Sports Engineering (2019) 22:1–9
https://doi.org/10.1007/s12283-019-0303-8

TECHNICAL NOTE

# Recommendations for estimating the moments of inertia of a tennis racket

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Published online: 27 February 2019


**Abstract**
Tennis racket properties are of interest to sports engineers and designers as it allows them to evaluate performance, review trends and compare designs. This study explored mathematical models that correlated to the mass moments of inertia of a tennis racket, both about an axis through the butt and about the longitudinal axis, using its dimensions, mass and centre of mass location. The models were tested on 416 rackets, dating from 1874 to 2017. Results showed that moments of inertia about the butt and longitudinal axis can be estimated to within −4 to 5% and −11 to 12% of measured values, respectively, using the proposed models on original rackets. When rackets were customised, with 30 g of additional mass, moment of inertia about the butt could be estimated within 6%, but the model for moment of inertia about the longitudinal axis was less accurate (largest error at 25%). A Stepwise Linear Regression model indicated that racket mass and then centre of mass location had the largest effect on moment of inertia about the handle, with head width having the largest effect on moment of inertia about the longitudinal axis.

**Keywords** Sport equipment characterisation · Modelling · Mass · Centre of mass · Mass moment of inertia

## 1 Introduction

Tennis equipment plays a critical role in player performance [1]. Tennis has evolved drastically from its origins in the 1870s, mainly due to developments of the racket. A racket has three moments of inertia (MOI) acting about the principal axes through the centre of mass (CoM), and changes in these affect the racket in play. The MOIs are defined as transverse ($I_x$), lateral ($I_z$) and polar ($I_y$), acting about the lateral in-plane axis, out-of-plane axis and longitudinal axis respectively, as detailed by Brody [2] and defined in Fig. 1. These MOIs effect the racket by determining its resistance to rotation about the principal axes. Increasing $I_y$, or "twist-weight", provides greater resistance to rotation for impacts away from the longitudinal axis [3]. Increasing $I_x$ can increase ball speed off the racket, or 'racket power', assuming racket swing speed remains unchanged [1, 4, 5]. The parallel axis theorem can be applied to calculate MOI about different locations, as MOI is often measured about, or moved to, an axis passing through the handle to be more representative of the axis about which the player swings the racket [2]. If the axis is located approximately 10 cm (4 inches) from the butt and parallel to the lateral in-plane axis, the MOI is typically defined as the 'swing-weight' ($I_s$) as it relates to how hard it is to accelerate the racket through a swing [6, 7].

Haake et al. [8] used a physics based model to predict that a player can serve almost 20% faster using modern equipment, compared to what was used in the 1870s. The predicted increase in ball speed and performance was largely due to an increase in swing speed and modern rackets can be swung faster as their swing-weight is lower [9, 10]. In contrast, Whiteside et al. [11] reported players to swing rackets with lower swing-weight faster when serving, but with similar ball speed. The effect of racket swing-weight on tennis is complex and

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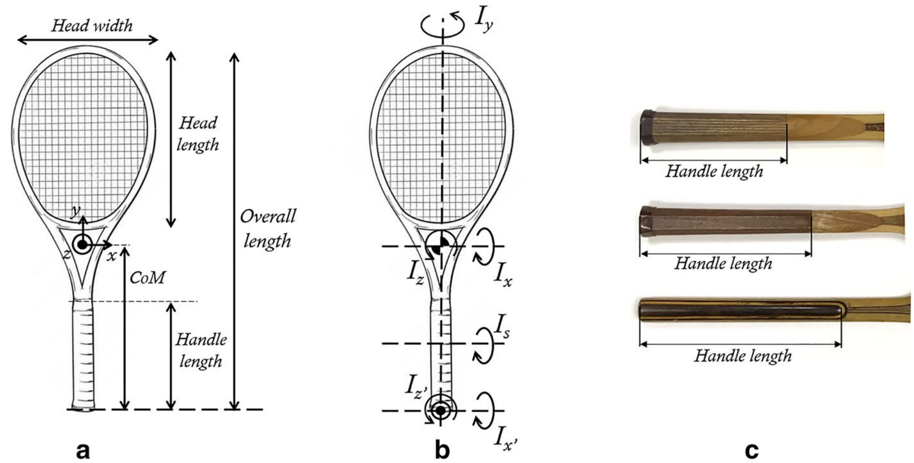


**Fig. 1** Diagrams showing **a** the measured geometric and mass properties and **b** moments of inertia of a tennis racket. **c** Methods for measuring handle length of old wooden rackets. Dates are: top 1916, middle 1911, bottom 1877

remains an open question, with differences between players and stroke types. Nevertheless, understanding swing-weight, as well as other MOIs, is important for those interested in racket performance and selection.

