Knowledge enrichment and conceptual construction: Which domain-general cognitive mechanisms are required for a three-year-old to learn natural number?

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ABSTRACT

Knowledge acquisition can be quick and easy, formulated in terms of concepts the learner already has (knowledge enrichment), or hard, requiring new representational resources to be created over years (conceptual construction). Previous research suggests knowledge enrichment requires receptive vocabulary, whereas conceptual construction requires executive functions (EFs), namely set-shifting and inhibition. This study investigates whether different domain-general cognitive mechanisms support these two types of knowledge acquisition in a new domain: learning natural number. Fifty-seven three-year-olds were tested on their count procedural knowledge (knowledge enrichment), understanding of the cardinal principle (requiring conceptual construction), receptive vocabulary, fluid IQ, and EFs (working memory, inhibition, set-shifting). Multiple linear regression analyses showed that neither receptive vocabulary, set-shifting nor inhibition predicted count procedural knowledge when controlling for age and other predictor variables, but working memory did. Logistic regression analyses showed that inhibition and exogenous set-shifting significantly predicted understanding of the cardinal principle when controlling for age and receptive vocabulary, as did working memory. The findings suggest that different forms of knowledge enrichment are supported by different cognitive mechanisms (here, working memory and not receptive vocabulary). Secondly, set-shifting and inhibition may have a role in conceptual construction, but this did not remain significant when controlling for all other predictor variables, unlike previous research. Finally, count procedural knowledge was the strongest, and only, significant predictor of cardinal principle understanding when including all predictors variables in the regression. The findings contribute theoretically to the field of knowledge acquisition in a new domain and set the stage for future research.

KEY WORDS:	CONCEPTUAL DEVELOPMENT	EXECUTIVE FUNCTIONS	NUMERICAL COGNITION	CARDINAL PRINCIPLE	KNOWLEDGE ENRICHMENT

Children learn vast amounts within their first few years of life. In some instances, new knowledge is quick and easy to learn. For example, new facts may be learned and remembered upon first encountering them (Bascandziev, Tardiff, Zaitchik, & Carey, 2018; Markson & Bloom, 1997), a phenomenon called “fast-mapping” (Carey, 1978). This new knowledge is formulated in terms of concepts the learner already has, and is the output of what are called “knowledge enrichment” processes (Carey, 2009). In contrast, other knowledge only emerges after years of exposure. Frequently, such knowledge requires “conceptual construction”, the creation of new representational resources that allow thoughts that were previously unthinkable (Carey, 2009). Previous research has established that different domain-general cognitive abilities underlie knowledge enrichment and conceptual construction. This study explores whether this is also the case in learning natural number: the meaning of positive integers.

The Conceptual Construction of Natural Number

Several kinds of evidence establish that acquiring concepts of integers requires conceptual construction. First, it is very difficult; concepts of integers arose over tens of thousands of years of cultural evolution and arise over years of development in children (Carey, 2009). The first system of representation capable of representing integers is verbal counting (Carey, 2009). When the count list (“one, two, three…”) is applied in one-to-one correspondence with individuals in a set, the last numeral in the count list provides a representation of the cardinal value (quantity) of that set. Children in numerate societies learn to count around the age of two-years old, yet this is initially a meaningless list of words. It takes children another 18-24 months after learning how to count to understand the meaning of these numerals, and thus how counting represents number (LeCorre, Brannon, Van de Walle, & Carey, 2006; Sarnecka & Carey, 2008; Wynn, 1990; 1992). This occurs in a stage-like process with around 3-6 months between stages. The child first comes to understand the number “one” (referred to as a “one-knower”), then “two”, then “three” and sometimes “four”. These children are known as “subset-knowers”, since they only understand a subset of their count range. After this, children understand the cardinal principle, and are then called “cardinal-principle knowers” (CP-knowers), because they understand that the final word when counting a set represents the quantity of that set. Coming to understand the cardinal principle shows a signature of conceptual construction since it is so protracted and difficult to learn.

Second, the best evidence for conceptual construction is an empirically supported description of the representational systems before and after the construction, in which it is possible to *see* how the earlier conceptual system cannot express the concepts that are represented in the later one. This shows that new representational resources must have been created to allow thoughts previously unthinkable. There are two innate systems that represent number (Feigenson, Dehaene, & Spelke, 2004). Firstly, the Analog Number System (ANS) allows the approximation of magnitudes, for example whether one set is larger than another (Dehaene, 1997). These representations are ratio dependent according to Weber’s Law. For example, six-month old infants can discriminate between sets of 8 versus 16, but not 8 versus 12 (Xu & Spelke, 2000). The second system is termed parallel individuation which enables the tracking of up to three or four individual objects in working memory. For example, two cats might be represented as “cat cat” (Feigenson & Carey, 2003; Trick & Pylyshyn, 1994). Neither of these systems, however, can represent positive integers, such as the number “seven”. Firstly, the parallel individuation system does not contain symbols for number, only symbols for the individuals. Furthermore, it has an upper limit of three or four. Secondly, the ANS representations are approximate and cannot even discriminate 11 from 12 (Carey, 2004). Since neither innate system can express the representation of positive integers, this must require conceptual construction.

Furthermore, adults without a count list, for example monolingual speakers of Amazonian languages and Nicaraguan deaf home-signers, show evidence of innate number systems but are unable to represent set sizes exactly beyond the range of parallel individuation (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004; Spaepen, Coppola, Spelke, Carey & Goldin-Meadow, 2011). This provides further evidence that the innate number systems cannot represent positive integers and suggests that the count list is vital in the conceptual construction of positive integers.

Count Procedural Knowledge – the Product of Knowledge Enrichment
The above analyses establish that mastery of the cardinal principle is a conceptual construction. An essential part of the process is mastery of the count list and the count routine, since understanding how counting represents number requires knowing how to count. Learning the count routine is not fast-mapped since it takes years for children to learn (Wynn, 1990). Nonetheless, learning the procedure of counting draws on knowledge enrichment alone, since it draws on cognitive resources even infants have. Both infants and non-human primates can learn short arbitrary serial lists (Gulya, Rovee-Collier, Galluccio, & Wilk, 1998; Terrace, Son, & Brannon, 2003), and infants can carry out the one-to-one correspondence procedure required for counting (Slaughter, Itakura, Kutsuki, & Siegal, 2011).

The Learning Mechanisms Underlying Conceptual Construction and Knowledge Enrichment
Having established the differences between conceptual construction and knowledge enrichment, the learning mechanisms underlying these should be quite different. Carey (2009) characterises a bootstrapping mechanism, “Quinian Bootstrapping”, that is implicated in every case of conceptual construction she has analysed. In the case of the CP-induction, the child needs both the meaningless count list and the meanings of the words “one” to “four”, which draw on resources from the parallel individuation system. The CP-induction requires making an analogy between the order of words in the count list and the sets labelled “one” to “four”, by noticing that a numeral next in the count list and the model next in the series (of “one”, “two”, “three”, “four”) are related by adding one (Carey, 2009). The numerals in the count list thus serve as placeholder structures that are filled with meaning over time, enabling new representational resources to be created. Knowledge enrichment, in contrast, does not require Quinian bootstrapping. It is subserved by straight-forward statistical learning mechanisms, associative mechanisms, and hypothesis testing mechanisms (Bascandziev et al., 2018).

