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Speculative bubbles or explosive fundamentals in stock prices? New evidence from SADF and GSADF tests

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Abstract

This paper uses recently developed sequential ADF tests to distinguish between rational speculative bubbles and explosive fundamentals in the US Stock market. The sequential ADF tests are shown to be more sensitive than the conventional ADF test. Results also suggest the more refined GSADF test may deliver more consistent results compared to the SADF test. We find strong evidence of explosive behavior in real stock prices that cannot be attributed to explosive fundamentals. We find renewed evidence of a stock market bubble during the dot com boom but no evidence of a bubble at other times.

Keywords: *Rational bubbles, SADF test, GSADF test, Fundamental value, Explosive behavior*

AMS Classification: *62F03; 62P20*

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1. Introduction

For centuries bubbles and crashes have fascinated both academics and the public at large (Reinhart & Rogoff, 2009). However, recent events such as the internet bubble and the subprime and Eurozone crises have given the subject a renewed sense of importance. Allied to their real-world significance modelling bubbles poses several difficult theoretical challenges (Cochrane, 2005). The available empirical evidence is also mixed (Gurkaynak, 2008). A common misconception is that bubbles necessarily imply mass irrationality. However, Blanchard and Watson (1982) introduced rational bubble models to account for the possibility that asset prices may deviate from fundamentals without assuming irrationality on the part of market participants. In addition to the subject's wider importance several new tests have recently been developed to detect speculative bubbles (see e.g. Al-Anaswah & Wilfling, 2011; Lammerding et al., 2013; Asako & Liu, 2013; Cheah & Fry, 2015). Here, we apply the sequential unit root tests developed in Phillips et al. (2015). These indirect stationarity tests have a long heritage in econometrics (see e.g. Diba & Grossman, 1988; Hamilton & Whiteman, 1985), are linked to a voluminous literature on rational bubble models and also have the advantage of being able to detect bubbles despite a potential misspecification of the bubble process. This approach has also begun to gain traction in empirical applications as diverse as exchange rates (Bettendorf & Chen, 2013), commodity markets (Xiaoli et al., 2014), metals markets (Figuerola-Feretti, 2015) and bitcoin (Cheung et al., 2015). Therefore this paper applies sequential unit root tests to shed new light upon the existence of rational bubbles in the US stock market. We find strong evidence of a bubble in US stock prices that cannot be explained by explosive behaviour in fundamentals. However, this appears to be confined to a single bubble episode during the dot com boom at the turn of the century. The layout of this paper is as follows. Section 2 describes the rational bubble model and sequential ADF tests used in this paper. Section 3 discusses empirical results. Section 4 concludes.

2. Rational bubbles and sequential ADF tests

The conventional unit root and cointegration tests, initially applied by Diba and Grossman (1988), are often unable to confirm the existence of bubbles when they are periodically collapsing. To overcome this problem, Phillips et al. (2011), Phillips and Yu (2011) and Phillips et al.

(2015) recently proposed a new test procedure to detect any bubble as well as its starting and ending points. According to these authors, testing for a mildly explosive departure from a unit root data-generating process can be a convenient method consonant with our goal of checking if there are bubbles in the financial data. These testing procedures have become increasingly significant for empirical applications. For exemple, El Montasser, Fry and Apergis (2016) have used such a procedure to test if there are bubbles in US-China exchange rate, while Chang et al. (2016) have applied it to BRICS stock markets. In other areas of application, El Montasser (2015) applied this method to test for bubbles in the ethanol–gasoline price ratio of Brazil, while Caspi, Katzke and Gupta (2015) have employed these right-tailed ADF tests for testing bubbles in a historical data of US oil price…etc…

