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AT THE INTERSECTION BETWEEN THE SUBJECT AND THE POLITICAL: A CONTRIBUTION TO AN ONGOING DISCUSSION

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Abstract. The issue of subjectivity has recently occasioned a lively discussion in this journal opposing socioculturalism and Lacanian psychoanalysis. By confronting Luis Radford's cultural theory with Jacques Lacan's psychoanalysis, Tony Brown sought to show the limitations of socioculturalism. This article takes advantage of that discussion to develop a critique of Radford's theory of objectification, taken as an exemplary sociocultural theorization of the teaching and learning of mathematics. It does so by extending the criticism made by Brown at the level of the Subject, namely by showing what is lost in socioculturalism when it reduces the Hegelian notion of dialectics to a relation between constituted entities; but mostly by exploring the possibility opened by contemporary theory to posit the discussion around subjectivity in the Political. While socioculturalism assumes the possibility of a synthesis between person and culture thus making education possible, it will be argued that a theory which assumes the impossibility of education is in a better position to, on the one hand, conceptualize the resistance of many towards the learning of mathematics, and on the other hand, to address the ongoing political failure in achieving the desired goal of "mathematics for all".

Keywords: Cultural theory of objectification, socioculturalism, dialectics, subject, political, school's credit system, Lacan, Marx, and Žižek.

1. Introduction

Socioculturalism and its plea for conceiving learning as more than the cognitive process occurring inside people's heads has entered mathematics education research with might and main during the last two decades. Recent analysis of the role of theory in mathematics education research (e.g., Cobb, 2007; Silver & Herbst, 2007) clearly recognizes that learning is related to the sociocultural context. This emphasis in the "sociocultural" brings up the fundamental encounter occurring in all

educational acts: the "clash" between individual agency and social demand. Thus, I concur with Lerman (2000) when he argues that:

> perhaps the greatest challenge for research in mathematics education (and education or social sciences in general) from perspectives that can be described as being within the social turn is to develop accounts that bring together agency, individual trajectories ..., and the cultural, historical, and social origins of the ways people think, behave, reason, and understand the world. (p. 36)

How might we conceptualize "agency" or "individuality" for people who from the beginning are immersed in a sociocultural frame? This question is at the centre of Luis Radford's *cultural theory of objectification*, arguably one of the best grounded theorizations, historically and philosophically, of the teaching and learning of mathematics.[1] The way Radford (2006a, 2006b, 2008a, 2008c) found to solve this impasse was through a "dialectical" relation between individual agency and the culturally constituted reality. Radford reserves some autonomy for an individual who reflects and modifies a historically and culturally constituted reality according to her interpretations and personal meanings (Radford, 2008a, p. 453). Because of this dialectic, a synthesis between person and culture is achieved, thus making education possible.

Such interpretation of the learning process (or, at a more general level, of the relation between the individual and the social) was the focus of a critique carried out by Tony Brown and published in this journal (Brown, 2008a). In it, Brown introduces Lacan's semiotic apparatus to address the conflicting relation between the cognitive psychology of Piaget and the more socially oriented model of Vygotsky (that partly substantiates Radford's theory). The discussion continued with Brown's (2008b) publication of an article where he analyses how subjectivity is variously understood in a Special Issue of this journal featuring work written from a semiotic perspective, and to which Presmeg and Radford (2008) later respond in turn. Apparently, as it was emphasized by Presmeg and Radford (2008), the Lacanian subject introduced by Brown (2008a) is caught in an all-encompassing social-psychic machinery, which makes it a marionette deprived of agency—contrary to the sociocultural approach where an autonomous subject actually intervenes and modifies its

[1] Recent theoretical developments in mathematics education research are also founded on the need to articulate the relation between individual and social. For instance, Anna Sfard (2008), through the exploration of Wittgensteinian philosophy and in order to surpass the traditional divide between thinking and speaking, introduces the notion of *commognition* as a way of conceptualizing the relation between cognition and communication; also, Walshaw (2004) and her search for a conceptualization in Lacan and Foucault that could aid the interpretation of subjectivity.

social reality. This critique continued with Roth's (2012) response to Brown's book *Mathematics Education and Subjectivity*, which Brown (2012) later responded in turn.[2]