Domain-General Cognitive Resources Supporting Conceptual Construction and Knowledge Enrichment
Further insight into the learning mechanisms underlying conceptual construction and knowledge enrichment derives from the investigation of the general cognitive abilities each draw upon. Domain-general cognitive resources include executive functions (EFs), the abilities underlying fluid IQ, and the abilities underlying lexical learning

(verbal learning, or crystallized IQ), as tapped by measures of receptive vocabulary. The EFs are a family of top-down mental processes that enable us to pay attention, think outside the box, or operate in a non-automatic way (Diamond, 2013). Confirmatory factor analysis has identified three core EFs: working memory; being able to hold information in memory and manipulate that information; set-shifting, being able to flexibly select among relevant sources of information; and inhibitory control, being able to inhibit potentially competing responses or representations (Miyake et al., 2000). EFs are strongly associated with academic ability throughout school (Diamond, Barnett, Thomas, & Munro, 2007; Espy et al., 2004; Gathercole, Pickering, Knight, & Stegmann, 2004) and are specifically implicated in measures of mathematical achievement (Blair & Razza, 2007). It is therefore plausible for them to play a role in conceptual construction since this underpins learning in many school subjects (Wiser & Smith, 2016).

Indeed, evidence suggests that EFs are required for the conceptual construction of a theory of physics (Baker, Gjersoe, Sibielska-Woch, Leslie, & Hood, 2011; Bascandziev, Powell, Harris, & Carey, 2016), and theory of mind (Benson, Sabbagh, Carlson, & Zelazo; Carlson & Moses, 2001). This is supported by a series of studies investigating the construction of a vitalist biology in 5-7 year olds, an episode that involves radical conceptual change (Carey, 1985). These studies found that receptive vocabulary, a test of knowledge enrichment, predicted mastery of a vitalist biology when controlling for age and EFs. However, they also found that EFs, specifically set-shifting and inhibition, but *not* working memory or fluid IQ, predicted vitalism when controlling for age and receptive vocabulary. This suggests that conceptual construction requires more than the accumulation of facts (Zaitchik, Iqbal, & Carey, 2014; Zaitchik, Tardiff, Bascandziev, & Carey, 2018). Importantly, Bascandziev et al. (2018) conducted a biology training study that compared learning the concepts of a vitalist biology from a bootstrapping curriculum with learning ten fast-mappable generic fun facts about animals. They found that EFs, specifically inhibition and set-shifting, predicted the improvement in vitalist biology understanding but not the learning of the fun facts. In contrast, measures of knowledge enrichment mechanisms (receptive vocabulary) predicted an increase in fun facts, but not vitalism. This double dissociation provides evidence that EFs (set-shifting and inhibition) are required for conceptual construction, and that learning factual knowledge (knowledge enrichment) does not require set-shifting or inhibition, but instead draws on different knowledge enrichment mechanisms (receptive vocabulary, in this case).

Performing procedural mathematic tasks, including those using a counting strategy, draws heavily on working memory, as shown by studies that deploy dual-task paradigms (Cragg, Richardson, Hubber, Keeble, & Gilmore, 2017). Furthermore, previous research highlights the importance of working memory as a predictor of later procedural mathematic achievement (DeStefano & LeFevre, 2004; Raghubar, Barnes, & Hecht, 2010). Counting itself plausibly draws on working memory in younger children, since it requires keeping track of where in the count list they are and updating this accordingly as new objects are pointed to. Working memory may also be important in memorising the count list, because children must monitor what is new information (e.g. the next number) to add to their memorised count list. If learning to count requires different learning mechanisms than the knowledge enrichment of learning facts, then receptive vocabulary may not predict how well children can count, but perhaps a different domain-general mechanism, such as working memory, will.

The Present Study

The present study extends the investigation of the role of domain-general cognitive abilities in conceptual construction and knowledge enrichment to a new domain, numeracy, taking the CP-induction as the case study of the former and count procedural knowledge as the case study of the latter. This extends the previous work to much younger children, whose EFs are very immature (Anderson, 2002), and to a different kind of knowledge enrichment. Besides the theoretical interest of understanding knowledge acquisition since the human knowledge repertoire is a unique phenomenon on earth (Carey, 2009), understanding the learning mechanisms underlying this conceptual construction is also of urgent social importance. Entering school without an understanding of the cardinal principle has disastrous consequences for children's subsequent mathematical learning (Geary, 2011; Jordan, Glutting, & Ramineni, 2010; Jordan, Kaplan, Ramineni, & Locuniak, 2009).

This study measures children's count procedural knowledge and understanding of the cardinal principle, as well as several domain-general cognitive abilities: fluid IQ, EFs (working memory, set-shifting, inhibition) and receptive vocabulary. This study seeks to begin to investigate the learning mechanisms that underlie the conceptual construction of the integers and the mastery of procedural knowledge of counting.

The study explores three broad hypotheses:

I. Whether the same domain-general mechanisms implicated in previous studies of conceptual construction (set-shifting and inhibition and *not* working memory and fluid IQ) also predict mastery of the cardinal principle. If all conceptual construction draws on the same learning mechanisms, they should, but since this episode of conceptual construction occurs when children are very young, with very immature EFs, perhaps they will not.

II. Similarly, whether the same domain-general mechanisms implicated in previous studies of knowledge enrichment, the acquisition of fast-mapped generic facts (receptive vocabulary, and *not* set-shifting, inhibition, working memory, or fluid IQ) also predict mastery of procedural knowledge of counting. If all forms of knowledge enrichment draw on the same mechanisms, it should, but since the count routine is difficult to master, and since counting draws on working memory, perhaps working memory will predict count procedural knowledge.

III. Since mastery of the count list is a necessary prerequisite for mastery of the cardinal principle, measures of count procedural knowledge will be strong predictors of being a CP-knower.

Method

Participants

Sixty-seven children aged between 36 and 48 months (M_{age} = 41.0, *SD* = 3.40; 32 girls) were tested but ten were excluded from analyses because they did not return for the second session (*n* = 6), were extremely distracted during tasks (*n* = 2) or obtained a knower-level of 5 (*n* = 2), leaving it unclear whether they were four-knowers succeeding by chance or CP-knowers failing by chance. The final sample was 57 children aged between 36 and 47 months (M_{age} = 40.8, *SD* = 3.38; 28 girls). Participants were fluent English speakers from the Cambridge, MA area, recruited from the Harvard University Laboratory for Developmental Studies database of middle and high SES families interested in participating in studies of conceptual development. Children received a small toy for participating, and parents were given $5 as travel compensation.

Design
The experiment was an individual difference study, with a correlational design. Testing occurred in two 20-30 minute sessions (mean time between sessions = 8.3 days, *SD* = 7.1). The tasks in the number session were administered in the following order: Fast Cards, Give-a-Number, Successor Function, Highest Count, Count Routine. Tasks in the cognitive resource session were administered as follows: Dimensional Change Card Sort, Stroop, Words Forwards, Words Backwards, Odd-One-Out, Receptive Vocabulary, Object Assembly, and lastly, Verbal Fluency. Fast Cards, Successor Function, Stroop, and Odd-One-Out are not analysed in this paper due to word limits or unconventional testing procedures.

Materials and Procedure
Prior to the study, parents were given a description of the study, the opportunity to ask questions, and a consent form to sign. Testing took place in a laboratory testing room.

Number Session (Outcome Variables)
Give-a-Number (Wynn, 1992; Sarnecka & Carey, 2008) assesses a child's "knower-level". The child was instructed to give a certain number of one type of toy (see Figure 1) to a stuffed animal on the table (e.g. "can you give "Mr. Lion" two ducks?") and was asked to check their answers for sets of two or more toys. Subset-knowers' knower-level was the highest numeral for which they gave the requested number on at least 67% of trials, and for which they did not give that number of items when asked for sets with cardinal values named by other numerals. CP-knowers could create the appropriate sets when probed with "one" to "six". Give-a-Number provides a second measure relevant to the distinction between subset-knowers and CP-knowers: what children do when asked to check. CP-knowers count, but subset-knowers, for whom counting is irrelevant to number representations, do not (LeCorre et al., 2006).