Measuring the properties of tennis rackets is an important step in evaluating racket design, selection and performance. Observing historical trends allows us to evaluate how materials and design have changed the racket, and hence the game [8], which could provide useful insights for product development, regulation and injury prevention strategies [12, 13], as well as spectator experience and education purposes [14]. Allen et al. [15] measured a range of parameters, including $I_x$, and $I_z$, in 100 rackets from different eras employing simple, low-cost and portable tools. They concluded that, since $I_x$, and $I_z$, were similar in magnitude, as detailed in Brody [2], measuring both is not always necessary when characterising a large number of rackets. In addition, they also presented that it is possible to estimate $I_x$, from models that use measurements of racket dimensions, mass and CoM location, which may be preferable in larger studies. Using a sample of ten, Brody [2] showed $I_y$ to be proportional to the product of a racket's mass and head width squared, but as values were overestimated a new model would be beneficial. Approximating MOIs from simple measures may also be useful to consumers who do not have access to measurement devices, as suggested by Brody [2].

The aim of this research was to assess the accuracy of the models for $I_x$, used by Allen et al. [15], as well as two models of $I_y$. This process identified the parameters that influence these MOIs and quantified their effects, using 416 diverse rackets.

## 2 Materials and methods

Data from 416 rackets was collected. 309 rackets were characterised from the Wimbledon Lawn Tennis Museum, where it was possible to test the oldest (early 1870s) and rarest (e.g. first steel racket) examples. Four rackets were from the International Tennis Federation (ITF), and three were from the Manchester Metropolitan University collection. Measurements of 100 rackets were available from [15]. Only strung rackets were selected in the study, although it was possible to have broken strings in old specimens. In these rackets (~20% of total), all strings were present and breaks tended to occur in the centre of the string bed. Moreover, string tension does not affect the parameters taken into account in the models for estimating the MOIs of the racket. While most rackets were originals, three were modern reproductions manufactured in the 1970s of wooden rackets dating back to the 1880s.

A measuring tape, to a resolution of 1 mm, was used for obtaining racket dimensions, including overall length, handle length, head length (internal/external) and head width (internal/external). Handle length was identified from the leather or rubber wrapping used as a grip on most rackets. For rackets without wrapping (less than a third of the population), handle length was identified according to grip enhancement features, such as a different wood colour, polished coverings, grooves or engravings (Fig. 1).

A digital scale (Smart Weigh Elite Series, 1 kg capacity, 0.1 g resolution) was used to obtain racket mass and CoM location (relative to butt). The tip of the racket was positioned on the scale with a support at the butt holding the racket horizontal. The mass (reaction) at the tip was measured, and CoM location was obtained from the product of overall racket length and the ratio of tip–racket mass. The test was repeated with the butt on the scale, and a mean value was obtained for CoM location. A check was performed to ensure that racket mass and the sum of the butt and tip masses were within 1 g of each other. The method was checked using a rod of known length (700 mm) and mass (289.9 g), similar to a racket. Seven points, symmetric about the centre and 50 mm apart, were identified on the rod and on these a mass of 50 g was positioned. For each point, rod CoM location was measured. The CoM measurements were within 1 mm (error < 1%) of predicted values.

$I_{x'}$ and $I_y$ were measured using a simple pendulum and bifilar pendulum, respectively, similar to [15]. Both techniques have been shown to measure the MOI of rods to within 2% of theoretical values [15, 16]. A frame assembled from aluminium profiles (Rexroth Bosch Group, section 30 × 30 mm) was used for the pendulum tests, and a calibration was performed to check the accuracy of the rig. Three measures were carried out on two rods of known theoretical $I_{x\prime}$ (0.0576 and 0.0198 kg·m$^2$) and two rods of known theoretical $I_y$ (0.00501 and 0.00185 kg·m$^2$), spanning typical values for rackets. A stopwatch to a resolution of 0.1 s was used for timing 25, 50 and 75 oscillations. The mean of the measured $I_{x\prime}$ was within 2% of the theoretical value, irrespective of how many oscillations were counted. For $I_y$, the mean of the measured value was within 2% of the theoretical value when 50 oscillations were counted. A mean period for the pendulum was obtained by dividing the measured time by the number of oscillations, while having more oscillations can reduce error in manual timing it can also increase the influence of damping [17]. For the rackets, 25 oscillations for $I_{x\prime}$ and 50 for $I_y$ were used. Each test was conducted twice and a mean MOI was calculated. If the difference between each repeat was more than 0.5 s (difference in $I_{x\prime}$ and $I_y$ of about 3%) the measurement was repeated keeping the closest values. It was not possible to measure $I_y$ for old asymmetric rackets with 'lopsided' heads, with the techniques employed here.