Taken together, these articles propose a novel theoretical and philosophical discussion of contemporary issues in mathematics education. Such a debate is already at the theoretical end of the spectrum of this journal's scope. Why then do I decide to pursue the debate, and to some extent, make it even more theoretical? The first reason has to do with what I consider to be a misreading of the Lacanian subject. I will argue for a different interpretation of Lacan's writings to the one provided by Presmeg and Radford. The ultimate horizon of Lacan's approach to subjectivity is not the reconciliation of person and culture, but the assertion of their very gap as the positive condition of education. This implies moving from conceptualizing the relation between individual and social as a *polarized* relation between constituted entities, towards a fundamental *antagonism* of the subject and its social-symbolic milieu. This discussion will be developed around the notion of Hegelian dialectics, where the opposites are not reconciled in a "higher synthesis"; it is rather that their difference is posited as such[3]. Yet, the most important reason for this engagement of mine has to do with the possibility opened by the response of Presmeg and Radford to situate the discussion around subjectivity in the political arena. Presmeg and Radford managed to colour Brown's discussion by positing it against the background of Marxism. They offer us an interpretation of Marx that matches their sociocultural conceptualization of the subject and opposes the Lacanian one. In doing so, they invite us to explore how the theories for teaching and learning deployed by people in a community are "informed" by broader ideologies that underlie their actions.

Despite their theoretical differences, Brown, Presmeg, Radford and Roth acknowledge that theoretical and analytical conceptualizations are associated with ideologies that escape self-awareness and dictate the kind of project researchers undertake. As posited by Brown (2008b), the theoretical and analytic lenses we deploy in our research "comprise particular choices in terms of the analytic filters that we apply, governed by underlying ideological motivations and trends of which we are not always aware" (p. 249). Analysing these ideological motivations, or "world-views" (Jablonka, Wagner & Walshaw, 2013, p. 41), has been a recent concern of researchers engaged in *researching research* (Pais & Valero, 2012). Using contemporary theory, and by teasing out the assumptions and discourses generated within research, researchers have been positing mathematics education research itself as an object of study (e.g., Lundin, 2012; Pais, 2013; Radford,

[2] In Pais (2015), I provide a critique of Brown's use of Lacan's apparatus in mathematics education research, which complements the critique of Radford's theory developed here.

[3] My use of Hegel's theory is informed by Žižek's works on Hegel.

2012). The focus of this paper is thus mathematics education *research*, and the examples analysed come from research rather than from concrete episodes of teaching and learning mathematics. It is my contention that such an approach, although not directly aimed at providing some kind of insight for action, can help us redefine the coordinates we use to make sense of the problems of the field.

This article is supported by a combination of Lacanian psychoanalysis and Marxist tradition within contemporary theory[4], as articulated in the work of Slavoj Žižek. It seeks to show how politics is present at the very heart of our most sophisticated theorisations about subjectivity, learning and society. My approach to politics assumes that capitalism, both as ideology and as economy, has become the "concrete universal" (Žižek, 2004, p. 3) of our historical époque, that is, "while it remains a particular formation, it *overdetermines* all alternative formations, as well as all noneconomic strata of social life" (p. 3). It follows that every position in education is also at the same time an implicitly or explicitly political stance on the nature of "late capitalism" (Jameson, 1991). The question to be posed, if we admit that capital is present as an *automatic subject* (Marx, 1976) in all classrooms, is: how does this affect our conceptions of education, learning and subjectivity that are being implemented there? By exploring the way capital is *overdetermining* our work as mathematics educators I seek to ensure that the way we conceptualise what it means to be a subject will not directly and simply coincide with the spirit of capitalism.

2. Socioculturalism and Radford's cultural theory of objectification

Sociocultural theories assume that by merely looking into the individual's cognitive development we just have a fragmented, "artificial" understanding of how she knows, and ultimately, of what she is. It is only by looking to the surroundings, to the social and cultural context, that we can truly understand someone (and furthermore, how she thinks, learns, and participates). In a sociocultural approach, an individual's actions are elements of a broader sociocultural system and cannot be adequately understood unless this relation is explicated.