Figure 1. Small toys used in Give-a-Number.

Highest Count assesses a child's procedural knowledge of number by assessing the length of their memorised count list. Children were asked to count as high as they can to the experimenter (first trial) and to a stuffed animal on the table (second trial). Scores were the highest number the child counted to correctly before making an error in their count list.

Count Routine assesses a child's procedural knowledge of number by assessing their knowledge of the count routine. Children were asked to count a row of toy ducks and point as they count. There were three trials which included three ducks in a row (first trial), five ducks in a row (second trial), and ten ducks in a row (third trial). Children's mastery of the count routine was scored using a coding schedule. For example, good knowledge of the count routine involved saying each number of the count list in one-to-one correspondence with each duck, whereas poorer knowledge of the count routine was reflected by double counting or skipping ducks.

Cognitive Resource Battery (Predictor Variables)
Dimensional Change Card Sort (Zelazo, 2006) primarily assesses a child's capacity to shift flexibly between rules when cued exogenously and to inhibit currently prepotent responses. There were six pre-switch trials in which children were instructed to sort cards (see Figure 2) depending on their colour. The experimenter then announced a new game and instructed children to sort cards by shape. Each card was labelled by its relevant dimension for each trial (e.g. pre-switch trials: "here is a blue one, where does it go?", and post-switch trials: "here is a boat, where does it go?"). A child had to correctly sort at least five out of six post-switch trials to pass the task.

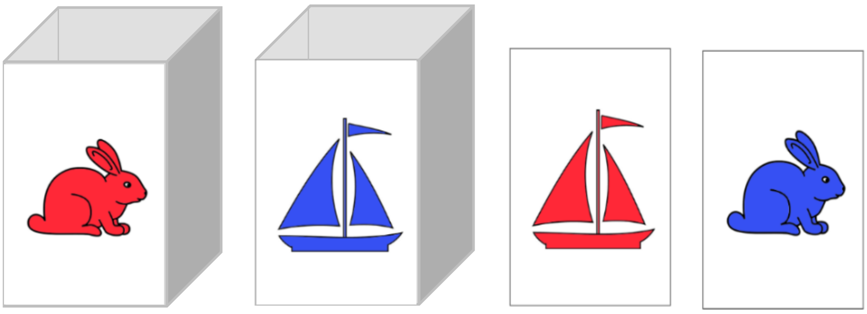

Figure 2. Target boxes and test cards used in the Card Sort task

Words Forwards (Alloway, 2007) assesses simple working memory span. The child was instructed to repeat sequences of words spoken by the experimenter (e.g. "repeat after me, 'two – five – three'"). The experimenter gave one example followed by three practice trials with correction and feedback if necessary. Test trials began with two words in a row and continued to six words in a row, increasing in length by one word on each trial. A child's forward span was the length of the largest list repeated correctly before making errors on two consecutive trials.

Words Backwards (Alloway, 2007) assesses updating working memory. The child was instructed to repeat sequences of words said to them in a *backwards* order. The child must maintain the ordered list in working memory, and then operate on this memory representation to select the last item, the penultimate item, and so on. Given its increased difficultly compared to words forwards, this was modelled twice with a puppet (e.g. "when I say 'horse – pig – duck', Mr. Lion says 'duck – pig – horse', see, he said the animals that I said, but in a backwards order"). There were three practice trials with correction and feedback if necessary. If a child failed all the practice trials, the test trials were not attempted. Test trials ranged from 2-5 words in a row. A child's backwards span was the length of the largest list the child correctly repeated back in reverse order before making errors on two consecutive trials.

Object Assembly taken from the Wechsler Preschool and Primary Scale of Intelligence – Third Edition (WPPSI-III) (Wechsler, 2002), is a normed and standardised test of fluid IQ. Only half of the Object Assembly trials were administered (even numbered trials) to save time. Previous analyses in the laboratory showed that performance on these trials highly correlated with performance on the whole task (r = .97, p < .0001). The child assembled seven sets of puzzle pieces into target objects (e.g. "these pieces go together to make a hand, put them together as quickly as you can"). Each trial ended when the child successfully completed the puzzle, or after 90 seconds. A child's score was the total number of correct junctures (the points in which puzzle pieces meet, see Figure 3) across all attempted puzzles.

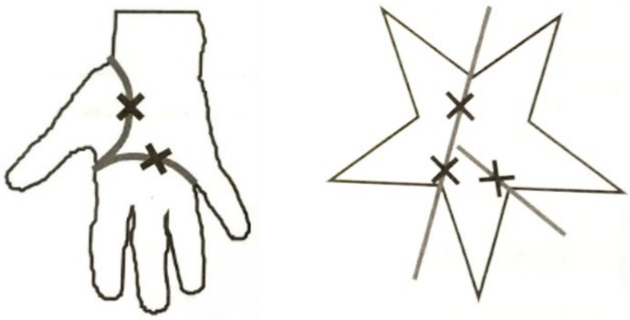

Figure 3. Junctures marked by crosses for "hand" and "star" Object Assembly trials.

Receptive Vocabulary (WPPSI-III; Wechsler, 2002), is a measure of the size of the child's lexicon. The words assessed are the output of knowledge enrichment mechanisms. Twenty-seven words were probed by asking the child to point to one of four images that depicts the meaning of the word (e.g. "point to the butterfly").

In the ***Verbal Fluency*** tasks (Snyder and Munakata, 2010), the child was asked to name as many different animals (first trial), then foods (second trial), as they could within 30 seconds. Scores were the total number of different, correct items. For older children, the task assesses endogenous set-shifting, because the child must monitor when a subcategory (e.g. pets) of a focused category (e.g. animals) has been exhausted and shift appropriately to a different subcategory (e.g. zoo animals). Children this young do not spontaneously use the strategy of systematically running through subcategories. However, the task still measures set-shifting and inhibition; the child must figure out how to search a huge database for animal or food names, and must inhibit the tendency to repeat a name already given.

Results

First, descriptive statistics for each task are presented, followed by correlations between predictor variables and outcome variables to motivate creating composite measures. These are followed by a series of regression analyses that test hypotheses 1-3. To assess which domain-general mechanisms predict count procedural knowledge, multiple linear regressions are presented because the dependent variable (DV) is continuous (count procedural knowledge). To assess which domain-general mechanisms predict CP-knower status, logistic regressions are presented because the DV is dichotomous (subset-knower or CP-knower). In all regressions, age and receptive vocabulary scores are included to control for overall opportunities for knowledge enrichment, since older children have come across more facts, which is a major source of variance in receptive vocabulary (Zaitchik et al., 2014). Preliminary analyses ensured no test assumptions for these regressions were violated. Gender differences were analysed but gender was not included in further analyses, since most research indicates there are no gender differences in EF development in childhood (Anderson, 2002). This prevented partialling out natural variance in this small sample.

Descriptive Statistics

Number tasks. Descriptive statistics for Give-a-Number are shown in Table 1. The distribution of CP-knowers and subset-knowers is consistent with the literature for children this age, as roughly half are CP-knowers (n = 25) and half are subset-knowers (n = 32) (Sarnecka & Carey, 2008). The literature also establishes a qualitative difference between CP-knowers and subset-knowers based on their understanding of how counting represents number (e.g. LeCorre et al., 2006). Our data are consistent with this generalisation, as seen in Table 2. CP-knowers almost always spontaneously counted when asked to check the number of toys requested, whereas subset-knowers almost never did ($X^2(1, N = 57) = 35.24, p < .0001$). These data confirm our designation of children as subset-knowers and CP-knowers.