## 3 Models for predicting moments of inertia

There are devices designed to measure racket swing-weight (e.g. babolat diagnostic racket centre, prince precision tuning centre), but they are specialist and high-cost. Devices for measuring $I_y$ are not common, and while this parameter can be inferred from $I_x$ and $I_z$ [2], combined uncertainty from two measures and the assumption that the racket is planar can introduce error (particularly as $I_y$ is much lower than the others). Rigs for measuring MOI tend to be both more bulky and specialist than the tools needed to obtain the model parameters (e.g. tape measure, pocket scale), and these parameters can often be obtained from books [18], catalogues or websites. MOI models could reduce the need for the researcher to access the racket, an advantage with rare specimens in a private, or small, collection.

Three models for estimating $I_{x'}$ of a tennis racket were assessed in [15], using 100 rackets: the two-section beam [19], the unequal two-section beam [17] and the five section beam model [17], with good agreement to experimental data (correlation coefficients $r^2$ equal to 0.939, 0.943 and 0.934, respectively). The three models were explored here by testing a larger and more diverse group of rackets (416, including 316 new models), which included different designs and old samples with lopsided heads and wooden frames. Following initial data collection and analysis, if an absolute difference of more than 15% was observed between a measured MOI and a model, the racket was retested, i.e. the racket was reset in the rig and two further measurements of MOI were collected to replace the original values.

As observed in [15], the three models estimated $I_{x\prime}$ with similar accuracy: the correlation coefficients r$^2$ were 0.953, 0.950 and 0.947 and also the root mean square errors (0.0013, 0.0013 and 0.0027 kg·m$^2$) and the error standard deviations (2.4, 2.3 and 2.4%) were close to each other. Following the recommendations of [15], the unequal two-section beam model (Fig. 2b) was preferred due to its accuracy and relative simplicity. The model simplifies the racket as a mono-dimensional element and assumes that one section is equal to the racket handle and the other to the frame (open/closed throat and head), as shown in Fig. 2b. The mass of each section is identified by taking into account that the total mass $m$ and the CoM location $c_{\mathrm{m}}$ should be identical for the model and the racket, that is:

$$m = m_{\mathrm{h}} + m_{\mathrm{f}} \tag{1}$$

$$c_m = \frac{m_h \frac{l_h}{2} + m_f\left(l_h + \frac{l_f}{2}\right)}{m} \tag{2}$$

where $m$ and $c_{\mathrm{m}}$ are the measured mass and CoM location of the racket, $\mathrm{m_h}$ and $m_{\mathrm{f}}$ are the masses of the handle and the frame, and $l_{\mathrm{h}}$ and $l_{\mathrm{f}}$ are the lengths of the handle and the frame. Having identified $m_{\mathrm{h}}$ and $m_{\mathrm{f}}$, $I_{x\prime}$ can be found using:

$$I_{x'} = \frac{m_{\mathrm{h}} l_{\mathrm{h}}^2}{3} + \frac{m_{\mathrm{f}} l_{\mathrm{f}}^2}{12} + m_{\mathrm{f}}\left(l_{\mathrm{h}} + \frac{l_{\mathrm{f}}}{2}\right)^2 \tag{3}$$

The model for estimating $I_y$ (Fig. 2c) is necessarily bi-dimensional and divides the frame into two sections, which correspond to the throat region and head of the racket. Using the mass of the frame $m_{\mathrm{f}}$ identified in the previous model, the masses of these sections are calculated according to a simple proportion, that is:

$$m_{\mathrm{t}} = m_{\mathrm{f}} l_{\mathrm{t}} / l_{\mathrm{f}} \tag{4}$$

$$m_d = m_f l_d / l_f \tag{5}$$

where $l_{\mathrm{t}}$ and $l_{\mathrm{d}}$ are the lengths of the throat and head (mean of internal and external head lengths), respectively.

The $I_y$ of the handle and throat is small (due to their limited width) if compared to the head (50–100 times smaller); and the thickness and depth of the frame can be ignored when calculating $I_y$, and a thin section formula can be employed instead.