Compared with constructivism—which many of us will agree to be the most influential theory of cognitive development used and expanded in mathematics education research—socioculturalism maintains two of its three fundamental premises (Radford, 2008b). Firstly, the idea that knowledge is not passively received but built up by the cognizing subject. Secondly, and in a move coherent with Kant's theory of knowledge, the idea that "objective reality" is not simply given "out there",

[4] Ernesto Laclau, Chantal Mouffe, Fredric Jameson and Alan Badiou are among the ones who have been articulating insights from Lacanian psychoanalysis with a Marxist critique of political economy.

waiting to be perceived by the subject, but an artificial composite constituted through the subject's active participation. However, socioculturalism rejects the third principle of constructivism: the assumption that knowing, in line with Piagetian theory, must be understood as an intrapsychic activity, where the subject possesses intrinsic, "personal" mechanisms that will allow her to know. These mechanisms are likely to be schematized so that we can study how people learn by looking at individual cognitive changes. In socioculturalism, on the contrary, the subject and the process of knowing cannot be separated from the culture in which they are immersed. As Radford explains:

> [S]ocioculturalists consider cognition as a cultural and historically constituted form of reflection and action embedded in social praxes and mediated by language, interaction, signs and artefacts. As a result, knowledge is produced by cognizing subjects who are, in their productive endeavours, subsumed in historically constituted traditions of thinking. (p. 11)

Against this background, Radford's *cultural theory of objectification*, which draws substantially on Vygotsky's and Leontev's theories, and also Husserl's and Peirce's phenomenological epistemologies, conceives learning as the reaching of a culturally-objective piece of knowledge that students attain through a social process of *objectification*. Signs, language, artefacts and social interaction mediate this process as people engage in cultural forms of reflecting and acting. The way the dichotomy between the subject and the object of knowledge is solved is by introducing the notion of *being*, as a dialectical process where learning is both a process of objectification (knowing) and subjectification (being or becoming); this plea invites us to understand learning as more than constructing logico-mathematical mental structures or picking up ready-made knowledge. It invites us to understand learning as knowing and *being*, which brings attention to the classroom as an ethical and political space where being is constantly renewed. Learning thus is not just about knowledge but also entangles an ethical and political dimension.

There are however two problems in the way Radford's theory conceives subjectivity that I will explore in the next section, in the light of Hegelian dialectics. These are: a) the tendency to "ontologize" culture, as if culture had an existence as such beyond the individual[5]; and b) the

[5] This tendency to allocate "cogitation" in the object (such as artefacts) was already criticized by Glasersfeld (1996). He denies that (radical) constructivism ignores the role of social interaction in the construction of knowledge, however the "other" "constitute 'exist' for the individual subject only to the extent to which they figure in that individual's experience" (p. 309). That is, the meaning of an artefact is not *deposited* in the artefact, as Radford argues (2008a, p. 451), but *posited* by the subject. Also recent research in situated cognition (Watson & Winbourne, 2008) has called

preservation of a notion of subject as a coherent rational self, its own source of meaning, knowledge and action[6]. These assumptions, I argue, keep us stuck with a conceptualization of the subject that although claiming to be dialectic, continues to deal with the individual and the social (or, to use Radford's terms, the person and its culture, the individual and the collective) as two merely relational entities.

3. Putting dialectics in the right place

For Lacan, according to Presmeg and Radford (2008), our "self" is not our own but the Other's; that is, our sense of the self is articulated in the socio-symbolic arena in advance. At a first glance, this move may appear as a surrender of the person to culture, as an immersion of the subject in the higher unity of society. As mentioned by Presmeg and Radford (2008), within this framework, we can easily fall into an all-encompassing discursivism where the subject never "speaks for itself" but is always made to speak by a network of discursive devices. If these are the conditions against which our existence as subjects is laid out, there seems to be no room for such notions as "autonomy", "agency" or "emancipation". According to Presmeg and Radford (2008), such alienated subject, coincidental with the Lacanian subject, negates agency to the individual: "this is why the subjectivity portrayed by Lacan is an agonizing and essentially alienated one" (p. 268).

