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Optimization of Indoor Hybrid PLC/VLC/RF Communication Systems

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Abstract: In this work, we propose a hybrid power line communication (PLC)/visible light communication (VLC)/radio frequency (RF) fronthaul with a fiber based wired backhaul system to support massive number of smart devices (SDs). Since, a signal-to-noise ratio (SNR) based access point (AP) association and bandwidth (BW) allocation for the SRC converges to an optimal value. Further, an analytical approximation for the BW allocated to each SD for a given AP association is derived using the Lagrangian multiplier method. The performance of the proposed system is evaluated through extensive numerical results. Moreover, the effect of the increased number of SDs on the optimal SRC is analysed.

1 Introduction

Connectivity in indoor environment has become one of the most important aspects of our modern society. Radio frequency (RF) based communication system is one of the most commonly used technology for indoor connectivity. Due to availability cost issues with RF spectrum, recently, several other technologies have come under consideration to achieve reliable and high data rate indoor connectivity. Light emitting diodes (LEDs), for instance, are widely used because of their enhanced color rendering capability, long life time, and huge energy savings [1]. These LEDs can be used for indoor wireless communications, known as visible light communication (VLC), for various smart applications as discussed in [2]. Power line communications (PLC) is another such technology that has gained considerable research attention in recent years due to the existing ubiquitous infrastructure. These PLC terminals can communicate through power lines, whereas, mobile devices have to connect via either RF or VLC interfaces [3].

The Internet of Things (IoT) has led to the deployment of millions of smart devices (SDs) due to their vast applications [4]. Moreover, providing high data rates for these large numbers of SDs, using the conventional RF based communication systems has become a challenging task, owing to the limited spectrum. Further, it has been shown that PLC, VLC, or RF alone cannot be an effective solution to provide high data rates [1]. To address this problem, the hybrid systems have been proposed in the literature with the coexistence of different communication systems.

The integration of PLC and RF for wireless relaying systems has been proposed for many applications including indoor, outdoor, and smart grids [5–9]. In [5], for instance, wireless relaying systems are integrated with PLC for long distance transmissions. This integration shows better robustness than using conventional PLC or RF for both long and short range communication. In addition, a hybrid wireless-broadband PLC (BPLC) system has been presented in [6] that utilizes the medium voltage power line cables for offering broadband services to remote areas. In [7], a hybrid PLC/RF system has been considered to improve the range of the RF signals in a multi-storey building. Here, PLC links work as a backbone to pass the RF signals between multiple rooms and floors. In [8], the authors have proposed a multi-channel receiver based hybrid PLC/RF communication system which has been analysed in terms of selection combining and maximal ratio combining methods for fading compensation. The capacity analysis of an amplify-and-forward (AF) relay based cascaded PLC/RF system has been carried out in [9], where, PLC and wireless users communicate with each other through these relays.

Integration of PLC and VLC technologies for various applications have been discussed in [10–14]. The first integration proposal has been discussed in [10] which utilizes ON/OFF keying for data modulation. Then, orthogonal frequency division multiplexing (OFDM) technique has been developed for hybrid PLC/VLC system to mitigate the interference and inter-symbol interference. In [14], a survey on opportunity of integrating PLC and VLC channels has been presented. The capacity analysis of an amplify-and-forward and decode-and-forward based hybrid PLC/VLC systems have been carried out in [15] and [16], respectively. Here, the source node first transmits the information to the relay through a PLC link and then the relay node amplifies or decodes and forwards it to the destination node through a VLC link. A hybrid VLC and RF system has been considered in [17] for indoor environments which improves the per user average and outage throughput. A cascaded PLC/VLC system has been considered in [18] that is compatible to provide multiple services with different quality of service requirements. In [19], an energy efficient wireless communication network has been developed using both RF and VLC technologies to solve the power and BW allocation problem.
problem. An optimization problem has been formulated in terms of power and bandwidth (BW) in order to maximize the energy efficiency of the heterogeneous network constrained by the maximum allowable transmit power from each access point (AP) and required data rates [19].

In [20], a cell breathing technique has been used for traffic transfer between the devices and APs in a cellular network. A pricing-based distributed algorithm has been used in [21] which considers congestion levels of the base stations (BSs) and the transmission environment of the mobile terminals as the parameters for load balancing in the heterogeneous network. The weighted sum-rate (WSR) maximization for a wireless cellular network has been addressed in [22]. In [22], the prioritized and randomized algorithms have been proposed which are based on the coordinated scheduling and discrete power control and require limited information exchange and processing at each BS.