Table 1

Frequency Table for Give-a-Number

Knower Status		Frequency
Subset	1-knower	9
	2-knower	11
	3-knower	9
	4-knower	3
Cardinal Principle		25

Table 2

Contingency Table for Knower Status and Counting to Check in Give-a-Number

		Checking Toys	
		Not Counting	Counting
Knower Status	Subset-Knowers	29	3
	CP-Knowers	3	22

The descriptive statistics for measures of count procedural knowledge are shown in Table 3. Children varied substantially in count procedural knowledge, for example memorised count lists ranged from 4 to 20. Children's knowledge of the memorised count list (highest count) and ability to use this (count routine) highly correlate ($r_s(51) = .646, p < .0001$). A Count Procedural Knowledge composite measure was therefore created by averaging the Z-scores for each task.

Table 3
Descriptive Statistics for Count Procedural Knowledge Tasks

	N	Min	Max	Mean	SD
Highest Count	53	4	20	11.40	4.81
Count Routine	53	0	5	3.14	1.76
Count Procedural Knowledge Composite	53	-1.52	2.28	0	0.89

Cognitive resource battery. The descriptive statistics of the predictor variable tasks are shown in Tables 4 and 5. Children showed poor performance in words backwards, with 43 children scoring 0, and only seven children producing a response. Performance in words forwards was better, with only three children scoring 0. Since examples, practice, and feedback were given for both tasks, the difficulty on the words backwards task suggests very poor updating working memory. These tasks reflect the immaturity of EFs in three-year-olds. Nevertheless, these measures significantly correlate ($r_s(46) = .399$, $p = .005$) and scores were therefore summed to create a Working Memory composite.

Table 4
Descriptive Statistics for Continuous Measures in the Cognitive Resource Battery

	N	Min	Max	Mean	SD
Words Forwards	49	0	5	3.47	1.19
Words Backwards	50	0	3	0.33	0.79
Working Memory Composite	49	0	8	3.78	1.63
Verbal Fluency – Animals	53	0	8	2.25	2.17
Verbal Fluency – Foods	53	0	7	1.66	1.91
Verbal Fluency Composite	53	0	6	1.95	1.86
Object Assembly	57	1	17	7.22	4.54
Receptive Vocabulary	57	5	28	17.14	5.52

Children were also poor at verbal fluency, with 16 children unable to produce a single animal name, and 25 unable to produce a food name. Since children this age know dozens of animals and foods, that more than a quarter of the children could not produce a single name highlights the EF demands of this task. Many three-year-olds could not even begin to search their vast knowledge base. These two measures significantly correlate ($r_s(51) = .724$, $p < .0001$), showing that neither animal or food names specifically were the problem. A Verbal Fluency composite was therefore created by averaging scores from each trial.

Table 5
Frequency Table for Card Sort

Post-Switch Performance	Frequency
Fail	15
Pass	40

As seen in Table 5, almost three quarters of the children passed the Card Sort task. This performance is much better than expected, since many studies of children this age find that less than 50% of 3-year-olds succeed (e.g., Zelazo, Frye, & Rapus, 1996, 40% pass).

Relations Among Predictor Variables
Correlations among predictor variables are shown in Table 6. Given the lack of significant correlations between EF measures, these were kept as separate measures in subsequent analyses. There is a weak, yet significant, correlation between Verbal Fluency and Receptive Vocabulary, $r_s(51) = .287$, $p = .037$. This most probably reflects the fact that both tasks draw on lexical knowledge. Verbal Fluency also taps EFs, and so these are kept separate in subsequent analyses on theoretical grounds.

Table 6
Bivariate Correlations Among Predictor Variables

	Working Memory	Card Sort	Verbal Fluency	Object Assembly	Receptive Vocabulary
Working Memory	–	.249†	.005	.086	.218
Card Sort		–	.269†	.116	.229†
Verbal Fluency			–	.162	.287*
Object Assembly				–	.003
Receptive Vocabulary					–

Notes: Calculated using Spearman's Rank Order Correlation due to non-normal distribution of data. The number of children who completed each task ranged between 49 and 57. Preliminary analyses were consistent with the assumption that these are missing completely at random (MCAR) using Little's MCAR test, $X^2(8, N = 57) = 6.645$, $p = .575$. Thus, pairwise deletion is used, meaning the number of participants in these analyses ranged from 47 to 57.
†$p \leq .10$, *$p \leq .05$, **$p \leq .01$, ***$p \leq .001$.

Since Object Assembly did not correlate with any of the predictor variables (Table 6), or with either outcome measure (Count Procedural Knowledge: $r_s(51) = .103$, $p = .464$; CP-knower status: $r_{pb}(55) = .169$, $p = .209$) it is dropped from subsequent analyses.

Predictors of Count Procedural Knowledge

Tables 7-9, display regression analyses that test individually, whether Card Sort, Verbal Fluency (measures of set-shifting and inhibition), and Working Memory predict variance in Count Procedural Knowledge, when age and receptive vocabulary are controlled for. As can be seen in Tables 7 and 8, just as in previous case studies of knowledge enrichment, inhibition and set-shifting are not related to Count Procedural Knowledge. However, as can be seen in Table 9, unlike the previously studied case of fast-mapped generic knowledge, Working Memory does predict Count Procedural Knowledge, when age and receptive vocabulary are controlled for (β = .336, *p* = .016).

Table 7

Multiple Regression Predicting Variance in Count Procedural Knowledge from Age, Receptive Vocabulary and Card Sort

	B	S.E. B	β
Constant	-4.713	1.415	
Age	0.120	0.036	.451**
Receptive Vocabulary	-0.027	0.022	-.169
Card Sort	0.282	0.273	.136

Notes: adj. R^2 = .168, $F(3, 47)$ = 4.370, p = .009

†$p \le .10$, *$p \le .05$, **$p \le .01$, ***$p \le .001$.

Table 8

Multiple Regression Predicting Variance in Count Procedural Knowledge from Age, Receptive Vocabulary and Verbal Fluency

	B	S.E. B	β
Constant	-3.956	1.507	
Age	0.098	0.039	.371*
Receptive Vocabulary	-0.022	0.022	-.135
Verbal Fluency	0.115	0.070	.238

Notes: adj. R^2 = .215, $F(3, 47)$ = 3.513, p = .002

†$p \le .10$, *$p \le .05$, **$p \le .01$, ***$p \le .001$.

Table 9
Multiple Regression Predicting Variance in Count Procedural Knowledge from Age, Receptive Vocabulary and WM

	B	S.E. B	β
Constant	-4.687	1.378	
Age	0.109	0.036	.414**
Receptive Vocabulary	-0.040	0.023	-.235†
Working Memory	0.206	0.082	.336*

Notes: adj. R^2 = .283, $F(3, 42)$ = 6.932, p = .001
†$p \le .10$, *$p \le .05$, **$p \le .01$, ***$p \le .001$.

The only significant predictors of Count Procedural Knowledge, when controlling for other variables (Tables 7-9), were Age and Working Memory. To test whether these remain significant predictors when controlling for all other predictor variables, a final multiple linear regression examined all predictor variables in a single model (see Table 10).

Table 10
Multiple Regression Predicting Variance in Count Procedural Knowledge Scores from Age, Receptive Vocabulary, Card Sort, Verbal Fluency and WM

	B	S.E. B	β
Constant	-3.765	1.645	
Age	0.082	0.044	.309†
Receptive Vocabulary	-0.037	0.025	-.210
Card Sort	-0.023	0.310	-.011
Verbal Fluency	0.094	0.077	.196
Working Memory	0.209	0.087	.350*

Notes: adj. R^2 = .275, $F(5, 37)$ = 4.181, p = .004
†$p \le .10$, *$p \le .05$, **$p \le .01$, ***$p \le .001$.

