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Investigating the impact of a Realistic Mathematics Education approach on achievement and attitudes in Post-16 GCSE resit classes

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Acknowledgements

We are grateful to the Nuffield Foundation for their funding and support.

The Nuffield Foundation is an endowed charitable trust that aims to improve social well-being in the widest sense. It funds research and innovation in education and social policy and also works to build capacity in education, science and social science research. The Nuffield Foundation has funded this project, but the views expressed are those of the authors and not necessarily those of the Foundation. More information is available at **www.nuffieldfoundation.org**

Nuffield Foundation

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Our thanks go to all the students and teachers who allowed us into their classrooms and gave their time to this project.

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Summary

Students who have not achieved an acceptable pass grade in GCSE Mathematics by the age of 16 (a grade C at the time of this project) are required to work towards this as part of a 16-19 study programme. As students with a history of repeated failure in mathematics, they have low confidence levels and a tendency to rely on mis-remembered rules applied without understanding. They are often disengaged from mathematics and many do not see it as relevant beyond a pass grade requirement for entry to further training or jobs. Re-sit examination success rates are often poor. The short duration of post-16 resit courses, from September to May, means that teachers face a number of challenges: they need to build students' confidence and improve achievement, but they do not have time to address gaps in knowledge while targeting the required curriculum coverage.

This project trialled the use of an alternative approach: Realistic Mathematics Education (RME) prioritises use of context and model-building to engage and motivate students, enabling them to visualise mathematical processes and make sense of what they are doing without resorting to rules and procedures which have no meaning. Using materials based on RME design principles, the research team developed and delivered two short modules, on Number and Algebra, employing a quasi-experimental design to assess impact on performance in four GCSE resit classes. The main objectives of the project were to assess:

- The impact of an intervention based on RME on students' achievements and attitudes in Post-16 GCSE resit mathematics;
- The potential of an RME-based approach to positively impact on students' understanding of mathematics and their engagement in class;
- The practical issues involved in the adoption of an RME approach with Post-16 GCSE resit classes;
- Teachers' perceptions of the role of RME in the context of GCSE resit classes;
- The implications for a wider application of an RME approach in this context, and teachers' professional support needs.

In addition to Number and Algebra pre- and post-tests, a range of quantitative and qualitative data was collected, including attitude questionnaires, student and teacher interviews, and classroom observations.

The project was independently evaluated by the Centre for Development and Research, Sheffield Hallam University (CDARE), who analysed the pre-/post-test comparisons, and pre/post attitude questionnaires. Their results showed small but significant gains for the intervention group in the Number module post-tests, although not in Algebra. In addition, there was a significant difference between intervention and control groups in terms of the use of RME methods to answer questions. There was no discernible impact on attitudes.

The Manchester Metropolitan University research team carried out a range of qualitative analyses to assess the impact of the intervention. Qualitative script analysis of Number post-tests suggested that impact of the RME intervention was significant, not only in terms of usage, but also in terms of enabling students to make progress. Analysis of the impact of RME on classroom interaction showed that RME has the potential to change classroom norms and encourage students to engage in discussion and meaning-making, despite some resistance to the approach as evidenced in interviews with both students and teachers. In the case of the Algebra module, various factors, notably time pressure, may have led to lack of gains in this topic. GCSE work requires that students reach some demanding formal levels of symbolisation and algebraic manipulation, and in some cases, the gradient of moving from the RME contexts written into the materials through to answering GCSE-type questions was too steep in the time available. Despite the challenges of bringing about change in this difficult context, both students and teachers expressed positive views about the ability of the RME approach to enhance understanding.

The project findings lead to recommendations for extending the RME intervention within one year courses, or extending to two year courses, and focusing on changing classroom cultures and improving students' learning skills in order to support new understandings of mathematics with RME. Recommendations are also made for sustained Continuing Professional Development (CPD) using lesson study which enhances teachers' pedagogic and subject knowledge while enabling them to successfully employ RME design principles to support students' developing understanding.

Key findings and recommendations

Key Findings

- The independent evaluation found that students receiving a short Realistic Mathematics Education (RME) intervention showed improved attainment on post-test performance in Number but not Algebra. There was also a significant difference between intervention and control groups in terms of the use of RME methods to answer questions.
- Qualitative script analysis of Number post-tests revealed the impact of the RME intervention in terms of:
    - use of the bar and ratio table models to find a route to a solution
    - allowing some students to re-engage with informal sense making strategies
    - providing a structure within which to organise and record thinking
    - encouraging flexibility and creativity
- Script analysis also revealed that students could recognise the unifying potential of the models for answering questions across a range of topics.

- Analysis of the impact on classroom interaction showed the potential of RME for encouraging students to engage in discussion and meaning-making.
- Students were responsive to RME but did show some resistance to new methods which they saw as slow.
- While successful in individual lessons, Algebra teaching suffered from time pressure in the resit context.
- Both students and teachers expressed positive views about the ability of the RME approach to enhance understanding.
- Teachers were positive about the RME approach but expressed concerns about pace in the context of the challenges of teaching GCSE resit.

Recommendations

- GCSE mathematics resit courses should ideally be extended to two years and incorporate an RME approach.
- Within one-year courses, an RME-based approach for the whole of term 1 would establish models and pedagogies on which to build a more effective term 2 and 3 focus on revision and past papers.
- Courses need to focus on changing classroom cultures towards new norms of discussion involving questioning, sharing and evaluating ideas in order to support new understandings of mathematics and encourage students to take ownership of the subject. Courses also need to work on enhancing students' learning skills by encouraging them to question their own and others' strategies.
- Teachers need sustained CPD in order to support the development they seek for addressing the challenges of GCSE resit teaching.
- A lesson study model of CPD would enhance teachers' pedagogic and subject knowledge while enabling them to successfully employ RME design principles to support students' developing understanding.

1 Introduction

1.1 The post-16 landscape

In 2011, the Wolf Report highlighted the importance of GCSE Mathematics and English at grades A*-C to a young person's employment and education prospects, noting that:

> *Less than 50% of students have both at the end of Key Stage 4; and at age 18 the figure is still below 50%. Only 4% of the cohort achieves this key credential during their 16-18 education.*

This statement of concern reflected a general trend: of the 36.4 percent of young people failing to gain grades A*- C in their Mathematics GCSE in 2009/10, only 17.6% (of the 36.4%) went onto re-enter for the examination during their 16 -18 education, less than half of whom went onto achieve a grade C or above - just 2.7% of the original KS4 cohort (DfE, 2016). The situation has not improved in subsequent years, with DfE (2016) reporting that although GCSE mathematics entries during 16-18 have increased since 2011/12, "there has been no increase in the proportion achieving A* to C" (p.1). This is illustrated in the most recent available statistics, shown in Table 1-1. Students included in Key Stage 5 cohorts are based on those finishing Key Stage 4 two years before.

End of Key Stage 4 in	2009/10	2010/11	2011/12	2012/13
Percentage of KS4 students not achieving grade C or above	36.4%	33.0%	29.1%	28.2%
End of Key Stage 5 in	2011/12	2012/13	2013/14	2014/15
Percentage of those leaving KS4 without A*-C entered for GCSE resit	17.6%	17.9%	19.6%	24.6%
Percentage of those leaving KS4 without A*-C who achieved a grade C or above in KS5	7.4%	7.0%	7.1%	7.1%
Percentage of original cohort	2.7%	2.31%	2.1%	2.0%

Table 1-1 KS5 GCSE mathematics resit entry and achievement based on KS4 achievement (Source, DfE 2016)

However, GCSE Mathematics and English have long been seen by employers as the accepted standard, and as such are the gateway to many careers both vocational and otherwise. It is perhaps unsurprising that in response to the findings of the Wolf report (2011) a national requirement was introduced in September 2013 that all students aged 16-19 who did not have Grade C passes in GCSE Mathematics and English must work towards this qualification.

This creates a number of challenges for the Post-16 sector: Firstly, students who have previously failed to gain an acceptable pass at GCSE, and who in the past would have opted to study functional mathematics courses, must currently follow a course which requires a repeat of the *same* content they previously failed at. Secondly, the subsequent impact on student numbers means that a significant proportion of non-specialist teachers will now be required to teach GCSE resit. These challenges are exacerbated by the policy and practice context of mathematics teaching in England.

1.2 Mathematics teaching in England: pressure towards early formalisation

Mathematics teaching in England is deeply embedded in its evaluation and accountability systems. The Office for Standards in Education (Ofsted) framework for inspections (Ofsted, 2012) emphasises expectations that pupils will typically make the equivalent of two whole levels of progress from one Key Stage to the next. Schools are required to evidence this, leading to the adoption of rigorous pupil tracking systems and regular testing in mathematics, with interventions for those who are not making the required progress.

At lesson level, the emphasis on student progress within predetermined time limits has increased the practice of setting lesson objectives and sharing these with students at the start of a lesson. The National Strategy documentation (DfE, 2013) provides teachers with lists of objectives, referring to specific mathematical content and focussing on the formal methods students need to learn. So, for example, the geometry and measures strand of the latest version of the Key Stage 3 (age 12 – 14) programmes of study states that "Pupils should be taught to derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms and trapezia, …" (p. 8). The Key Stage 2 programmes of study require students to "use formulae for area" and "calculate" (p. 43). Consequently, most teachers in England have adopted the practice of setting lesson objectives which refer to acquisition of a formal process.

Teachers in England are consequently under pressure to move towards formal mathematics as quickly as possible, and this has an impact on how they use context to support teaching. Any contexts which may have been introduced are quickly dropped to allow for abstraction and development of the desired formal methods. Progression is seen in terms of students' acquisition of these methods, their ability to use them in more complicated situations (often 'bigger' numbers), and finally to apply them in order to answer 'contextual' questions. So, for example, in the teaching of fractions, formal notions of equivalence though 'doing the same to numerator and denominator' are quickly developed with halves and quarters and then extended to thirds, fifths, etc. The idea of a common denominator is also introduced early in the curriculum, and becomes the sole method for comparing and ordering fractions and then for addition and subtraction. Even given recent moves towards spending more time on a topic, and working with the issue of 'mastery' (NCETM, 2014), there are at the time of writing very few examples of willingness to slow down the process of formalisation.

1.2.1 Mathematics classroom cultures: students' expectations and experiences

English mathematics education traditions have had a well-documented impact on classroom cultures and on student experiences and expectations. The emphasis on student performance in public examinations frequently leads to classroom cultures that emphasise getting right answers over understanding (Noyes, Drake, Wake & Murphy, 2010; Wake & Burkhardt, 2013). Consequently, many young people see mathematics as a question of learning rules which lead to answers based on received wisdom and the authority of the teacher (De Corte, Op 't Eynde, & Verschaffel, 2002). It is seen as irrelevant to everyday life, and as meaningless and abstract (Boaler, 2002).

It is widely acknowledged that many students become disaffected with school mathematics (Swan, 2006; Nardi & Steward, 2003; Lewis 2011). The Smith Report (Smith, 2004) expressed concern about the negative attitude and disengagement of many students, and it highlighted in particular the fact that many students found GCSE Mathematics irrelevant and boring. Disengagement is compounded for Post-16 GCSE resit students by the fact that they have already experienced failure, known to have a detrimental effect on students in terms of motivation levels, confidence and attitude (Boaler, Wiliam & Brown, 2000; Dalby 2013; Hannula, 2002). Resit students are also highly likely to have been taught in lower ability groups, but many studies report that ability grouping has adverse effects on lower groups (Francis et al, 2017). Higgins et al. (2015) report that low levels of self-confidence are responsible for their finding that low attaining learners drop behind by one or two months a year in comparison with similar students in mixed ability classes, particularly in mathematics. In lower ability groups, they are likely to experience a reduced curriculum, which limits exposure to mathematics and the grades they can attain in public examinations at age 16 (Boaler & Wiliam, 2001; Boaler, Wiliam & Brown, 2000; Hallam & Ireson, 2007; Solomon, 2007), and lower expectations from their teachers (Horn, 2007; Zevenbergen, 2005).