The sum rate maximization using a signal-to-interference-plus-noise ratio (SINR) approximation and the max-min weighted SINR optimization for a wireless network has been proposed in [23]. A study on the linear filter design for maximizing the WSR in the multiple input multiple output broadcast channel (MIMO-BC) has been carried out in [24]. The WSR problem in this study is solved as a weighted sum minimum mean square error-problem with optimized mean square error-weights. The robust beamforming approach has been used in [25] to calculate the WSR for a multi-cell massive MIMO downlink system. In [4], the authors have investigated achievable data rate of a hybrid system comprising of cascaded PLC/VLC system in coexistence with the RF system for a constrained transmit power. In this study, the authors have developed an algorithm that optimally divides the maximum allocated power among all three communication links such that the achievable data rate can be maximized [4]. Further, an investigation of power allocation problem has been carried out in [1] for hybrid PLC/VLC system in coexistence with RF in an indoor environment. In [1], the authors have analyzed the transmit power minimization problem with a constrained generalised quality of service requirement. Then, for a constrained data rate requirement, an analytical approximation for the BW allocated to each SD for a given AP association is derived.

An analytical approximation for the BW allocated to each SD for a given AP association is derived. A numerical analysis is presented on the achievable sum rate capacity (SRC) with AP association and BW allocation is an open problem [19] and to the best of our knowledge, the sum rate capacity (SRC) with AP association and BW allocation is an open problem [19] and to the best of our knowledge, it has not been carried out for the hybrid PLC/VLC/RF system. Thus, in this work, a joint distribution algorithm based on (1) worst device reshuffling and (2) load balancing techniques are proposed, in order to maximize the achievable SRC for the fronthaul hybrid PLC/VLC/RF communication system with a fixed transmit power from all APs. The key contributions of this work are as follows:

- The AP association and BW allocation for the hybrid PLC/VLC/RF system has been formulated as an optimization problem.
- The presence of impulsive noise in the PLC system adds non-linearity to the optimization problem compared to conventional cellular systems. Thus, a hierarchical decomposition method is considered to convert the resultant non-linear optimization problem into a set of convex optimization problems.
- A joint distribution algorithm based on (1) worst device reshuffling and (2) load balancing techniques are proposed, in order to maximize the achievable SRC.
- An analytical approximation for the BW allocated to each SD for a given AP association is derived.
- A numerical analysis is presented on the achievable SRC to study the effect of increased number of PLC-disabled SDs.
- Finally, the effect of increased number of SDs on the optimal SRC is analysed numerically.

The rest of the paper is outlined as follows. The channel models corresponding to PLC, VLC, and RF are presented in Section 2. The proposed joint association and BW allocation is discussed in Section 3 and the performance of the proposed joint distribution algorithm is evaluated through extensive simulations in Section 4. Finally, Section 5 provides concluding remarks along with possible future scopes.

### 2 System model

We consider an indoor downlink scenario with 1 RF AP, k VLC APs, l PLC APs, and N SDs. Let \( a \in A = \{0, 1, \ldots, k, k + 1, \ldots, k + l\} \) denotes the set containing the indices of RF, VLC, and PLC APs, where, \( a = 0 \) denotes the index of RF, \( a = 1, 2, \ldots, k \} \) denote the indices of VLC AP, and \( a = k + 1, k + 2, \ldots, k + l \} \) denote the indices of PLC AP. For convenience, let \( n \in U = \{1, 2, \ldots, N\} \) denote the set of SDs. In this work, we assume two kinds of SDs such as PLC-enabled and PLC-disabled as shown in Fig. 1. The PLC-enabled SDs have signal reception interfaces for PLC, VLC, and RF, whereas, the PLC-disabled SDs have reception interfaces for VLC and RF only [1], [4]. Further, we assume that a SD can connect to only one AP using only one communication link at a given time. The channel models corresponding to PLC, VLC, and RF are presented in the following subsection. Moreover, we consider the effect of both attenuation and fading in the channel for the SRC analysis.