The motivation behind this set of tests is as follows. Rational bubble models are derived from the present value theory of finance whereby prices are determined by the sum of the discounted present values:

$$P_t = \frac{1}{1+R} E_t\left(P_{t+1} + D_{t+1}\right), \tag{1}$$

where R is the constant interest rate. A log-linear approximation of equation (1) (Campbell & Shiller, 1989) gives

$$\log P_t = p_t = p_t^f + b_t,$$

where the fundamental component p_t^f is completely determined by expected dividends:

$$p_t^f = \frac{\kappa - \gamma}{1-\rho} + (1-p) \sum_{i=0}^{\infty} \rho^i E_t d_{t+i+1},$$

where $\gamma = \log(1+R)$ and $\kappa = -\log(\rho) - (1-\rho)\log\left(\frac{1}{\rho} - 1\right)$. The double component b_t satisfies

$$b_t = \lim_{i \to \infty} \rho^i E_t p_{t+i},$$

$$E_t(b_{t+1}) = \frac{1}{\rho} b_t = \left(1 + \exp\left(\overline{d-p}\right)\right) b_t, \tag{2}$$

where $\overline{d-p}$ denotes the average log dividend-price ratio. See e.g. Phillips et al. (2011) for full details. From equation (2) the bubble component b_t has the stochastic representation

$$b_t = (1+g) b_{t-1} + \varepsilon_{b,t}, \; E_{t-1}(\varepsilon_{b,t}) = 0, \tag{3}$$

where $g = \exp(\overline{d-p}) > 0$ is the growth rate of the (log) bubble. The stochastic properties of the p_t are determined by equation (3) (see e.g. Phillips et al., 2011). In particular, equation (3) shows that explosive bubbles will lead to a right-sided departure from a unit root for $g > 0$.

Motivated by the above Phillips et al. (2015) consider an ADF-type regression in a rolling window for a given time series y_t. An ADF regression is run over a rolling interval beginning with a fraction r_1 and ending with a fraction r_2 of the total number of observations. Therefore the size of the window as a proportion of the whole sample is $r_w = r_2 - r_1$. The time series model at the root of Phillips et al. (2015) can be presented as follow:

$$y_t = m + \lambda y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \varepsilon_t\ ,\ \varepsilon_t\ iid\ N(0,\sigma^2), t=1,....,T. \quad (4)$$

As suggested by equation (3) the usual null hypothesis H_0: $\lambda = 1$ applies but Phillips et al. (2011) suggest an unconventional alternative defined as follows: $H_1 : \lambda > 1$, therefore focusing attention on the right side of the distribution. Explicitly, we reject the null if the test statistic of Phillips et al. (2011) is greater than the critical value— determined by the authors through the use of Monte Carlo simulations. Consequently, if the null is rejected, we have an explosive process to take account of the bubble phenomenon. The number of observations considered by (4) is $T_w = [r_w T]$ where T is the total number of observations and [.] denotes the integer part. The ADF statistic associated to equation (4) is denoted $ADF_{r_1}^{r_2}$.

As noted by, *inter alia*, El Montasser *et al*. (2015, 2016), bubbles generally collapse periodically, and it is often observed that the traditional unit root tests have low power in detecting them. To overcome this shortcoming Phillips *et al*. (2011) and Phillips and Yu (2011) propose to use recursive sequences of right-tailed ADF-type tests based on a forward expanding sample and then take the supremum of these. Homm and Breitung (2010) point out that this test delivers a *fairly efficient bubble-detection* technique in one or two bubble alternatives. Accordingly, Phillips et al. (2015) show that although the procedure in Phillips et al. (2011) consistently estimates the start date of the first bubble in any sample in case of two bubble alternatives, it may fail to identify the second bubble. *Inter alia* this implies that in the presence of two bubbles, the second bubble may not be detected if it is dominated by the first bubble. This motivated Phillips *et al*. (2015) to formulate a backward sup ADF test where the endpoint of the subsample is fixed at a fraction r_2 of the whole sample and the window size is expanded from an initial fraction r_0 to r_2; for more details, see El Montasser *et al*. (2015, 2016).

In summary, the backward sup ADF (SADF) statistic is defined as

$$SADF_{r_2}(r_0) := \sup_{r_1 \in [0, r_2 - r_1]} ADF_{r_1}^{r_2}. \quad (5)$$

Table 1

Summary statistics of the log-returns of the series P_t and $P_t - F_t$

Series	P_t	$P_t - F_t$
Mean	0.020	0.017
Median	0.028	0.018
Standard Deviation	0.177	0.013
Skewness	-0.611	-0.286
Kurtosis	3.522	2.500
Jarque – Bera	10.389	3.389