The patterns of classroom interaction that are fostered by a traditional transmissionist approach to teaching mathematics can lead students to have lower expectations of themselves as well as of mathematics. Zevenbergen (2005) argues that lower performing students' awareness of the restrictions on them in terms of curriculum and pedagogy leads them to develop a predisposition towards mathematics as negative and to behave in ways that contribute further to their reduced participation. Addressing this situation can be challenging, since interventions which aim to change mathematics pedagogy may be rejected by students who have become used to particular mathematics classroom cultures; while they might not like them, they are at least predictable situations in which they have developed strategies for coping. An approach which asks students to explain their thinking and make connections, ask questions and generally take more risks instead of simply 'learning the rules' needs to take this into account (Brantlinger, 2014; Lubienski, 2007).

1.3 The challenge of GCSE resit

Teachers of Post-16 GCSE mathematics resit face particular pedagogic challenges as they seek to raise achievement in a potentially disaffected student body under pressure to succeed in order to pursue further training and career pathways. 'Success' is dominated by a final examination only months away, and inevitably a large proportion of teaching is focussed on examination practice, favouring transmission teaching and memorisation of rules and procedures. Teachers feel a tension between covering all the content (but at an even quicker pace than when their students first learned it) and taking the time to develop understanding (Swan, 2006).

The aim of this project was to address the multiple challenges of GCSE mathematics resit – for both teachers and students - by intervening with a different approach. We saw the potential of a Realistic Mathematics Education approach for tackling gaps in students' understanding and replacing poorly retained algorithms with more meaningful approaches that could support students' engagement and ultimately raise achievement.

1.4 A note on the GCSE context of this project, and the new GCSE specification

This project took place in the context of the GCSE specification which was last examined in Summer 2016. The examination could be taken at Foundation or Higher Tier levels, enabling students to achieve grades C to G (Foundation Tier) and A* to D (Higher Tier). GCSE resit students do not necessarily take Foundation Tier, and some of the students in this project were entered for the Higher Tier. Although any grade above a U is a pass grade, a grade C - termed a 'good pass' - is required for entry to further training and jobs. The new 9-1 GCSE specification, first examined in Summer 2017, includes changes which are particularly pertinent in the context of the issues discussed in this report, namely: coverage of broader and deeper mathematical content; a focus in Foundation Tier on core mathematical understanding and skills; a greater focus on problem-solving; and additional requirements to provide clear mathematical arguments. At the time of writing, indications are that students who achieve grade 4 will be classed as gaining a 'standard' pass, the minimum level that they must achieve in order not to be required to continue studying mathematics post-16. Grade 5 will be described as a "strong pass". This terminology will replace the description of the grade C as a "good" pass (TES, 2017).

2 The Realistic Mathematics Education approach

The aim of Realistic Mathematics Education (RME) is to enable students to visualise mathematical processes by careful use of context and model-building which is always present and accessible to the student. The RME approach differs from regular teaching in that it moves more slowly towards formalisation, and does so in such a way that students can maintain a link back to the original context that they worked with, and how it has been modelled. In so doing so it aims to enhance students' understanding of, and facility with, mathematical processes which have meaning rather than being rote-learned and subsequently forgotten or mis-remembered. A potential 'side-effect' of more meaningful mathematics is enhanced student engagement and interest.

2.1 The background to Realistic Mathematics Education

Realistic Mathematics Education is a pedagogical theory developed in the Netherlands over the last 40 years, shown to lead to greater student engagement, increased understanding of the underlying concepts and improved problem solving skills (Van den Heuvel-Panhuizen & Drijvers, 2014). It is internationally recognised, and materials based on RME are used in many countries (De Lange, 1996) and by over 80% of schools in the Netherlands itself, which is considered one of the highest achieving countries in the world in mathematics according to TIMSS and PISA comparisons (TIMSS, 1999, 2007, 2010; OECD, 2017). PISA's most recent comparisons showed the Netherlands ranking between 10th and 14th among all participating countries/economies compared to the UK's 21st-31st place (OECD, 2016), while an international comparison of numeracy levels amongst 16-18 year olds by OECD showed the Netherlands in 2nd position and the UK in 17th (Department for Business, Innovation and Skills, 2013). Based initially on the ideas of Hans Freudenthal, the RME approach is significantly different to those used in England in a number of respects. Here we focus on three of these: the use of context, the use of models, and the notion of progressive formalisation. We demonstrate why these are particularly pertinent to the post-16 sector and to GCSE resit students.

2.2 Use of context

The use of context in mathematics teaching is not new. Contexts are often used as a means of providing interesting topic introductions, and then for testing whether or not pupils can apply their knowledge. In RME, however, context is used not only to apply previously learned mathematics, but also to construct *new* mathematics (Fosnot & Dolk, 2002). In this respect, context is seen as both the starting point and as the source for learning mathematics (Treffers, 1987), and contexts are carefully chosen to encourage students to develop strategies and models which are helpful in the mathematizing process. These contexts need to be experientially real to the students, so that they can engage in purposeful mathematical activity (Gravemeijer, Van den Heuvel & Streefland, 1990). Post-16 students bring a significant amount of 'life-experience' to the mathematics classroom which can be drawn on through the use of carefully chosen contexts which connect it to mathematics; strategies and procedures are more likely to make sense and there is less need to resort to memorising rules and procedures. In pilot work, students working out the 'best buy' in a supermarket already had a number of informal strategies which could be modelled in a ratio table, ultimately leading to more formal ideas.

2.3 Use of models and two ways of 'mathematising'

In RME, models are given the role of bridging the gap between informal understanding connected to 'reality' on the one hand, and the understanding of more formal systems on the other. Although some 'models' are instantly recognisable as such (for example, the empty number line), the meaning also extends to "materials, visual sketches, paradigmatic situations, schemes, diagrams, and even symbols" (Van Den Heuvel-Panhuizen, 2003, p. 13). Models and contexts support the process of formalisation while retaining the 'sense-making' element, allowing the formal and informal to 'stay connected' in the minds of the students. Models also allow students to work at differing levels of abstraction, so that those who have difficulty with more formal notions can still make progress and will still have strategies for solving problems. Consequently, teachers feel less pressure to replace students' informal knowledge with formal procedures. RME identifies two ways in which students engage with mathematics; at one level, they solve the contextual problem under consideration ('horizontal mathematisation'), on the other, they work within the mathematical structure itself by reorganising, finding shortcuts, and recognising the wider applicability of their methods: this is called 'vertical mathematisation' (Treffers, 1987) and is further explained and exemplified in section 4.1.

Vertical mathematisation, and particularly the recognition that the same model can be used to approach a variety of problems, is particularly beneficial for students attempting to cover a syllabus in a short time, as in the post-16 context. It enables unification of elements of the curriculum which previously they would have perceived as 'different'. In the pilot study for this project, it was noticeable how students began to recognise that the model of the ratio table could be used to answer questions on fractions, percentages, 'best buy' comparisons, conversions and so on, which in the UK are traditionally taught using a range of algorithmic methods which rely on memory.

2.4 Multiple strategies and formalisation: redefining 'progress'

RME's emphasis on building on informal strategies does not mean that formal methods and procedures are ignored. Teachers are always aware of the need for students to develop mathematically, and to become more mathematically efficient and sophisticated over time. What RME does do, however, is offer a very different story of how students and teachers work towards this aim. While formal notions are there, they are seen as being 'on the horizon' (Fosnot & Dolk, 2002) or the 'tip of the iceberg' (Webb, Boswinkel & Dekker, 2008) as illustrated in Figure 2-1. If teachers are not to teach formal procedures, however, they must be given an alternative, and materials based on RME provide this.

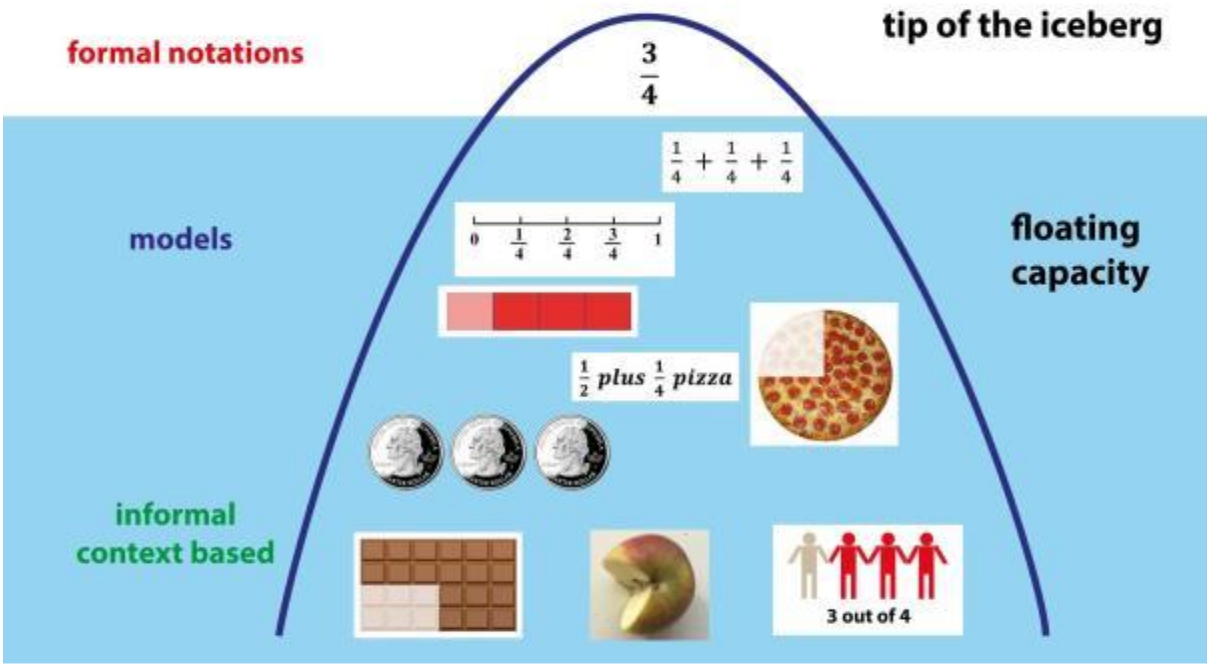

Figure 2-1 The tip of the iceberg, taken from Webb, Boswinkel and Dekker (2008)

Here, for example, while $\frac{3}{4}$ is the formal notation, within the 'main body' of the iceberg are a range of informal representations and pre-formal strategies which students could work on and develop. These are not only seen as desirable but as essential under RME – it is through them that students are able to 'make sense' of formal mathematics. This is particularly important in GCSE resit classes, where students arrive with a range of mathematical experiences and a variety of previously taught methods and procedures. It is clearly beneficial if a teacher can develop these, rather than imposing new methods - RME gives a structure within which this can happen.

Rather than seeing progress as a matter of taking away context in order to work on more formal mathematics, in RME progress is defined through the progressive formalisation of models (Van den Heuvel-Panhuizen, 2003), and in particular the progression from 'model of' to 'model for' (Streefland, 1985). Initially, the model is very closely related to the specific context being considered, but eventually becomes a model which can be applied in numerous mathematical situations (Van den Heuvel-Panhuizen, 2003).

In terms of fractions, this can be seen in Figure 2-2. A problem about how to share 'sub-sandwiches' is initially represented by a drawing of a sandwich, but eventually becomes represented by a model for the formal comparison of fractions.