#### 2.1 PLC channel model

In the PLC system, in addition to the distance dependent signal attenuation, fading and impulsive noise also affects the transmitted data. Thus, in this work, we consider the effect of both channel impairments. The fading gain of the PLC channel can be modeled by a log-normal distribution [26–28]. Let \( P_{f} \) (in dB), \( P_{g} \) (in dB), and \( \alpha \) (in dB/length) denote the transmit power, the gain, and the attenuation factor, respectively.
Table 1 Summary of important parameters and their typical values used in this work.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Set of all APs</td>
<td></td>
</tr>
<tr>
<td>A_d</td>
<td>Area of the photodetector in VLC</td>
<td>$10^{-4}$ $m^2$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Index of an AP</td>
<td></td>
</tr>
<tr>
<td>$B_{a,n}$</td>
<td>BW allocated for the link between AP $a$ and SD $n$</td>
<td></td>
</tr>
<tr>
<td>$B_{a,max}$</td>
<td>Maximum available BW at AP $a$</td>
<td></td>
</tr>
<tr>
<td>$B_n$</td>
<td>Noise bandwidth</td>
<td>100MHz</td>
</tr>
<tr>
<td>$B_p$</td>
<td>Allocated BW in PLC</td>
<td>10MHz</td>
</tr>
<tr>
<td>$B_r$</td>
<td>Allocated BW in RF</td>
<td>10MHz</td>
</tr>
<tr>
<td>$B_v$</td>
<td>Allocated BW in VLC</td>
<td>100MHz</td>
</tr>
<tr>
<td>$C$</td>
<td>Sum rate capacity (SRC)</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>Set of PLC-disabled SDs</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Distance between AP and SD</td>
<td></td>
</tr>
<tr>
<td>$f_c$</td>
<td>Carrier frequency in RF</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>$G_{h,r}$</td>
<td>Channel gain in RF</td>
<td>1</td>
</tr>
<tr>
<td>$G_p$</td>
<td>Channel gain in PLC</td>
<td>1</td>
</tr>
<tr>
<td>$G_{r,n}$</td>
<td>Path gain in RF</td>
<td></td>
</tr>
<tr>
<td>$g(\beta)$</td>
<td>Gain of the concentrator in VLC</td>
<td>1</td>
</tr>
<tr>
<td>$H(\beta)$</td>
<td>DC channel gain in VLC</td>
<td></td>
</tr>
<tr>
<td>$I_2$</td>
<td>Noise BW factor</td>
<td>0.562</td>
</tr>
<tr>
<td>i</td>
<td>Background current</td>
<td>5100$\mu$A</td>
</tr>
<tr>
<td>k</td>
<td>Number of VLC APs</td>
<td>4</td>
</tr>
<tr>
<td>l</td>
<td>Number of PLC APs</td>
<td>3</td>
</tr>
<tr>
<td>N</td>
<td>Number of SDs</td>
<td>40, 0-80</td>
</tr>
<tr>
<td>$N_p$</td>
<td>Noise power spectral density in PLC</td>
<td>$10^{-14}$ watts/Hz</td>
</tr>
<tr>
<td>$N_r$</td>
<td>Noise power spectral density in RF</td>
<td>$3.89 \times 10^{-21}$ watts/Hz</td>
</tr>
<tr>
<td>$N_v$</td>
<td>Noise power spectral density in VLC</td>
<td>$10^{-21}$ watts/Hz</td>
</tr>
<tr>
<td>$n$</td>
<td>Index of an SD</td>
<td></td>
</tr>
<tr>
<td>$P_{a,b}$</td>
<td>Noise power due to ISI</td>
<td>0</td>
</tr>
<tr>
<td>$P_R$</td>
<td>Received power at an SD</td>
<td></td>
</tr>
<tr>
<td>$P_T$</td>
<td>Transmit power at an AP</td>
<td>10 mWatt</td>
</tr>
<tr>
<td>q</td>
<td>Electronic charge</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>Responsivity of the photodiode in VLC</td>
<td>1</td>
</tr>
<tr>
<td>$T_e(\beta)$</td>
<td>Gain of optical filter in VLC</td>
<td>1</td>
</tr>
<tr>
<td>$U_a$</td>
<td>Set of all SDs that are associated to AP $a$</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Attenuation factor with distance in PLC</td>
<td>40 dB/KM</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Parameter of Bernoulli random variable $n_b$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Power ratio of impulsive noise to the background noise</td>
<td>10</td>
</tr>
<tr>
<td>$\Gamma_p$</td>
<td>SNR of PLC</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Order of Lambert index in VLC</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The angle between the light emitting direction and light source normal direction in VLC</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Incident angle of radiation in VLC</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Field of view of the receiver in VLC</td>
<td>$60^\circ$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>LED’s semi-angle at half power</td>
<td>$60^\circ$</td>
</tr>
<tr>
<td>$\Gamma_v$</td>
<td>SNR in VLC</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>Path loss in RF</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_{a,n}$</td>
<td>Binary indicator that defines the association of $SD$ $a$ to $AP$ $n$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_{a,n}$</td>
<td>SNR at SD $n$ from AP $a$</td>
<td></td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Lagrange multiplier</td>
<td></td>
</tr>
</tbody>
</table>

power, received power, and the attenuation factor with distance, respectively. Then, $P_R$ can be expressed as [29]

$$P_R = P_T - ad,$$  \hspace{1cm} (1)

where, $d$ is the distance in the power line between the PLC AP and SD.