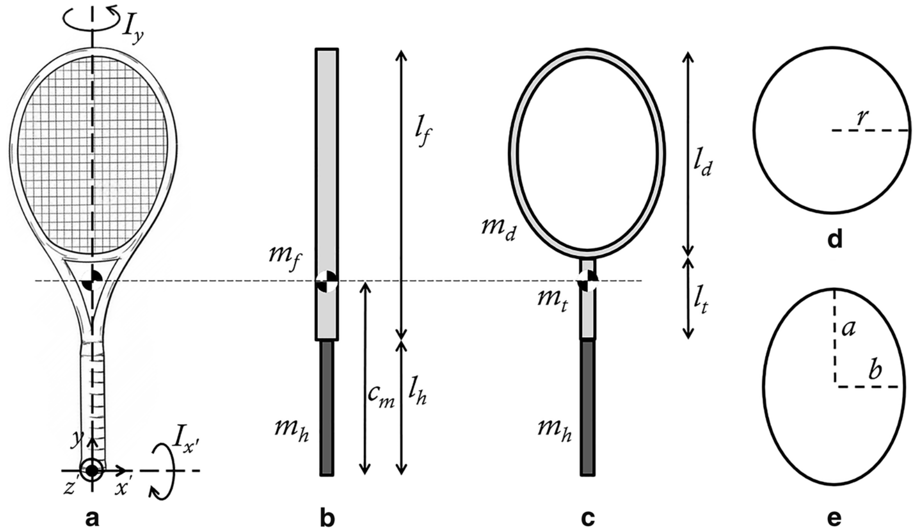


**Fig. 2** Diagrams of the models for predicting $I_{x'}$ (**a**–**b**) and $I_y$ (**c**–**e**) MOIs; **d** circular head, **e** elliptical head

Two formulas can be used for calculating $I_y$. The first formula corresponds to a circular hoop of mass $m$ and radius $r$ [20], as the racket head was at first approximated to a circle (Fig. 2d):

$$I_{y^c} = \frac{mr^2}{2} \tag{6}$$

where $m$ is equal to the mass of the head $m_d$ and $r$ is equal to half of the external head width. The external head width was used as this gave the lowest mean error, when compared to either the internal or the mean of the internal and external head widths.

The second formula corresponds to an elliptical hoop of mass $m$, semi-major axis $a$ and semi-minor axis $b$ [21], the geometry of the head of the racket being approximated to an ellipse (Fig. 2e). The MOI about the semi-major axis $a$ is:

$$I_{y^e} = \frac{1}{3}mb^2\left(2 - \frac{b^2 S}{c^2\left(a^2 - S\right)}\right) \tag{7}$$

where $a$ is half of the mean of internal and external head lengths, $b$ is half of the mean of internal and external head widths, $c^2 = a^2 - b^2$. The head width used in this formula (for $a$) was different from the head width used for the circle (for $r$, in Eq. 6) as this gave the lowest error. $S$ is the sum of a rapidly convergent series of $a$ and $b$ [21]:

$$S = \sum_{n=0}^{\infty} 2^{n-1} c_n^2 \tag{8}$$

where $c_n^2 = a_n^2 - b_n^2$, $a_{n+1} = \frac{1}{2}\left(a_n + b_n\right)$ and $b_{n+1} = \sqrt{a_n b_n}$, with $a_0 = a$ and $b_0 = b$. The index of summation n went from 0 to 10 [21].

A sensitivity analysis on the effect of a variation in input parameters, due to measurement uncertainty, on the outputs of the models was carried out. The input parameters (overall, handle and head lengths, head width, mass and CoM location) were increased, one at a time, by 1% and the variation of outputs was checked. The parameters that most affected estimates of $I_{x'}$ were mass and CoM location (with a difference of 1.0 and 1.3%, respectively), whereas an uncertainty in handle length had a low effect (0.02%). Similar results were found for estimates of $I_y$, but the parameter that most affected the output of the model was head width, with a difference of 2.2%.

Two Stepwise Linear Regression models were constructed on all dimension and mass variables used in the models, to examine the racket parameters that best predicted $I_{x'}$ and $I_y$. In both cases, a pairwise Pearson's Correlation was first used to see which variables were correlated to the experimental values of $I_{x'}$ and $I_y$. These variables were then introduced to the Stepwise Linear Regression models in order, starting with those which were best correlated to $I_{x'}$ and $I_y$ based on their Pearson's Correlation coefficient.

A Mann–Whitney $U$ test was used to compare the difference between the measured and predicted values of $I_{x'}$ and $I_y$ for more modern rackets (from 1990) and older rackets manufactured before 1990. This date separated the modern rackets manufactured from fibre-reinforced composites ($n = 65$) from their older predominately wooden counterparts ($n = 351$ with 79% wood, 6% metal & 16% fibre-reinforced composite). Fibre-reinforced composites provided engineers with greater design freedom, which may have led to more non-uniform mass distribution in modern rackets.