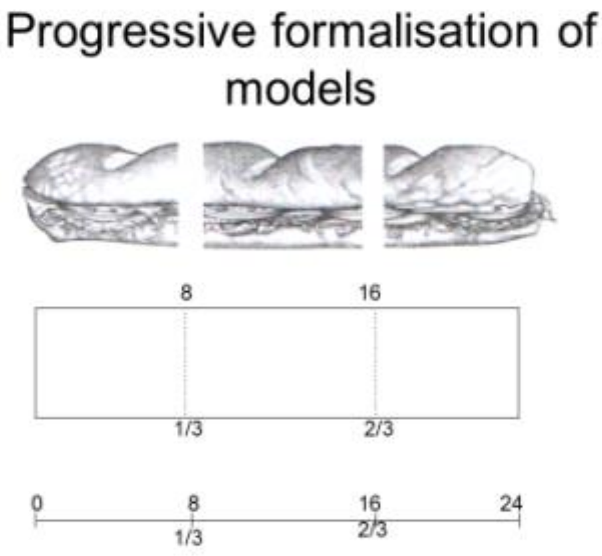

Figure 2-2 Progression from 'model of' to 'model for'

2.5 Applying RME design principles for GCSE resit

Designing materials for work with post-16 students poses particular problems, since students must undergo a formal examination at the end of a course that is in practice eight months long. Hence, it was crucial that host teachers could see a learning trajectory that gave students the opportunity to work towards formal content coverage. It was also important, from a design standpoint, that the process of formalisation would not come at the expense of understanding. Contexts were chosen which allowed mathematical representations to emerge which supported the formalisation process, and which were realisable to older students.

So, for example, a piece of work looking at fractions begins with a survey of eating habits in a local college, with students being put in the position of a new canteen chef. A section of the results from the survey is shown in Figure 2-3.

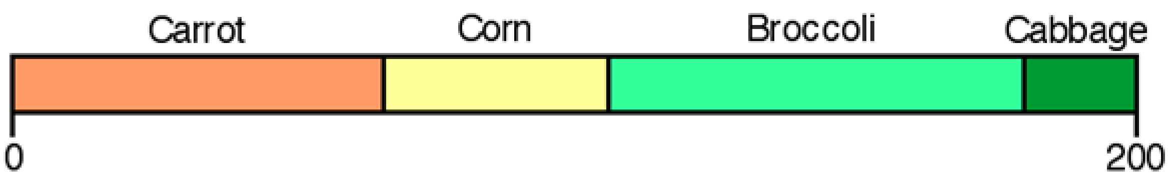

Figure 2-3 Early section of material in the survey bar lesson

Students then produce their own survey, shading fraction bars appropriately. In addition to the main focus on fractions, this activity also leads to work on pie charts in a typical example of how, within RME, models can help to unify different elements of the curriculum.

Subsequent questions and activities aim to develop more formal fraction strategies while retaining the context and model. So, for example, as we move towards adding fractions, the following problem is posed. “At a staff meeting, Jan asked people how often they used the canteen; $\frac{1}{3}$ said every day, $\frac{1}{4}$ said three or four times a week, and $\frac{1}{10}$ said once or twice a week. How many people do you think Jan asked?” This is one strategy for introducing the notion of a common denominator. At another point, when $\frac{1}{4}$ and $\frac{1}{6}$ emerge from a survey, the question asked is “Jan seems to think she asked 40 people for their opinion. Is this possible? If not, suggest some numbers of people she might have asked”. Later in the lesson, as we move towards addition and subtraction, the question is “How many segments should you use in your bar if you want to work out $\frac{1}{3} + \frac{1}{8}$?” Finally, students are asked to use a ‘segmented bar’ to show that $\frac{2}{5} + \frac{1}{2} = \frac{9}{10}$ and addition and subtraction of fractions follows. The major design aim is that students see the purpose of needing a ‘common denominator’, and that they come to the notion themselves in order to be able to solve the problems posed. At no point is the idea of a common denominator ‘taught’ to the students.

A further significant issue at post-16 is the amount of prior knowledge that students have. In order to take this into account, materials were designed so that models could serve a dual purpose: supporting the development of new mathematics for some students, while also strengthening the understanding of those students who had already met formal methods and were reasonably secure with them. The segmented fraction bar is an example of how this works in practice, giving a visualisation of the fraction and enabling some students to develop the notion of a common denominator, while allowing others to make sense of and gain further insights into mathematics they already know.

The tension for designers is to maintain the guiding principles of RME within a relatively short, examination-focussed course. The nature of GCSE resit courses creates challenges to all the main principles of RME: maintaining the use of context throughout comes under pressure from the need to ensure students’ ability to answer more abstract examination questions. This challenge is even more in evidence with the principle of ‘progressive formalisation’: within an RME approach, contexts and models guide students slowly towards more formal mathematics. The danger for GCSE resit is that students are pushed towards the formal too quickly, and connection with ‘reality’ may be lost. A more detailed explanation of how activities were designed in relation to the principles of RME can be found in Appendix 2.

3 The GCSE re-sit project

This study built on the findings of a small pilot study carried out in 2013 (see Appendix 1). The pilot identified a number of potential benefits of the use of an RME approach with GCSE Mathematics resit classes, including:

- Improvement from pre- to post-test performance;
- Willingness to 'have a go' at a new problem;
- Greater understanding of connections within mathematics;
- Improved ability to explain and justify strategies and solutions;
- Teachers' recognition of the wider application of models;
- Teacher reports on the positive impact of RME, including on their own mathematics understanding.

The current study aimed to extend our understanding of the application of RME in the post-16 context with a more rigorous evaluation of the intervention in terms of student test performance. Alongside this, we aimed for a more detailed qualitative investigation of the impact of RME on student engagement with mathematics in terms of their approach to problem-solving, and their perceptions of mathematics and of their own learning. We also sought to understand the implications of the RME approach for (1) students' perceptions of and participation in classroom practice; and (2) our host teachers' perceptions of how the challenges of teaching GCSE resit classes might be met.

3.1 Research questions

The study set out to address the following research questions:

RQ 1: How does an intervention based on RME impact on students' achievement and attitudes in Post-16 GCSE resit mathematics?

RQ 2: How does an RME-based approach affect student understanding and engagement with mathematics in terms of (a) problem-solving strategies (b) experience of and aspirations in mathematics and (c) classroom participation?

RQ 3: What issues arise in practice in the adoption of an RME approach with Post-16 GCSE resit classes? How do teachers perceive the role of RME in the context of the challenges of working with GCSE resit classes?

RQ 4: What are the implications of students' and teachers' experiences of an RME-based approach for its wider application to the mathematics curriculum in this context and teachers' professional support needs?

3.2 Methodology

We drew on a variety of quantitative and qualitative methods in this project. To address RQ 1, we employed pre- and post-tests (including both mathematical content and attitude measures) in a quasi-experimental design devised in collaboration with an independent evaluator (The Centre for Development and Research, Sheffield Hallam University (CDARE)). CDARE carried out the analysis required for RQ1, as detailed below. CDARE also observed one teaching session and interviewed host intervention teachers as part of their process evaluation of the quasi-experimental study. Their interviews with teachers also aimed to provide an additional independent perspective in relation to RQ 3. We employed a case study methodology to address RQs 2, 3 and 4 through a mix of qualitative analysis of students' answers in the pre- and post-tests, semi-structured interviews with students in both control and intervention groups, semi-structured interviews with our host teachers (both control and intervention class teachers), and lesson observation.

3.2.1 The study sites

Three institutions in two cities in the North-West of England hosted the study, providing 4 intervention/control pairs. All were delivering GCSE resit in 9 months. We refer to them as follows in this report:

SFC: SFC is a sixth form centre which is part of a large multi-site FE College. At the time of our study, it was running several resit classes for 16-17-year-olds, limited by college policy to those with a grade D. Both classes were taught by experienced teachers. One class hosted our intervention (SFCP) and the other acted as a control (SFCC).

SFS: SFS is the 6^{th} form of a large 11-18 comprehensive girls' school. It ran two resit classes catering for its Year 12 and 13 students, covering the full range of GCSE grades D – U. Both classes were taught by experienced teachers, although one was new to resit that year. This teacher hosted our intervention (SFSP) and the other acted as control (SFSC).

FEL: FEL is a large multi-site FE college. It ran resit classes for school leavers and younger students, and mixed age range (17-60 years old) classes. Two paired school leaver classes hosted our intervention (FELLP) and acted as control (FELLC), and two paired mixed age classes acted as intervention (FELAP) and control (FELAC). Classes were taught by two teachers new to resit (FELLP and FELLC), and two with resit experience (FELAP and FELAC).

3.2.2 The host teachers

All the host teachers were specialist teachers, with the exception of one, who had an engineering background and had taught GCSE mathematics for the past 5 or 6 years (SFCC). We refer to them in this report as in Table 3-1.

Pseudonym	Class	Experience
David	SFCC	Entered teaching in FE in 2005, not trained as a mathematics teacher but has an engineering background. First taught English but has taught GCSE mathematics for the past 5 or 6 years.
Mike	SFCP	Trained as a mathematics teacher, in his 6th year in the college.
Asad	SFSC	Trained as a mathematics teacher, has been teaching GCSE resit for more than 5 years.
Tanviha	SFSP	Trained as a mathematics teacher, in her first year of teaching resit, but has taught Yr 11 for some years.
Lucy	FELLC	Trained as a mathematics teacher, new to the college but has experience in adult education. She has taught resit for a short time in a previous job, and taught GCSE some time ago. In the intervening years has taught A-level standard mathematics to international students.
Peter	FELLP	Trained as a mathematics teacher, in his first year of teaching resit, normally teaches functional skills
Kate	FELAC	Trained as a mathematics teacher, new to FE, previously worked for 5 years in a 6^{th} form college including teaching resit.
Carol	FELAP	Trained as a mathematics teacher, has been teaching resit for six years.

Table 3-1 The host teachers

3.2.3 The students

Seventy-five students participated in the intervention classes and 72 in control classes. The final student sample is presented in Table 3-2.

Control Classes	Student numbers	Intervention classes	Student numbers
SFCC	13	SFCP	17
SFSC	23	SFSP	22
FELLC	13	FELLP	13
FELAC	23	FELAP	23
Total	72		75

Table 3-2 The students

3.2.4 The intervention

The intervention consisted of two modules covering the Number and Algebra strands of the curriculum, using approximately 12 hours and 9 hours of teaching time respectively. The full lesson schedule for each module can be found in Appendix 3, and drew on our experiences in the post-16 pilot described in Appendix 1. See also Appendix 2 for an explanation of the design principles. The Number module was taught in the Autumn term 2014, and the Algebra module in the Spring term 2015, as in Table 3-3. Exact timing was negotiated with host intervention teachers according to their overall plan of work, and also taking into account control group teachers' plans.

Phase	Pre-Test (beginning of module)	Post-test (end of module)
Phase 1: Number, Autumn 2014 – 12 hours	Number	Number
Phase2: Algebra, Spring 2015 – 9 hours	Algebra	Algebra
April/May 2015	Delayed post-test: Number, algebra and other mathematics	

Table 3-3 Phases and testing

Teaching was delivered by three members of the research team, two taking responsibility for one intervention class each (FELAP and FELLP), and the third taking responsibility for two classes (SFCP and SFSP). All are experienced mathematics teachers with longstanding RME experience. They took sole responsibility for planning and teaching in their classes, with host teachers sometimes present and sometimes not; if they were present, they were encouraged to observe but not participate in order to avoid the impact of an increased teacher-student ratio. Intervention teaching followed the research team's agreed planning of the modules, but team members made their own decisions on the pace of delivery according to local conditions. These included timetabling (for example, FEL ran 3-hour classes once a week whereas SFS and SFC ran 90 minute or 2 hour classes twice weekly), and differences in student progress, as exemplified in the delivery of the survey bar lesson in section 4.2.1.

3.2.5 Data collection

We collected a variety of data during and after the intervention. Details of the pre-post tests, questionnaires and interview topic guides are presented in a Technical Appendix, available from the authors on request. We obtained informed consent from all students and teachers at the beginning of the project. All were promised confidentiality and anonymity, and assured of their right to withdraw at any time. We obtained specific permission from students for videoing classes, on the assurance that there would no focus on specific individuals. Pseudonyms are used throughout this report.

