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Measurement as relational, intensive and analogical: Towards a minor mathematics



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ABSTRACT

Minor mathematics refers to the mathematical practices that are often erased by state-sanctioned curricular images of mathematics. We use the idea of a minor mathematics to explore alternative measurement practices. We argue that minor measurement practices have been buried by a ‘major’ settler mathematics, a process of erasure that distributes ‘sensitivity’ and formulates conditions of mathematics dis/ability. We emphasize how measuring involves the making and mixing of *analogies*, and that this involves attending to *intensive relationships* rather than *extensive properties*. Our philosophical and historical approach moves from the archeological origins of human measurement activity, to pivotal developments in modern mathematics, to configurations of curriculum. We argue that the project of proliferating multiple mathematics is required in order to disturb narrow (and perhaps white, western, male) images of mathematics—and to open up opportunities for a more pluralist and inclusive school mathematics.

1. Introduction

Through repeated or rhythmic activity, children develop strategies for negotiating distances and carrying weights, inventing measurements that are ground in their body and familiar objects (finger, step, rhyme, rock, etc.). Measurement always has this embodied lineage, tied directly to a body’s first attempts to ‘sense’ the world autonomously, through movement, touch, sound, vision, and even taste. One first develops ‘more or less’ relational responses through these sensory encounters, allowing one to create quantitative relationships between different media – one measures a bowl *with* water, or water *with* rocks, or rocks *with* hardness. This kind of activity explores the *emergent* coupling by which different modal beings are bound together in ‘minor’ measuring activity. Measurement begins in this relational engagement with the material, where what ‘matters’ is the specific co-relation between two or more material processes. In this pluralist ‘with’ world, these acts of measurement are diverse and situational.

The measurement curriculum is often considered the most accessible part of school mathematics, because it pertains to the sensible, material world—and relates to relevant and “real-world” uses of mathematics. As such, measurement curricular standards lend themselves to corporeal kinds of investigation, sometimes employing tools and technical instruments. The measurement standards often have a ‘common-sense’ feel to them that can betray normative assumptions regarding what can be sensed by particular bodies, and therefore what ‘makes sense’ to those particular subjects. This ‘common sense’ is a powerful way of formatting the curriculum and excluding divergent ways of making sense, which poses unique challenges for students with disabilities (Sinclair & de Freitas, 2019).

This raises important political questions about the different kinds of corporeal capacities and material practices that are entailed

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when we perform measurements, since different kinds of bodies engage the physical environment from different vantage points. Theories in Disability Studies demand that we attend more closely to the political and historical framing of embodiment in these kinds of material practices (Snyder & Mitchell, 2001; Siebers, 2008). Such work attends to the biases built into our conceptions of mathematical ability, inviting us to destabilise taken-for-granted notions of the body's capacity to measure and to be measured (Tan, Lambert, Padilla, & Wieman, 2019).

In this paper we delve into the under-examined complexity of measurement as an ever-changing material practice that entails corporeal mobilities of all kinds. We show how measurement is more than simply 'covering' spatial objects with standardized units. We submit that the notion of 'a unit' as that which is used to *cover* an extended object comes 'after' the dynamic co-relational aspect of measuring activity. We argue that measurement be understood in terms of an intensive relational ontology, and that measurement activity be studied for how it mobilizes *analogical* thinking. Analogies are partially captured in the teetering balance of proportions (A : B is to C : D). We first elaborate our theoretical framework, drawing on the work of Michel Serres, Vicki Kirby, Jasbir Puar, Gilles Deleuze and Felix Guattari. We then present archaeological and historical perspectives on the emergence of measurement within human cultures, focusing on the legacy of settler geo-metry (earth measurement) as expressed in two themes (floods and cosmos). We use the ideas of Deleuze and Guattari (1987) to critique the covering tendencies of a settler measurement that segments land and society, and controls movement. We track different kinds of minor nomadic mathematics that become buried by a major settler mathematics, a process that distributes 'sensibility' and formulates conditions of dis/ability. Minor mathematics is the term that Deleuze and Guattari use to describe mathematical practices that go against the grain of major (or state) mathematics (de Freitas, 2016a).

This kind of historical and philosophical approach allows us to raise open questions such as: How are matter and measure related? How is measurement part of settler and/or nomadic practices? How is dis/ability inscribed into these practices? Finally we show how measure theory has evolved in complex ways in recent years, and that the shallow treatment of measurement found in curriculum policy reflects an over-emphasis on measurement as "covering". State and national curricular standards for students K-12 reveal a tendency to focus on identifying measurable attributes of objects, translating systems of units, and applying formulae (Goldenberg, Clements, Zbiek, & Dougherty, 2014). Our critique of the US mathematics curriculum engages with curriculum as a text that reflects certain biases – we treat curriculum as a kind of archive of collective practices and ideas endorsed by the state or the national body. In this we follow other Curriculum Studies scholars who focus on the alchemical nature of the mathematics curriculum (Popkewitz, 2004) and on the possibility for other ways of doing curriculum (Appelbaum, 2016). Our aim is to attend to the relational, intensive and analogical mode of measuring which is lost in curricular erasures.¹

2. Theoretical framework

This paper builds on our previous work on mathematical behaviour and our program of Inclusive Materialism (de Freitas & Sinclair, 2014; de Freitas, 2016b, 2016b, de Freitas & Sinclair, 2018; de Freitas, Sinclair, & Coles, 2017) – where we explore the cultural-material practices of mathematical activity of all kinds, be it expert, recreational, school-based, or non-human. One of our aims has been to reveal how mathematical dis/ability is tied to particular kinds of material-semiotic practices, and that differently abled bodies pursue different kinds of mathematics. This paper continues that work, by digging into the diverse practices of measurement that are not well represented in school mathematics. We argue that the project of proliferating multiple mathematics is required in order to disturb narrow (and perhaps white, western, male) images of mathematics—and to open up opportunities for a more pluralist school mathematics.² As we have done elsewhere, we turn to the history, archeology, and philosophy of mathematics to help shake up staid assumptions within education research about what constitutes measurement activity. Although our work draws from different theories, we see our project as allied with those who explore indigenous forms of knowledge while seeking to "avoid the trap of Western versus 'other' mathematics ..." (Gutiérrez, 2017, p. 26).

