Book Review: The limits of pluralism. Bharath Sriraman (Ed.) (2017) Humanizing mathematics and its philosophy: Essays celebrating the 90th birthday of Reuben Hersh

Basel: Birkhäuser. 363 pages. doi:10.1007/978-3-319-61231-7
Hardcover: ISBN: 978-3-319-61230-0. eBook: ISBN: 978-3-319-61231-7

Alexandre Pais

I remember being a masters student at the University of Lisbon in the early 2000s when, in one of the classes, the teacher, João Pedro da Ponte, introduced, not without reluctance, Philip J. Davis and Reuben Hersh's The Mathematical Experience as the textbook for the course. His reluctance was due to the idea that the book might be a bit too Bnon-academic^ and Bpopulistic^ for a postgraduate course. For me, having just come from a highly formalistic mathematical training, where mathematics was presented as a finished science, and all the work consisted of proving theorems with no attempt to problematise the implications, the history, or the meaning of such results, this book brought a breath of fresh air into my mathematics education, and sparked a different way of engaging with this science. I realised that one can talk about mathematics outside the straitjacket of deductive proof and formalism. Mathematics has a history, it has (sometimes controversial) applications, it can be approached philosophically, and the work of a mathematician is much broader than proving theorems. In the words of Davis and Hersh, the book humanises mathematics, by exploding this science in many unforeseen directions. At the time, no other book offered such an approach to mathematics, and I am grateful to João for having had the courage, possibly against some reactionary voices within the same faculty, for having adopted this book for the course.

Thirty-six years after the publication of The Mathematical Experience, Hersh is now considered one of the main figures in the popularisation of mathematics, having contributed decisively to move the philosophy of mathematics from an aseptic discussion of its fundaments, towards a humanistic philosophy, where the purpose is not (only) to study mathematics

in itself, but as an activity, developed by humans in a variety of different settings. This posits Hersh within a lineage of Bmavericks^ (Kitcher & Aspray 1988, p. 17, quoted by Carlo Cellucci,[1] p. 223), along with Imre Lakatos, George Pólya, Thomas Tymoczko, Philip Kitcher, Ubiratan D'Ambrosio, Ole Skovsmose, Tânia Cabral, and Roberto Baldino, who have been, through their work, displacing mathematics and its philosophy, opening new and unexpected ways of philosophically engaging with this science. Hersh's work, in particular, provided those working in education with the knowledge and the authority to present a more vivid and complex image of mathematics when working with their students.

It is thus not surprising that the book that is being reviewed here, Humanizing mathematics and its philosophy: Essays celebrating the 90th birthday of Reuben Hersh, was edited by a mathematics educator. Bharath Sriraman has collected in this book 27 texts, comprising five chapters from Hersh himself (including one interview with the author), and 22 texts from authors whose work has been influenced by Hersh. These are mathematicians, philosophers, logicians, and linguistics, and altogether they offer the reader a cornucopia of lives, visions, positions, and contradictions about the nature of mathematics and its role in science and society. Moreover, the chapters provide a privileged window into what Hersh calls the Bback^ side of mathematics,[2] that is, mathematics as it appears among working mathematicians, in the making, involving creative and discovery work, but also failures, errors, ethical doubts (e.g., the chapter by Michael Harris entitled BDo Mathematicians Have Responsibilities?^) and controversies (e.g., the chapter by Doron Zeilberger where the author goes as far as to posit today's mathematics as a religion, with its doctrines, dogmas, priests of publishing, and community of believers). This picture of mathematics is not recognised by the mainstream philosophy of mathematics, in which the sole purpose is to present and analyse mathematics in its finished form, as a self-subsistent entity—the Bfront^. This distinction is at the core of Bonnie Gold's chapter, one of the two chapters in the book addressing school mathematics (the other being that by Alexandre Borovik). The author reiterates what is already well known in mathematics education research: that students need to be confronted with mathematics in the making, by solving problems (instead of routine exercises), doing investigations that involve different concepts and areas of mathematics, by modelling reality, communicating their ideas with others, and so on. The author, however, fails to address the question of why, notwithstanding our better knowledge, school mathematics continues to be a caricature of mathematics. Perhaps one has to be a mathematics education researcher to ask these questions—as Nel Noddings, the only recognisable education researcher in this collection, does in her chapter BA Gift to Teachers^.