However, whereas Presmeg and Radford (2008) conceive this socio-symbolic substance—what Lacan calls the big Other, the field of symbolic order, culture—as a positive entity orchestrating individuals' lives as if they were puppets, for Lacan (as well as for Hegel) such entity lacks any ontological consistency. It has a purely virtual status: it does not exist out there, it is not "deposit" in artefacts, but *posited* by the subject. This is precisely what Lacan has in mind when he claims that "the big Other doesn't exist": it is not that culture, shared meanings, and "embodied intelligence" (Radford, 2006a, p. 52) do not "exist", it rather means that their existence is completely correlative to the subject who supposes them to exist—they exist only insofar as people believe their existence. As emphasized by Žižek (2006) the point is not that there is no "objective"

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attention to the way some studies within socioculturalism overestimate the formative influence of artefacts and situational configurations on mental functions, "as if these were embodied in tools" (Stech, 2008, p. 20).

[6] Walshaw (2011) and Walkerdine (1998) made the same critique apropos of the theory developed by Cobb and Hodge (2007) and Lave (1988) respectively. They take the position that their work leaves intact the characterizations of identity, consciousness and agency put forward by traditional social science. These understandings, in turn, provide a limited perspective of how mathematical identities are constituted within the realities of the classroom and the wider sociopolitical context. As a way to surpass these limitations, both Walshaw and Walkerdine introduce the Lacanian subject, as one where there is more at stake than the rational links between individual and social. Namely, the Freudian unconscious.

reality, only our subjective representations of it, but that this "objective" reality, in its opposition to subjective representations, is posited by the subject. Culture, in this sense, is a subject's presupposition. Therefore, when Presmeg and Radford (2008) argue that the Lacanian subject is a "Kafkaesque subject alienated by strangulating systems and their cold impersonal rules" (p. 269), they fall short to grasp this "other side" of Lacan: that those "cold rules", although seen as impersonal by the individual—beyond her control, and determining her—are indeed a result of the frenetic and reified activity of all of those who, like her, believe in them.[7]

Subjects are always already implicated in the constitution of the same reality that is perceived as being imposed on them. Radford seems to be aware of this identity of person and culture, as when he says that "in this encounter with the other and cultural objects, the seeking individual finds herself" (2008c, p. 225), or when he equates knowing with being at the core of his ethics (Radford, 2012). However, instead of taking to the end the consequences of this identity—namely, by giving away the idea of an autonomous subject that intervenes in a socially constituted reality—, there is the assumption that the individual and the social are distinctive entities that relate to each other through "cultural significations" (e.g., artefacts): "[t]hese cultural significations … function as links between the individual consciousness and his or her cultural objective reality. They are prerequisites and conditions of the cognitive activity of the individual" (Radford, 2008a, p. 454). We thus have two entities that, although inseparable, are distinct. They conserve some kind of autonomy, otherwise there will be no need to consider a "link" between them. Although Radford emphasises the mediated way in which they are related—"we originate thinking, but at the same time become subsumed by it (Eagleton, 1997)" (2008c, p. 219)—he falls into a vulgar use of dialectics: "[t]hinking as re-flection means rather a dialectical process *between* a historically and culturally constituted reality and an individual who reflects and modifies it according to his or her interpretations and personal meanings" (Radford, 2008a, p. 453, my emphasis).[8]

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[7] What one should render problematic here is that the big Other, precisely as pure artificial schemata it has effects on reality: "precisely insofar as it is a 'dead scheme', we must presuppose it as an ideal point of reference which, in spite of its inexistence, is 'valid', i.e. dominates and regulates our actual lives" (Žižek, 2008b, p. 61). Culture has thus this paradoxical nature: although it is not to be found anywhere in "reality", is nonetheless effective and regulate our lives. When responding to Brown's book, Roth (2012) raised the issue of *corporeality*. Roth does so in order to emphasise how the body precedes any given language or symbolic system. Yet more interesting than exploring how every "language" needs a body, is to investigate how the immaterial, the fictions, affect the body. The typical Marxian example of a system without a body is capital. Marx's characterization of capital as an "*automatischem Subjekt*", an oxymoron uniting living subjectivity and dead automatism (Žižek, 2012, p. 250), presupposes a "subject" that does not exist as a living-one, but nonetheless has real (automatic) effects on reality.