3.2.5.1 Pre- and post-test, and the delayed post-test

All project and control students took a short test prior to and at the end of each module, and a delayed post-test. Test items were devised by the research team with final test design undertaken with advice from the CDARE evaluators, particularly in relation to question construction which would enable identification of evidence of the use of RME intervention methods (see Section 4.1 for further details). The tests contained a range of typical GCSE questions on the target subject matter, which had been trialled during the pilot project and amended in order to reveal differences in levels of student understanding of the underlying mathematical concepts. The delayed post-test covered both number and algebra, and a selection of other GCSE areas. This test was delivered as late as possible after the phase 2 teaching intervention in each site, subject to negotiation with host teachers. The tests were blinded with respect to site and membership of control versus intervention groups, and were marked and internally moderated by the Manchester Metropolitan University researchers. A random selection of papers was sent to CDARE for checking as part of their external evaluation process, and CDARE carried out the analysis of impact as described in their report in Appendix 5.

Three students from each intervention class were selected on the basis of their post-test solutions for both number and algebra and asked to elaborate on their solution strategies in short video-recorded post-test interviews. These data informed our general understanding of student engagement with the RME materials.

3.2.5.2 Student attitudes and experience

All project and control students completed an attitude to mathematics questionnaire prior to the intervention and after the second module delivery. This was adapted from the Understanding Post-16 Participation in Mathematics and Physics Project (Reiss, 2012), with additional items designed to identify beliefs about mathematics.

A sub-sample of students (13 intervention students and 14 control students, spread across the 8 classes) participated in semi-structured interviews during the delivery of the first module for intervention students and at a comparable point in time for control students. Discussion focused on prior and current experiences of mathematics learning, perceptions of mathematics and aspirations to study it in the future. Project students were also asked about their experience of the intervention modules. In addition, seven intervention students provided 'echo smartpen' data on one or in some cases two occasions. Smart pens record written work in addition to speech; when using the pens a member of the research team was present in order to elicit student accounts of their thinking.

3.2.5.3 Teachers' experiences of GCSE resit

All eight teachers were interviewed during the number module phase. One (Mike, the intervention teacher at SFC) participated in an additional interview during the algebra delivery, partly in response to his clear interest in the project but also in response to his anxiety about time pressure. Discussion focussed on the challenges of teaching GCSE re-sit, their usual pedagogical approach, and their perceptions of the RME intervention in terms of their views on students’ responses, issues such as pace and coverage, the potential impact of the intervention on their own practice, their experience of working with the Manchester Metropolitan University team, and their anticipated future practice and professional needs.

3.2.5.4 Lesson observation and recording

We video-recorded a large majority of intervention lessons, using a fixed camera focused on the teacher. Most lessons were also observed by one of the research team, who noted students’ responses to the materials and to the central elements of RME. We were most interested in the ways in which students reacted to the different socio-mathematical norms of an RME approach, and on their participation in different forms of interaction and sense-making in mathematics. We explore this issue in Section 4.2.

3.2.6 Independent evaluation

An independent evaluation of the RME intervention was led by Mark Boylan from the Centre for Development and Research in Education (CDARE), Sheffield Hallam University, assisted by Tim Jay of Sheffield Hallam University. The aims of the evaluation were:

(i) To provide an independent evaluation of the impact of the RME approach as operationalised in the project, on Post-16 GCSE resit students' achievements and attitudes in mathematics, by contributing to addressing RQ1.

(ii) To advise the Manchester Metropolitan University team on issues of fidelity and teacher CPD, so contributing to addressing RQ3.

(iii) To advise on scalability of the intervention for a larger efficacy trial using a randomised controlled trial methodology.

The evaluation report is reproduced in Appendix 5, where details of the pre-/post-test analysis process and outcomes can be found. We draw on these findings in section 4.1 of this report, where the main results of CDARE’s analysis are presented prior to analysis of the qualitative data collected and analysed by the research team. We also draw on CDARE’s reflections and recommendations in sections 8.5.6-8.5.7 in formulating our own implications and recommendations in section 6.

4 Research findings: the impact of RME

4.1 Overall test performance and script analysis of the Number test

As the independent evaluators, CDARE took responsibility for analysis of the test data, and their full analysis is presented in detail in Appendix 5. Their analysis of scores from 75 (intervention) and 73 (control) students showed small but significant gains for the intervention group on the Number module post-test scores ($F_{1,93}$=4.55, p=0.035, Cohen's d = 0.26). There was no effect on performance in the Algebra module at test result level; we comment on this finding in Sections 5 and 6.

In Number, the scores for both groups increased and revealed a similar level of performance in post-tests, but a lower level of performance by the intervention group at the pre-test stage. Further analysis led the evaluators to suggest that the intervention had been effective and had resulted in the intervention group 'catching up' with the control group.

In addition, there was a significant difference between intervention and control groups in terms of the use of RME methods to answer questions. Of the 49 students in the intervention group who took the Number post-test, 36 students were seen to use a bar or a ratio table at least once. There was also a significant correlation between students' improvement from pre- to post-test in Number and the extent to which they used an RME approach (r = .258, n = 86, p = .016).

In order to investigate these findings further, we carried out additional in-depth script analysis on pre- and post-test performance in Number, as this was where intervention students made most progress. Our aim was to analyse what strategies students found useful and make comparisons with the 'Concepts in Secondary Mathematics and Science' study (CSMS) from the 1970s. In particular, we examined how the Realistic Mathematics Education (RME) approach enabled students to make progress. Our analysis involved:

- Comparing marks gained from pre-test to post-test for each question;
- Categorising and quantifying the types of responses given for each question;
- Scrutinising how the use of RME methods impacted on individual student approaches.

(As noted in the external evaluation, missing data was an issue. In this analysis, we are able to report on 54 intervention students and 39 in the control, ie, students for whom we had both pre-test and post-test Number scripts.) The basic marks comparison is shown in Table 4.1.

Question	1. Photocopier		2. Sharing in given ratio		3. 17% of £3300	
	Pre	Post	Pre	Post	Pre	Post
Intervention	42%	47%	27%	56%	40%	54%
Control	51%	45%	44%	55%	43%	54%
Question	**4. $\frac{5}{8}$ of £600**		**5. Comparing speeds**		**6. Reverse percentage**	
	Pre	Post	Pre	Post	Pre	Post
Intervention	31%	41%	20%	42%	6%	18%
Control	33%	27%	24%	42%	0%	0 %

Table 4-1 Percentage of marks gained per question by intervention and control groups in Number pre- and post-tests

The Number test was designed to consist of GCSE-type questions covering grades B to E involving the topics ratio, proportion, finding a percentage of an amount, finding a fraction of an amount, comparing two rates and a reverse percentage calculation. Quantities were chosen which students could be expected to manipulate without the use of a calculator. Two out of the six questions analysed were similar to questions used in the Ratio and Proportion test taken from the 'Concepts in Secondary Mathematics and Science' (CSMS) 1970's programme which was revisited thirty years later, as part of the 'Increasing Confidence and Competence in Algebra and Multiplicative Structures' (ICCAMS) programme. A detailed analysis of student performance on these two questions forms the basis of this section.

4.1.1 Methods used by the students to answer question one

Question 1: It takes a photocopier 18 seconds to produce 12 copies. How long would it take at the same speed to produce 30 copies?

4.1.1.1 Use of formal methods

There are well-known formal approaches to solving problems of this type. One is to use a formula of the type $\frac{a}{b} = \frac{c}{d}$ substituting in the three known values and solving to find the fourth. The second is the unitary method. i.e.

12 copies takes 18 seconds 1 copy   takes $\frac{18}{12}$ seconds 30 copies takes $\frac{18}{12}$ x 30 seconds

None of the students in this study presented solutions using the formula $\frac{a}{b} = \frac{c}{d}$ and only three students attempted the formal unitary method. This finding is consistent with the analysis of secondary students' performance on ratio and proportion items carried out by the CSMS study where use of either of the two formal methods was extremely rare. The CSMS study commented that one class consistently and correctly used the $\frac{a}{b} = \frac{c}{d}$ rule but it is worth noting that these were a group of high attainers. As with the CSMS study there was little overall evidence that this cohort of GCSE resit students were familiar with the formally taught methods. They will no doubt have seen these methods in school textbooks or teacher demonstrations or various revision support resources, but on the evidence of this study, they have had little impact.

4.1.1.2 Informal versions of the unitary method

Some of the GCSE resit students did attempt solutions which involved working out the value of one 'unit', but these were structured in a less formal way than in the unitary method shown above. Around 10% of intervention students used informal unitary type methods in their pre-test and all were successful in this, but interestingly these students all elected to use a bar or ratio table in their post-test. (When interviewed these students spoke about how they found it easier to draw a bar or a ratio table because they could do that to answer lots of questions. It would appear in the case of these students that exposure to an RME approach has encouraged them to move away from multiplicative reasoning (30 copies in 30 x 1.5 seconds) to building up approaches involving doubling, halving and adding amounts. This could be seen as a backwards step and yet these students all gained more marks in the post-test because they were able to see the generalisability of the RME strategies to answer a range of question types.)

For the control groups, just over 20% of students used informal unitary type methods in pre-test and a similar amount in the post-test, although this was not always the same students. The success rate for control students using this method was 8% percent in pre-test rising to 13% in post-test. Most students were able to identify a need to do 18 ÷12 to find the time for one copy, but either left this blank or re-wrote the division formally as $12\overline{\smash{)}18}$, at which point they then became stuck. Dividing one whole number by another proved to be a sticking point for many students throughout the study.

4.1.1.3 Building up an answer

'Build up' methods (Hart 1981; Küchemann 1981) involve doubling, halving, adding combinations of these, trebling, scaling by 10 or other scale factors. Such strategies were in frequent use in the CSMS study, with students tailoring their approaches to suit the numbers given in the question. Likewise, the GCSE resit students favoured a 'building up' approach. In the case of the photocopier question, where 12 copies are produced in 18 seconds, the most common version was to double these quantities, halve the quantities and then to add: 24 copies + 6 copies takes 36 seconds + 9 seconds = 45 seconds.

Of the intervention students, 61% used a building up approach in the pre-test rising to 74% in the post-test. The corresponding figures for the control are 49% (pre-test) rising to 59% (post-test). However, in both the intervention and the control groups, not all of these students were able to build to a correct solution. In some cases, students doubled only, or doubled and trebled and stopped or, having exhausted the possibility of reaching 30 copies through use of whole number multipliers, resorted to an additive strategy. The incorrect use of an additive strategy for quantities which are in a 'rated' relationship is well documented (Hart et al, 1981; Lamon, 1999; Küchemann, Hodgen & Brown, 2011), and of the students who attempted a 'build up' strategy, around one quarter demonstrated this error.

4.1.1.4 Evidence of impact

The external evaluation suggests that the intervention had been effective in enabling the intervention group to 'catch up' with the control. In order to investigate this further we calculated the percentage of marks achieved by each group in pre- and post-tests on each question. The decision to look at marks as opposed to numbers of student who were able to achieve the correct answer reflected the need to account for students who were able to achieve one of the two marks awarded to this question. The results for the photocopier question are shown in Table 4-2:

Photocopier question	Percentage of marks gained in pre-test	Percentage of marks gained in post-test
Intervention group	42%	47%
Control group	51%	45%

Table 4-2 The photocopier question results

Clearly, the gain in percentage marks for the intervention group is only slight, as is the corresponding drop in marks for the control, suggesting that there had been little significant impact of either the intervention or the control teaching phases on a student's ability to gain marks on this question. However, the trend for the intervention group to increase the percentage of marks gained from pre- to post-test by more than the control group was replicated in every question (see Appendix 5), particularly when their initial marks were considerably lower than the control groups, which validates the 'catching up ' effect.