The robustness of the models was checked using four customised rackets made of different materials (wood, metal and fibre-reinforced composite). A racket can be customised by adding mass, with the amount and location

**Table 1** Geometric properties

| Overall length (m) | Handle length (m) | Head length (m) | | Head width (m) | |
|---|---|---|---|---|---|
| | | External | Internal | External | Internal |
| 0.686 ± 0.010 | 0.196 ± 0.036 | 0.315 ± 0.032 | 0.292 ± 0.031 | 0.234 ± 0.021 | 0.209 ± 0.021 |
| 0.652–0.811 | 0.148–0.355 | 0.240–0.493 | 0.226–0.455 | 0.185–0.308 | 0.157–0.285 |

Mean ± standard deviation, range

**Table 2** Mass and inertial properties

| Mass (kg) | COM location (m) | $I_{x\prime}$ (kg m$^2$) | $I_y$ (kg m$^2$) |
|---|---|---|---|
| 0.360 ± 0.035 | 0.336 ± 0.016 | 0.055 ± 0.0053 | 0.0011 ± 0.00022 |
| 0.220–0.427 | 0.30–0.43 | 0.041–0.0745 | 0.00058–0.0018 |

Mean ± standard deviation, range

influenced by factors such as player preference, style, level and experience, as well as the inertial properties of the unmodified racket. To the best of the authors' knowledge, there are no publications related to how often and to what extent rackets are customised, therefore, two customisations equal to the ones described in [19] were reproduced. In one configuration, two 15 g masses were added at the widest points of the racket head; in the other, 30 g was added at the tip. Tests on customised rackets were carried out with the methods previously described, to assess whether the models could accurately estimate MOI's, without knowing whether, or to what extent, the racket has been customised. This is a likely scenario when testing used, or otherwise customised, rackets i.e. the tester may not be aware that the racket has been customised.

## 4 Results

Tables 1 and 2 summarize the parameters used in the models, with the mean, standard deviation and range reported. From these values, it is clear that a diverse range of rackets were measured, particularly in terms of handle and head lengths, head width and mass.

Figure 3a shows experimental values of $I_{x\prime}$ against the estimated $I_{x\prime}$, using the unequal two-section beam model. The experiment and model showed similar outputs, the correlation coefficient ($r^2$) was 0.950, the regression line slope was close to 1 (0.928) and the values of the intercept (0.004 kg m$^2$) and root mean square error (RMSE) (0.0013 kg m$^2$) were low. Figure 3b shows a Bland–Altman plot [22] between experimental and estimated values. The mean difference (bias) between the experiment and model is represented by the solid line, while the dashed lines show the limits of agreement. The bias was 0.00026 kg m$^2$, indicating the model tended to overestimate the experimental values. The limits of agreement ranged from − 0.0022 to 0.0027 kg m$^2$ (corresponding to − 4 to 5% based on the mean $I_{x\prime}$ of 0.055 kg m$^2$), which means the model can be used when the acceptable difference from the experimental value is with this range. The variation between the model

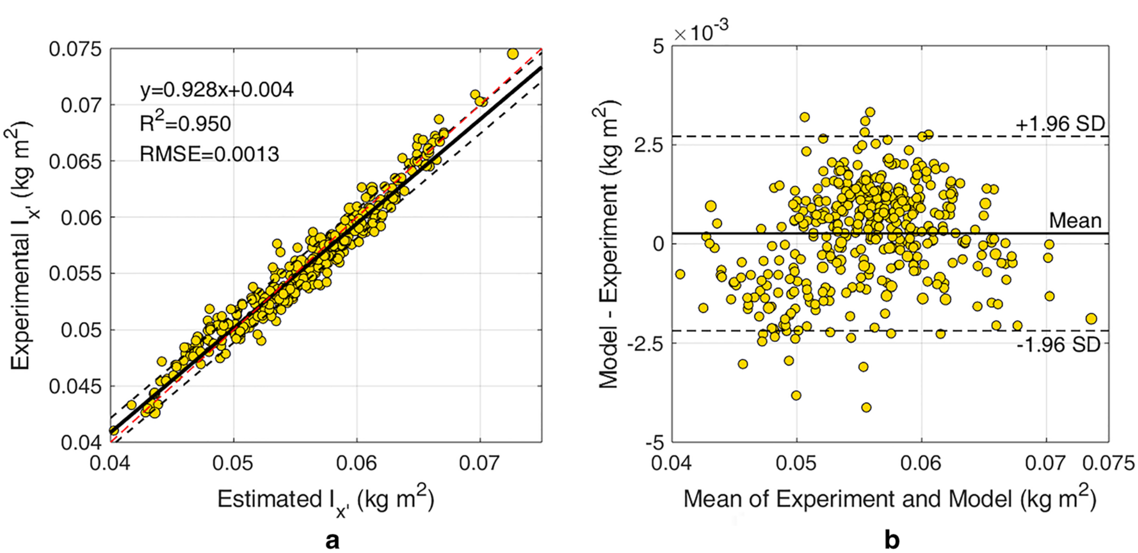


**Fig. 3** $I_{x\prime}$: **a** experimental versus estimated values and **b** Bland–Altman plots between experimental and estimated values

and experiment depended on the magnitude of $I_x$, such that medium values (~ 0.05–~ 0.06 kg m$^2$) tended to be overestimated, with low and high values underestimated. A Stepwise Linear Regression model indicated that mass [partial correlation (pC) = 0.970], CoM location (pC = 0.906), overall racket length (pC = 0.324), and handle length (pC = 0.216) all significantly contributed to predicting $I_x$, ($R^2 = 0.949$, $p < 0.001$). The model for $I_x$, significantly predicted older rackets (1874–1989) better than newer rackets (1990–2017) ($z = -5.116$, $p < 0.001$). The absolute error of the model on older rackets had a mean of 1.8%, compared to 3.0% in the newer rackets.

