation to represent the function machine?

Explain how you can find the input.

Fill in the missing box. Is that the only possible solution?
Explain how you know.

Guided Learning

Solve:

$$2x + 4.2 = 16.5$$

Solve:

$$34 - 2x = 9$$

INDEPENDENT

a) x = 4
b) b = 6
c) t = 6
d) r = 4
e) t = 8
f) d = 10
g) m = 7
h) g = 17

a) a = 3
b) p = 20
c) q = 3
d) k = 8
e) n = 20
f) y = 35
g) h = 49
h) a = 12

Aron has some balloons.

Fiona has 12 more balloons than Aron.

In total they have 40 balloons.

How many balloons has Fiona got?

x + 12 = 40
x = 28

- The length of a rectangle is $2x + 3$ and width is $x - 2$
  The perimeter is 17cm.

  Find the area of the shape.

LENGTH
2x+3 when x=2.5
2x+3 = 8

WIDTH
x-2 when x=2.5
x-2 = 0.5

2x + 3 + 2x + 3 + x - 2 + x - 2 = 17
6x + 2 = 17
6x = 17
x = 2.5

Area = length x width
= 8 x 0.5
= $4cm^2$

an