What is significant for this question is how many intervention students shifted to using an RME type approach in their post-test – 63%. Comparing the control group pre- and post-test scripts, there was no such shift from one strategy to another and no emergence of a particular strategy which could be aligned with the way a teacher had taught during the control phase. With the intervention groups it was clear that several students were attempting to answer this question using either a bar model or a ratio table model. In the next section, we look at specific examples of how students applied these models and how using these approaches helped students to make progress.

4.1.1.5 Examples of RME strategies enabling students to make progress

Student A was a 17 year old resit student who entered Year 12 having gained a grade E in her GCSE. The school's policy was to enter most resit students for higher level GCSE in the belief that it was easier to gain a grade C. In her resit at the end of Year 12, student A gained a low grade D. Student A was attentive in lessons, but like many of the group lacked confidence in her ability to do mathematics.

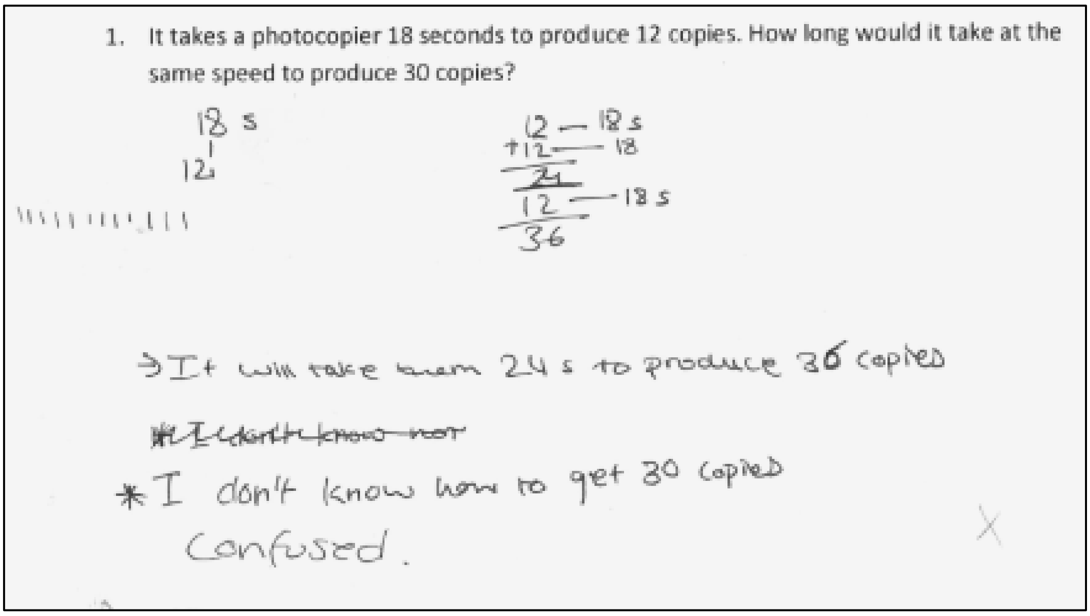

Figure 4-1 Student A, photocopier question, pre-test attempt

In her pre-test (Figure 4-1), Student A appears to be making a genuine attempt to engage with the meaning of the statement 'it takes a photocopier 18 seconds to produce 12 copies'. At one stage, she draws 12 tally marks. Her use of the words 'it will take them…' implies an attempt to make sense of her doubled values in terms of the context of time and copies. However, she incorrectly swaps the copies and seconds and a close inspection of her vertical arithmetic would suggest that her 36 has come from adding three lots of 12 rather than doubling 36. In her own words, she is ‘confused’.

In her post-test, Student A draws a ratio table and is now able to use a ‘build up’ strategy. She realises that 30 copies can be made from the time taken for 24 + 6 copies, but proceeds to make an error when adding 36 and 9 seconds. The crossings out would imply there is still a good degree of uncertainty, but some progress afforded by the structure imposed by use of a ratio table (Figure 4-2).

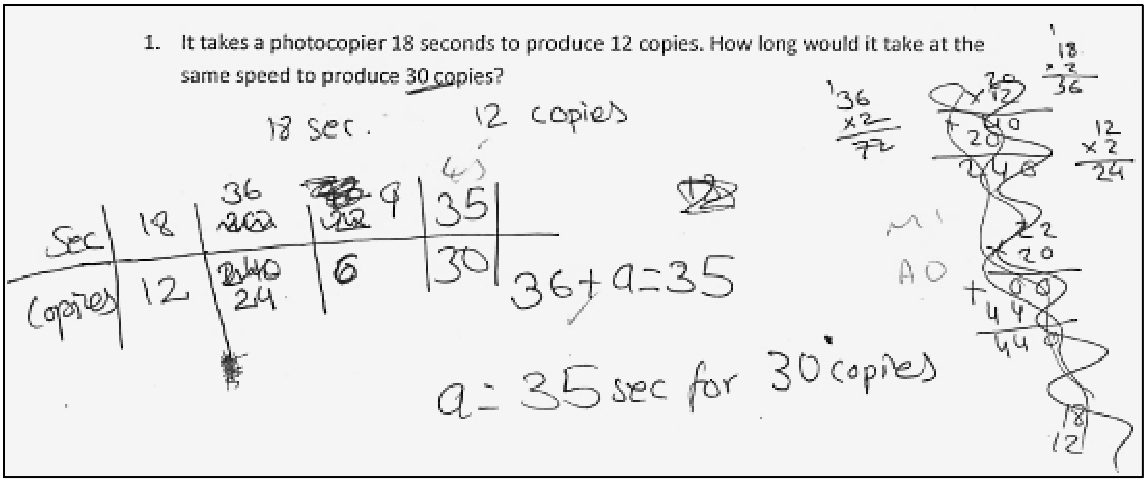

Figure 4-2 Student A photocopier question, post-test attempt

Student A scored zero marks in her pre-test, where she used a variety of half-remembered rules and referred to not being able to ‘remember it’ (ratio); don’t know how to do these types’ (percentage); ‘not sure’ (fractions) and ‘don’t have a clue’ (comparing speeds). In her post-test she made use of the ratio table as shown in Figure 4-2 and was able to represent the other questions using a bar model. By accurately portioning her bars, she was able to figure out $\frac{5}{8}$ of £600 and find 10% and 1% of £3300. This only amounted to 4 marks, but we see this as a good deal of progress in that she has moved away from trying to remember a separate rule for each question and perhaps more importantly is beginning to work with a structure from which she can develop her thinking in a way that makes sense to her. Student A would need further exposure to this methodology, but it provides her and her teacher with something to work from as opposed to re-visiting what for this student appeared to amount to meaningless routines.

Student B also entered Year 12 with a grade D at GCSE. She was under the impression that she had narrowly missed the grade C and believed that she just needed to ‘brush up’ on a few topics. She lacked motivation in lessons and was initially reluctant to engage with the RME approaches.

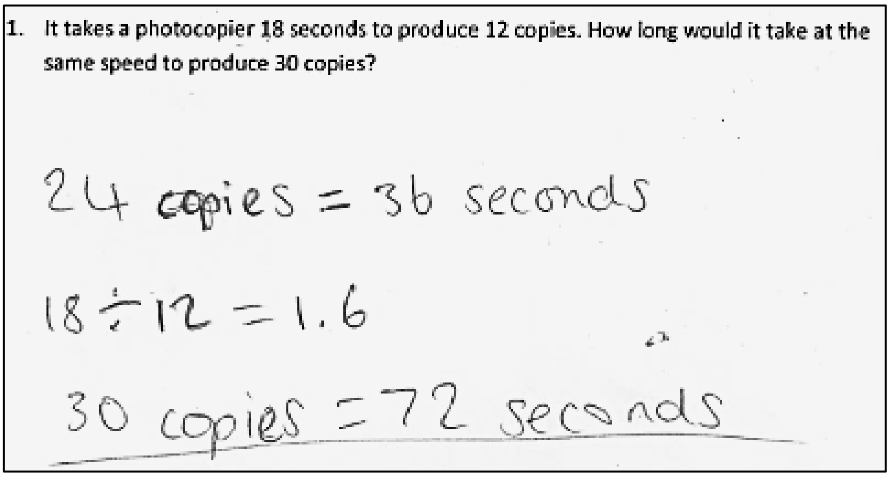

Figure 4-3 Student B, photocopier question, pre-test attempt

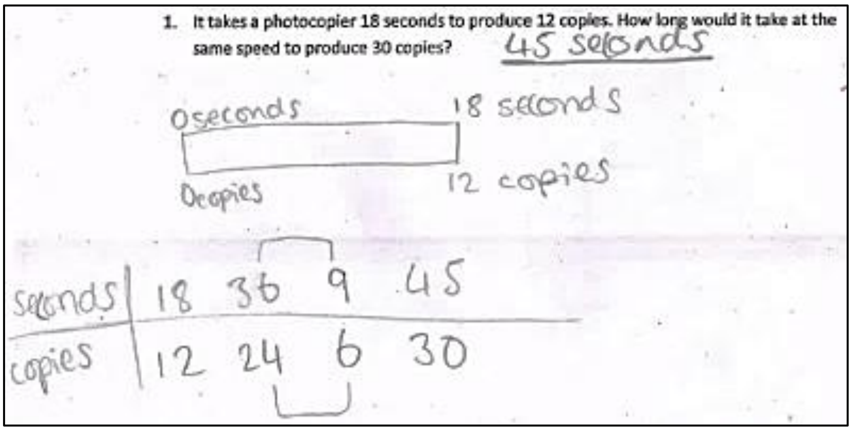

Figure 4-4 Student B, photocopier question, post-test attempt

In her pre-test attempt at the photocopier question (Figure 4-3), Student B correctly identifies 18 ÷12 but is unable to accurately compute this, or know how to use this rate to find the time for 30 copies. In her post-test (Figure 4-4), she initially represents the information on a bar then proceeds to use a ratio table. Her 'build up' strategy is the same as that of Student A, and she is able to complete this with accuracy and so achieve full marks. Student B missed out most of the other questions in the pre-test or used a mis-remembered rule, i.e. for $\frac{5}{8}$ of £600 she wrote 600 ÷5 = 120; 120 ×8 = 960. In her post-test, she used an RME method for four out of the six questions and as a result gained nine marks from pre- to post-test.

A noticeable issue for many of the GCSE resit students was their lack of ability and confidence to accurately and consistently operate with two and three digit numbers. Inappropriate application of vertical algorithms, as seen in Student A's solutions, plus errors in mentally adding, subtracting and dividing numbers by 10 were quite common. There were many other examples where drawing an RME model liberated students in terms of what operations to perform, but a lack of fluency in basic number meant they were not able to successfully compute the required operations.

The ratio table was the more popular choice of method for the photocopier question with twice as many students using a ratio table compared to those using a bar. In the next section we illustrate how use of the ratio table encouraged students to take ownership of their work and develop individual pathways towards solving the problems.

4.1.1.6 The ratio table

Previous studies (e.g. Middleton & Van den Heuvel-Panhuizen, 1995) have noted the flexibility of the ratio table as an open computational tool, which can give rise to a range of student approaches. In the post-test examples in Figures 4-5, 4-6 and 4-7, students use combinations of multiplicative and additive strategies, use differing numbers of steps and create larger and smaller entries. In some cases, the quantities are presented in size order, similar to a scaled bar, in others the entries are entered from left to right.