When Hersh was asked by Sriraman what he would like contributors[3] to address, he referred to the prediction made by Paul Cohen that, at some unspecified future time, mathematics would be replaced by computers. Hersh objected to this view, creating a dispute concerning the status of artificial intelligence (AI) in performing mathematical activity. As a result, the phantasm of AI haunts some of the chapters, either by directly engaging with Hersh and Cohen's dispute, thus providing a rich overview of the state of the art concerning mathematics and AI (e.g., the chapters by Dimitri Yu, Manin & Yuri Manin; and by Sven Delarivière & Bart Van Kerkhove), or by concluding with implications for the prospect of

[1] References solely by the name of the author refer to chapters in the book being reviewed.

[2] A wonderful example of this is the piece by Elena Marchisotto, exploring the Bbackside^ of an episode of the history of mathematics—the Bézout theorem—in order to illustrate the social nature of mathematics and the different avenues that emerge because of it.

[3] Suggested by Hersh, comprising friends, colleagues, and scholars.

computer-generated mathematics (e.g., the chapter by William Byers). Michael Harris goes further in questioning the potentialities, but also the risks involved in increasing automation. According to the author, the emphasis on AI signals an instrumental view of human activity, Bwith little sympathy for democratic decision-making^ (p. 110). The author calls the reader's attention to the mathematics informing the four main contemporary challenges posed by scientific development: big data and the manipulation of public opinion, the surveillance industry, financial mathematics, and artificial intelligence. This issue is further explored in Borovik's paper in relation with education. The increasing mathematisation that characterises our information society poses important educational challenges, as most of the mathematics informing social, scientific, technological, and economic decisions are not known or understood by the majority of the population—the Busers^, while crucial decisions concerning aspects that affect our common lives are informed by a small group of experts, the Bmakers^. Within both groups, there is a drive towards the flattening of mathematical research, by prioritising Bwhat works^, instead of asking Bwhy it works^. This drive towards ignorance is behind the recent cases of Bfake news^, of the electoral manipulation of data (e.g., Cambridge Analytica), and Google's hegemonic control of information and knowledge. The author calls mathematics education's attention to the necessity of remerging with computer science.

In his interview with Sriraman, Hersh also indicated his desire to break with convention and to be different (p. vi). This was achieved in a masterly fashion. The reader will struggle to find in the mathematics philosophy literature a book that displays such a diversity of texts, both in terms of the form, content, and size. Starting with the last, we have some very short texts (less than 5 pages, such as those from Chandler Davis, William Labov, and Nel Noddings), some of medium size (around 10–20 pages, the majority), and some quite long texts (e.g., those of Delarivière & Kerkhove, Cellucci, and Judy Azzouni whose was nearly 40 pages). The form in which contributions are presented is also diverse. Some of them one can read as if reading the newspaper, since they put aside academic protocols and freely discourse about issues related to Hersh's work and life (e.g., the initial interview with Hersh, and the later chapters by Noddings, Labov, and Davis, and the quasi-satiric texts by Zeilberger and by Ian Stewart). Others were made not to be just read but to be studied, as they engage with theoretical discussions that require some knowledge of philosophy or cognitive sciences (e.g., the chapter by Emily Grosholz on Kant, geometry and number theory; the chapter by Manin and Manin on cognitive networks; or the chapter by Paul Livingston on Wittgenstein and mathematics).[4] The book starts with an interview with Hersh by Sriraman, and displays a Bphotographic passage^ with 12 pictures depicting different moments in Hersh's life over the last nine decades. Some chapters use dialogues between invented personas, thus perpetuating the tradition initiated in The Mathematical Experience (the chapter by Delarivière & Kerkhove), others play with humour to show the absurdity of some results (e.g., the chapter by Stewart on how to communicate with aliens), while others are written as a letter to Hersh (e.g., Labov's). This unruliness is also present at the level of content, with chapters exploring topics as diverse as education, cognition, philosophy, history, artificial intelligence, Google, surveillance, communism, dogs, logic, aliens, genetics, religion, computer science, baboons, failed revolutions, and many others. Concerning quirkiness, one can hardly go further than this collection.