Much of the disability research in mathematics education has drawn on either bio-medical theories or socio-cultural theories, the choice between these often based on the degree to which the individual or the collective is assigned causal prominence for the disability (Lambert, 2019). Various attempts to think beyond this binary trap (individual/social) have involved further destabilizing the normative and exclusionary category of the human itself, by turning to posthumanism (Goodley, Lawthom, & Runswick-Cole, 2014; Goodley, Runswick-Cole, & Liddiard, 2016; Mitchell, Antebi, & Snyder, 2019). This work takes up the micro-politics of materiality and bodies, but often continues to privilege white, hetero-male, cis-gendered subjects, which has made some scholars concerned with the ways in which a 'posthuman flight' might simply sustain and even shore up previous inequalities (Colebrook, 2019; King, 2017). Some scholars prefer the ecological "more-than-human" or the somewhat humbler "inhumanisms" as a way of emphasizing forms of life that are *the least* en/abled in milieus that are characterized by radical power differentials (Chen, 2012; Singh, 2018). We are interested in how this theoretical approach helps us think differently about mathematical ability and the geo-politics of dis/ability, attending more carefully to the ways in which agentic capacities are distributed historically across populations and places (Snyder & Mitchell, 2010). This approach explores the intersections between humanism, colonialism, nationalism, and ableism, and sheds light on the complex political friction entailed in state-sanctioned forms of 'inclusion'.

We turn first to the philosopher Michel Serres (2017) who argues that measurement is not simply *derived* from matter, nor is measurement the mere covering of matter with units. Matter and measurement are not detachable, but rather reciprocally

¹ Our approach traces some threads from ethnomathematics and indigenous studies, such as Urton and Llanos (1997) and Verran (2001).

² Zalamea (2012) shows how a more pluralist image of mathematics in all its current diversity would represent the field more accurately.

presupposed and imbricated, bound together in a metamorphic mixture. It is this reciprocal implication that makes measurement a practice which is ultimately paradoxical in that it is both objective *and* subjective, abstract *and* concrete, collective *and* singular. Rather than embrace measurements as accurate containers and coverings of material objects, and rather than dismiss all measure as a distorted misconstrual of nature, we seek ways to better understand the imbrication of matter and measurement. Serres (2012) directs our attention to how the body ‘knows’ mathematics through micro-movements and mimicry of material processes. This raises the odd question of “what kind of mathematics can a body do?” (Sinclair & de Freitas, 2019), a provocation to consider the possibility of radically different sensing bodies, pursuing radically different mathematics. These bodies might be technology-enhanced or differently organ-ized, contesting the assumption of pre-given sensory modalities with definitive capacities, while destabilizing an absolute and universal mathematics.

The anthropologist Vicki Kirby (2011) provokes us to consider corporeality more generally as ‘calculating and thinking material through and through’ so much so that the *very nature of corporeality* is “to mathematize, represent, or intelligently take measure of itself” (p. 63). In other words, we are asked to imagine the way that matter and measurement are part of a metamorphic mixture, open to remixing, reformulating, and altered modes of bodying. This ensures that mathematics remains in the world (rather than transcends it), and emphasizes a pluralist mathematics engendered through diverse assemblages and encounters. Otherwise the body remains subordinate to an immaterial mathematics, while the material world remains inert, mute, and passive. Such separation of body from mathematics is problematic, in part because the ‘immaterial’ mathematics is then treated as apolitical and neutral.

Kirby (2011) critiques dualisms that separate out mathematics from the material world. She characterizes these as anthropocentric in two different ways: the first imagines that the earth is there to serve humans, whose magical mathematics aims to control it (reductive scientism); the second imagines that the earth as entirely socially constructed by the cultural veil of human mathematics (social constructivism). Kirby suggests instead an approach that dethrones the anthropocentrism of these two traditions, and seeks a new empiricism of life, earth, and the more-than-human.

Measure would then not *only* be the anthropocentric habit inscribed in Protagoras’ aphorism ‘man is the measure of all things’ nor reflect a unique human capacity. Instead, measure would be a tendency or potentiality of matter. Geometry, for instance, would be a more material mingling of *geo* and *metric*. For Kirby, too much of socio-cultural theory forecloses this possibility by defining geometry against geology, language against matter, mathematics as a *representation* that codes matter from without (de Freitas, 2016b, p. 656).

These attempts to study measuring practices as more-than-human engagement have been further elaborated by queer post-colonialist theorist Jasbir Puar (2017) who seeks out the cultural-material measuring practices that distribute dis/ability across a ‘population’ and determine how and whether bodies ‘belong’ and perform accordingly. She emphasizes the provisionality of dis/ability, stating: “disability is not a fixed state or attribute but exists in relation to assemblages of capacity and debility, modulated across historical time, geopolitical space, institutional mandates, and discursive regimes” (p. xiv). Some scholars critique Puar for the way this work undermines disability identity, compromising affirmative projects that mobilize through strategic essentialism. We recognize the tensions here, but appreciate the way that Puar looks transversally across the geo-historical. Our project here is similarly trans-local, while remaining cautious of the tensions within “universalizing and locating impulses” (Luciano & Chen, 2015, p. 192).

Like Puar, we draw on the work of Deleuze and Guattari (1987) to suggest that minor measuring practices contest the striating and controlling gesture of enclosure, resisting the ‘covering unit’ that contains and apports (p. 388).³ Rather than the territorial drive to divide up space or distribute space to various claimants, they argue that the minor (or nomadic) is that which distributes itself in space, and in so doing, creates an opportunity for entirely different measurements to emerge (de Freitas, 2016a). These would be measurements *immanent* to the space, rather than imposed from without. As such, these measuring habits have the potential to be relational, intensive, and situated (rather than absolute), and better attuned to a pluralist mathematics of difference. Minor mathematical measurements emerge from a more situated and analogical form of thinking, engaging with metamorphic relational ecologies. We use this approach to help us identify the maverick and counter practices that bubble up within (or outside) the state-sanctioned curricular images of mathematics. Our method in this paper is to excavate the intensive and analogical measuring practices that point to the generative plasticity of the matter-measurement mixture, opening up space for more minor mathematical abilities to emerge alongside the state-sanctioned ‘covering’ tendencies of the measurement curriculum. In the next section, we discuss examples of minor measurement practices found and lost in the historical record. Other excavations of minor mathematics can be pursued through historical and cultural studies of practices that have been displaced by pedagogical desires, technological innovations, colonial erasures, and various ableisms.

3. Settler habits uncovered

According to the archaeological record, measurement practices emerged to structure group relations, distribute resources and commodities, navigate journeys and comprehend time’s passage and cycles (Renfrew & Morley, 2010b). Large amounts of archaeological evidence of measurement practices have been dated as part of the emergence of widespread sedimentism, about 10,000 years

³ Deleuze and Guattari often point to the 17th century infinitesimal as an example of a fruitful but controversial mathematical concept, controversial in part because it resisted measurement – it was considered at times “blasphemous” and at others lacking in rigour or logical foundation, before being formally legitimated by non-standard analysis in the 1950s.

ago, when most humans invested in permanent dwellings and agriculture. And so most of our understanding of how measurement figures in human culture comes from the term that archaeologists call “tectonic”, which designates a period of time of intense making and constructing in fairly stable ‘settler’ locations. Direct archaeological evidence for measurement prior to this period is sparsely identified (Renfrew & Morley, 2010a). This has implications for us today, as we ‘inherit’ a particular historical account of what and how human measurement practices evolved.