Working Memory (β = .350, p = .021) was the only significant predictor of Count Procedural Knowledge in the model. For every 1-unit increase in Working Memory scores, there is a 0.350 increase in Count Procedural Knowledge scores when controlling for all other factors in the model. Receptive Vocabulary did not predict Count Procedural Knowledge in any of the regressions presented in Tables 7-10. This contrasts with measures of generic factual knowledge that are the output of knowledge enrichment mechanisms (Bascandziev et al., 2018; Zaitchik et al., 2014; 2018). Thus, the domain-general mechanisms associated with the mastery of count procedural knowledge differ from those that underlie learning vocabulary and generic facts that are fast-mapped. Receptive vocabulary predicts variance in generic factual knowledge, controlling for age, and working memory does not, where the reverse is true for procedural knowledge of counting.

Predictors of CP-Knower Status

Parallel analyses explored which measures of domain-general cognitive mechanisms predict CP-knower status. It was first assessed whether, as in other cases of conceptual construction, the measures of set-shifting and inhibition did so. As can be seen from Table 11, Card Sort is a significant predictor of CP-knower status (Odds Ratio = 7.378, p = .031) when controlling for age and receptive vocabulary. That is, the odds of being a CP-knower were 7.4 times higher for those that passed the Card Sort than for those that failed. This finding is consistent with the conclusion that the conceptual construction that results in the CP-induction draws on the same EFs as do older children's conceptual constructions. However, unlike previous work on older children's theory constructions, Verbal Fluency failed to predict CP-knower status in a parallel analysis (Table 12). This may be due to the fact that Verbal Fluency does not engage set-shifting in children this young.

Table 11

Logistic Regression Predicting Likelihood of CP-Knower Status from Age, Receptive Vocabulary and Card Sort

	B	S.E. B	Wald	Odds Ratio	95% C.I. for Odds Ratio	
					Lower	Upper
Constant	-16.057	4.783	11.270***			
Age	0.352	0.116	9.208**	1.422	1.133	1.785
Receptive Vocabulary	-0.007	0.064	0.013	0.993	0.876	1.125
Card Sort	1.998	0.927	4.650*	7.378	1.200	45.372

Notes: Nagelkerke's R^2 = .434, $X^2(3, N = 55)$ = 21.557, $p < .0001$

†$p \leq .10$, *$p \leq .05$, **$p \leq .01$, ***$p \leq .001$. Wald df = 1.

Table 12
Logistic Regression Predicting Likelihood of CP-Knower Status from Age, Receptive Vocabulary and Verbal Fluency

	B	S.E. B	Wald	Odds Ratio	95% C.I. for Odds Ratio Lower	Upper
Constant	-11.972	4.397	7.412**			
Age	0.277	0.111	6.247*	1.319	1.062	1.640
Receptive Vocabulary	0.013	0.059	0.049	1.013	0.902	1.138
Verbal Fluency	0.148	0.186	0.628	1.159	0.805	1.669

Notes: Nagelkerke's $R^2 = .286$, $X^2(3, N = 53) = 12.781$, $p = .005$
†$p \leq .10$, *$p \leq .05$, **$p \leq .01$, ***$p \leq .001$. Wald $df = 1$.

Another logistic regression tested whether Working Memory predicts CP-knower status, controlling for age and receptive vocabulary (Table 13).

Table 13
Logistic Regression Predicting Likelihood of CP-Knower Status from Age, Receptive Vocabulary, and WM

	B	S.E. B	Wald	Odds Ratio	95% C.I. for Odds Ratio Lower	Upper
Constant	-18.968	5.836	10.565***			
Age	0.360	0.130	7.668**	1.433	1.111	1.848
Receptive Vocabulary	0.016	0.067	0.056	1.016	0.890	1.159
Working Memory	1.035	0.443	5.468*	2.815	1.182	6.703

Notes: Nagelkerke's $R^2 = .536$, $X^2(3, N = 49) = 25.162$, $p < .0001$
†$p \leq .10$, *$p \leq .05$, **$p \leq .01$, ***$p \leq .001$. Wald $df = 1$.

Working Memory was a significant predictor of CP-knower status (Odds Ratio = 2.815, $p = .019$) when controlling for age and receptive vocabulary (Table 13), unlike previous case studies of theory constructions. For every 1-unit increase in WM scores, children were 2.8 times more likely to be a CP-knower. A final difference between the pattern of results in the previous studies and this one is that Receptive Vocabulary predicted theory construction, controlling for age and EFs, whereas it is unrelated to CP-knower status in any of the above analyses.

A final logistic regression assessed whether Card Sort and Working Memory remain significant predictors of CP-knower status when controlling for all other predictors—Age, Receptive Vocabulary, and Verbal Fluency (Table 14).

Table 14
Logistic Regression Predicting Likelihood of CP-Knower Status from Age, Receptive Vocabulary, Card Sort, Verbal Fluency, WM

	B	S.E. B	Wald	Odds Ratio	95% C.I. for Odds Ratio	
					Lower	Upper
Constant	-20.875	7.580	7.584**			
Age	0.381	0.177	4.620*	1.464	1.034	2.073
Receptive Vocabulary	-0.033	0.076	0.187	0.968	0.834	1.123
Card Sort	1.530	1.142	1.793	4.617	0.492	43.320
Verbal Fluency	0.116	0.258	0.204	1.124	0.678	1.862
Working Memory	1.162	0.540	4.635*	3.197	1.110	9.208

Notes: Nagelkerke's R^2 = .614, $X^2(5, N = 45) = 27.756$, $p < .0001$
†$p \leq .10$, *$p \leq .05$, **$p \leq .01$, ***$p \leq .001$. Wald $df = 1$.

Age (Odds Ratio = 1.464, $p = .032$) and Working Memory (Odds Ratio = 3.197, $p = .031$) are significant predictors of CP-knower status when controlling for all other factors in the model. For every 1-month increase in age, children were 1.5 times more likely to be a CP-knower, and for every 1-unit increase in WM scores, children were 3.2 times more likely to be a CP-knower. Card Sort did not remain a significant predictor when controlling for all other factors in the model.

This is a very different pattern of results from that seen in the studies of conceptual construction within vitalist biology. In those studies, like this one, measures of fluid IQ (here Object Assembly) did not predict the outcome measure, but in those studies working memory did not either. Rather, in those studies, the EFs that predicted progress in the conceptual construction were set-shifting and inhibition alone. While Table 11 provides some support for the hypothesis, that set-shifting and inhibition are required for the transition to CP-knower status, the predictive relation between success on Card Sort and CP-knower status controlling for age and receptive vocabulary, does not survive the full regression examining which variables predict CP-knower status when controlling for all other variables. Also, receptive vocabulary does not emerge as a significant predictor of CP-knower status in any of the regressions in Tables 11-15, unlike the findings concerning construction of vitalist biology.

Count Procedural Knowledge as a Predictor of CP-Knower Status
Age is a significant predictor of CP-knower status in all of the above analyses. While it may seem obvious that older children will be more likely to be CP-knowers than younger ones, this fact still calls out for explanation. The above analyses show that there is something other than the fact that older children have greater working memory, set-shifting and inhibition ability, and accumulated factual knowledge (as measured by receptive vocabulary), that is correlated with age and important to CP-

knowledge. An obvious hypothesis concerning what that "something" is, is Count Procedural Knowledge, for the CP-induction simply is understanding how counting represents number. Up to now, Count Procedural Knowledge has been used as an outcome variable, but will now be used as a predictor variable. A final logistic regression tests whether Count Procedural Knowledge is a significant predictor of CP-knower status, when controlling for all other predictor variables (see Table 15).