The question of whether mathematics needs mathematicians relates to the question of humanism, and, more generally, to what mathematics Bis^. Chapters in the book, albeit in very different ways, revolve around these core questions. It is interesting to notice that those

[4] Some of the latter are authentic treatises on the philosophy of mathematics.

authors who engage with Hersh's work often assume a position that, cordial as it may be, contests Hersh's ideas. The essays presented in this book are less a eulogy of Hersh's work than an opportunity to debate different views on the philosophy and nature of mathematics: The essay from Julian Cole attempts to highlight Hersh's humanism, while maintaining that mathematical objects are Bunchanging abstract objects that are necessary and atemporal existents^ (p. 165)—hence in clear contradiction to Hersh's conception of mathematical objects as Bequivalence classes of mental models^ (p. 43). The (long) essay by Carlo Cellucci discusses, in close articulation with Hersh's work, the different modalities of proof, of what counts as mathematical activity and the nature of mathematical objects, and the equally long essay by Michèle Friend complicates Hersh's view of mathematical theories as models. This book is thus also a cacophony of dissonant voices and contradictory approaches. The question of whether this cacophony sits well with Hersh's pluralism is an open one. It does, however, offer the reader a fresco of the multiplicity and richness (but also confusion and lawlessness) that characterises the way people talk about mathematics.

Hersh's pluralism, a commonplace in four of his chapters and a topic debated by other authors in the book, is based on the assumption that contradictory theories on the philosophy of mathematics can peacefully co-exist (p. 19). Pluralism is a kind of a Bmetaphilosophy^, where different theories are like different models that one uses to better suit a given purpose. According to Hersh, he derives his pluralism from mathematics itself, where, and contrary to what happens in other areas of knowledge (chiefly humanities and social sciences), pluralism is the rule.

However, some of the chapters in the book problematise this picture. Heated discussions on what is valued as a Bproof^ have divided mathematicians between, for instance, those who are willing to accept empirical and/or heuristic evidence, and those who require axiomatic rigour (see the three chapters from Zeilberger, Azzouni, and Cellucci). There is also a clash between pure mathematicians and applied mathematicians, who, in the words of Stewart, Bconsidered each other to be a hotbed of misguided idiots who didn't have a clue^ (p. 72); as well as the insularity of many mathematicians within their narrowly specialised areas of study, up to the point of accusing others of Bwasting their time on pointless frivolities^ (p. 72); and the list goes on.[5] The mathematics community is not only Ban elitist and exclusive club^ (Zeilberger, p. 148), it is also a corporation and, as in any corporation, a distance has to be kept between the internal and often sanguinary conflicts, and the public image. While the latter might be pluralistic and synchronic, chapters on this book show a different Bback side^ of mathematics.

The fact that Hersh uses mathematics as a model for his pluralism signals a certain reluctance in his engagement with philosophy. Notwithstanding that he was one of the most important scholars in humanising mathematics and its philosophy, his writings show a tendency to posit mathematics as the exemplary case, not only of knowledge and science, but also on how one should position oneself in the face of different theories. The idea that transpires after reading Hersh's chapter on pluralism (see p. 19) is one where in mathematics we all live a peaceful co-existence, accepting one another in our differences, while philosophers are always in disarray, with themselves and with the world. Reuben Hersh wants to bring peaceful co-existence to the philosophy of mathematics. His pluralism implies that Bdisagreement and conflict are sometimes fruitful and instructive, but often they are unproductive and futile^ (p. 25). Notwithstanding the

[5] Exemplary of the disputes within the mathematical cosmos is Grigori Perelman's withdrawal from mathematics, due to the derision of ethical standards in the community. See Nasar, Sylvia; Gruber, David (August 21, 2006). BManifold Destiny: A legendary problem and the battle over who solved it.^ The New Yorker. Archived from the original on March 19, 2011. Retrieved January 21, 2018. https://www.newyorker.com/magazine/2006/08/28/manifold-destiny

fact that this book itself, as well as the history of science in general, demonstrates the contrary, that is, how disagreement and conflict are the cornerstones of knowledge development, Hersh remains here a true humanist and, perhaps unwillingly or without realising it, aligning himself with a philosophical tradition that goes against his communist roots (see chapters by Labov and Chandler Davis concerning the political inclinations of Hersh).