[8] By posing dialectics as a relation *between* two entities, Radford misses the full potentiality of this notion. If we recall how Hegel refers to the dialectical process, we can say that there is no dialectic between parts. Rather, dialectics is itself the motor that generates the parts, it takes precedence over the poles, it does not occur "between" constituted entities.

In order for Radford, (and to that matter, the whole sociocultural approach) to conceive the individual and social as two distinct (albeit "dialectically" related) entities, he has to presuppose, on the one hand, an entire system of language, signs and objects that are bearers of an "embodied intelligence" (Radford, 2006a, p. 52), and on the other hand, an autonomous individual who reflects and modifies these cultural artefacts. This conceptualization frames "dialectics" as the process by which these two entities relate with each other in a continuous course of transformations.

Within Lacanian theory, however, the subject does not relate with culture, but stands precisely for that which resists total alienation in culture. This happens because Lacan's symbolic order is inconsistent or lacking. The non-All character of symbolic reality means that there is always a mismatch between the demands of society, the call for the individual to become a socially well integrated being, and a feeling of misplacement, as if the individual was always out of joint: this—the symbolic identities conferred upon me in the different settings of my life as a teacher, student, and/or mother—is not really "me", but without it I am no one. Far from emerging as the outcome of interpellation—that is, as a product of social alienation as Presmeg and Radford (2008) interprets it—the subject emerges only when and insofar as interpellation ultimately *fails*. Not only does the subject never fully recognize itself in the interpellative call: its resistance to interpellation (to the symbolic identity provided by interpellation) *is* the subject:

> the subject is not only always displaced, and so on, it is this displacement. What this means is, again, that the above-described dialogical structure [in our case, the individual/social divide] is inscribed into the very being of subject: the subject aims at representing itself; this representation fails; the subject is this failure of its own representation. (Žižek, 2012, p. 538)

The subject is thus not directly cancelled in culture, it is rather inscribed into culture as its own gap or failure. The move I am proposing, in the words of Žižek (2006, p. 382), is the move "from the gap between two 'somethings' to the gap that separates a something from nothing, from the void of its own place". In this sense, the division planted by socioculturalism between a culturally constituted reality (in Lacanese, the symbolic) and a person who can autonomously reflect and intervene in that reality, that is, the division between two "contents", gives place, in a Lacanian framework, to a division between something and nothing, between the feature of identification and its immanent failure.

This conceptualization diverges from Radford's where individual and social become united: "it is in the realm of *meaning* that the essential *union* of person and culture and of knowing and knowledge are realized" (Radford, 2006a, p. 54, my emphasis). Radford's theory thus presupposes an external "third", a shared medium in which the opposition between person and culture is "synthetized". But Hegel's dialectics is radically groundless, a process of self-relating of the "two", which lacks any "third": "[f]or Hegel, the goal is thus not to (re)establish the symmetry and balance of the two opposing principles, but to recognize in one pole the symptom of the failure of the other" (Žižek, 2102, p. 303). In the very failure of our endeavour to develop a consistent theoretical synthesis between individual and social, we reach the fundamental deadlock constitutive of the subject as such: the only possible mediation between individual and social is the one who already takes into account the impossibility of such mediation. This is what Hegelian "synthesis" is all about: the opposites are not reconciled in a "higher synthesis", rather their difference is posited as such.

The "nothingness" of the Lacanian subject signals an impossibility in culture. It stands for a *real*[9] that prevents all efforts of symbolization becoming closed, totalized—for instance, that which prevents the students from becoming the mathematically successful learners fantasized by curricula. Contrary to the subject of socioculturalism, who autonomously objectifies culturally embodied meaning, the Lacanian subject is a purely negative one. My suggestion is that instead of trying to bridge the gap between person and culture, we should formulate it as such, to conceive it properly.