Similar graphs were generated for $I_y$, comparing experimental values to our models that represent the racket head as circular or elliptical (Fig. 4a, c, b, d, respectively). It was not possible to measure $I_y$ for 45 rackets, mainly from the 1870s and 1880s, that had asymmetric or 'lopsided' heads, therefore, they were removed from the analysis of $I_y$. Figure 4a, b show experimental versus estimated $I_y$. The equation of the linear regression line, the correlation coefficients r$^2$ (0.916 and 0.910) and the root mean square errors ($6.3 \times 10^{-5}$ and $7.3 \times 10^{-5}$ kg m$^2$) showed that the two models, circular and elliptical, were similar. The outlier at the top-right corner of the graphs was a racket from the 1990s with no throat region (Head, Ti.S7), the string bed extended to the handle and the head length exceeded the legal limit. Figure 4c, d show Bland–Altman plots between experimental and estimated values for the two models. The bias for the model that approximates the racket head to a circle was $3.0 \cdot 10^{-6}$ kg m$^2$, compared to $-3.3 \cdot 10^{-5}$ kg m$^2$ for the model that employs an elliptical shape. The limits of agreement ranged from $-0.00012$ to 0.00013 kg m$^2$ for the circular head model (corresponding to $-11$ to 12% based on the mean $I_y$ of 0.0011 kg m$^2$) and from $-0.00016$ to 0.00010 kg m$^2$ for the elliptical head model (corresponding to $-15$ to 9% based on the mean $I_y$ of 0.0011 kg m$^2$). Eighteen rackets with an absolute difference of more than 15% between the measured $I_y$ and one or both models were identified and re-tested, and for 13

