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Appendix 2

HERMENEUTICS AND MATHEMATICAL ACTIVITY

ABSTRACT. This paper sets out to explore how hermeneutics might offer an approach to describing the nature of developing understanding in mathematical activity. In discussing the relationship between mathematics and mathematical activity it shows how hermeneutical understanding provides an opportunity to avoid positivistic descriptions that draw a hard distinction between the process and content of learning mathematics. Further, it suggests that personal interpretation underlies all mathematical understanding.

Hermeneutics

Early writers in hermeneutics (eg Dilthey), described a hard distinction between explanation as might be offered within the natural sciences and understanding (or more specifically an interpretation) as might be offered in the human sciences such as history (fig a). The former could be offered as a statement of fact whilst the latter could always be subject to personal interpretation. More recent writers (eg Ricoeur (1971, 1976), Habermas (1982)) have challenged this, bringing understanding and explanation into a more complementary relation under the umbrella of interpretation (fig b).

This later view might be summarised by the following statements which might be seen as characterising the two arcs of the hermeneutic circle.

1. Understanding to explanation.

Statements about historical events may be forever subject to review but in order to act it is necessary to suspend doubt and act as if our current reading is correct. Such a closure might be seen as complementing the phenomenologist's suspension of belief whilst thinking (see Schutz 1962).

2. Explanation to understanding

'Hard' statements about scientific phenomena are always viewed by individual humans in a particular context who make decisions about where these statements apply and choose where to use them.

Mathematics, which is often characterised as a subject comprising 'hard' statements, only ever finds expression in human activity. It can thus be seen as a subject of hermeneutic understanding if the emphasis is placed on interpreting mathematical activity, which itself might embrace the generation of mathematical statements. However, the making of these statements might cause a change in our perception of the context in which we see them arising. Such circularity emphasises the individual relation of the learner to mathematics

where the learner can only see mathematical phenomena from an individual perspective. Thus it might be suggested that the individual's view of mathematical 'content', as represented in the statements he makes, necessarily retains a residue from the 'process' through which it has been approached.

Any notion of a correct universal meaning does not arise within hermeneutic understanding. The way in which an expression is seen and used is always in a state of flux, being modified as the life experience of the individual affects the contexts in which it is seen as being appropriate. In reporting on mathematical activity we may choose to make a statement about it but may not be able to claim that it is an 'exact' representation of the phenomena described. Expressions offered by an individual are necessarily approximations to that which he means, speaking from the perspective of his individual life context. Habermas (1982) suggests that the gap between such an expression and what is meant by it can only be closed by interpretation. It is this very tension between statements and the meaning assigned to them that locates the hermeneutic circle. This moves away from notions of understanding developing in the mind, as might be offered by disciples of Piaget but, like Walkerdine (1989), focuses more on understanding arising in the social use of linguistic (or mathematical) forms in signifying phenomena. The issue being not so much arriving at concepts in a world that is knowable but rather converging to conventional usage of linguistic expressions.

As an example, I recently witnessed a lesson where eight year olds were working on the program "Reflect" (SMILE) as part of some work on "symmetry". This program allows the children to generate symmetrical shapes. The children were quite able to offer statements on the subject:

"If it goes up there, the other side goes up the other way"

"It's the same both sides, like it's cut in half"

"That line up the middle is like the line of symmetry"

"It's straight in that it divides"

Here it seems the children are not so much arriving at the 'concept' of 'symmetry' but rather offering a succession of statements that might be seen as being under the umbrella of this label. Children of this age seemed unable to offer anything approaching a formal definition. By seeing the commonness in the collection of statements the student might move towards more abstract notions of 'symmetry' and recognise the appropriateness of the term in other situations. But whatever this commonness might be it is always subject to modification as the set of experiences seen to be embracing it increases. A formal statement thus can only ever be seen as a report on the current view. However 'fact-like' a statement might appear it is always subject to humans deciding where and when to use it. Some might assume their own understanding of such a term is shared with others but this might simply mean that they are sufficiently close in their usage of it for them to be able to say they agree on its meaning. Any permanence supposed here is perhaps an illusion. Surely, it is no more than a way of describing that allows the individual to cope for the time being, until the linguistic categories employed become inadequate in describing the situation in which he perceives himself to be acting.

Mathematical activity

The same might be said of expressions generated in mathematical activity. Even though people may believe there are mathematical expressions that mean the same to all people, each individual takes that expression and places it in the context of their experience, cultural perspective and current intentions. Expressions are used in a particular way which may indicate the intended meaning. For example, in an infants class I witnessed a six year old child engaged in a partitioning exercise using counters. He wrote down the equation $2+1+1+1=5$. Perhaps it was the first time he had ever written this particular expression. I feel his perception of it was rather different to my own since we were bringing different things to it. The contexts in which we see such statements being used and thus our understanding of them necessarily develop as our experience grows. Such an expression is simply a label we attach to a certain class of situations we recognise in our individual life context.