4. The limitations of the sociocultural Marx

I now take advantage of Presmeg and Radford's exploration of Marxism as an opportunity to posit the discussion around subjectivity in the political arena. Presmeg and Radford's appropriation of Marx resembles post-Marxist tendencies that read Marx in ways that disavow the constitutive determinacy of economy (Eagleton, 2001; Jameson, 1991; Žižek, 2008a). What is in fact a structural problem, endemic to a mode of production, is transformed into an abstract problem of *greed* (Özselçuk & Madra, 2011), which could be solved through "a joint of ethical engagement, a collectively motivated activity based on trust and responsibility" (Radford & Roth, 2011, p. 244). It follows that emancipation "is considered as an ethical and political project that can only be put in

[9] Towards the end of his life, Lacan (e.g., 2007) moved from an analysis of the psychic apparatus centred on the Imaginary and Symbolic, to an apparatus revolving around the Symbolic and the Real. The latter is only slightly mentioned in the response of Presmeg and Radford (2008) to Brown's article. This is also why they fail to catch the radical emancipatory spirit of the Lacanian subject.

motion through the encouragement of forms of solidarity, trust, sharing, and a commitment to improving the quality of human life" (Radford, 2012, p. 108). Painted this way, Marxism uncannily resemblances a catechism, with charity as the main safe guard of Humankind. Economic exploitation, the fundament of capitalism, is reduced to political domination. As posed by Souza, Linardi and Baldino (2002), presented in this way "Marxism becomes completely palatable to the Bourgeoisie, transvestite of humanist ideology, castrated from its fundamental concept, the 'surplus-value'" (p. 28). That is, exploitation is not a structurally determined feature of capitalism, it is not the exploitation of labour by capital, but the abstract exploitation of "man by man", which can be solved through the efforts of well-intended human beings engaged in the amelioration of their praxis.

At stake here are two different readings of Marx. The *earlier* or *humanist* Marx (Althusser, 2005; Žižek, 2008a)—compounding the works previous to *Das Capital*—is closer to Kant's liberal-rationalist humanism than to the Hegelian subject-substance dialectics. This means that, for the earlier Marx, only the essence of man makes history, and this essence is freedom and reason. Human freedom is neither a narcissistic caprice, nor the determinism of interest, but as Kant meant it, autonomy, obedience to the inner law of reason. Man, in this sense, is the lord and master of its own thought and action, and relates with culture by means of transforming it through its labour. This characterisation fully depicts Presmeg and Radford's sociocultural subject (2008): "a concept of the autonomous person that is sensitive to the importance of history, the contexts and others, and where autonomy is both self-fulfilment and social commitment" (2008, p. 272). *Change* is conceived in terms of the fixed Kantian dualism, (which Radford uses as dialectics) as a reciprocal interaction between individual and instituted culture, aimed at achieving equilibrium between individual and collective interests.

We can, however, interpret Marx in a different way—what Althusser (2005) refers to as the *mature* Marx: "Marx's writings gradually developed a historically overdetermined notion of subjectivity and offered a new reading of class antagonism as an irreducible *limit* of the social" (Özselçuk & Madra, 2010, p. 329): this reading of Marx contrasts with Radford and Presmeg's in one crucial aspect. It envisions a world based on antagonism, as opposed to the polarized world view of socioculturalist theory, which is in pace with "tendencies within political economy that read Marx in ways that contain and even annul the constitutive negativity of class antagonism" (Özselçuk & Madra, 2010, p. 333). Instead of confronting antagonism, socioculturalism partakes in the fantasy of a proper balance between individual and collective interests. This is achieved through

the creation of a *space of joint action* (Radford & Roth, 2011, p. 229) in which collective enterprises and individual commitments organically supplement each other. Radford and Roth guarantee that different individual egotisms work for the common good by introducing the analytical category of *togethering* used "to capture joint practical activity that has the purpose of realizing a collectively motivated object [learning mathematics]" (p. 236).