Since nomadic measurement practices have gone undetected in the archaeological record, we are left to guess what these might have been. For instance, Farr (2010) speculates on Neolithic measurements associated with navigation practices in the Adriatic sea, since obsidian volcanic glass found in different seafaring locations suggests this must have occurred. Considering tidal patterns and currents, and the fact that hominids were seafaring in other areas as early as 50,000 years ago, Farr suggests that time and space might have been measured in very different, fluid ways, as open water demands a very distinctive sense of temporality and movement. This is precisely the type of measuring approach used by the Marshall Islanders for navigational purposes; instead of providing an external, flattened map of the islands and atolls, the *mattang* measuring device was used *in the water* as a kind of land/sea interface, responding to the swells of the ocean, the location, the body of the navigator (Ascher, 1995).

When we turn to historical and ethnographic records, there is a pluralism of diverse measuring habits that indicate a wide range of measuring devices “such as bows, chain links, and goads for driving oxen, as well as spans of the body such as finger-widths, hand-breadths, and arm-lengths” (Cooperrider & Gentner, 2019, p. 1).⁴ Many practices include measures that reference the body (the foot, the hand, the thumb, etc.). Medium-scale measures tend to reference events—such as the ‘bow shot’ used in the Andaman Islands and the ‘stone’s throw’ used in Morocco, or the sonic measures used in Burma, which were based on the distance at which one could still hear a person’s voice. Larger-scale measures, which were much rarer, were also often temporal, based on protracted events such as days spent travelling or, for the Saami, the number of coffee stops required and for the Mi’kmaq people, the number of capes traversed. Early North American Ojibwe used the body to measure how much of a day it would take to travel a certain distance, by superimposing an outstretched hand on the arc of the sun: in this analogical measuring, “one ‘hand-stretch’ was considered one fourth of the arc from sunrise to zenith (Cooperrider & Gentner, 2019, p. 5). In the remaining parts of this section, we turn to two settler geometry themes (floods and cosmos) to look for more evidence of minor measurement practices.

3.1. Floods

Serres (2017) explains how the concept of proportion figured prominently in early Egyptian measurement practices. After considering how Anaximander, Thales, Euclid, Pythagoras and Zeno were involved in the judicial, political, discursive, ethical, astronomical and arithmetic origins of geometry, Serres (2017) circles back to an important text by the 5th century BCE Greek historian Herodotus. Herodotus describes how the king of Egypt Sesotris ordered his men to measure *the land lost* to the Nile at every exceptional rising of the water, which would enable the King to *proportionally* reduce the taxes paid by the farmers whose lands had been inundated. The annual fluctuations of the river caused a loss of silt and good soil for growing, thereby redistributing territory at each bend and year. These men were called rope-stretchers, or harpedonaptai, because of the knotted chords they used to measure, stretched just enough to not sag.

For Serres (2017), it is this forever changing “abstract space” (of more or less land) that allows us to inhabit an earth where “the agrarian zone fits into the laws of the state” (p. 199). In the ancient Egyptian tax system, one always starts in the middle of comparative measurement – it’s always a matter of more or less. In other words: “pro-portion precedes the portion” (p. 323). What matters is the changing relation or, more precisely, the changing ratio of land lost (through flooding) to taxes paid *over time*: the difference in arable land, from one year to the next, becomes associated with the tax to be paid. Note how this act of measuring attends to the economic differential, creating an analogical conjunction of materialities (water and land and time). Measurement in this case involves a kind of abstraction, but not in reductive or absolute terms; proportional measurement continues to inhere in the transports of material effluvia that link land and river and tax, tracing the relations “that bridge and compensate their variations” (p. 201).

These practices were clearly part of an imperialist population control, but also ‘belonged’ to processual matter, to earthly transformations, to climatic regimes and fluctuating floods. For our purposes, this flood story is important because it provides insight into the minor threads of Western mathematics ‘origin’ stories, linked as they are to both imperial conquest and humbler agrarian habits (Alder, 2002). Tax collection was in part a form of subjection; consider how the Inkan empire used a complex system of knotted colour cords called Khipus to record census data and resources, as well as accounts of conquest and debt. Measurement in these examples serves the effort of empire building, but it does more than that. The brilliance of using chords and knots and colours for the Inkan registry is found not simply in its ability to represent place value and difference in kind, but in its reusability—knots can be untied and retied (Urton, 2010). This account also draws attention to the important role of technology in measurement, where ‘techniques’ serve to regulate populations, monitor community participation, and control value (and e/valuation). Geometry, in this context, was pursued in ‘outsider’ artisan practices, using techniques for exploring proportionality that were always already political and material.

In the case of ancient Egypt, Serres’ account makes evident that it is not simply a focus on relation (of wheat berries to rice, for instance) but a focus on material-cultural *analogy*, whereby measurement is the medium of transport laid down, so that a certain diplomacy and commensurability can be pursued. This co-mensuration or co-relationality is partially captured in our formal notion of

⁴ See the Human Relations Area Files (HRAF) ‘World Cultures’ database (<http://ehrafworldcultures.yale.edu/ehrafe/>).

proportion $a/b : c/d$. We wonder if this see-saw analogy is the way that measurement lives in the material world, embodying a relational ontology. Accordingly, analogy can be considered the origin of a minor and nomadic abstract thought, only to be covered over, later in the Greek tradition, by deduction and other images of reason. According to such a reading, analogy furnishes a chain that links bodies ‘in parallel’ (in fact, analogy engenders the very notion of parallel), tracking what transits across encounters and what falls away in the rhythm of loss (of more or less). Measurement, as Kirby (2011) and Puar (2017) argue, is not merely socially constructed by political regimes, but is more-than-human in its material base. In Egypt, we start in the middle, in the medium, where variance flows and the river floods. It’s this changing area of the alluvial beds that inspires the cultural collective contract of measurement. We might even say that measurement is always fumbling with *difference in itself*, rather than the difference between two prior portions or bodies. We are so accustomed to thinking that measurement rests fundamentally on a standardized and de-territorialized unit, we forget that cultures across the globe invented their units in response to reciprocally varying processes. Consider how units have often had to stretch to suit our changing relationships. The sundial, for instance, like the gnomon, tracks twelve units every day, regardless of the ‘length’ of day. Consequently, the unit “hour” actually stretches or shrinks to match the day: “Always twelve, in spite of everything, like an invariant count of quantities that are variable everyday” (Serres, 2017, p. 196).