The generalized sup ADF (GSADF) is then constructed by repeatedly implementing the SADF test procedure for each $r_2 \in [r_0, 1]$. The GSADF statistic can be written as

$$GSADF\ (r_0) := \sup_{r_2 \in [r_0, 1]} SADF_{r_2}(r_0). \quad (6)$$

3. Empirical analysis

In this paper, we test the null hypothesis of no rational speculative bubbles in the US stock market against the alternative that such bubbles do exist. Data consist of annual observations for the S&P 500 index for the years 1871-2013 and consist of the stock price index P_t (detrended by dividing by a factor proportional to the long-run exponential growth path), and its ex-post rational counterpart F_t (proxy of fundamental value) – the discounted present value of the actual subsequent real dividends. Summary statistics for the series P_t and $P_t - F_t$ are shown below in Table

Table 2

Sequential ADF tests results

Variable	ADF test	SADF test	GSADF test
P_t	-2.018	4.649**	4.649**
$P_t - F_t$	-3.125	2.863**	2.863**
1% Critical Value	-0.240	0.964	1.404
5 % Critical Value	-0.899	0.408	0.951
10% Critical Value	-1.253	0.174	0.709

Notes: For the SADF and GSADF tests the initial window size r_0 is set as three years (36 observations). Critical values are based on Monte-Carlo simulations with 1000 replications. ** denotes significance at the 1% level.

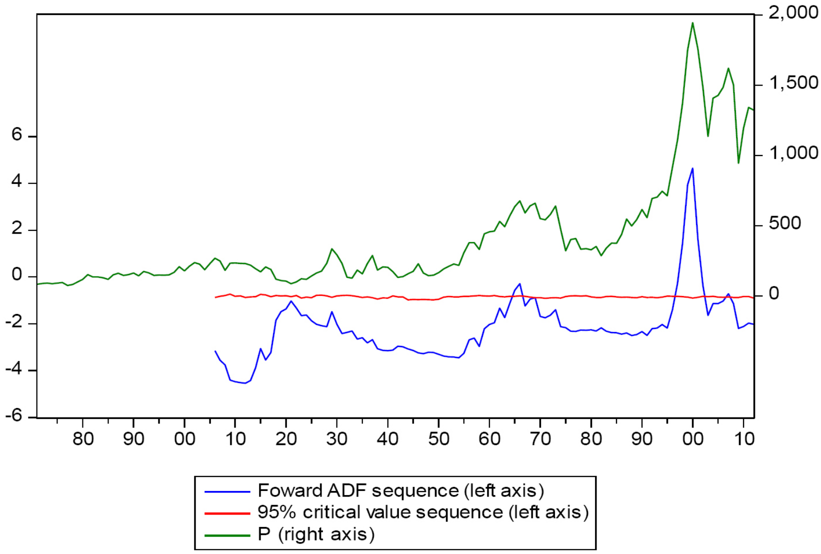

Figure 1

The SADF test graphic of the stock price index

Notes: This graph shows the series of the difference between the real stock price and its fundamental (green line, right axis) and its corresponding sequence of ADF statistics (blue line, left axis). The red line (left axis) represents the 5% critical values of the GSADF test.

1. Real stock prices appear to show excess volatility (Shiller, 1981). The two series P_t and $P_t - F_t$ do appear quite different in nature although both series appear to be negatively skewed. The series $P_t - F_t$ is thinner tailed, less volatile and closer to being normally distributed than the series P_t.

The results of the sequential ADF tests are shown in Table 2. The standard right-sided ADF test gives no evidence of explosive behaviour in stock prices and the series $P_t - F_t$, as well. As mentioned above, this result could be misleading if periodically collapsing bubbles occur during the given period (Evans, 1991). The SADF and the GSADF tests help to counter this shortcoming. Both the SADF and GSADF test give conclusive evidence for explosive behaviour in both real stock prices and the difference between real stock prices and estimated fundamental value. Thus, the SADF and GSADF tests give convincing evidence for rational speculative bubbles beyond purely explosive behavior in fundamentals.