The fading amplitude $h_p$ of the PLC channel is modeled as an independent and identically distributed (i.i.d.) log-normal random variable with probability density function (PDF) given as [30]

$$f_{h_p}(\nu) = \frac{1}{\nu \sqrt{2\pi\sigma_{h_p}^2}} \exp\left(-\frac{(\ln \nu - \mu_{h_p})^2}{2\sigma_{h_p}^2}\right), \nu \geq 0,$$  \hspace{1cm} (2)

where, $\mu_{h_p}$ and $\sigma_{h_p}^2$ are the mean and variance of the normal random variable $\ln(h_p)$, respectively. Thus, the $n^{th}$ moment of $h_p$ can be obtained as [31]

$$E[h_p^n] = \exp(\frac{m\mu_{h_p} + m^2\sigma_{h_p}^2}{2}),$$  \hspace{1cm} (3)

where, $E[\cdot]$ represents the expectation operator. Then, the average channel gain of the PLC channel is given as [32]

$$G_p = E[h_p^2] = \exp\left(2\mu_{h_p} + 2\sigma_{h_p}^2\right).$$  \hspace{1cm} (4)

Unlike the other communication systems, power lines are subject to impulsive noise along with background noise. The prior noise occurs due to the switching transients at irregular intervals whereas the later occurs from the household appliances such as televisions, computers, etc. [33], [34]. This mixture of noise can be well modeled by a Bernoulli-Gaussian process and its samples can be obtained as

$$n_p = n_g + n_b n_i,$$  \hspace{1cm} (5)

where, $n_g$ and $n_i$ represent the zero mean additive white Gaussian noise (AWGN) random variables with variances $\sigma_g^2$ and $\sigma_i^2$, respectively, and $n_b$ represents the Bernoulli random variable with parameter $\xi$. Furthermore, as all samples have different origins, they are assumed to be independent. Therefore, the noise power spectral density, $N_p$ (watts/Hz), can be obtained as

$$N_p = E[n_p^2] = \sigma_i^2(1 + \xi\gamma),$$  \hspace{1cm} (6)

where, $\gamma = \sigma_i^2/\sigma_g^2$ represents the power ratio of impulsive noise to the background noise. With this in mind, the corresponding SNR is given as [35]

$$\Gamma_p = (1 - \xi)\Gamma_1 + \xi\Gamma_2,$$  \hspace{1cm} (7)

where, $\Gamma_1$ and $\Gamma_2$ are given as

$$\Gamma_1 = \frac{P_R G_p}{\sigma_g^2 B_p},$$  \hspace{1cm} (8)

$$\Gamma_2 = \frac{P_R G_p}{\sigma_i^2(1 + \gamma) B_p},$$  \hspace{1cm} (9)

respectively, where, $B_p$ is the allocated BW.
2.2 VLC channel model

In the VLC system, there exist point-to-point and diffusing links between the LED and photodetector. In the primary links, the AP directly communicates with the devices and requires no obstacles present between them. Thus, in point-to-point links, the beams are directly pointed in the right directions. This in turn reduces the interference, attenuation, and results in higher data rate transmissions. Further, these links are very sensitive to blocking and shadowing effects as they require line-of-sight (LoS) transmission. In the secondary links, the signal radiates in accordance with a wide angle that is similar to RF links. Thus, these links suffer a loss in data rate and introduce multipath induced signal distortion, which significantly degrades the overall channel capacity. The multipath fading, which is common in traditional RF channels, does not significantly impact the VLC channel. This is due to the fact that the signal wavelength in VLC is only hundreds of nanometres, and the size of commonly used photodetectors is in the order of few centimetres, which is sufficiently large to achieve effective space diversity for VLC signals, thereby mitigating the multipath fading [36].

In this work, we assume that all the VLC links are point-to-point for VLC signals, thereby mitigating the multipath fading [36]. Which is sufficiently large to achieve effective space diversity between the LED and photodetector. In the primary links, the obstacles present between them. Thus, in point-to-point links, the AP directly communicates with the devices and requires no cooperation with only one AP that belongs to only one communication technology. Thus, the SRC can be obtained as [43]

\[
C = \sum_{a=0}^{N} \sum_{n=0}^{N} \sum_{b_{an}} \sum_{b_{an}} \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) + \sum_{a=k+1}^{N} \sum_{b_{an}} \sum_{b_{an}} \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) \left( 1 - \xi \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) \right),
\]

where, \( B_{a,n} \) represents the BW allocation for RF AP (a = 0), or VLC AP (a = \{1, 2, ..., k\}), or PLC AP (a = \{k+1, ..., N\}).

3 Joint association and BW allocation

In this section, we present the SRC, C, as a function of association and BW allocation. Wherein, we formulate a constrained optimization problem of C with AP association and BW allocation as the optimization parameters. After that, hierarchical decomposition method is used to solve the optimization problem. Finally, we present the proposed joint distribution algorithm that improves the overall SRC of the proposed system.