3.2. Cosmos

Aside from the pragmatic goals of mercantile trade and accounting, there are also underlying complex cosmological themes that traverse different cultural investments in measurement. The cosmological link is particularly interesting, because it reminds us that speculation and measurement go hand in hand. Cosmology and measurement have always been partners, as seen in the alignments of Stonehenge or the calendars of Mesoamerica. Justeson (2010) notes that Mayan calendar specialists used zero rather than simple container metaphors for their number systems, precisely because they were working with mythic time scales evoking a time before man, a negative time. Cosmology is, in part, about seeking and creating a kind of order in the universe, and measurement lends itself to this objective. There is a reciprocal determination - *measuring activity encounters an order in the world and humans desire an ordering of the world*. Here we insist on the mutual reciprocity of that encounter and that desire, as an ontological entanglement.

This cosmological link shows how measurement comes to figure prominently in human spirituality (Urton & Llanos, 1997). Measurement plays a crucial role in cultural and theocratic regimes of power, as a method for taming the unruly earth. We invest measurement with the capacity to *uncover* an ordered universe, which serves the human tendency to imagine an underlying stability and explanatory continuum. Although measurement seems to deal principally with finite substance, its fumbling practice points to the existence of an intensive infinite that lies beyond the limits of current measurement and even current perception.⁵ Lugli (2019) recounts how European Medieval measurement standards were built into the churches and public squares, and how the standards became quasi-objects, or “ambivalent entities that exist between physical objects and ideal ratios” (p. 30). Eventually, measurement practices – enforced by friars and statutes whenever possible – were used to both control everyday life as well as make the meta-physical palpable: “measuring served as an ideal interpretative channel to recuperate the body of Christ” (p. 146). We wish to underscore the philosophical point that measurement always has one leg in the finite world of actual dimension, and one in an indeterminate virtual realm, as characterized in the work of Deleuze and Guattari (1987).⁶ It is this mixture that makes measurement political and plastic, as well as speculative, despite its apparent fixity or definitive nature.⁷

Related to measurement’s speculative stretch is the practice of indirect measurement—that is, measurement that cannot be achieved by current measurement instruments. Indirect measurement has always played a generative role in the creation of new mathematics. While our contemporary measurement curriculum is overly focused on the concept of unitizing and then covering space with the unit, indirect measurements rely principally on the act of analogy and proportion. Eratosthenes in 200 BCE detects the circumference of the earth using only the difference in the angle of the mid-day sun in two locales, while Thales uses proportion to discover the height of the Great pyramid of Cheops in Egypt (600 BCE). These were measurement feats that seemed well beyond the capacities of the ancient Greeks, and were achieved by enlisting the sun’s rhythms and the starry transits of the night sky. Brown (2005) argues that the Babylonian gnomon should be seen as a kind of automatic inscription device that *knows*; it seems to measure on its own, with no need for human interpretation (unlike, for example, a telescope). For the Greeks, the gnomon “discerned, distinguished, intercepted the light from the Sun, left lines on the sand as if it were writing on a blank page and, yes, understood” (Serres, 1995, p. 80). We witness in these early historical accounts the technique of proportionality as immanent to the relationality of the material world.

Serres (2012) makes the argument that proportionality was a cornerstone of ancient Greek cosmology and mathematics, suggesting that the discovery of the incommensurability of the square’s diagonal was a monumental socio-cultural crisis as well as a

⁵ This point is well exemplified in recent news: in 2018, the lump of metal that has rested in the International Bureau of Weights and Measures in Paris for over a century, defining the weight of one kilogram, was dethroned. This physical, vulnerable specimen will be replaced by what scientists called a more fundamental and precise measure, defined in terms of an electric current and, more specifically, in terms of the Planck constant. Similarly, the length of a metre has now been defined in terms of the speed of light, and the second in terms of the vibrations of the caesium atom. Any increase in precision should not be mistaken for an escape from error.

⁶ Please see de Freitas and Sinclair (2014) on the virtual, and note that we are not taking up distinctions between the continuous and the discrete in this paper (Fazi, 2019).

⁷ Again, regarding the metre, desire for absolutes is found in the 1793 official report demanding platinum be used precisely because it was “the least susceptible to be altered” by climate; Platinum was not in circulation (like gold), and taken from Spanish mines in Ecuador (the ‘new’ world), while the desired aesthetic was bare and unmarked (purity), aspiring to mask the earthly materiality of measure (Lugli, 2019, p. 36).

mathematical crisis. He suggests that rationality and ratio were bound together in the ancient Greek logocentric image of thought and cosmology. The fact that the diagonal of a square did not comply with commensurable (rational) numbers meant that there were magnitudes beyond the realm of analogy – this fact troubled a basic cosmological belief that the universe was rationally ordered. The legend of Hippasus’ murder points to the significance of this fact, and the perturbations it created across their socio-material world. Book X of Euclid was meant to address this crisis (Stillwell, 2010), where Eudoxus uses an algorithmic method to determine these irrational magnitudes as closely as possible using rational lengths that were recursively closer and closer to that which were sought. This algorithmic approach to approximating the measure of a diagonal rescued the theory of proportions, which had become crucial to Greek mathematics.

Wong, Lipka, and Andrew-Ihrke (2014) show how proportionality is used by the Yup’ik Eskimos of Alaska, where the human body is central to linking measuring practices with the cosmos. For example, in the context of sewing, the Yup’ik Elders show how the approach to measuring begins as proportion—the relative height of two people involved in the making, which is calculated by lining the people up at their centre. Again, proportion precedes portion, and measuring practices emerge as forms of relationality. While the scaling and proportional measuring of the Yup’ik shares many aspects of the Davydov (1991) approach to measurement, it differs in at least one important way; the Yup’ik state “measuring begins in the centre” (p. 663), which in this case means that measuring is always intensive, and based on repeated halving, rather than on covering. The cosmological implications are huge: the centre (qukaq) is the “beginning [ayagnek] of everything” (Lipka, Adams, Wong, Koester, & Francois, 2019, p. 114), as one Yup’ik Elder said – it is that which produces upriver/downriver, sidedness, and the frame of reference. It is difficult to overstate the cosmic consequences of such an orientation, as this act of measuring is said to situate the human body in a worlding and centering relation to a totality.

4. Contemporary measure theory

The nineteenth century brought intense interest in measure theory, as many mathematicians developed new integration methods within the calculus. Relevant to our focus is Bernhard Riemann’s (1826–1866) revolutionary work on manifolds and curvature.⁸ O’Shea (2007) describes Riemann’s contribution in terms of a new conceptualisation of space that dislocates it from an absolute geometry and opens up the possibility of multiple geometries. Plotnitsky (2012) suggests that Riemann’s contribution takes up the problem of measure, and moves away from conventions of covering, towards a more immanent attention to the structure of space qua space. One of the problems that Riemann explored was how to determine the surface area of a variegated geographical region. He developed a new way to study the changing curvature of a surface without having to reference the external space in which it was embedded. Riemann studied curvature by tracing the *changing* angle sum of a moving triangle on a given surface. By tracking the changing sum of the interior angles of a triangle, as it moved across negatively and positively curved surfaces, Riemann was able to create a differential ‘measure’ of the surface without reference to the metric of an enveloping space around it. Durie (2006) suggests that this “consisted in the proposal that a surface can be conceived as a space in itself, rather than being embedded within a higher-dimensional space [...]” (p. 177). As Plotnitsky (2006) points out, curvature is then determined internally, “rather than in relation to an ambient space, Euclidean or not” (p. 200).