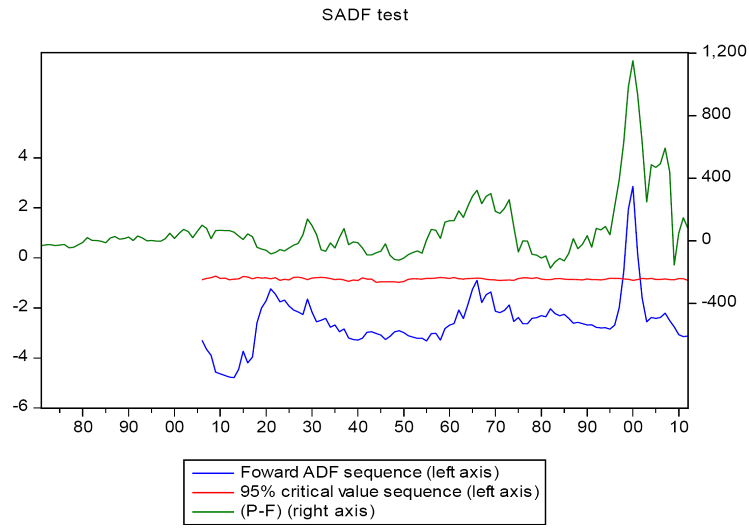

Figure 2

The SADF test graphic of the difference between real stock price and fundamental value

Notes: See Figure 1.

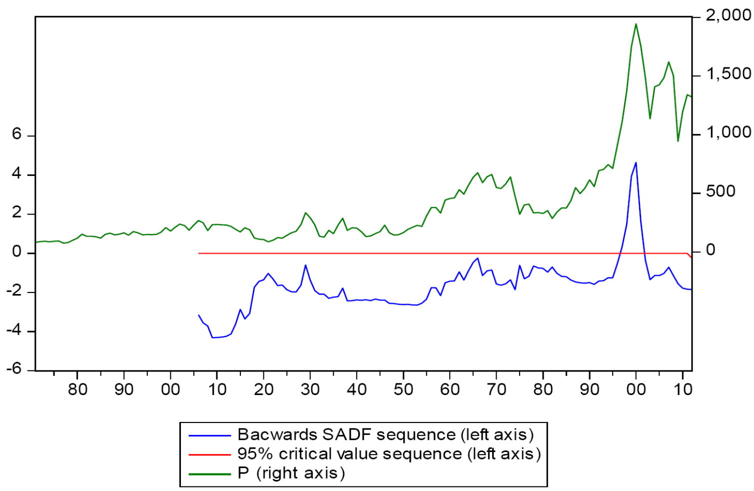

Figure 3

The GSADF test graphic of the stock price index

Notes: This graph shows the series of the real stock price (greenline, right axis) and its corresponding sequence of ADF statistics (blue line, left axis). The red line (left axis) represents the 5% critical values of the GSADF test.

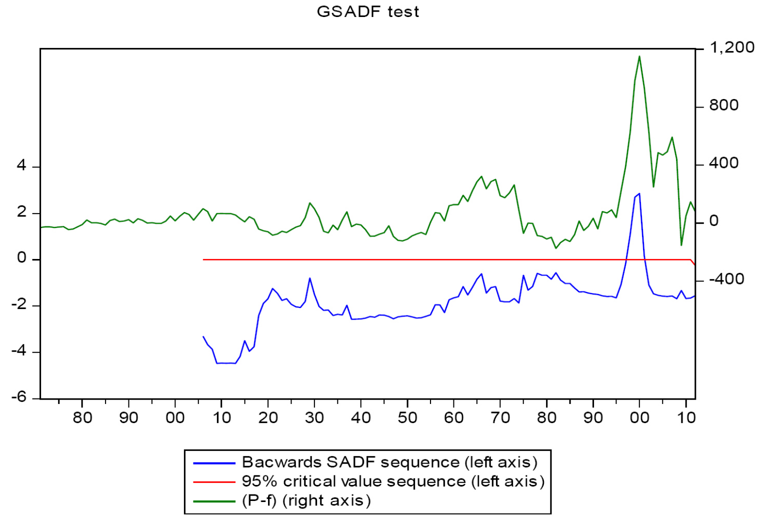

Figure 4

The GSADF test graphic of the difference between real stock price and fundamental value

Notes: See Figure 3.