Table 15
Logistic Regression Predicting Likelihood of CP-Knower Status from Age, Receptive Vocabulary, WM, Card Sort and Count Procedural Knowledge

	B	S.E. B	Wald	Odds Ratio	95% C.I. for Odds Ratio	
					Lower	Upper
Constant	-19.161	7.696	6.200*			
Age	0.303	0.161	3.536†	1.354	0.987	1.857
Receptive Vocabulary	0.035	0.089	0.152	1.035	0.870	1.232
Working Memory	1.300	0.722	3.241†	3.671	0.891	15.121
Card Sort	1.957	1.263	2.401	7.081	0.596	84.207
Count Procedural Knowledge	1.766	0.740	5.688*	5.847	1.370	24.953

Notes: Nagelkerke's R^2 = .721, $X^2(5, N = 44)$ = 34.243, $p < .0001$
†$p \leq .10$, *$p \leq .05$, **$p \leq .01$, ***$p \leq .001$. Wald df = 1.

When controlling for all other predictors, Count Procedural Knowledge (Odds Ratio = 5.847, p = .017) was the *only* significant predictor of CP-knower status. For every 1-unit increase in Count Procedural Knowledge scores, children were 5.8 times more likely to be a CP-knower, when controlling for all other factors in the model. This is consistent with the hypothesis that knowledge of the count routine is the major source of variance in understanding the cardinal principle.

Discussion

This study is the first to explore the role of domain-general cognitive resources in preschool children's construction of representations of positive integers. Two milestones in the process of integer learning were explored: learning the *procedure of counting*, which requires knowledge enrichment alone, and learning *how counting is a representation of integers*, which requires conceptual construction. Several important results emerged from this study. First, there is suggestive evidence that in this domain, and at this young age, knowledge enrichment and conceptual construction draw on different resources, since the Card Sort predicted the likelihood of being a CP-knower, but not variance in Count Procedural Knowledge. Furthermore, the EFs differentially implicated in conceptual construction, set-shifting and inhibition, are the same ones as in all previous studies. Second, the cognitive resources that predict knowledge of counting differ from those shown to be implicated in previous studies of knowledge enrichment such as generic factual knowledge. Receptive vocabulary alone predicts knowledge enrichment in previous studies, but here, working memory, and not receptive vocabulary, predicts procedural knowledge of counting. Third, the domain-general mechanisms that predict progress in constructing a representation of integers also differ in an important respect from those involved in previously studied conceptual constructions; here working memory, as well as set-shifting and inhibition, was an even more important predictor of conceptual construction, whereas working memory does not predict conceptual construction in previous studies. Finally, as in previous studies, the output of knowledge enrichment mechanisms has an important role to play in conceptual construction. The likelihood of being a CP-knower is predicted by Count Procedural Knowledge, when controlling for every other predictor variable and age, and no other predictor variables reached significance in this analysis.

Predictors of Count Procedural Knowledge

This study provides the first evidence that there may be different learning mechanisms supporting different types of knowledge enrichment. Working memory (WM), as opposed to receptive vocabulary, predicted Count Procedural Knowledge. WM is needed for the procedure of counting itself, like in the task where the child must count a row of ducks. Children must keep track of where in the count list they are and update this in one-to-one correspondence with successively counted objects. Furthermore, when learning the memorised count list, an ability tested by the highest count task, children must monitor for new information to add to their memorised list, thus drawing on WM.

Unlike previous case studies of knowledge enrichment (Bascandziev et al., 2018; Zaitchik et al., 2014), receptive vocabulary did not predict variance in Count Procedural Knowledge. These previous studies of knowledge enrichment concern learning facts that can be fast-mapped. For example, generic facts like "the ears of crickets are on their legs" and idiosyncratic facts like "my grandmother gave me this purse" (Bascandziev et al., 2018; Markson & Bloom, 1997). Lexical knowledge of the sort measured by the Receptive Vocabulary task is also fast-mapped; all of these measures of fast-mapped verbal knowledge are highly intercorrelated, and probably reflect verbal learning mechanisms that are highly constrained by structural knowledge of language (Bascandziev et al., 2018). Learning the count routine, however, is very different. Learning to count takes years of practice, as evident by the short count lists, the limited ability to enumerate objects, and variance of counting ability in this sample of three-year-olds.

Count Procedural Knowledge as a Predictor of CP-Knowledge

This study found that the only significant predictor of being a CP-knower was variance in count procedural knowledge, when controlling for all other predictor variables. That understanding *how* counting represents number (CP-induction) depends on how well a child can count and enumerate, is unsurprising. Knowledge that is the output of knowledge enrichment has many necessary roles to play in conceptual construction, and this is particularly clear in the present case. The bootstrapping process proposed to underlie conceptual construction involves explicit symbols which act as placeholder structures. In the case of constructing representations of the positive integers, the placeholder structure is the count list of verbal numerals (Carey, 2009). In this bootstrapping process, the child must notice that successive numerals among "one", "two", "three", "four" refer to sets that are related by +1, and induce that all successive numerals in the count list are so related (Carey, 2009). If a child has poor count procedural knowledge, her ability to notice that all successive numerals in the count list are related by +1 may be reduced, since the placeholder structures may not be stable. For example, if the child's count list changes then the relations among placeholder structures cannot be consistently explored.

That Count Procedural Knowledge predicts CP-knower status may explain the finding that Working Memory predicts CP-knower status. When controlling for all other cognitive resource predictor variables, only WM is a significant predictor of the CP-induction and similarly, WM is the only significant cognitive resource predictor of Count Procedural Knowledge. Thus, it is possible that the variance in WM predicting CP-knower status is mediated by Count Procedural Knowledge, since WM is the strongest predictor of Count Procedural Knowledge, which is the strongest predictor of CP-knower status. Future research with a larger sample could test this by running a mediation analysis. The conceptual construction resulting in a count list representation of integers may therefore differ from previous studies (e.g., Baker et al., 2011; Bascandziev et al., 2018; Zaitchik et al., 2014) in not drawing on EFs in the construction process itself.

However, these data also suggest that procedural knowledge of counting as the sole predictor of CP-induction may *not* be the whole story. Card Sort, measuring inhibition and set-shifting, predicts the likelihood of CP-knower status, but not variance in Count Procedural Knowledge, even when controlling for age and receptive vocabulary. This is consistent with previous research that suggests that conceptual construction, but not knowledge enrichment, is predicted by measures of set-shifting and inhibition (Bascandziev et al., 2018; Zaitchik et al., 2018). This inference in the present case is arguably weak, since Card Sort does not remain significant when controlling for all other predictor variables, possibly because Card Sort and CP-knower status are two dichotomous variables, or because Card Sort marginally correlates with Working Memory. Since Working Memory is a strong predictor of CP-knower status in the full regressions, variance in Card Sort may be partialled out. Nevertheless, it still survived controlling for age and receptive vocabulary, which given this small sample and that both variables are dichotomous, is a strong result. There are multiple possibilities as to why inhibition and set-shifting may be involved in the CP-induction.

Firstly, set-shifting, or cognitive flexibility, as measured by the Card Sort, has been linked to the ability to think abstractly. It is possible that the conceptual construction of the cardinal principle requires abstract thinking since the meaning of positive integers are themselves abstract concepts. For example, Kharitonova, Chein, Colunga, and Munakata (2009) tested three-year-olds on the Card Sort and added novel cards after the post-switch phase. These novel cards were similar to test cards on their abstract rule (colour or shape). Children were instructed to sort novel cards using whichever rule they had used in the post-switch phase. They found that only children who passed the Card Sort could sort these novel cards, and that children who failed the Card Sort could not sort novel cards using *any* rule. They concluded that children who pass the Card Sort generate rules based on abstract representations (colour/shape), whereas those who fail generate rules based on stimulus specific rules (e.g. red ones go here). Kharitonova and Munakata (2011) extended this work and found that successful Card Sort switchers generalised this abstraction to a third unrelated dimension. This suggests that set-shifting relates to an ability to think abstractly, potentially necessary for the CP-induction. For example, if a child generates abstract rules about the verbal numerals (e.g. all numerals are part of a single count list), then her ability to notice relations among these numerals, necessary for the bootstrapping process, may be enhanced since the count list is viewed as a unified whole rather than just a series of individual words.