3.1 Problem Formulation

Let \( \Lambda_{a,n} \) be the binary indicator that denotes the association of the SD to the RF channel for a = 0, VLC channel for a \{k+1, ..., k\}, and PLC channel for a \{k+1, k+2, ..., k+l\}. Thus, the relation is obtained as

\[
\sum_{a=0}^{k+1} \Lambda_{a,n} = 1, \forall n \in \mathbb{U},
\]

which represents the consideration that each SD should associate with only one AP that belongs to only one communication technology. Thus, the SRC can be obtained as [43]

\[
C = \sum_{a=0}^{N} \sum_{n=0}^{N} \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) + \sum_{a=k+1}^{N} \sum_{n=0}^{N} \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) \left( 1 - \xi \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) \right) + \xi \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right)
\]

where, \( B_{a,n} \) represents the BW allocated by RF AP (a = 0), or VLC AP (a = \{1, 2, ..., k\}), or PLC AP (a = \{k+1, ..., k+l\}).
Algorithm 1: Lagrange multiplier method to obtain the optimal BW allocation

Require: The minimization function \( f(B_a) \), equality constraint \( g(B_a) \)
Ensure: Optimal value of BW allocation

\( L(B_a, \Omega) = f(B_a) + \Omega g(B_a) \) with \( \Omega \) as the Lagrange multiplier

Compute \( \nabla_{B} L(B_a, \Omega) \) and \( \nabla_{\Omega} L(B_a, \Omega) \)

Assume \( \nabla = \nabla_{B} \)

Solve \( \nabla L(B_a, \Omega) = 0 \) to obtain the optimal values of \( B_{a,n} \) for each \( n \in U_a \) and \( \Omega \)

\((a = \{k + 1, k + 2, \cdots, k + l\})\), to the SD \( n \in U \). In (20), the first summation term corresponds to RF and VLC and the second summation term corresponds to PLC. Since both RF and VLC share the same capacity expression, we have used a single set to denote both. However, we denote the PLC with a different set as it shares a different capacity expression from the VLC and RF due to the presence of impulsive noise. Thus, the optimization problem for achieving the maximum SRC can be formulated as

\[
P : \max_{A,B} C (A_{a,n}, B_{a,n}) \quad (21)
\]

subject to

\[
\sum_{\Omega} A_{a,n} = 1, \forall n \in U, \quad (22)
\]

\[
\sum_{n=1}^{N} (A_{a,n} B_{a,n}) \leq B_{a,max}, \quad \text{for each} \quad a \in A, \quad (23)
\]

\[
B_{a,n} \geq 0, \quad (24)
\]

The constraint in (22) is required as we consider the case that a SD can only associate to either of PLC/VLC/RF links. The constraint in (23) upper bound the BW available with each AP, respectively. The constraint in (24) ensures that the allocated BW to a SD has to be non-negative. The optimization problem in (21) is a non-linear programming problem with association and BW allocation as the optimization parameters [44]. It should be noted that it is difficult to solve this problem. Thus, in the following subsection, we discuss the hierarchical decomposition method [45] to solve the above non-linear optimization problem in (21).

3.2 Hierarchical decomposition method

We can relax \( A_{a,n} \) in \( P \), which gives

\[
0 \leq A_{a,n} \leq 1 \forall (a,n) \in (A, U), \quad (25)
\]

where, the fractional value of \( A_{a,n} \) denotes the partial AP association with different APs during AP association period. Initial step of this method is an upper level primal decomposition that decomposes \( P \) into \( P_1 \) and \( P_2 \). Here, \( P_1 \) is the optimization problem of SRC with association as the optimization variable for a given BW. Further, \( P_2 \) is the optimization problem of SRC with BW as the optimization variable for a given association. For a given \( B_{a,n} \), the optimization problem \( P_1 \) can be obtained as

\[
P_1 : \max_{A_{a,n}} C (A_{a,n}, B_{a,n}^{*}) \forall (a,n) \in (A, U) \quad (26)
\]

subject to

\[
\sum_{n=0}^{k+1} A_{a,n} = 1, \forall n \in U, \quad (27)
\]

Similarly, for a given \( A_{a,n} \), the optimization problem \( P_2 \) can be derived as

\[
P_2 : \max_{B_{a,n}} C (A_{a,n}, B_{a,n}) \forall (a,n) \in (A, U) \quad (28)
\]

subject to

\[
\sum_{n=1}^{N} B_{a,n} \leq B_{a,max}, \quad \text{for each} \quad a \in A \& n \in U_a, \quad (29)
\]

\[
B_{a,n} \geq 0, \quad (30)
\]

where, \( B_{a,max} \) denote the maximum available BW with each AP \( a \in A \). Furthermore, for a given association, each AP present in the scenario gets the information about the number of SDs that are associated to it and has to optimally divide the BW, \( B_{a,max} \) available with it. Hence, the problem \( P_2 \) can be further simplified into \( P_{21} \), for each \( n \in \{0, 1, \cdots, k\} \) and \( P_{22} \), for each \( n \in \{k + 1, k + 2, \cdots, k + l\} \) which are defined, respectively, as

\[
P_{21} : \max_{B_{a,n}} \sum_{n \in U_a} B_{a,n} \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) \quad (31)
\]

subject to

\[
\sum_{n \in U_a} B_{a,n} = B_{a,max}, \quad (32)
\]

\[
B_{a,n} \geq 0, \quad (33)
\]

\[
P_{22} : \max_{B_{a,n}} \sum_{n \in U_a} B_{a,n} \left[ (1 - \xi) \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) + \xi \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{kn}} \right) \right] \quad (34)
\]

subject to

\[
\sum_{n \in U_a} B_{a,n} = B_{a,max}, \quad (35)
\]

\[
B_{a,n} \geq 0. \quad (36)
\]