Next, we address the timing of the bubble. Here, we stress that it is sometimes difficult to specify the nature of the bubble—its name and type—that corresponds to the time interval identified by our tests. So, upstream, we will focus on the chronological coincidence as a criterion. In this sense, our analysis will be based on the history of bubbles and identify subsequently those that fit our time interval. Downstream, we will explain by what mechanism this bubble identified affects the US stock prices in the United States. The SADF test shown in Figure 1 suggests explosive behaviour in prices over the years 1964-69 (Johnson's escalation of the Vietnam war– a result consistent with findings in Brune *et al.*, 2015), 1997-2001 (dot com boom and its aftermath) and from 2006-2008 (crash of 2008). However, once it is applied to the difference between observed and fundamental prices, the SADF test suggests that in 1964-69 and 2006-2008, we have explosive fundamentals rather than a stock market bubble (see Figure 2). This approach also suggests clear evidence of a bubble during the dot com boom (1997-2001). In contrast, the more advanced GSADF gives more consistent results. The GSADF test suggests explosive behaviour in stock prices only from 1997-2001 (see Figure 3). There is also explosive behaviour in the difference between the real stock price and the fundamental value from 1998-2000 (see Figure 4). Thus, we conclude that we have evidence for only one bubble (1998-2000) that cannot be explained by explosive fundamentals. Still, how to show this is an Internet bubble? Or in other words, through what channels we end up with such a bubble type? To explain this point, remember what happened in that time interval. At that point, the ferocity and frequency of the initial public offerings of internet companies have indeed created euphoria in US stock market. Investors blindly grapping in every new issue without even looking at a business plan and many of them foolishly ignored the fundamental rules of investing in the stock market, such as analyzing P/E ratios and studying market trends. The bursting of the bubble precipitated the 2001 stock market crash even more so than the September 11, 2001 terrorist attacks.

As we have used historical data on US stock prices, it is advantageous to divide the studied sample into various sub-samples to get a clearer idea of the bubble behavior. In resorting to such an approach, we can expect such that a bubble is present in a sub-sample and no longer present in the overall sample, and vice versa. This sample subdivision is ensured by the Bai and Perron's (1998) test identifying multiple unknown break dates therein. This procedure allowed us to identify two unknown break dates, namely 1956 and 1992. This suggests the study of three sub-samples: from 1871 to 1956, from 1957 to 1992, and from 1992 until 2013. Since the last sub-sample contains a limited number of observations—and given that the Phillips *et al*. (2015) procedure requires a rather considerable initial window— we decided to eliminate it and focus attention on the first sub-sample and the second extended one running from 1956 till 2013. In view of Table 3, and using the three sequential tests, we don't find any bubble in the stock prices, nor in the difference between real stock price and fundamental value, if the study involver the first sub-sample.

However the situation will clearly change in the second sub-sample, since both SADF and GSADF tests indicate that there are a number of bubbles in the stock prices and withal the series devoid of the fundamental component. To save space, we will focus on the GSADF test results since it is more powerful than the SADF test, especially in the multiple bubble-scenario, as we have mentioned above. That is why we are introducing only GSADF figures for the two series. At this level, Figure 5 shows that there are two bubbles: a relatively wide one dating from 1996 until 2003, and the other more or less reduced covering the period 2008-2009. On the other hand, Figure 6 shows for the series devoid of its fundamental 3 bubbles: the first from 1996 to 2001, the second from 2001 to 2003, and

Table 3

Sequential ADF test results for the first sub-sample (from 1871 to 1956)

Variable	ADF test	SADF test	GSADF test
P_t	-2.079	-1.028	-0.597
$P_t - F_t$	-3.028	-1.028	-0.789
1% Critical Value	-0.430	0.760	1.164
5 % Critical Value	-1.053	0.228	0.600
10% Critical Value	-1.342	-0.077	0.386

Table 4

Sequential ADF test results for the second sub-sample (from 1957 to 2013)

Variable	ADF test	SADF test	GSADF test
P_t	-1.908	4.210**	3.098*
$P_t - F_t$	-1.664	2.442**	2.738*
1% Critical Value	-0.244	1.606	3.952
5 % Critical Value	-0.867	0.723	2.225
10% Critical Value	-1.181	0.449	1.765

Notes: * indicates significance at the 5% level. See Table 2.

the third covering a shorter period (2008-2009). So the difference from the previous figure lies at the second bubble detected. This seems to be the aftermath of the Internet bubble. More specifically, many technology companies realized good business, but investors have mightily exaggerated the importance of the far long-term in their estimates, and neglected to calculate that some of the companies consumed too fast their capital to hope one day reach the balance point. It is remarkable withal that compared to the overall sample results, the test GSADF detected a second bubble in the both studied series. This might argue for the overall sample subdivision, especially when working on historical data. In point of fact, such a subdivision may disclose other informative elements hidden in the data.