Secondly, the role of EFs in the conceptual construction of the CP-induction may be related to analogical reasoning. The prefrontal cortex (PFC) underlies executive functioning (Bunge & Zelazo, 2006; Diamond, 2013) and has also been found to support integrating abstract information to form analogies, or abstract relations between items or categories (Badre, Kayser, & D'Esposito, 2010; Green, Fugelsang, Kraemer, Shamosh, & Dunbar, 2006; Speed, 2010). Furthermore, Richland and Burchinal (2013) found that four-year-olds' composite EF skills and inhibitory control were significant predictors of verbal analogy skills at 15 years old. The bootstrapping process underlying the CP-induction involves the child making an analogy between the words in the count list and the models of set sizes "one" to "four" by noticing that a numeral next in the count list and the model next in the series are related by adding one (Carey, 2009). It is possible that this analogy, which forms part of the conceptual construction of number, draws on cognitive processes, such as inhibition, subserved by the PFC and measured by the Card Sort.

Practical Implications

Understanding learning mechanisms underlying the CP-induction is important for educational practice. Preschool numeracy skills that reflect learning the cardinal meanings of verbal numerals predict later mathematic achievement, and entering school without understanding the cardinal principle is detrimental to future mathematic ability (Geary & vanMarle, 2016; Jordan et al., 2009; 2010). The present study suggests limitations in working memory capacity may be one source of children's failure to understand how counting represents number by the time they enter primary school. Furthermore, inhibition and set-shifting may also be needed for the CP-induction. EFs are malleable and can be trained, even in pre-schoolers, using a curriculum such as "Tools of the Mind" (Diamond et al., 2007). Thus, improving children's EFs could facilitate readiness for conceptual change. This would be especially useful for children from low-income families, who have disproportionately poor EFs and sometimes enter primary school without understanding the cardinal principle (Geary, 2011; Noble, McCandliss, & Farah, 2007).

Strengths and Limitations of the Present Study

A limitation of this study, and of all the studies to date investigating the role of EFs in conceptual construction and knowledge enrichment, is that they have been carried out on children from middle- and high-income households. Future research investigating the role of EFs in the CP-induction should be done in lower-income samples. Research suggests the development of mathematical concepts differs between lower- and middle-income backgrounds (Starkey & Klein, 2000), highlighting the importance of future research in diverse samples.

Furthermore, the sample size of 57 is small considering the number of predictors analysed. Tabachnick and Fidell (2007) advise a sample size ≥50+8M where M is the number of explanatory variables. Since the maximum number of predictors in regressions was five, this guideline recommends at least 90 children for this study.

Moreover, not all children completed every task, reducing the number of children in some regressions even further. It is possible this study therefore did not have enough power to detect smaller effects in the data and is a reason why the laboratory is continuing this work with a sample size four times the size, half from the same population and half from low-income populations. Despite this obvious limitation, the data show enough variance in every measure (each was correlated with at least something else) and the sample size was large enough to provide preliminary results with respect to the hypotheses.

A major limitation of this study is its correlational nature. Finding significant correlations between predictor and outcomes variables does not provide evidence of *causal* relationships between them. It is possible there is a common factor underlying the variation in both number knowledge and domain-general learning mechanisms. One possibility could be quality of the child's learning environment. However, this suggestion can be partially ruled out, since analyses controlled for receptive vocabulary, which measures general input from the environment, an accumulation of factual knowledge. Another possible explanation for the correlations between predictor and outcome variables is that measures of number knowledge are driving EF development. The direction of influence could be explored using training studies which test whether individual differences in domain-general cognitive abilities predict improvements from an episode of learning, involving either knowledge enrichment or conceptual change (Bascandziev et al., 2016; Bascandziev et al., 2018). Future training studies in the domain of number are therefore needed. Nevertheless, these correlations set the stage for future research.

This study also cannot rule out the possibility that EFs are needed solely for the *expression* of existing knowledge necessary to complete tasks, rather than for conceptual *construction*. For example, the Give-a-Number task likely requires EFs, for example holding the requested numeral in working memory, and inhibiting the response to grab toys or guess. This could explain the correlations between Card Sort and WM with Give-a-Number. However, that WM and Card Sort correlate with Give-a-Number simply due to task demands can be partially ruled out, since knower-status categorisation from Give-a-Number is consistent with tasks that require very different processing demands (e.g. Gelman, 1993; LeCorre et al., 2006; Wynn, 1990). For example, LeCorre et al. (2006) used a puppet task which did not require children to count, thus minimizing processing demands, and found remarkable consistency with Give-a-Number: subset-knowers failed the puppet task, whereas CP-knowers robustly succeeded. Since data from Give-a-Number in this study reflect the qualitative difference found in the literature, it is likely that a different task with fewer EF demands, such as the puppet task, would still produce the same categorisation of subset-knowers and CP-knowers.

A wide range of tasks was administered which is a strength of this study, since many different domain-general cognitive mechanisms could be analysed. Furthermore, these predictor variables were analysed with respect to two different outcome variables (the procedure of counting, and how counting is a representation of integers). This provides a broader picture of the domain-general cognitive mechanisms required for learning number. The measures of number knowledge are also reliable, since the Count Procedural Knowledge composite was formed from two tasks, and data from

Give-a-Number strongly replicated the qualitative difference between subset-knowers and CP-knowers in the literature.

Conclusions

Despite some limitations, this study is the first to assess the domain-general mechanisms supporting knowledge enrichment in a new domain: count procedural knowledge. Unlike previous case studies of knowledge enrichment that involve generic factual knowledge that can be fast-mapped, receptive vocabulary did not predict count procedural knowledge, but working memory did, suggesting that different forms of knowledge enrichment are supported by different cognitive mechanisms. It is also the first study to assess the domain-general mechanisms supporting conceptual construction in a new domain: the meaning of natural number. Count procedural knowledge was the strongest predictor of this conceptual construction, and possibly the reason working memory correlated with being a CP-knower. However, this may not be the whole story, since inhibition and exogenous set-shifting predicted CP-knower status, but not count procedural knowledge, when controlling for age and receptive vocabulary. This, like previous case studies of conceptual construction in other domains of knowledge, suggests that inhibition and set-shifting have a role in conceptual construction. The findings make important theoretical contributions to the field of knowledge acquisition and set the stage for future research investigating the learning mechanisms that support the conceptual construction of number.

References

Alloway, T. P. (2007). *Automated Working Memory Assessment: Manual*. London: Pearson.

Anderson, P. (2002). Assessment and development of executive function (EF) during childhood. *Child Neuropsychology*, *8*(2), 71-82.

Badre, D., Kayser, A. S., & D'Esposito, M. (2010). Frontal cortex and the discovery of abstract action rules. *Neuron*, *66*(2), 315-326.

Baker, S. T., Gjersoe, N. L., Sibielska-Woch, K., Leslie, A. M., & Hood, B. M. (2011). Inhibitory control interacts with core knowledge in toddlers' manual search for an occluded object. *Developmental Science*, *14*(2), 270-279.

Bascandziev, I., Powell, L. J., Harris, P. L., & Carey, S. (2016). A role for executive functions in explanatory understanding of the physical world. *Cognitive Development*, *39*, 71-85.