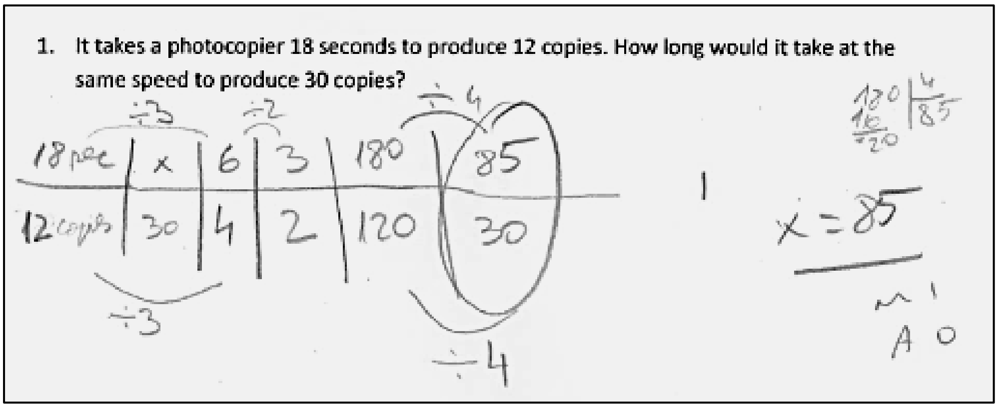

Figure 4-5 Student C's ratio table strategy for the photocopier question

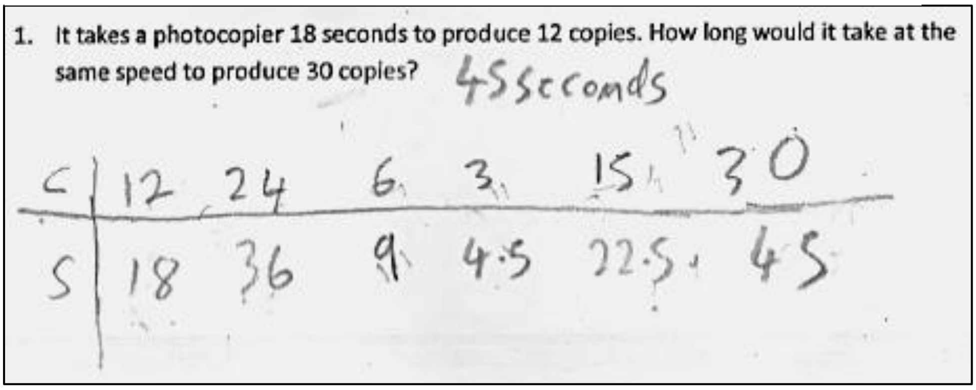

Figure 4-6 Student D's ratio table strategy for the photocopier question

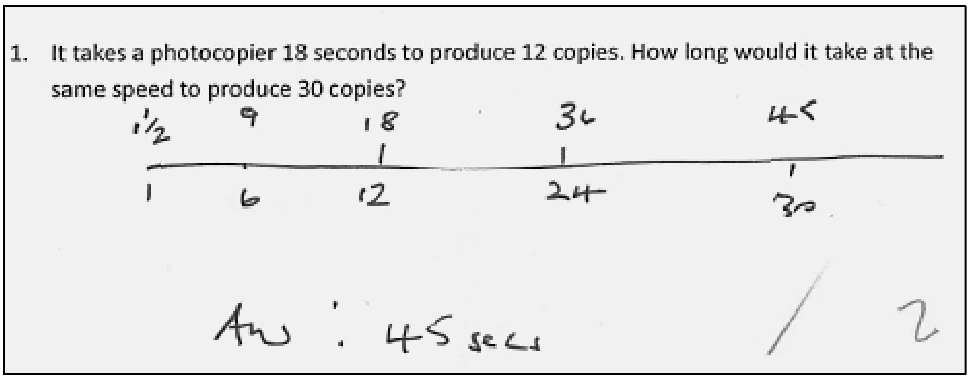

Figure 4-7 Student E's ratio table strategy for the photocopier question

The ratio table provides a medium for students to organise their thinking and keep track of operations and results. It encourages students to be creative; there were no examples of the novel 'build up' routines shown above in scripts where students did not use an RME method. It also leaves an evidence trail of the thought processes of the students, which is useful to learners and teachers alike: it is easy to identify that Student C made a calculation error as opposed to a process issue. In the classroom situation, it provides material for students to compare and contrast their approaches: Student D did not see the 'popular' strategy of combining 24 and 6 copies, but instead went on to find and then double 15 copies. This kind of novel thinking inspires questions and debate. It is not clear whether Student E combined 6 and 24 copies to make 30 copies, or whether she scaled up from 1 copy to 30, but her ratio table demonstrates that not only are several different routes available within the course of one solution, but that these lead to the same answer.

One disadvantage of this flexible use of the ratio table is that it may encourage students to work less efficiently, and increase the number of entries made; this increases their chance of making errors. As previously mentioned, this is a particular concern for GCSE resit students where their facility for number operations is often low. However, this issue has to be balanced against the flexibility it promotes, enabling students to 'own' their method. Once they have set up their ratio table the learner is encouraged to fill in 'what else do you know' before focusing on the requirement of the question. This is empowering and confidence boosting for GCSE resit students who have struggled for many years to remember and replicate the precise steps of a particular method owned by their teachers and their textbooks, not by them.

Middleton and Van den Heuvel-Panhuizen (1995) describe the ratio table as 'a simple tool.for developing students' conceptual understanding of rational number'. In other words, it enables students to develop a number sense around seeing the connections across fractions, decimals, percentages and ratios in terms of how their individual notations link and how they are embedded in a variety of different situations. Certainly, fractions, decimals, percentages and ratios can all be represented in a ratio table, so it does provide a model where seemingly different notations and the contexts leading to those notations can be seen to be equivalent. In terms of this study, it is difficult to judge how much the intervention helped to develop students' conceptual understanding of rational number, but at the very least many post-test students were able to recognise that questions which they had attempted in pre-test using a variety of different methods and which traditionally would be classed under different topic headings, could now be answered using the same model.

Student D, who successfully used a ratio table to answer the proportionality question (see figure 4-6), was also able to apply this approach to compare two rates in question 5. In the pre-test he had correctly identified the division required to work out the speed of the lion, but had been unable to proceed further as in Figure 4-8. Figure 4-9 shows his post-test attempt.

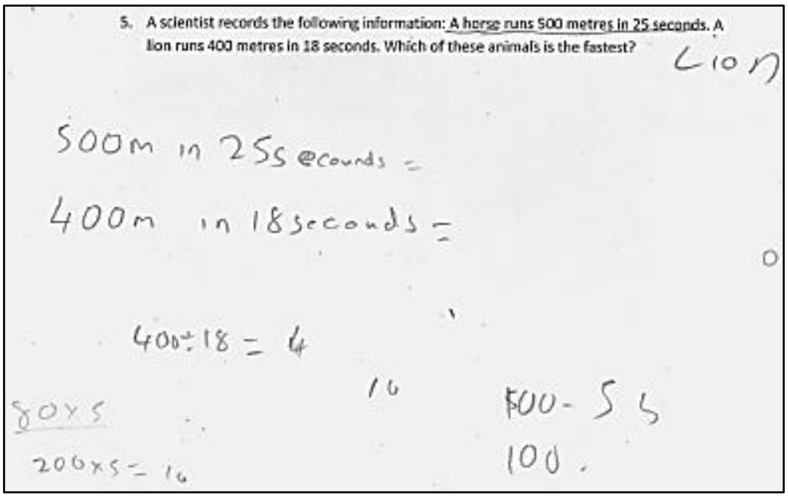

Figure 4-8 Student D's pre-test attempt to compare two speeds.

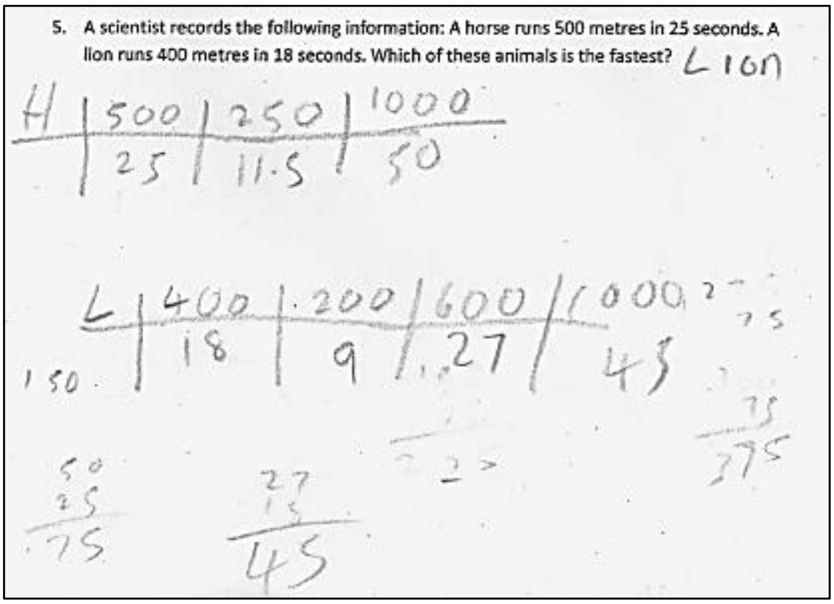

Figure 4-9 Student D's post-test attempt to compare two speeds.

It is worth noting that several other intervention students shifted to a ratio table solution strategy for questions that they had successfully solved in their pre-tests, using other methods. When interviewed, some of these students clearly recognised the power of the ratio table and/or number bar for answering questions across a number of topic areas. In a 'eureka moment' during the third lesson, one intervention student, recognising the tremendous potential of the ratio table raised her hand and said almost in disbelief, 'What, I can use this for percentage as well? I don't have to remember all those other concoctions?'

4.1.2 Methods used by the students to answer question two

> Question 2: Pat and Julie share £140 in the ratio 2 : 5. How much money does Julie get?

The latest statutory programmes of study for mathematics (DfE, 2013) indicate that students should be introduced to the idea of ratio including the a: b notation in year 6 and then work on dividing quantities in a given ratio a year later in Key Stage 3. The results of the 1970's CSMS study for a similar problem to the one shown above (Hart, 1981) gave the percentage of students obtaining the correct answer as 44% (12-13 year olds), 46% (13-14 year olds) and 57% (14 –15 year olds). By way of comparison, the percentage of GCSE resit students obtaining the correct answer to the share in a given ratio question is shown in Table 4-3.

Share in a given ratio question	Percentage of students correct pre-test	Percentage of students correct post-test
Intervention group	20%	44%
Control group	38%	51%

Table 4-3 Performance in the given ratio question

It is interesting to note that, even after the intervention, the facility of the post-16 students to answer this problem is still below the facility of the CSMS 14-15 year olds and serves to remind us how difficult these problems are for lower attaining students, despite revisiting the topic of ratio over a number of years of schooling. Prior to the intervention, their facility was lower than that of CCSMS 12-13 year olds and particularly so in the case of the intervention group. Further script analysis was completed in order to identify the types of errors.

4.1.2.1 Exemplification of errors and omissions

In the pre-test, 31% of control students left the question out, the same figure as for the intervention group. If they could not do a question, students were asked to give reasons. Their comments included: 'forgotten the method'; 'can't do ratio'; 'need more work on this'; 'never was any good at ratios so always try to avoid, they look too complicated to be done'; 'not sure what ratio is'; 'found it hard to learn probably why I don't remember how to do it'; 'need to practice ratio, never got one of these correct'. The use of the word 'ratio' in the question seemed to evoke reactions similar to that to the words 'algebra' or 'fractions' as known difficult topics for these students which they would prefer not to study; there is an awareness that there is 'a method' and an acknowledgement of the role of memory in reproducing this rule. The comparable CSMS question did not make use of the word ratio or the a : b notation, suggesting that language may be part of the barrier. After re-visiting the topic, the incidence of non-attempt by post-16 students in the post-test reduced to 21% for the control, but more so for the intervention group at 11%.

The most common error made by the CSMS students was to equally share the total amount (240 hours) between the number of people involved in the CSMS problem (3 people). Very few post-16 students equally distributed the £140, the most common errors being made by students who appeared to be using half-remembered rules. These included 140 ÷2 and 140 ÷5 or even 140 ×7, using what looks like a concoction of the numbers given in the question linked by the mathematical operations usually involved in answering questions of this type. An attempt at 'doing something with the numbers' irrespective of the meaning of those numbers leads to a rich array of procedural malfunctions.