4. Conclusion

In this paper, we used the new right-tailed tests introduced in the literature by Phillips *et al*. (2011) and Phillips *et al*. (2015) to test if there are bubbles affecting the US stock market. In doing so, we used historical

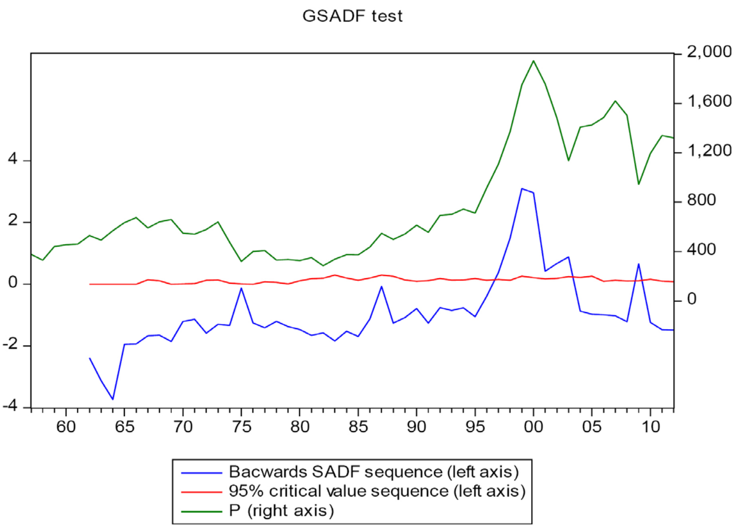

Figure 5

The GSADF test graphic of the stock price index in the second sample

Notes: See Figure 3.

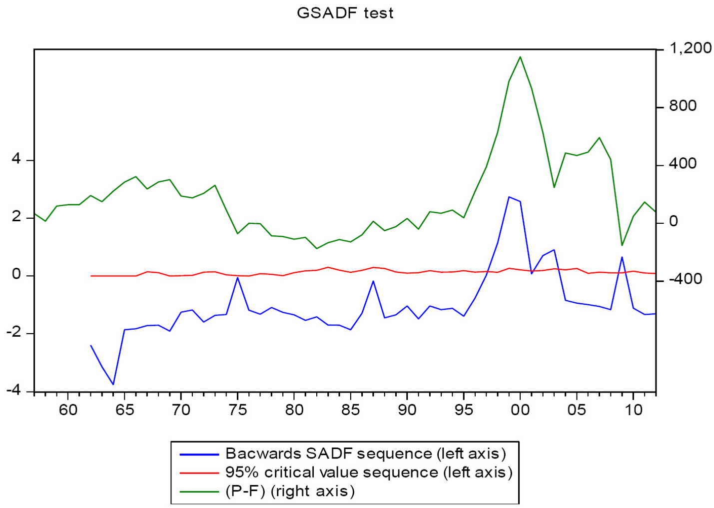

Figure 6

The GSADF test graphic of the difference between real stock price and its fundamental value in the second sample

Notes: See Figure 3.

data series of the stock price index—from 1871 till 2013— and the latter series devoid of its fundamental value. Given that Bubbles are, in general, decisively identified in retrospect, we will build on the chronological coincidence with the bubble time interval suggested by the tests and the stylized facts reported in the literature to define the nature of the bubble in question. This paper has found evidence of a bubble in the US stock market likely at the time of the dot com boom. We find explosive behavior in both the real stock price and the difference between real stock price and the fundamental value (bubble). This behavior cannot be attributed to explosive fundamentals. The evidence in favor of bubbles at other times appears limited. The GSADF test identifies only one bubble. The SADF test labels attributes explosive behavior in prices, coincident both with the escalation of the Vietnam war and the 2008 crash, to explosive fundamentals. However, by dividing the whole sample into two subsamples, the GSADF test will detect another bubble of a limited size (2008-2009) in the second subsample---from 1957 till 2013--- for both studied series. Evidence suggests that the GSADF gives more consistent results than the SADF test and may constitute a meaningful improvement in applications. Our results may thus hold practical implications for investors and financial market authorities alike and reinforce the importance of analyzing fundamentals when identifying bubbles.

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