Bascandziev, I., Tardiff, N., Zaitchik, D., & Carey, S. (2018). The role of domain-general cognitive resources in children's construction of a vitalist theory of biology. *Cognitive Psychology*, *104*, 1-28.

Benson, J. E., Sabbagh, M. A., Carlson, S. M., & Zelazo, P. D. (2013). Individual differences in executive functioning predict preschoolers' improvement from theory-of-mind training. *Developmental Psychology*, *49*(9), 1615-1627.

Blair, C., & Razza, R. P. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child Development*, *78*(2), 647-663.

Bunge, S. A., & Zelazo, P. D. (2006). A brain-based account of the development of rule use in childhood. *Current Directions in Psychological Science*, *15*(3), 118-121.

Carey, S. (1978). The child as word learner. In J. Bresnan, G. Miller and M. Halle (Eds.), *Linguistic theory and psychological reality* (pp. 264-293). Cambridge, MA: MIT Press.

Carey, S. (1985). *Conceptual change in childhood.* Cambridge MA: MIT Press.

Carey, S. (2004). Bootstrapping & the origin of concepts. *Daedalus*, *133*(1), 59-68.

Carey, S. (2009). *The origins of concepts.* New York: Oxford University Press.

Carlson, S. M., & Moses, L. J. (2001). Individual differences in inhibitory control and children's theory of mind. *Child Development*, *72*(4), 1032-1053.

Cragg, L., Richardson, S., Hubber, P. J., Keeble, S., & Gilmore, C. (2017). When is working memory important for arithmetic? The impact of strategy and age. *PloS one*, *12*(12), 1-18.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics.* New York: Oxford University Press.

DeStefano, D., & LeFevre, J. A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology*, *16*(3), 353-386.

Diamond, A. (2013). Executive functions. *Annual Review of Psychology*, *64*, 135-168.

Diamond, A., Barnett, W. S., Thomas, J., & Munro, S. (2007). Preschool program improves cognitive control. *Science*, *318*(5855), 1387-1388.

Espy, K. A., McDiarmid, M. M., Cwik, M. F., Stalets, M. M., Hamby, A., & Senn, T. E. (2004). The contribution of executive functions to emergent mathematic skills in preschool children. *Developmental Neuropsychology*, *26*(1), 465-486.

Feigenson, L., & Carey, S. (2003). Tracking individuals via object-files: evidence from infants' manual search. *Developmental Science*, *6*(5), 568-584.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*(7), 307-314.

Gathercole, S. E., Pickering, S. J., Knight, C., & Stegmann, Z. (2004). Working memory skills and educational attainment: Evidence from national curriculum assessments at 7 and 14 years of age. *Applied Cognitive Psychology*, *18*(1), 1-16.

Geary, D. C. (2011). Consequences, characteristics, and causes of mathematical learning disabilities and persistent low achievement in mathematics. *Journal of Developmental and Behavioral Pediatrics*, *32*(3), 250-263.

Geary, D. C., & vanMarle, K. (2016). Young children's core symbolic and nonsymbolic quantitative knowledge in the prediction of later mathematics achievement. *Developmental Psychology*, *52*(12), 2130-2144.

Gelman, R. (1993). A rational-constructivist account of early learning about numbers and objects. In D. L. Medin (Ed.), *The psychology of learning and motivation. Advances in research theory* (pp. 61-96). San Diego: Academic Press.

Gordon, P. (2004). Numerical cognition without words: Evidence from Amazonia. *Science*, *306*(5695), 496-499.

Green, A. E., Fugelsang, J. A., Kraemer, D. J., Shamosh, N. A., & Dunbar, K. N. (2006). Frontopolar cortex mediates abstract integration in analogy. *Brain Research*, *1096*(1), 125-137.

Gulya, M., Rovee-Collier, C., Galluccio, L., & Wilk, A. (1998). Memory processing of a serial list by young infants. *Psychological Science*, *9*(4), 303-307.

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences*, *20*(2), 82-88.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*(3), 850-867.

Kharitonova, M., Chien, S., Colunga, E., & Munakata, Y. (2009). More than a matter of getting 'unstuck': flexible thinkers use more abstract representations than perseverators. *Developmental Science*, *12*(4), 662-669.

Kharitonova, M., & Munakata, Y. (2011). The role of representations in executive function: Investigating a developmental link between flexibility and abstraction. *Frontiers in Psychology*, *2*(347), 1-12.

LeCorre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology*, *52*(2), 130-169.

Markson, L., & Bloom, P. (1997). Evidence against a dedicated system for word learning in children. *Nature*, *385*(6619), 813-815.

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology*, *41*(1), 49-100.

Noble, K. G., McCandliss, B. D., & Farah, M. J. (2007). Socioeconomic gradients predict individual differences in neurocognitive abilities. *Developmental Science*, *10*(4), 464-480.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, *306*(5695), 499-503.

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences*, *20*(2), 110-122.

Richland, L. E., & Burchinal, M. R. (2013). Early executive function predicts reasoning development. *Psychological Science*, *24*(1), 87-92.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition*, *108*(3), 662-674.

Slaughter, V., Itakura, S., Kutsuki, A., & Siegal, M. (2011). Learning to count begins in infancy: evidence from 18 month olds' visual preferences. *Proceedings of the Royal Society of London B: Biological Sciences*, *278*(1720), 2979-2984.

Snyder, H. R., & Munakata, Y. (2010). Becoming self-directed: Abstract representations support endogenous flexibility in children. *Cognition*, *116*(2), 155-167.

Spaepen, E., Coppola, M., Spelke, E. S., Carey, S. E., & Goldin-Meadow, S. (2011). Number without a language model. *Proceedings of the National Academy of Sciences*, *108*(8), 3163-3168.

Speed, A. (2010). Abstract relational categories, graded persistence, and prefrontal cortical representation. *Cognitive Neuroscience, 1*(2), 126-137.

Starkey, P., & Klein, A. (2000). Fostering parental support for children's mathematical development: An intervention with Head Start families. *Early Education and Development*, *11*(5), 659-680.

Tabachnick, B. G., & Fidell, L. S. (2007). *Using Multivariate Statistics* (5th ed.). New York: Allyn and Bacon.

Terrace, H. S., Son, L. K., & Brannon, E. M. (2003). Serial expertise of rhesus macaques. *Psychological Science*, *14*(1), 66-73.

Trick, L. M., & Pylyshyn, Z. W. (1994). Why are small and large numbers enumerated differently? A limited-capacity preattentive stage in vision. *Psychological Review*, *101*(1), 80-102.

Wechsler, D. (2002). *Wechsler preschool and primary scale of intelligence* (3rd ed.). San Antonio, TX: The Psychological Corporation.

Wiser, M., & Smith, C. (2016). How is conceptual change possible? Insights from science education. In D. Barner & A. S. Baron (Eds.), *Core Knowledge and Conceptual Change* (pp. 29-52). New York, NY: Oxford University Press.

Wynn, K. (1990). Children's understanding of counting. *Cognition*, *36*(2), 155-193.

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, *24*(2), 220-251.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, *74*(1), B1-B11.

Zaitchik, D., Iqbal, Y., & Carey, S. (2014). The effect of executive function on biological reasoning in young children: an individual differences study. *Child Development*, *85*(1), 160-175.

Zaitchik, D., Tardiff, N., Bascandziev, I., & Carey, S. (2018). *The role of executive functions and fluid intelligence in young children's reasoning about biology.* Manuscript submitted for publication.

Zelazo, P. D. (2006). The Dimensional Change Card Sort (DCCS): A method of assessing executive function in children. *Nature Protocols,1*(1), 297-301.

Zelazo, P. D., Frye, D., & Rapus, T. (1996). An age-related dissociation between knowing rules and using them. *Cognitive Development*, *11*(1), 37-63.