Here, \( P_{21} \) corresponds to RF or VLC and \( P_{22} \) corresponds to PLC as they share different SRC expression. The optimization problems defined in \( P_{21} \), for each \( n \in \{0, 1, \cdots, k\} \) are concave optimization problems the dual of which is a convex optimization problem. Thus, the duality of the optimization problem \( P_{21} \) can be obtained as

\[
f(B_a) = \min_{B_{a,n}} \sum_{n \in U_a} -B_{a,n} \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) \quad (37)
\]

subject to

\[
f(B_a) = \sum_{n \in U_a} B_{a,n} - B_{a,max} = 0, \quad (38)
\]

where, \( B_{a} \) is a vector of BWs allocated to each SD \( n \in U_a \), i.e., to each SD that is associated to AP \( a \). In this work, the Lagrange multiplier method is used to solve the minimization of convex optimization problem defined in (37) with equality constraint on the BW defined in (38) [46]. Algorithm 1 shows the steps involved in the Lagrange multiplier method. Let \( L_1(B_a, \Omega_1) \) denotes the Lagrangian of the convex optimization problem which is obtained as

\[
L_1(B_a, \Omega_1) = \sum_{n \in U_a} B_{a,n} \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) + \Omega_1 \left( \sum_{n \in U_a} B_{a,n} - B_{a,max} \right) \quad (39)
\]

where, \( \Omega_1 \) denotes the Lagrangian multiplier. Then, obtaining the partial derivatives of \( L_1(B_a, \Omega_1) \) with respect to each variable in \( B_{a} \) as well as \( \Omega_1 \) and equating the result to zero which

\[
\sum_{n \in U_a} B_{a,n} \leq B_{a,max}, \quad \text{for each} \quad a \in A \& n \in U_a, \quad (29)
\]

\[
B_{a,n} \geq 0, \quad (30)
\]
are obtained as
\[ \nabla_{B_{a,n}} L_2(B_a) = \]
\[ \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) - \frac{\Gamma_{a,n}}{\Gamma_{a,n} + B_{a,n}} + \Omega_1 = 0, \quad (40) \]
\[ \nabla_{\Omega_2} L_2(B_a) = \sum_{n \in \mathcal{U}_a} B_{a,n} - B_{a,n,max} = 0, \quad (41) \]

Finally, by solving the linear equations defined in (40) and (41), with an assumption that \( \Gamma_{a,n} / B_{a,n} \gg 1 \), the optimal values of \( B_{a,n} \) and \( \Omega_1 \) are obtained as
\[ B_{a,n} \approx \frac{\Gamma_{a,n} B_{a,n,max}}{\sum_{i \in \mathcal{U}_a} \Gamma_{a,i}}, \quad n \in \mathcal{U}_a, \quad (42) \]
\[ \Omega_1 \approx \log_2 \left( \frac{\exp(1) B_{a,n,max}}{\sum_{i \in \mathcal{U}_a} \Gamma_{a,i}} \right), \quad (43) \]

respectively, as shown in Algorithm 1. Similarly, from P22, derived in (34), it is observed that the first and second terms in the summation are also concave functions. The dual of each of these terms is a convex function. Thus, according to the additive property of convex functions [46], P22 is also a convex minimization problem. Therefore, similar to P21, P22 can also be solved using the Lagrange multiplier method and the corresponding Lagrangian is defined as
\[ L_2(B_a) = \sum_{n \in \mathcal{U}_a} B_{a,n} \left( 1 - \xi \right) \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) + \xi \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) + \Omega_2 \left( \sum_{n \in \mathcal{U}_a} B_{a,n} - B_{a,n,max} \right), \quad (44) \]

where, \( \Omega_2 \) is the Lagrangian multiplier. Then, the partial derivatives are obtained as
\[ \nabla_{B_{a,n}} L_2(B_a) = \]
\[ \left( 1 - \xi \right) \log_2 \left( 1 + \frac{\Gamma_{a,n}}{B_{a,n}} \right) - \frac{\Gamma_{a,n}}{\Gamma_{a,n} + B_{a,n}} + \frac{\Gamma_{a,n}}{\Gamma_{a,n} + B_{a,n}} + \Omega_2 = 0, \quad (45) \]
\[ \nabla_{\Omega_2} L_2(B_a) = \sum_{n \in \mathcal{U}_a} B_{a,n} - B_{a,n,max} = 0. \quad (46) \]

Finally, the analytical approximations to optimal values of \( B_{a,n} \) and \( \Omega_2 \) are obtained by solving the linear equations defined in (45) and (46) with an assumption that \( \Gamma_{a,n} / B_{a,n} \gg 1 \) and \( \Gamma_{2,n} / B_{a,n} \gg 1 \) and are given as
\[ B_{a,n} \approx \frac{\Gamma_{1,a,n} \Gamma_{2,a,n}}{\sum_{i \in \mathcal{U}_a} \Gamma_{1,i} \Gamma_{2,i}}, \quad n \in \mathcal{U}_a, \quad (47) \]
\[ \Omega_2 \approx \log_2 \left( \frac{\exp(1) B_{a,n,max}}{\sum_{i \in \mathcal{U}_a} \Gamma_{1,i} \Gamma_{2,i}} \right), \quad (48) \]

respectively, as shown in Algorithm 1.