While it is tempting to dismiss these ideas as being beyond the purview of the k-12 curriculum, we are arguing that the study of surfaces and curvature is one that has occurred and been repeated across many cultures and eras, and it is precisely its focus on relationality that makes it worthy of curricular attention. Riemann’s work helps us challenge the currently dominant “covering” approach in curriculum policy, which feeds into a particular settler, logocentric, cognitivist and narrow image of measurement—narrow in the sense of containing measurement within a very restricted kind of engagement.

Measure theory has developed in response to the various problems that emerge when bringing the continuous line together with the discrete number.⁹ This paradoxical ‘aporia’ at the heart of measurement has been an engine of mathematical invention for centuries (de Freitas, 2018). Consider how Legendre’s (1823) and, later, Dedekind’s (1872) arithmetisation of geometry achieved a correspondence between the line segment and number, in an attempt to bring rigour to analysis (Buckley, 2012; Hartshorne, 2000). These developments can be considered part of a concerted effort to find new ways to confidently measure the world. Moreover, the idea of “covering” has undergone mathematical developments to ensure that it might be relevant in higher dimensional spaces. The Lebesgue measure, for instance, is a kind of covering technique, introduced to address some of the paradoxes associated with the mathematical continuum. In 1911, Lebesgues (1875–1941) proposed that we define dimension in terms of the number of open sets used to “cover” the given object: a set is dimension n if for every refined covering of the set, there exist elements in the set that belong to $n + 1$ open sets of the covering. Refining the covering is a technical process which means keeping the number of open sets constant, but reducing their ‘size’ so that every point in the original set belongs to a minimum number of covering sets.

Measure theory and Dimension theory continued to be important parts of 20th century advanced mathematics, becoming pivotal domains in the field. According to Zalamea (2012), the arithmetisation movement of the 19th century is now giving way to a new geometrisation programme where the arithmetic question of ‘what is the measure of X?’ shifts to a “transitory” question of ‘What kinds of new measures are made possible under these transformations?’. Zalamea (2012) draws extensively on contemporary mathematics to show how conceptual and technical developments are fuelled by the ongoing feed-back between contemporary

⁸ See de Freitas (2016a) for more.

⁹ From at least the time of Zeno, but also in relation to the discovery of irrational numbers, and ongoing work on the continuum hypothesis. Longo (2019a) mentions the significance of Riemann’s attempt to distinguish continuous and discrete manifolds in his own development of measure theory.

physics and mathematics. Through continuing transits between physics and mathematics, new forms of 20th century “mixed mathematics” have opened up generative problems of measurement; new theories of space-time formulated in the last century called for new kinds of mathematics, which in turn further elaborated new physical theories (Ferreirós, 2019; Longo, 2019b). These more recent developments, much like the others we’ve discussed, are no doubt linked to new relational mixtures of matter-meaning, and to particular habits of human movement, although we haven’t room to explore this claim here. The point is that measurement is, as we show below, entirely undervalued in the mathematics curriculum, demoted to a set of formulae and their application, when in fact these examples make evident that it is a rich domain of contemporary mathematics.

5. Measurement curriculum

Although there are important international differences in mathematics curriculum, we decided to focus on the U.S. National Council of Teachers of Mathematics (NCTM) and Common Core State Standards (CCSS). There is no mention of measurement in the NCTM 9–12 curriculum standards. Measurement is also absent in the CCSS document for grades 9–12—but it is additionally absent for grades 6–8, so that all the measurement standards appear only in the k-5 grade range. Curriculum policy thus demotes measurement as being for younger students only, and reduces it to tasks that involve unitizing and covering. We also see a delayed adoption of tools in the early curriculum, which reduces them to being *merely* efficient for the process of producing numerical values, when in fact, using tools involves certain ways of moving the body to express conceptual understanding—as Zacharos (2006) shows in the case of area measurement.

The first attempts at developing a standardized curriculum in the USA, which date to the end of the 19th century, specifically mention measurement as a suitable and important area of study for junior high school students, and one that was seen as being connected to “concrete geometry” (rather than the definition/theorem/proof geometry of Euclid’s Elements). Early on, the National Education Association stipulated measurement skills in which students “should learn to estimate by the eye, and to measure with some degree of accuracy, lengths, angular magnitudes, and areas” (1894, p. 24). Despite this practical push, no mention was made of the important role of tools and instruments in measuring activities, reflecting a longstanding bias against the material practices of mathematics (Rotman, 2008). This speaks to the general prejudice at the time against the applied, time-consuming nature of actual measuring. Indeed, early criticism of the laboratory schools that had emerged at the turn of the 20th century complained that learning to use instruments was distracting students from a more ‘proper’ mathematics: “Too much time spent on experimental and graphical work is wearisome and of little value to intelligent pupils. They can’t appreciate the logical training of theoretical geometry, while experiments and measurements of far greater interest can be made in the physical and chemical laboratories” (Davison & Richards, 1907, p. v).

Early 20th century measurement curriculum saw a tug-of-war between ‘real-life’ applications and geometric applications. By 1975, concerns emerged over the disappearance of geometry and its subordination to measurement tasks (National Advisory Committee on Mathematical Education (NACOME), 1975). Shortly after, there was a move to separate geometry from measurement, in order to ensure that the former was given its due attention. The NACOME report articulated a list of ten basic skills needed by students who “hope to participate successfully in adult society,” a list that was eventually adopted by many groups, including TIMMS, NAEP, and the 1989 NCTM Standards. The list reified the trend of separating geometry from measurement. This may have orphaned measurement in the curriculum as a stale, isolated part of mathematics, while also cutting it off from its diversely embodied lineage, as measurement outcomes became more narrowly focused.

In the 1989 NCTM k-4 Standards (National Council of Teachers of Mathematics, 1989), measurement is interpreted as covering objects with units: children must first engage in “comparing objects directly, covering them with various units, and counting the units” (National Advisory Committee on Mathematical Education (NACOME), 1975). Measurement is also presented as relevant only to length, weight, area, and time. In grades 5–8, the authors insist that “Measurement activities can and should require a dynamic interaction between students and their environment” (p. 116) and repeat the claim about measurement being useful in everyday life. In this grade band, more emphasis is placed on students’ use of instruments, but principally ones that more efficiently cover a given object with a unit. In the 2010 CCSS, the concept of unit and of iteration of units is presented as a method of covering—indeed it is offered as *the fundamental measurement concept*. Unlike the NCTM Standards, where practices of relational comparisons (superposition, shearing, transforming) are mentioned, the CCSS approach is fundamentally formulaic and alpha-numeric.