We are approaching a level of discussion where politics is intermingled with knowledge and science. As with Hersh and other (maverick) philosophers of mathematics, every way of engaging with mathematics presupposes its own philosophy (either implicit or explicitly) on what mathematics is and what it means to do mathematics, one can also ask about the political position implied when one assumes a given philosophical tradition.[6] The problems with Hersh's pluralist position are explored in the chapter by Byers (p. 45). According to Byers, Bif you want to generate a new creative insight into the nature of mathematics, then perhaps the thing to do is to highlight the incompatibilities and not hide them^ (p. 56). The author asks BDo we even want to create a world where all the possible philosophies can live together in harmony?^ (p. 56). He adds: Bthis would be nothing more than the dream of an ultimate theory^. Indeed, behind the idea of consensus and pluralism rests an assumption about society based on a corporative model, where everybody lives happily within their proper designated places. Lurking on the horizon lies a society where all agree with one another in their personal perspectives, and where there is space for multiple contradictory positions without the need for one to compromise oneself. This depiction characterises today's hedonistic society, where everyone can choose their own lifestyles as long as they do not harass others. That is, as long as everyone is left alone to their own solipsism. The other side of the coin of pluralism is solipsism: each enjoying their own particular, private, models and theories, and lifestyles, without disturbing the other.

Hersh's pluralism has to be supplemented with a dose of singularity. Yes, it is acceptable to use different Btheories^ to understand particular phenomena. We can (wrongly) use, for instance, Piaget to understand a student working alone, and Vygotsky to understand students working with peers. Mathematics education is full of research that bricolages different approaches. However, one has to have a singularity quilting together all these different approaches. Something that brings all together in a political position that compromises each one. In the past, Hersh found it unacceptable for a mathematician to oscillate between formalism and Platonism—the famous sentence he used to describe the philosophical astuteness of the mathematician: BThe typical mathematician is a Platonist during the week and a formalist on Sundays^ (p. 27). But this is no longer Hersh's position. The ambiguity is gone, Bjust call it pluralism^ (p. 27). Would this line of thought still hold if, for instance, applied to the refugee smuggler who says BI am a NGO worker in a refugee camp during the week, and a modern slave trade entrepreneur during the weekend?^; or to a businessman who exploits the work of others during the week, and practises philanthropy during the weekend? Can we still overlook and live at peace with these contradictions? Can we assume the mathematical mode, and behave as Bmost mathematicians who live in [Georg] Cantor's paradise in spite of [Bertrand] Russell's paradox; they simply learn to avoid making certain moves which have been shown to lead to contradictions^ (Edwards, p. 62)?[7]

[6] This was Louis Althusser's main argument concerning the role of philosophy as politics in the field of theory.

[7] Is this not a wonderful metaphor for political life? We all live in the middle-class, academic, bourgeoisie paradise, in spite of knowing the exploitative nature of our economic system; that our privileges come with a cost, that outside the wall people are living in misery, etc. We learn to avoid thinking about these issues, looking to certain landscapes, or going to certain places.

Sometimes in life, not everything goes, and one has to make decisions, to assume positions, and to struggle for them. This is why pluralism is not a desirable epistemological, political, or philosophical position. Philosophy's business is not fixing things, where we can Bapply^ different philosophical theories depending on their usefulness. While mathematics has thrived within modernity, mostly because of its utility for science, society, and technology, philosophy has no Buse^ whatsoever, and, as such, seen as superfluous within a society that praises immediate results, ready to be applied, and also lucrative. Philosophy implies a life commitment. It implies the realisation that you, as a person, are part of what you are trying to understand; that what you say about mathematics is also what you say about yourself, and your personal and political commitments. These are, for me, the limits of Hersh's pluralism, which I see as a reflex of the contemporary ideological mode. That this book itself raises these issues and allows space for discordance and debate is only a step in the right direction.

Reference

Kitcher, P., & Aspray, W. (1988). An opinionated introduction. In W. Aspray & P. Kitcher (Eds.), History and philosophy of modern mathematics (pp. 3–57). Minneapolis, MN: The University of Minnesota.