The statements arising out of mathematical activities may not always be so precise as the equation arrived at by this child. For example, problem solving or investigational tasks often lead to the generation of many sorts of statements (eg Evans and Billington (1987), Mason, Burton and Stacey (1982)). Such activities, which might be characterised by the teacher being less prescriptive in terms of requiring particular methods and answers, permit the students to take a more general overview of the activity in which they are engaged.

Elsewhere (Brown 1990), I have described a lesson with a class of ten year olds in these terms. Here the children were exploring the areas of the 'lawn' and 'path' in 'gardens' comprising a rectangular lawn surrounded by a path. Initially, the children made models of the gardens out of plastic squares and counted the squares to find the appropriate areas each time. The statements about 'gardens' referred to the plastic models. However, in due course the construction of the plastic models became cumbersome and the children readily transferred their work on to squared paper which permitted a more efficient way of producing representations of 'gardens'. For a while this proved successful but as bigger drawings were produced the limitations of the paper became evident. The possibility of tabulating the areas then seemed an appropriate way of gathering together the data that had been generated. In doing this, number patterns were suggested which allowed new data to be produced without the need for making or drawing new models. As more tables, were generated, more general statements could be made about them. For some children these statements were translated to a shorthand in the form of a more conventional algebraic symbolisation. During this work verbal and written statements from the children included:

"You add the top to the bottom and then add on the two sides"

"The area of the path goes up in twos"

"The path is always bigger than the lawn"

"Area of Path = $M \times N - M - 2 \times N - 2$ "

Whilst some of these statements may lack the precision of formal mathematical statements they suggest real attempts by the children to represent the mathematical phenomena they are dealing with.

Mason (1989a) proposes a model which seems useful in describing such activity. This comprises a helix where the experience of a mathematical situation is seen as passing repeatedly through the "getting a sense of", the "manipulating of" and the "articulating of" the problem. This has much in common with the notion of the 'Hermeneutic Circle' which might be used here to describe the tension between interpreting a problem and making statements in respect of it, which in turn influence subsequent interpretations. Mason himself suggests "the process of abstracting in mathematics lies in the momentary movement from articulating to manipulating. Articulation of a seeing of generality, first in words or pictures, and then increasingly tight and economically succinct expressions, using symbols and perhaps diagrams, is a pinnacle of achievement, often achieved only after a great struggle. It becomes a mere foothill as it becomes a staging post for further work with the expression as a manipulable object". I take this to mean that the student, in gaining understanding, moves between emphases; e.g. from following through a chain of thought to placing it in to some context, or from talking about some situation to declaring an algebraic pattern, or from proposing a formula to checking it out with an example. In working on a problem one may become engrossed in the procedures and restrictions but after a while see a pattern; moving from work with particular cases to a recognition of the general. Thus the task of understanding might be seen as a mixture grasping relations internal to the mathematics and of seeing the problems in some context.

Assessing mathematical activity

With such a view of mathematical understanding there seems to be a need to develop an appropriate way of talking about mathematical achievement. Most assessments seem to be concerned with the production of correct mathematical statements as evidence of broader mathematical understanding. An alternative to this suggested by Mason (1989b) places emphasis on the 'story' told about the event of a mathematical activity. Such a story might be no more than the set of statements offered by the children under the label of 'symmetry'. Here assessment is not so much based on the proportion of correct statements but rather, on the quality of understanding demonstrated in giving an account of the activity. Thus the assessment might be more like one normally associated with a piece of writing.

This suggests a possible reorientation of the teaching relation. Whilst the teacher may have selected the work she can ask the student to describe it in his own terms and then enter into a dialogue. This enables the student to articulate aspects of his thinking which may serve to help him clarify this. It also enables the teacher to gain some insight into the student's view and the language he uses. The resulting dialogue might be seen as an attempt to communicate in a shared language. However, the teacher might see part of her task as guiding the student towards conventional usage of certain expressions.

In conclusion, the 'content' of the mathematical activity might be seen as the outcome of the 'process' as described by the individual learner. Whilst modifying notions of mathematics which underlie syllabi constructed from a content-oriented point of view, traditional mathematical content still has a home here. However, the syllabus cannot be seen as remaining intact as the student progresses through it since the content of such a syllabus is flavoured by the activities that give rise to it. A residue remains of the experience in any identification of content covered which will be present in statements made by the student. Assessing the student's understanding of his mathematical work through the statements he makes in respect of it necessarily requires personal interpretation from his teachers in deciding how these statements signify the student's understanding. This does not rely solely on the student's production of correct mathematical statements. Commentary on the sense the student's make of the experience cannot necessarily be reduced to such a form.

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