In the 1992 *Handbook of research on mathematics teaching and learning*, there was no chapter on measurement *per se*; however, it was mentioned in the chapter on rational numbers, especially in relation to estimating lengths and weights. In the second 2007 Handbook, measurement appears only in the same chapter on rational numbers, but this time the authors emphasise the importance both of indirect measurement (like the height of the pyramid), which they claimed was not often addressed by teachers, and the “measurement concepts as opposed to acts of measuring” (p. 651). By measurement concepts, the authors elaborate, for example, the inverse relation between the size of a unit of measure and the number of times that unit covers some fixed quantity. In this same 2007 Handbook, measurement appears in the geometry chapter, with a particular focus on area and volume. A theory for the three levels of measurement reasoning is also offered, which focuses entirely on how students use iterating units when measuring (Battista, 2007). Finally, in the Third Handbook (called the Compendium) of 2017, measurement has a chapter of its own. Much of the research surveyed in this chapter by Smith and Barrett (2017) emphasizes covering by units, with a strong focus on identifying and iterating units in relation to the measure of length, area, volume, angle and time. Notably, the authors state that there is currently “no consensus on the place of indirect comparison in the transition from qualitative comparison to metric length measurement” (p. 362).

The literature on measurement practices and disability is sparse, but there are a few articles that explore some related issues. Cawley, Foley, and Hayes (2009) suggest that students with learning disabilities might benefit from studying “the relationships that

things have in space” instead of only using “measurement activities involving formal measures such as telling time, the length of a string, or the weight of an object” (p. 31). Güven and Argün (2018) recommend geometric approaches to measurement, as well as the use of informal measures that are context-relevant. These recommendations direct educators’ attention towards the bodily, intensive, and relational nature of measurement. They also shift attention away from formula-driven and numerical aspects of measuring activity, both of which can over-emphasise memorization and calculation. This is a crucial point because researchers are increasingly aware of the links between learning disabilities and anxiety induced by limited, short-term memory (Geary, 2011; Moore, McAuley, Allred, & Ashcraft, 2014).

Hunt (2015) suggests that intervention research into mathematics learning disability overemphasizes the part-whole partition approach (shading parts of circular wholes, for example), and argues that the concept of ratio and relationships between different kinds of quantities would better develop number sense. She discusses tasks that involve proportion and relationality—precisely the kind of analogical mixture of materialities that Serres describes — that are less focused on the numerical value of the measurement than on the relations of relations (proportions). Instead of drawing on the additive thinking of unit iteration (counting up units that cover space), analogy is fundamentally multiplicative. Other kinds of tasks might also be considered. For instance, tasks that involve dissection and shearing are didactically powerful ways of thinking measurement (Ng & Sinclair, 2015; Proulx & Pimm, 2008; Zacharos, 2006), but are also more resonant with the archeological practices we have discussed, which are fundamentally temporal and dynamic in their treatment of shape.

6. Conclusion

Our aim in this paper has been to show how measuring involves the making and mixing of *analogies*, and that this involves attending to the political and plastic nature of *intensive material relationships* rather than *extensive properties of isolated objects*. This approach shows how measurement is itself modulated by that which it measures. In the move within schools to develop skills with standardized units, measurement becomes a matter of *coverings* in which the relational aspect becomes hidden under the veneer of non-relational quantity, which ignores the underlying variability and plasticity of the material world. In this manner, measurement is seen only as a process of imposing standards, and the instruments of measuring are treated as detachable prosthetic devices that can be discarded after use. This in turn produces a school mathematics curriculum in which only certain attributes are seen as measurable (the extensive ones), and where the unitizing of such attributes becomes the primary conceptualisation of measurement.

We are cautious and wary, however, that emphasis on relationality and plasticity might be seen to inherit some of the political legacies of liberal humanism (Colebrook, 2019). In other words, one must avoid declaring “everything is related” at the expense of misrecognized others, who are forcefully made commensurate by such grand gestures. For instance, Schuller (2018) suggests that past investments in a sentimental relationality of “impressibility” have often been a means for separating the ‘sensitive’ civilized white and able-bodied subject from the abject other. For this reason, it is essential to grapple with the hierarchical intimacies that structure any relational space, and to note the non-innocence of claims for relational ontology. As we argued in de Freitas and Sinclair (2016), regarding the cognitive labour of number sense dis/ability, most deficit models deny any temporal becoming on behalf of both learners and concepts. Clare (2017) shows how many interventions aim to correct disability (i.e. treat disproportionality as deficit), while Kafer (2013) describes the false innocence of many claims for inclusion as contributing to a “compulsorily hypernormative” that refuses any future to those who are presently disabled (p. 44). Our proposals here for a minor mathematics are not intended as a way to correct or cure. We don’t contest the need for interventions that minimize pain and suffering, nor the provisional temporality of dis/ability, but we do contest the proposal that ability is the pure state of perfect proportion to which we must return.

Our focus on relational and intensive measures directs attention to the disappearance of minor mathematics, whether through cultural erasure, settler and colonial habits, geographical changes, metaphysical assumptions or mathematical developments. These are all forces at work in the political shaping of mathematics ability. When mathematics education aims to produce numerate citizens of the 21st century, it participates directly in ablenationalism:

ablenationalism involves the implicit assumption that minimum levels of corporeal, intellectual, and sensory capacity, in conjunction with subjective aspects of aesthetic appearance, are required of citizens seeking to access the ‘full benefits’ of citizenship. (Snyder & Mitchell, 2010, p. 124).

We follow other disability theorists who have used the ideas of Deleuze and Guattari to think differently about the all-too-racist, sexist and ableist concept of ability (Goodley et al., 2014; Overboe, 2009). We have argued that our excavations of minor measuring practices can provide insight into potential non-normative conceptions that may be more inclusive, not as recuperative and assimilationist, but as a way of pluralizing practices. We emphasise that for the Egyptians, Babylonians, Greeks and Inkans, ratios and proportions were processed by machines, or the “automatic knowledge”, as Serres (2017) calls it, of the gnomon, the knotted ropes, the tables of chords, etc. Compared to instruments such as rulers and protractors, the knotted ropes of the harpedonaptai provide a different vision of an analogical approach, where the measuring device itself embodies the varying proportionality of a relational world. Analogical approaches that invite early use of machines or tools create an image of mathematical practice that is more materially diverse. Moreover, this approach recognizes measuring tools, be they fingers and feet or rulers and protractors, as part of the material distribution of mathematical concepts, rather than treating these devices simply as that which objectively counts units to be ‘read-off’. In closing, we emphasise that our analysis of mythic origin stories and speculative archeology is not intended as an argument that the teaching and learning of measurement should follow its historical development. Rather, we have used these accounts to unearth the significance of material practices associated with analogical and intensive measurement.