Such a display of students' engagement with mathematics, however alluring in prospect, conceals a major question: what will make students work for a common goal amidst a school system that grades individually? In his critical review of Brown's book, Roth (2012) rightly calls our attention to the importance of having in mind not only the Aktivität—the particular tasks students and teachers do in their school life—but also the Tätigkeit—the totality of the activity that gives the ultimate motive and meaning to the tasks developed by teachers and students. It is not by changing the task conditions that the fundamental conditions of mathematics education will change (p. 454). The particular problems teachers and students experience are always coloured by the role of school as "credit systems" (Vinner, 1997), where "students *still* seek diplomas, for which they have to take certain courses, receive grades or pass/fail marks" (Roth, 2012, p. 454, my emphasis). If today's ideology is one that supports capitalism as an economic system, it follows that the ultimate Tätigkeit framing students' engagement in mathematics is the need for approval, materialized in school credit (grades, pass/fail, etc.). Yet, when positing the object of the activity, socioculturalism emphasizes not the economic but mathematics as the ultimate cultural object guiding students' engagement: "learning how to model contextual and familiar situations in an efficient mathematical way was in fact the object of the activity" (Radford & Roth, 2011, p. 232). The problem arises when we conceive the Tätigkeit not as the realisation of a collectively motivated goal through continuous ethical commitment, but as the strictly individualistic goal of passing the course or achieving the highest grade. The school system is inherently individualistic, and this feature is completely obliterated in socioculturalism through the illusion that students are indeed working for a collective purpose. The antagonism that perpetrates schooling is disavowed by the fantasy of a collective of learners that does not match the real conditions of today's schooling.

5. A theory of failure

Within the Lacan-Žižek axis, the notion of *fantasy* is a necessary counterpart to the concept of antagonism: "fantasy is precisely the way the antagonistic fissure is masked" (Žižek, 2008a, p. 142). Socioculturalism can be seen as a fantasy formation: it enables researchers to conceive as

meaningful something that is inherently out of joint. In Radford's theory, the antagonistic character of the subject (in the sense that it never equals its symbolic identifications) is foreclosed, and the fantasy of "meaning" (Radford, 2006a, p. 53, 54) is created precisely to tame such antagonism. The fact that people fail in school mathematics means that students do not always, (and in some cases, almost never) identify themselves with the mathematical learner envisaged by the curriculum and pursued by the teacher. Contrary to the examples explored by Radford (2006a, 2006b, 2008c) where pupils always seem to get the knowledge the teacher desires them to learn, there are situations where students repeatedly do not learn what is intended by the teacher, no matter how different are the didactical approaches deployed by her (e.g., Baldino & Cabral, 2005). By painting the relation between individual and social as a polarized relation of common desires, socioculturalism ends up dismissing learning situations where such polarization does not occur. Indeed, if we seriously take the Freudian insight, as posed by Lacan, that "the very foundation of interhuman discourse is misunderstanding" (2000, p. 184), such congruence of desires never occur. Something gets in the way between the student who learns and the mathematical content to be learned: something screws their "interaction". Lacanian theory is, in this sense, a theory of "negativity", whereas Radford's theory is a theory of "positivity"; that is, from the point of view of a teacher, Radford's theory provides literature for a situation where students fully endorse the identity conferred upon them by the symbolic (that of being a student, with the task of learning). While Lacan's theory gives us tools to address the blockages, the violence, and the negation of students in complying with the symbolic mandate conferred upon them.

At this point, the reader can reproach that although we may know about the difficulties students experience in mathematics classrooms, together with the worldwide problem of failure in this school subject, we do not need to reinforce even more such depressing reality. What we need, instead, is "positive" research exploring successful experiences, where apparently students learn meaningful mathematics for their lives. To develop and broadcast successful experiences seems to be the aim of research (Gutiérrez, 2010; Presmeg & Radford, 2008; Sriraman & English, 2010). This is why Presmeg and Radford (2008, p. 269) argue that Lacanian subjectivity effectively encapsulates the disenchantment of capitalism. One cannot agree more. And one only has to add that it is precisely this encapsulation that makes Lacanian theory a powerful construct to address today's schooling. It is insofar as mathematics education research has to address the problems of the practitioners that it cannot afford dismissing what are the real conditions of their work. Instead of running after an ideal—of a union between student and mathematics, or the ideal of a harmonious

social whole where mathematics is for all—, Lacanian theory invites us to take reality as it is. Students never fully "unite" or "identify" with a specific "culture", not because we lack a proper theory of learning, which would allow the complete realization of a "mathematical learner", but because the mismatch between person and culture is *real*, that is, there is no "formula" that "describes" how people learn. Socioculturalism, along with other learning theories used in mathematics education, are ultimately all impossible attempts to circumscribe this kernel of real, which forever eludes our symbolic grasp. This is why Lacanian theory is not a theory of how the relation between individual and social is or should be, but a theory of the everlasting *failure* of such relationship.