3.3 Joint distribution algorithm

Fig 2 shows the flowchart of the joint distributed algorithm that solves \( P \) in an iterative manner. Let \( D \) represents the set of PLC-disabled Ss in the system and \( \mathcal{D} = \{ 0 \} \) represents the case where all Ss are PLC-enabled. An SNR based initial association is considered for \( \Lambda \) where the SD associated to an AP from which it receives maximum SNR, this solves the problem P1. Therefore, for a known \( \Lambda \), the problem is simplified to solving \( P2 \) which is equal to obtaining the optimal values of BW allocation and SRC, \( S_2 \), with new association.

Algorithm 1

**Start**

1. Initialize \( \Lambda \) based on \( D \) and maximum values of \( \Gamma_2 \).
2. Compute the optimal BW allocation and \( S_2 \).
3. Read \( k, l, \Gamma, N \).
4. Identify the SD, \( n \), that needs to be reshuffled based on (V1) or (V2) and the AP, \( a \), to which it is associated to.
5. Compute the optimal BW allocation and \( S_2 \), with new association.
6. Is \( S \) converged?
7. Yes, stop.
8. No, yes.
9. Is \( n \) PLC-enabled?
10. Yes, reshuffle \( n \) to each \( i \in \{ 1, 2, \ldots, N \} \) \( a \), and compute the optimal BW allocation and SRC, \( S_2 \), with new association.
11. No, update the SRC and associate SD, \( n \), to AP, \( b \).

**Fig. 2.** Optimal resource allocation.
and the AP \( a \) to which it is associated. In case \( n \in \mathcal{D} \), partially associate \( n \) to all the APs in the set \( \{0, 1, \cdots, k\} \setminus a \). Otherwise, partially associate \( u \) to all the APs in the set \( \mathcal{A} \setminus a \). In each case find the optimal values of BW and corresponding SRC. Let \( C_{t+1} \) be the maximum of all the sum rate capacities obtained in each partial association. In case \( C_{t+1} > R_t \), then associate the SD to the AP that results in maximum SRC. Repeat the above procedure until the SRC converges to a stable value.

4 Numerical Results

Numerical results are presented in this section to compare the performance of the proposed hybrid communication system with hybrid PLC/VLC, PLC/RF, and VLC/RF systems consider in this work with worst device reshuffling and load balancing techniques. We also comment on the change in the association from initial to final (the association at which the SRC converges at a maximum value) in both the reshuffling techniques. Further, we present the numerical results to analyse the effect of increased number of SDs on the optimal SRC. Finally, we consider a practical scenario where there is a possibility for the PLC-disabled SDs that are not capable to connect to PLC APs. We consider a \( 10 \times 10 \times 3.5 \text{ m}^3 \) indoor scenario [41]. In case of PLC, we choose \( \zeta = 0.1 \), \( \gamma = 10 \), \( G_{s,1} = 40 \text{ dB/KM} \), \( N_p = 10^{-14} \) watts/Hz and \( B_p = 10 \text{ MHz} \) [47]. For VLC, we choose \( \psi = 60^\circ \), \( \delta = \beta = 0 \), \( A_f = 10^{-4} \text{ m}^2 \), \( T_f(\beta) = g(\beta) = 1 \), \( N_v = 10^{-21} \) watts/Hz and \( B_v = 100 \text{ MHz} \). Finally, in the case of RF, the carrier frequency \( f_c \) is considered to be 2.4 GHz, \( G_{s,v} = 1 \), \( N_v = 3.89 \times 10^{-21} \) watts/Hz and \( B_r = 10 \text{ MHz} \). Moreover, the transmit power, \( P_T \), is considered as 10 mW for each AP belonging to any of the communication technology [1]. Further, the list of all parameters and corresponding values are tabulated in Table 1.