References

- Alder, K. (2002). *The measure of all things: The seven-year odyssey and hidden error that transformed the world*. New York: Simon & Schuster.
- Appelbaum, P. (2016). Mathematics education as a matter of curriculum. In P. M. A. (Ed.). *Encyclopedia of educational philosophy and theory* (pp. 1–6).
- Ascher, M. (1995). Models and maps from the Marshall Islands: A case in ethnomathematics. *Historia Mathematica*, 22, 347–370.
- Battista, M. (2007). The development of geometric and spatial reasoning. In F. K. Lester (Ed.). *Second handbook of research on mathematics teaching and learning* (pp. 843–908). Charlotte, NC: Information Age.
- Brown, S. (2005). The theatre of measurement: Michel Serres. In C. Jones, & R. Munro (Eds.). *Contemporary organization theory* (pp. 215–227). Oxford: Blackwell Publishing.
- Buckley, B. L. (2012). *The continuity debate: Dedekind, Cantor, du Bois-Reymond, and Peirce on continuity and infinitesimals*. Boston, MA: Docent Press.
- Cawley, J. F., Foley, T. E., & Hayes, A. M. (2009). Geometry and measurement: A discussion of status and content options for elementary school students with learning disabilities. *Learning Disabilities: A Contemporary Journal*, 7(1), 21–42. <https://doi.org/10.1080/1034912X.2011.548476>.
- Chen, M. (2012). *Animacies: Biopolitics, queer mattering, and racial affect*. Duke University Press.
- Clare, E. (2017). *Brilliant imperfection: Grappling with cure*. Durham: Duke University Press.
- Colebrook, C. (2019). A cut in relationality: Art at the end of the world. *Angelaki: Journal of the Theoretical Humanities*, 24(3), 175–195.
- Cooperrider, K., & Gentner, D. (2019). The career of measurement. *Cognition*, 191, 1–12. <https://doi.org/10.1016/j.cognition.2019.04.011>.
- Davison, C., & Richards, C. (1907). *Plane geometry for secondary schools*. Cambridge: University Press.
- Davydov, V. V. (1991). On the objective origin of the concept of fractions. *Focus on Learning Problems in Mathematics*, 13, 13–83.
- de Freitas, E. (2018). The mathematical continuum: A haunting problematic. *The mathematics enthusiast*, 15(1-2), 148–158.
- de Freitas, E. (2016a). Deleuze, ontology and mathematics. In M. Peters (Ed.). *Encyclopedia of educational philosophy and theory*. Singapore: Springer.
- de Freitas, E. (2016b). Number sense and calculating children: Multiplicity, measure and Mathematical monsters. *Discourse Studies in the Cultural Politics of Education*, 37(5), 650–661.
- de Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. Cambridge, UK: Cambridge University Press.
- de Freitas, E., & Sinclair, N. (2016). The cognitive labour of mathematics dis/ability: Neurocognitive approaches to number sense. *International Journal of Education Research*, 79, 220–230.
- de Freitas, E., & Sinclair, N. (2018). The quantum mind: Alternative ways of reasoning with uncertainty. *Canadian Journal of Science Mathematics and Technology Education*, 18(3), 271–283.
- de Freitas, E., Sinclair, N., & Coles, A. (Eds.). (2017). *What is a mathematical concept?*. New York: Cambridge University Press.
- Dedekind, R. (1872). *Stetigkeit und Irrationale Zahlen*. Braunschweig, Germany: Friedrich Vieweg.
- Deleuze, G., & Guattari, F. (1987). *A thousand plateaus: Capitalism and schizophrenia*, trans. Brian Massumi. Minneapolis, MN: University of Minnesota Press.
- Durie, R. (2006). Problems in the relation between maths and philosophy. In S. Duffy (Ed.). *Virtual mathematics: The logic of difference* (pp. 169–186). Bolton, UK: Clinamen Press.
- Farr, R. H. (2010). Measurement in navigation: Conceiving distance and time in the Neolithic. In C. Renfrew, & I. Morley (Eds.). *The archaeology of measurement: Comprehending heaven, earth and time in ancient societies* (pp. 19–27). Cambridge, UK: Cambridge University Press.
- Fazi, M. B. (2019). Digital aesthetics: The discrete and the continuous. *Theory Culture & Society*, 36(6), 3–26.
- Ferreiros, J. (2019). Wigner's "unreasonable effectiveness" in context. In M. Pitici (Ed.). *The best writing on mathematics 2018*. Princeton: Princeton University Press.
- Geary, D. C. (2011). Consequences, characteristics, and causes of mathematical learning disabilities and persistent low achievement in mathematics. *Journal of Developmental and Behavioral Pediatrics*, 32(3), 250–263. <https://doi.org/10.1097/DBP.0b013e318209edef>.
- Goldenberg, E. P., Clements, D., Zbiek, R. M., & Dougherty, B. (2014). *Developing essential understanding of geometry and measurement for teaching mathematics in Pre-K-Grade 2*. Reston, VA: NCTM Publishing.
- Goodley, D., Lawthom, R., & Runswick-Cole, K. (2014). Posthuman disability studies. *Subjectivity*, 7(4), 342–361. <https://doi.org/10.1057/sub.2014.15>.
- Goodley, D., Runswick-Cole, K., & Liddiard, K. (2016). The DisHuman child. *Discourse: Studies in the Cultural Politics of Education* ISSN, 37(5), 770–784. <https://doi.org/10.1080/01596306.2015.1075731>.
- Gutiérrez, R. (2017). Living mathematx: Towards a vision of the future. *Philosophy of Mathematics Education Journal*, 32(1), 2–26.
- Güven, N., & Argün, Z. (2018). Width, length, and height conceptions of students with learning disabilities. *Issues in Educational Research*, 28(1), 77–98.
- Hartshorne, R. (2000). Teaching geometry according to Euclid. *Notices of the AMS*, 47(4), 460–465.
- Hunt, J. (2015). Notions of equivalence through ratios: Students with and without learning disabilities. *The Journal of Mathematical Behavior*, 37, 94–105.
- Justeson, J. (2010). Numerical cognition and the development of 'zero' in Mesoamerica. In C. Renfrew, & I. Morley (Eds.). *The archaeology of measurement: Comprehending heaven, earth and time in ancient societies* (pp. 43–53). Cambridge, UK: Cambridge University Press.
- Kafer, A. (2013). *Feminist, queer, crip*. Bloomington, IN: Indiana University Press.
- King, T. L. (2017). Humans involved: Lurking in the lines of posthumanist flight. *Critical Ethnic Studies*, 3(1), 162–185.
- Kirby, V. (2011). *Quantum anthropologies: Life at large*. Durham, NC: Duke University Press.
- Lambert, R. (2019). Political, relational, and complexly embodied; experiencing disability in the mathematics classroom. *ZDM*, 51(3), 279–289.
- Legendre, A. M. (1823). *Elements de Geometrie, avec des notes* (12th ed.). Paris: Firmin Didot.
- Lipka, J., Adams, B., Wong, M. M., Koester, D., & Francois, K. (2019). Symmetry and measuring: Ways to teach the foundations of mathematics inspired by Yupiaq Elders. *Journal of Humanistic Mathematics*, 9(1), 107–157.
- Longo, G. (2019a). Quantifying the world and its webs: Mathematical discrete vs continua in knowledge construction. *Theory Culture & Society*, 36(6), 63–72.
- Longo, G. (2019b). Letter to Turing. *Theory Culture & Society*, 36(6), 73–94.
- Luciano, D., & Chen, M. Y. (2015). Has the Queer ever been human? *GLQ: A Journal of Lesbian and Gay Studies*, 21(2–3), 183–207. <https://doi.org/10.1215/10642684-2843215>.
- Lugli, E. (2019). *The making of measure and the promise of sameness*. Chicago: University of Chicago Press.
- Mitchell, D. T., Antebi, S., & Snyder, S. L. (2019). *Introduction The matter of disability* (kindle edition). 1–36. Available from Amazon.co.uk.
- Moore, A., McAuley, A., Allred, G., & Ashcraft, M. (2014). Mathematics anxiety, working memory, and mathematical performance. In S. Chinn (Ed.). *The Routledge international handbook of dyscalculia and mathematics learning difficulties* (pp. 326–336). London, UK: Routledge.
- National Advisory Committee on Mathematical Education (NACOME) (1975). *Overview and analysis of school mathematics, grades K-12*. Washington, DC: Conference Board of the Mathematical Sciences.
- National Education Association (1894). *Report of the committee of ten on secondary school studies*. New York: American Book Company.
- Ng, O., & Sinclair, N. (2015). "Area without numbers": Using Touchscreen dynamic geometry to reason about shape. *Canadian Journal of Science Mathematics and Technology Education*, 15(1), 84–101.
- O'Shea, D. (2007). *The Poincaré conjecture: In search of the shape of the universe*. New York: Walter and Company.
- Overboe, J. (2009). Affirming the impersonal life: A different register for disability studies. *Journal of Literary & Cultural Disability Studies*, 3(3), 241–256.
- Plotnitsky, A. (2006). Manifolds: On the concept of space in Riemann and Deleuze. In S. Duffy (Ed.). *Virtual mathematics: The logic of difference* (pp. 187–208). Bolton, UK: Clinamen Press.
- Plotnitsky, A. (2012). Experimenting with ontologies: Sets, spaces, and topoi with Badiou and Grothendieck. *Environment and Planning D Society and Space*, 30, 351–368.
- Popkewitz, T. S. (2004). The alchemy of the mathematics curriculum: Inscriptions and the fabrication of the child. *American Educational Journal*, 41(4), 3–34.
- Proulx, J., & Pimm, D. (2008). Algebraic formulas, geometric awareness and Cavalieri's principle. *For the learning of mathematics*, 28(2), 17–24.
- Puar, J. (2017). *The right to maim: Debility, capacity, disability*. Durham, NC: Duke University Press.
- Renfrew, C., & Morley, I. (2010a). Measure: Towards the construction of our world. *The Archaeology of measurement: Comprehending heaven, earth and time in ancient*