6. Conclusion

Socioculturalism (as displayed in the work of Radford) conceives the subject as a self-conscious, autonomous one that through its sociocultural interactions strives for the common good. Radford's theory assumes the possibility of a union between person and culture, and the idea that, with time and the efforts of engaged individuals, mathematics for all will be possible. In order for education to be conceived as possible one has to define the conditions of its possibility. How to decide when a student effectively learned mathematics? How to determine when mathematics is finally reaching all students? If the criteria is one of attending the maths classes, then at least in the countries where school is compulsory, almost every person learned mathematics. But attendance per se is not accepted as a guarantee of learning. One has to pass the exam in order to be entitled with a mark that attests that he or she learned mathematics. Thus the importance of the school's credit system: it installs a measurement of possibility—a student learns mathematics when he or she passed the exam (or any other evaluative procedure), and mathematics will be for all when all students achieve school success.

By introducing a measure of possibility, the school's credit system simultaneously conceals the impossibility of education and creates a division between those who fail and those who succeed. As I have been showing in my work (e.g., Pais, 2014), this division is not contingent but necessary, that is, from the moment we introduce into school a promotional criteria, there will always be people who fail. We are then left in a situation where not only the ideal goal of mathematics for all cannot be actualised, this same unrealisation is disavowed by the presence of the ideal. The ideal discourse functions as the proper staged discourse that makes the real sustainable, accepted, and reproduced.

As a result, school's credit system is completely buried in oblivion, naturalised[10], and we start to conceive as external hindrances that which thwarts a full meaningful mathematics education—deficient teaching, poor curriculum, not enough realistic mathematics, too much realistic mathematics, etc. That is, people fail not because the system sets them to fail, but because of some particularities that could be ameliorated through better research and school practices. This creates an entire academic and educational industry (from conferences, journals, or international assessment mechanisms to companies specialised in the production of didactical materials, teacher training or private coaching) aimed at solving the problem of failure in school mathematics. The focus is not the entire system where failure is necessary, but particular faults likely to be solved through expert engagement. The school's credit system is sustained by the illusion that success is possible, if only obstacles could be removed. What we fail to recognise is how these external hindrances are there precisely to create the illusion that without them, mathematics education will be possible. What these hindrances thereby conceal is the inherent impossibility of education.

Ultimately, one can also argue that the credit system as such is one of these external hindrances that, once removed, will make education possible.[11] However, if we withdraw credit from schools, how can we decide whether someone learned mathematics? Since the credit system is the system that installs possibility into education, its removal will make us face education in its radical impossibility—a "positivized" impossibility, one that clears up the space for something new to emerge. If a person truly "unites" with its culture, as advocated in Radford's theory, there will be no need for education—we will all naturally become the place assigned for us in the cultural milieu. In the conditions in which education would finally be possible, it will no longer be needed. It is because education ultimately fails—in the sense that there will always be a gap preventing us from a direct immersion in our surroundings—that makes it necessary. Education is thus simultaneously impossible and necessary. It is the necessary condition for someone to become part of a certain culture or society, but it never completely succeeds because, as explored before, the lack that is the subject is the lack of culture. The challenge that I am advocating here is precisely to engage with

[10] Less than a handful of people take into consideration the economy of schools in mathematics education research (Pais, 2014).

[11] Indeed, to withdraw promotion from schools it is suffice that teachers stop stamping people with letters and numbers. Technically, this can be done tomorrow, saving a huge amount of resources and people's time (a significant part of all the labour that revolves around schooling is spent in assessing and controlling students' performance). Why is it not so? Because there is more at stake in school than just the transmission of knowledge and competences. People in school learn the economic rules for the production and seizure of surplus-value (Baldino & Cabral, 2013, 2015). To question school's credit system is to question an entire economic structure in which we all participate. Such a radical change is often experienced as impossible.

the impossibility of education, not as a negative deadlock, but as the condition of possibility of any alternative education (or politics).

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