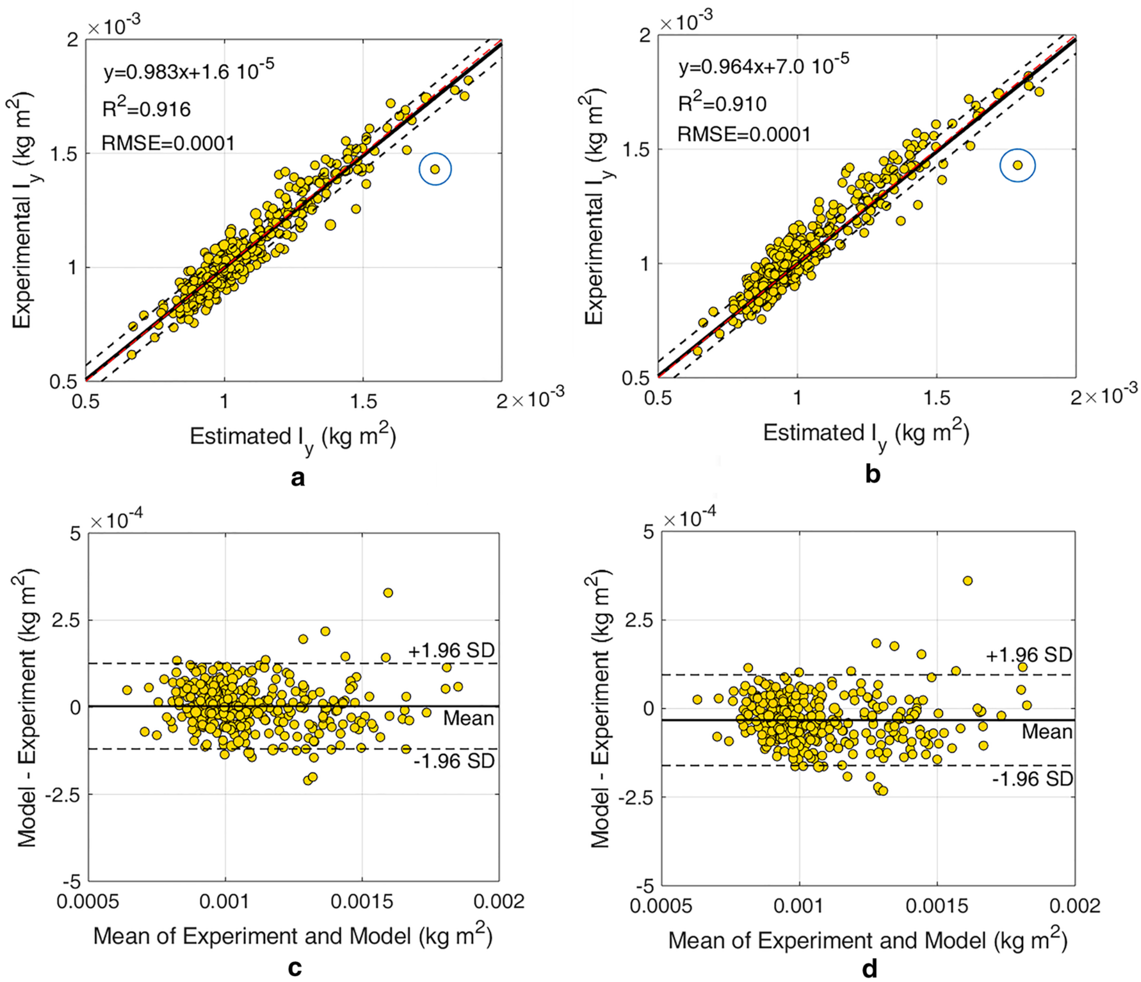


**Fig. 4** $I_y$, experimental versus estimated values for **a** circular head model and **b** elliptical head model, and Bland–Altman plots between experimental and estimated values for **c** circular head model and **d** elliptical head model

of these the difference decreased to below 5%, becoming similar to those of the other rackets. In three cases, the difference decreased to between 5 and 10%, while in two cases the difference remained close to 20%. A Stepwise Linear Regression model indicated that external head width [partial correlation (pC) = 0.893], mass (pC = 0.646), CoM location (pC = 0.444), mean head length (pC = 0.261) and overall racket length (pC = − 0.176) all significantly contributed to predicting $I_y$ ($R^2 = 0.894$, $p < 0.001$). Mean head width could not be included in the regression model, despite being included in the geometric model, because it was highly correlated to the external head width, and therefore, excluded by the Stepwise Regression process. The absolute error of the models for $I_y$ were not significantly different between the older (1874–1989) and newer rackets (1990–2017) (circular head $z = -0.207$, p. 836; elliptical head: $z = -0.836$, p. 403).

The error between measured values and model estimates for customised rackets is shown in Table 3. The model for $I_{x\prime}$ showed an error within 3% when 15 g was added at the points of maximum head width, and presented the largest errors (~6%) when 30 g was added at the tip, which was the customisation that highly affects $I_{x\prime}$. $I_y$ was underestimated when two 15 g masses were added at the points of maximum head width, with errors up to 17% as this customisation has a large impact on $I_y$. $I_y$ was overestimated (up to 25%) when 30 g was added at the tip of the frame; in this position, the added mass increased the mass of the head estimated by the model, but only slightly increased the measured $I_y$ (up to 6%) of the racket as it lies close to its longitudinal axis.

## 5 Discussion

MOI measurements of 416 rackets, dating from 1874 to 2017, were in the range 0.041–0.0745 kg m$^2$ for $I_{x\prime}$, and 0.00058–0.0018 kg m$^2$ for $I_y$. An unequal two-section beam model for $I_{x\prime}$ [17], presented a bias of − 0.00026 kg m$^2$ in comparison to the measured values. The limits of agreement indicate that this model can be used if the acceptable level of accuracy falls between −0.0022 and 0.0027 kg m$^2$. Two different geometric models, circle and ellipse, were assigned to the head of the racket to estimate $I_y$. These models were in similar agreement with the measured values, although the bias was lower for the circular head model at $3.0 \cdot 10^{-6}$ kg m$^2$, compared to $-3.3 \cdot 10^{-5}$ kg m$^2$ for the elliptical head model. The circular model may be preferential, due to its relative simplicity. Indeed, our regression model indicates that head width, more so than length, is a good predictor of $I_y$, thereby lending support for using a circular model. Regression analysis indicated that $I_y$ could be predicted by maximum head width, mass, CoM location, mean head length and overall racket length, in that order. The limits of agreement indicate that the circular head model can be used if the acceptable level of accuracy is within −0.00012 to 0.00013 kg m$^2$. The models of $I_{x\prime}$ and $I_y$ were robust over a wide range of different racket designs, which means that they may continue to have similar levels of error when applied to future similarly diverse rackets. Moreover, the model of $I_y$ had similar levels of accuracy when applied to old (pre-1990) and new (post 1990) rackets, indicating its suitability of use on modern rackets. While the model of $I_{x\prime}$ performed slightly worse on new (post 1990) compared to older (pre-1990) rackets, the mean difference in absolute error was only 1.2%.