- societies. Cambridge, UK: Cambridge University Press 1–6.
- Renfrew, C., & Morley, I. (2010b). *The archeology of measurement: Comprehending heaven, earth and time in ancient societies*. Cambridge, UK: Cambridge University Press.
- Rotman, B. (2008). *Becoming beside ourselves: The alphabet, ghosts, and distributed human beings*. Durham: Duke University Press.
- Schuller, K. (2018). *The biopolitics of feeling: Race, sex, and science in the nineteenth century*.
- Serres, M. (2017). *Geometry*. London, UK: Bloomsbury Press.
- Serres, M. (2012). *Variations on the body*. Minneapolis, MN: University of Minnesota Press.
- Serres, M. (1995). *Gnomon: The beginnings of geometry in Greece. A history of scientific thought (originally published 1989)*. Oxford: Blackwell.
- Siebers, T. (2008). *Disability theory*. Ann Arbor: University of Michigan Press.
- Sinclair, N., & de Freitas, E. (2019). Body studies in mathematics education: Diverse scales of mattering. *ZDM: International Journal of Mathematics Education Research*, 51(2), 227–237.
- Singh, J. (2018). *Unthinking mastery: Dehumanism and decolonial entanglements*. Durham, NC: Duke University Press.
- Smith, J., & Barrett, J. (2017). Learning and teaching measurement: Coordinating quantity and number. In J. Cai (Ed.). *Compendium for research in mathematics education* (pp. 355–385). Reston, VA: National Council of Teachers of Mathematics.
- Snyder, S., & Mitchell, D. (2001). Re-engaging the body: Disability studies and the resistance to embodiment. *Public Culture*, 13, 367–391.
- Snyder, S. L., & Mitchell, D. T. (2010). Introduction: Ablenationalism and the geo-politics of disability. *Journal of Literary & Cultural Disability Studies*, 4, 113–125. <https://doi.org/10.3828/jlcds.2010.10>.
- Tan, P., Lambert, R., Padilla, A., & Wieman, R. (2019). A disability studies in mathematics education review of intellectual disabilities: Directions for future inquiries and practice. *The Journal of Mathematical Behavior*, 54. <https://doi.org/10.1016/j.jmathb.2018.09.001>.
- Urton, G. (2010). Recording measure(ment)s in the Inka khipu. In C. Renfrew, & I. Morley (Eds.). *The archaeology of measurement: Comprehending heaven, earth and time in ancient societies* (pp. 54–68). Cambridge, UK: Cambridge University Press.
- Urton, G., & Llanos, P. (1997). *The social life of numbers: A Quechua ontology of numbers and philosophy of arithmetic*. Austin: TXL University of Texas Press.
- Verran, H. (2001). *Science and an African logic*. Chicago, IL: University of Chicago Press.
- Wong, M., Lipka, J., & Andrew-Ihrke, D. (2014). Symmetrical measuring: An approach to teaching elementary school mathematics informed by Yup'ik Elders. In J. Anderson, M. Cavanagh, & A. Prescott (Vol. Eds.), *Curriculum in focus: Research guided practice: 2014*, (pp. 661–668). Sydney: MERGA Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia.
- Zacharos, K. (2006). Prevailing educational practices for area measurement and students' failure in measuring areas. *The Journal of Mathematical Behavior*, 25, 225–239.
- Zalamea, F. (2012). *Synthetic philosophy of contemporary mathematics*. New York, NY: Sequence Press.

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