4.1.2.2 Procedural malfunction

An over-reliance on the use of procedures and a tendency for students to mis-represent those procedures is well-documented (Foster, 2014; Ofsted, 2008; Plunkett, 1979; Swan, 2006). In this study, we found many other examples of students attempting to apply a previously taught procedure including: trying to find 17% of £3300 by writing it as $\frac{3300}{17}$ ×100; working out $\frac{5}{8}$ of £600 by replacing $\frac{5}{8}$ with 0.58; using long multiplication with two of the three quantities given in the photocopier question to give an unrealistically high amount of seconds. Not only does this reveal their inability to remember a formal procedure, but also how little sense they have of how or why the rule delivers a sensible answer to the question. As commented by Hart (1981, pg. 73), rather than looking at a problem and saying 'what does this mean?', instead the student thinks 'what do I do when that sign appears?'

It was noticeable in previous RME-based projects how many students relied on the use of formal procedures and how many, particularly in the case of the lower attaining students, over-generalised or mis-remembered those procedures (Dickinson, Eade, Gough & Hough, 2010). The encouraging finding was that where students had experienced an RME intervention, they gradually shifted to using strategies which made sense to them in terms of answering the question, with much greater gains from pre- to post- tests (Barmby, Dickinson, Hough & Searle, 2011). Analysing the GCSE resit post-test scripts revealed several examples of students engaging in sense making, using RME strategies. This is illustrated by post-test intervention group solutions to question two.

4.1.2.3 Attempting to make sense of the problem

Student F's initial attempt (Figure 4-10) may be an example of a mis-remembered procedure, or instead an attempt to hand out 2 lots of £140 to one person and 5 lots to the other. In the post-test (Figure 4-11), she draws a bar split into 7 parts, and uses shading to distinguish the portions. Her bar is labelled as a continuum from 0 to £140. This time she selects a division procedure which when interviewed she was able to justify because 'there's 7 boxes, it's 140 for the whole thing'.

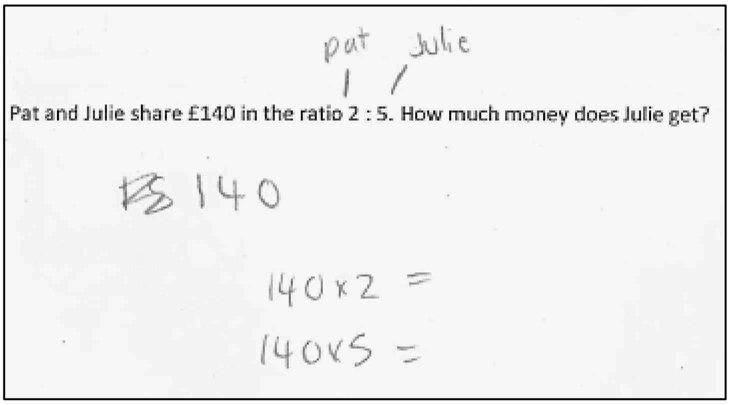

Figure 4-10 Student F, ratio question, pre-test attempt

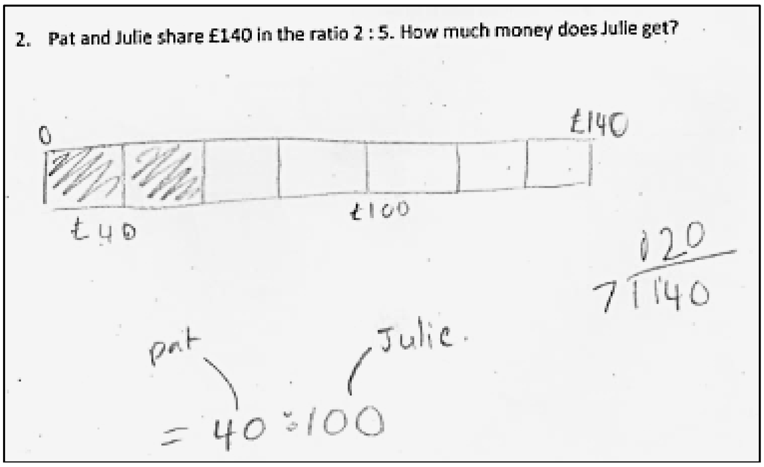

Figure 4-11 Student F, ratio question, post-test attempt

In his pre-test (Figure 4-12) Student G is able to complete the first two stages in the standard procedure, but then says he has forgotten how to do ratio.

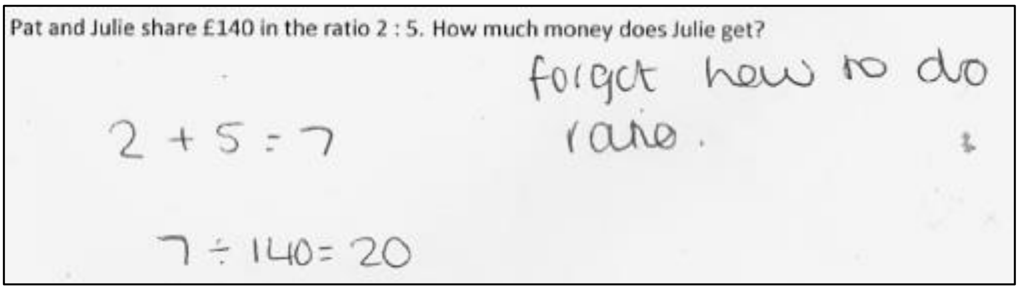

Figure 4-12 Student G, ratio question, pre-test attempt

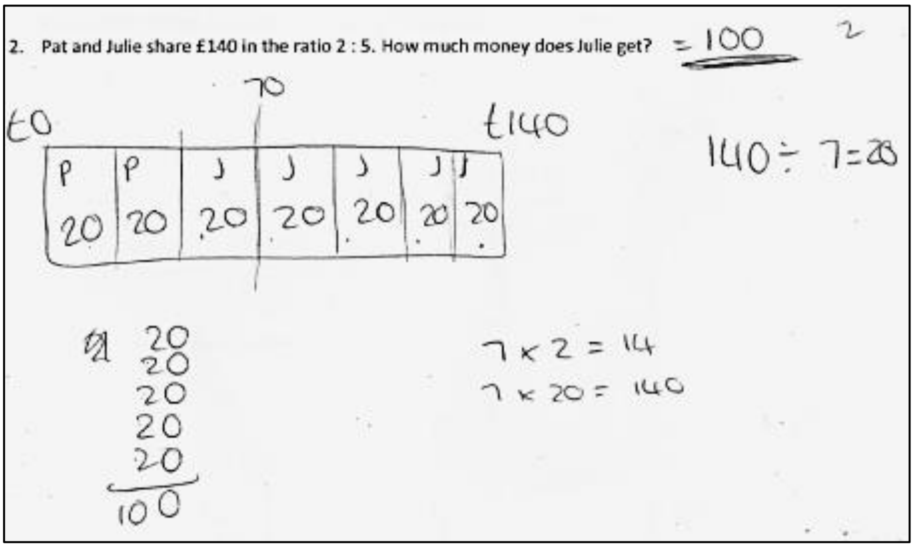

Figure 4-13 Student G, ratio question, post-test attempt

In his post-test bar (Figure 4-13), he uses the initials P and J to distinguish the parts and is able to represent the 20 in the context of his picture. The presence of dots within the blocks would suggest he has touch counted to reach Julie's total, in addition to writing it vertically as a repeated addition sum. A concern is the way he has marked 70 as though it is in the middle of his bar and yet it appears at the end of three of his seven pieces. This was an issue for a few students who over-generalised the strategy of marking in what they perceived to be the middle of the bar.

The third example comes from a student who, while scoring one mark in both her pre-test and post-test attempts, shows a great deal more engagement with the meaning of the problem in her second attempt (Figures 4-14 and 4-15).

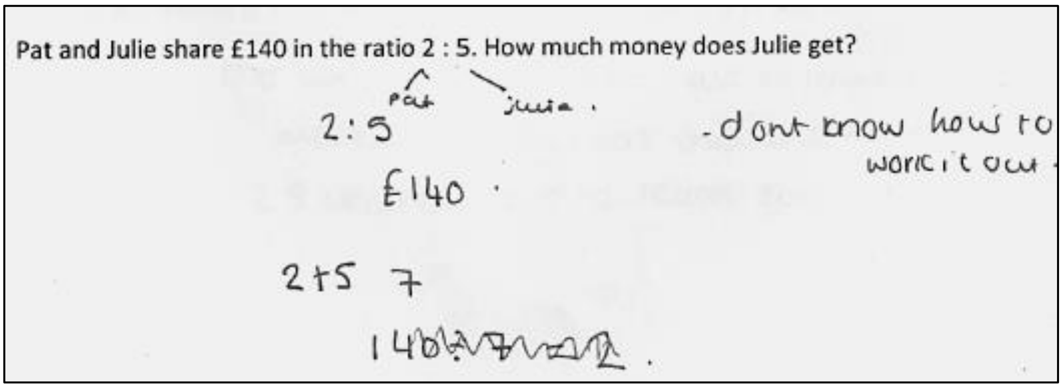

Figure 4-14 Student H, ratio question, pre-test attempt

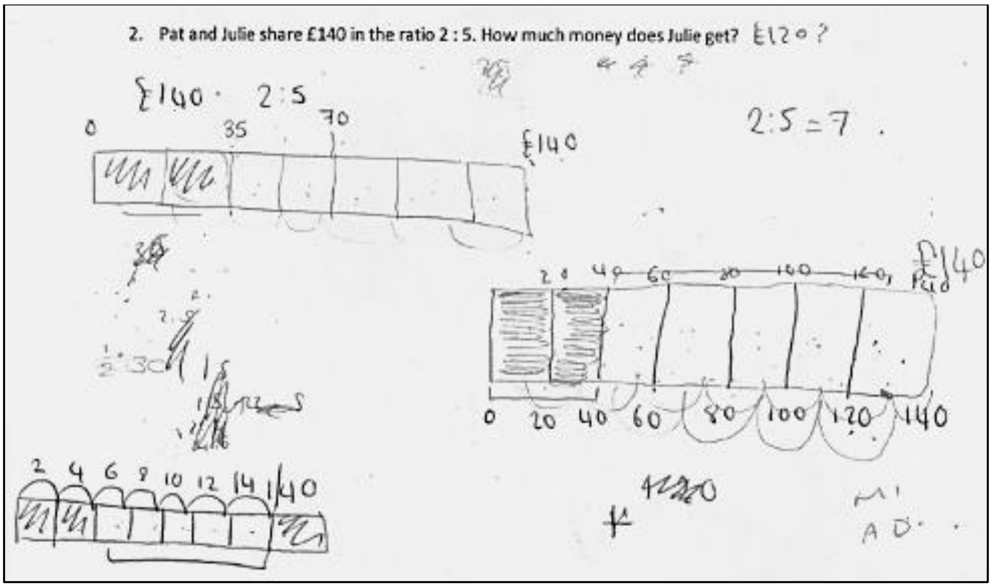

Figure 4-15 Student H, ratio question, post-test attempt

Student H was vocal in lessons about never having been able to do division. In her initial attempt, she writes 140 ÷7 = 2 but crosses it out. Drawing a bar affords her various other ways of trying to figure out the worth of one piece, including halving and halving again, counting up in twos on a bar to make 14 and guess and check in 20’s to make 140.

The examples above show that the transition to using a bar model to answer this question is not straightforward nor a magic fix. In lots of ways, it serves to expose even more of the gaps in the students’ understanding However, the intervention students did make more progress on this question than the control in terms of marks gained (from 27% to 56% for the intervention compared with 44% to 55% for the control). The fact that 37% of the intervention students used a bar post-test, 90% of whom gained marks, suggests that there is value in using this approach.