The models allow estimation of the MOIs of a racket from simple and quick measurements, and without the need of specialist equipment. It should, therefore, be possible to estimate MOIs using dimensions, mass and CoM location from catalogues, or a combination of catalogues and images. Indeed, future work could explore image processing to identify racket dimensions from photographs. The errors shown by the models was compared to the distribution of $I_{x\prime}$ and $I_y$ to check that the error due to the prediction was small if compared with the differences observed in the racket population. The comparison showed that the models are not effective at measuring rackets with similar properties, as the error might affect their relative positioning within the group more than their actual properties. For that reason, the models could be useful tools for monitoring trends, but for characterising the specific behaviour of individual rackets traditional measurement techniques are recommended.

It was necessary to re-test eighteen old rackets (> 90 years, and ~4% of total) that at first presented a large difference (> ± 15%) with the outputs of the model for $I_y$. Difficulties with measurement of $I_y$ may have been due to manufacturing inconsistencies, warping, wear or asymmetry

**Table 3** Error (%) between experimental and predicted values of MOIs of customised rackets

| Racket | Date | No customisation | | 15 g at the points of maximum head width | | 30 g at the tip | |
|---|---|---|---|---|---|---|---|
| | | $I_{x\prime}$ | $I_y$ | $I_{x\prime}$ | $I_y$ | $I_{x\prime}$ | $I_y$ |
| Donnay, Rod Laver | 1965 | 0.5 | − 3.0 | 2.6 | 10.4 | 5.1 | − 24.7 |
| Head, Arthur Ashe | 1975 | 0.8 | 1.6 | 2.7 | 15.7 | 5.6 | − 11.8 |
| Donnay, Pro 25 | 1987 | 0.8 | − 2.2 | − 0.7 | 16.9 | 4.9 | − 22.3 |
| Prince, EXO3 Rebel 98 | 2011 | 1.6 | − 3.3 | 2.3 | 15.1 | 6.3 | − 18.4 |

of the frame in these old rackets, although this was not measured specifically. While timing oscillations in the $I_y$ test was repeatable, asymmetry of the frame in an old racket can present a challenge when setting it up to rotate about its longitudinal axis consistently. In addition, Spurr et al. [16] reported that non-parallel wires can also produce error when measuring $I_y$ with a bifilar pendulum. Using the models may help to reduce experimental and data-entry errors, that arise during normal experimental data collection. Indeed, the $I_y$ model could be useful for highlighting human error when testing rackets. While measurements of $I_y$ (and $I_{x\prime}$) would likely change for some rackets if all of them were retested, the correlations with the models would not be expected to change substantially due to the large number of samples in this study. A sensitivity analysis showed that mass, CoM location and head width (for $I_y$) had the largest effect on the prediction of the models, therefore, these variables must be measured accurately during testing. Handle length had a low effect on the output of the models, which is an advantage as a handle may not be original or well-defined, especially in older specimens. Regression analysis supported these finding, indicating that $I_{x\prime}$ could be predicted by mass, CoM location, overall racket length and handle length in that order.

When applied to customised rackets, the model for $I_{x\prime}$ had errors up to ~6%, with the model for $I_y$ showing lower accuracy (absolute error > 10%). While this should not represent an issue when characterising the unmodified rackets sold on the market, more care should be taken when measuring players' rackets, as they can customise their equipment. Further work is needed to investigate the possibility of using models on customised rackets. While the models can be considered adequate for characterizing a large number of rackets, measurements of MOIs are recommended if a specific case is to be studied in detail, such as if MOIs are required for finite element modelling [23] or performance analysis of a particular design. Future work could look at relating the magnitude of the errors in the MOI model estimates to racket performance metrics, to evaluate their effectiveness in monitoring how design changes influence the game.

## 6 Conclusions

A group of 416 diverse tennis rackets were tested to develop and validate models for estimating $I_{x\prime}$ and $I_y$. $I_{x\prime}$ can be estimated using an unequal two-section beam model if the acceptable level of accuracy is between − 0.0022 and 0.0027 kg m$^2$. Two shapes, circle and ellipse, were assigned to the head of the racket for calculating $I_y$. The circle worked better, and this model can be used to predict $I_y$ if the acceptable level of accuracy is between − 0.00012 and 0.00013 kg m$^2$. In the presence of rackets customised with up to 30 g of additional mass, $I_{x\prime}$ can be estimated to within 6%, but the models for $I_y$ did not perform well (absolute error > 10%). MOI models can be useful tools for quickly characterising a large number of diverse rackets or monitoring trends, but for studying the specific behaviour of individual rackets, traditional measurement techniques are recommended.

**Acknowledgements** The authors wish to thank Matthew Glaze and the Wimbledon Museum Office staff for their kind and useful support and for providing access to the museum store. The authors wish to acknowledge the students William Dawber and Frank Hopkins, who benefited from two ISEA students engagement awards, for the assistance in measuring the rackets of the Wimbledon Lawn Tennis Museum. The authors wish to acknowledge the University of Rome "La Sapienza" for providing funding for this work.



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