Figs. 3a and 3b show the SRC of all the considered hybrid communication systems with worst device reshuffling and load balancing techniques, respectively. From both, Figs. 3a and 3b, we observe that the proposed mechanism doubles the achievable SRC compared to the SNR based AP association and BW allocation for most of the hybrid systems consider in this work. Further, the maximum SRC is achieved by the hybrid PLC/VLC/RF system among all communication systems under consideration. This observation is valid as, the aggregate BW of the hybrid PLC/VLC/RF system is higher than that of others. Moreover, we observe that the SRC of the hybrid VLC/RF system is higher than that of hybrid PLC/VLC and PLC/RF systems. This happens due to the combined effect of high SNR of the RF link and high BW of the VLC link for a fixed transmit power. Moreover, it is also evident that the initial SRC of hybrid PLC/VLC/RF and PLC/RF systems is higher compare to the other two hybrid systems under consideration. This is due to the fact that in these hybrid systems, the SDs are connected to...
both RF and PLC during the initial association as both RF and PLC provide relatively high SNR. However, in the case of hybrid VLC/RF and PLC/VLC systems, the SDs should associate to only RF and PLC, respectively, as both provide high SNR compared to the VLC system. This in turn reduces the achievable SRC in the initial association as all SDs associate to only RF and PLC in VLC/RF and PLC/VLC, respectively.

Fig. 4a shows the initial association of all SDs to the respective APs from which they receive higher SNR. Figs. 4b and 4c show the final association, at the saturated SRC, of the SDs with both worst device reshuffling and load balancing techniques, respectively. From Fig. 4a, it is observed that the number of SDs associated to PLC, VLC and RF are 3, 0, and 37, respectively. It is seen from the figure that all the SDs are connected to only PLC and RF during the initial association due to the provision of higher SNR by both PLC and RF compare to VLC. It has changed to 4, 7 and 29 with worst device reshuffling technique and 7, 1 and 32 with load balancing technique as shown in Figs. 4b and 4c, respectively. This is due to the availability of excess BW and more APs has led to the increase in the number of SDs associated to the VLC APs at the saturated SRC.

Fig. 5 shows the numerical results that correspond to the variation of saturated SRC for the proposed hybrid system with respect to the increase of SDs. From figure, it is observed that the achievable saturated SRC increases linearly with the number of SDs and is maximum at $N = 60$. Further increase in the number of SDs decreases this value as can be observed from Fig. 5. It is also seen that the saturated SRC is slightly higher with load balancing technique compared to the worst device reshuffling technique. In the later technique, the focus is on the reshuffling of the worst SD (worst individual sum rate) of all the SDs present in the system. In this case, the system can be left with a heavily loaded AP whose associated SDs can be further reshuffled in order to improve the achievable SRC. This is observed from Fig. 4b in which 7 SDs are associated to RF at the saturation which can be further reshuffled to improve the achievable SRC.

In the former technique, the focus is on balancing the load of each AP present in the system to achieve the saturated SRC as observed from Fig. 4c in which the number of SDs associated to RF has changed to 1.

Figs. 6a and 6b show numerical and simulation results that correspond to the SRC variation with worst device reshuffling and load balancing techniques, respectively, with increased number of PLC-disabled SDs ($D$) from a total number of 30 SDs. From both Figs. 6a and 6b, we observe that the achievable SRC is maximum in the absence of PLC-disabled SDs ($D = 0$) as all the SDs can connect to any communication system which improves the achievable SRC, whereas, the increase in the number of PLC-disabled SDs, $D$, reduces the SRC due to the inefficient utilization of the PLC system. Finally, the SRC is at its minimum value when all SDs are PLC-disabled as the hybrid system does not utilize PLC. Thus, it can be concluded that the addition of PLC in the system results in improved performance of the hybrid PLC/VLC/RF system in indoor scenarios.

5 Conclusion

In this paper, a hybrid PLC/VLC/RF fronthaul with a fiber based wired backhaul system has been proposed to improve the achievable SRC. Wherein, an optimization problem for the SRC has been formulated for the hybrid system in terms of AP association and BW allocation as the optimization parameters and a hierarchical decomposition method has been considered to convert the non-linear optimization problem into a set of convex optimization problems. Then, efficient AP association and BW allocation strategies have been proposed to solve the optimization problem in an iterative manner till the SRC converges to an optimal value. This iterative algorithm uses two reshuffling techniques such as the worst device reshuffling and the load balancing to obtain the saturated capacity. Further, an analytical approximation for the BW allocated to each SD for a given AP association has been derived using the Lagrangian multiplier method. Through extensive numerical results, it has been shown that the proposed algorithm doubles the achievable SRC compared to the SNR based AP association and BW allocation.
has also been concluded that the proposed hybrid PLC/VLC/RF system can considerably enhance the SRC in comparison to other hybrid combinations reported in the literature. Further, a practical scenario has also been considered, where, there exists PLC-disabled SDs that are only capable of connecting to either RF or VLC. Then, a numerical analysis has been presented on the achievable SRC to study the effect of increased number of PLC-disabled SDs. Finally, the effect of the increase in number of SDs on the optimal SRC is analysed numerically. It has been observed that the SRC is a concave function of the number of SDs. In future, we will analyse the system performance of the hybrid PLC/VLC/RF system with fiber optics as backhaul.

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7 References