4.1.3 The bar model

The bar model has huge potential as a device for enabling students to think mathematically. By representing situations on a bar, students can visualise, make connections, deduce information and so make progress towards solving a wide variety of problems in Number and in Algebra. The use of bar modelling has gained momentum in England in the last few years as a result of looking to learn from the success of high performing jurisdictions, in particular Singapore (DfE, 2013). In Singapore, students are introduced to the method gradually as a natural part of working with Number. Use of the 'Singapore bar' is thought to account for why their students perform so well in problem solving (Englard, 2010). It is important to note that the way students in this study were introduced to the bar model, through the use of Realistic Mathematics Education, is subtly different to the approaches in Singapore and in England.

4.1.3.1 Realistic Mathematics Education and the associated bar models

The role of context is a fundamental part of RME. One of the main reasons for choosing a particular context for students to work with is that when students make drawings to represent that context, they produce a 'model of' the context which the designer knows has a potential for developing mathematical thinking. As a result of years of experience, the Dutch curriculum designers are aware of contextual situations which will lead to bar-like representations. A subway sandwich becomes a rectangular bar when you draw it with the ends squared off. Sharing that sandwich fairly, marking cuts on the rectangular representation of the sandwich and labelling the pieces with fractions, leads to a bar model picture, which the Dutch would call a fraction bar. Other contexts such as shading a rectangular shaped theatre to represent the percentage of seats filled would lead to a percentage bar type of bar model. For some contexts, i.e. marking bottle stops on a race route, it may be more appropriate to draw a line showing distance on one side and bottle stop position on the other. This is sometimes described as a 'double number line' but would still be classed as a type of bar model, where the bar has been flattened to look like a line. In RME, students are exposed to many contextual situations which can be represented by a 'model of' that particular situation. Students are seen to make progress when they start to see the similarities in the 'model of' situations, enough to be able to generalise the use of these models and apply them to other problems. When students have met enough of the specific context-related bar models as described above, then they may be in a position to draw and use a bar to represent a situation which is not obviously 'bar like'. For example, students who drew a bar to represent the photocopier problem are not drawing a bar as a 'model of' the photocopier itself, but are drawing a bar as a 'model for' solving that problem. In RME, this is described as vertical mathematisation (Treffers, 1987), where students can recognise the mathematical sameness of different problems and are able to choose an appropriate model to solve the problem.

Designing the material for the Number module required choosing contexts that would naturally lead to a bar model drawing. One such problem required students to show how to cut up a rectangular pizza so that one person had 4 slices and the other person had 5, and share the cost according to what they eat. Two strategies for representing this problem are shown below:

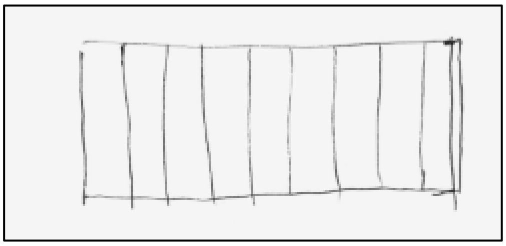

Figure 4-16 Student images for cutting a pizza into 9 equal slices: example 1

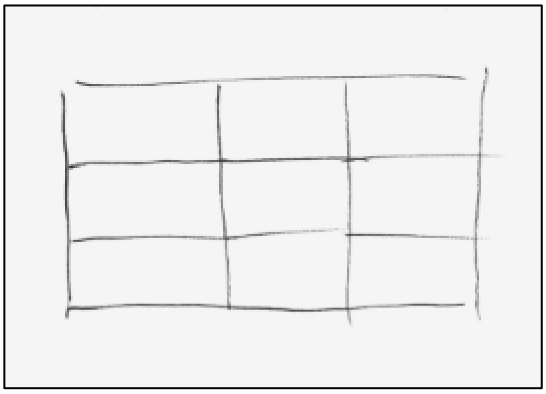

Figure 4-17 Student images for cutting a pizza into 9 equal slices: example 2

In the first drawing (Figure 4-16), some students drew the pizza first and then portioned it into 9 slices; their strategy for doing this tended to be guess the size of one slice, draw 8 same sized slices and then extend or reduce the last slice. Others started by drawing one slice, repeatedly adding slices until they got up to a 9 sliced pizza. Others split the pizza into 3 parts by drawing 2 vertical lines and then split each of the 3 slices into 3 pieces by drawing 2 horizontal lines (Figure 4-17). Students then develop their own systems for indicating which person gets which slices (one of the post-16 students always used his own initials, rather than the names given in the question) and how much each person should pay. The actions of drawing and splitting their bars can prompt mathematical thinking around the processes of division; that is, when students are considering how to distribute the £10.80 pizza cost amongst the people eating the pizza, they may see it as the whole amount split into 9 (£10.80 ÷9); or as guess a price for one slice and check it builds up to make £10.80; or split the cost into 3 parts (£10.80 ÷3 = £3.60) and then 3 parts again (£3.60 ÷3 = £1.20). In this example, the context of the rectangular pizza and apportioning the cost leads to a bar model which is very close to the context, a bar model picture which is at the ‘model of’ stage. In a later lesson, students were presented with problems similar to question 2, where they had to share money in a given ratio. One student said ‘it’s like the pizza problem’, another commented ‘what, is that what it means when I get one of those questions, that’s all I have to do?’. These students would appear to recognise that the bar which had represented a pizza in an earlier lesson could now be used as a ‘model for’ solving the sharing money problem.

By the time students were completing the Number module, the intention was for them to be able to see that the bar model and the ratio table could be used as a 'model for' solving a variety of different problems. In RME, the transition between the role of models from a 'model of' to a 'model for' is usually seen as a long term process. The fact that many of the intervention group were able to solve a range of problems in the post- test by drawing a bar or a ratio table is encouraging, considering that the intervention was for only 12 hours. However, we would see a case for a much longer intervention, to enable more students to see the power and the potential of RME based models for unifying their approaches and for developing their thinking.

4.1.4 The problems with division

There were many examples in both intervention and control scripts of the difficulties students encounter with division. In many cases, students were able to identify a division, re-write the sum using the standard 'bus stop' notation but could proceed no further. In other examples, students opted to find percentages such as 1% of 3300 by working out 3300 ÷100, using the standard division method, seemingly unaware of how inappropriate this is as a method for dividing by 100. Others applied the algorithm to finding $\frac{1}{8}$ of £600, when informal approaches linked to repeated halving may have proved much easier to perform. Anghileri, Beishuizen & van Putten (2002) refer to this as pupils failing to recognise the number relationships involved, and instead reaching for a formal procedure as soon as they know a division operation is required.

The bar model representation was helpful in the respect that it enabled some students to see relationships between numbers and, hence, re-engage with informal approaches. Students unable to find $\frac{5}{8}$ of £600 in their pre-tests drew a bar model representation, from which they could then target the value of $\frac{4}{8}$, $\frac{2}{8}$, $\frac{1}{8}$ and $\frac{5}{8}$ as illustrated in Figure 4-18.

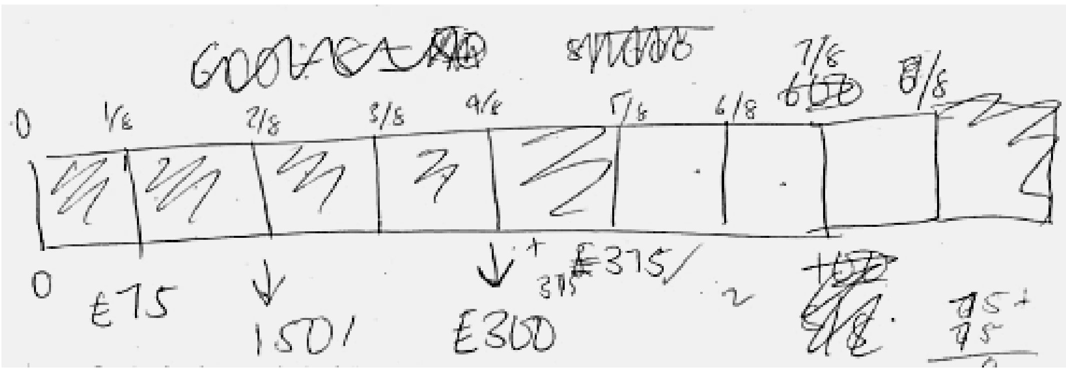

Figure 4-18 Using a bar model representation to find 5/8 of £600

The last test question required students to find the original price of a car, when the current price of £6820 was 20% less than the original price. None of the control students gained marks on this question in either the pre- or the post-test. By representing the problem on a bar, intervention students were able to see a straightforward route to the solution. An example of this strategy is shown in Figure 4-19.

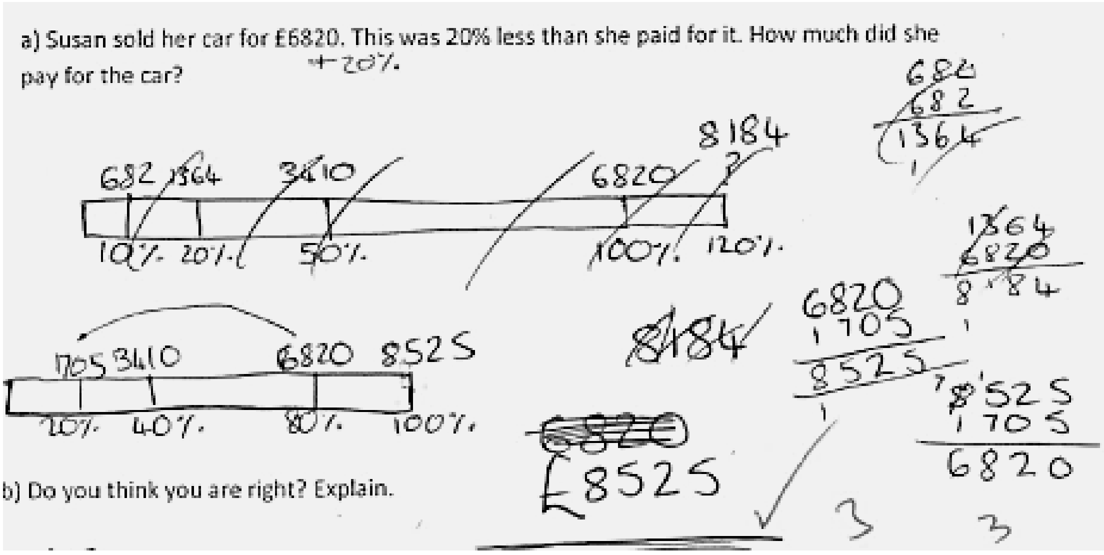

Figure 4-19 Using a bar model representation to solve a reverse percentage question

The bar model gave students access to strategies such as repeated halving as a means of working out the answer to division problems. Halving is particularly useful when the divisor is 4 or 8, but also helps reduce the size of the numbers for any even number divisor. There were examples in lessons where students had drawn a bar representing, say, £180, segmented into 12 pieces. By marking in the middle (£90 and 6) and the middle again (£45 and 3) they could see that 3 segments needed to be worth £45. Even then, the value of one segment was not always immediately obvious: some students used a grouping (quotitive) metaphor for division and began to count how many 3's they needed to get up to 45. Others used a sharing (partitive) metaphor, which amounted to give each segment an amount (say £10), check the total (£30) and adjust until the overall total is reached. Although these strategies were long-winded and inefficient, they were necessary and exposed just how little knowledge these students had of basic number relationships.

Equally alarming was the insistence by some students on always filling in the half and the quarter way point of their bars. Helpful as this was for 8 segments, it became much less so when their bar was cut into 7 pieces. Student J in Figure 4-20 went through several cycles of halving and combining chunks to fill in as far as the $\frac{1}{16}^{\text{th}}$ way point on his bar, but given that the bar is split into 7 pieces, these calculations served no purpose in finding one part. Student J was a very interesting case in that he scored zero marks in his pre-test, in which he put down answers with little or no working out. In his post-test he was able to achieve half marks, through the consistent application of drawing scaled bars, and yet his knowledge of number facts and number relationships was very poor. For him, the bar provided a means by which he could make better sense of what the question was asking, a way of recording interim calculations as opposed to storing them in his head. In the questions where halving strategies were beneficial, he was able to make good progress.

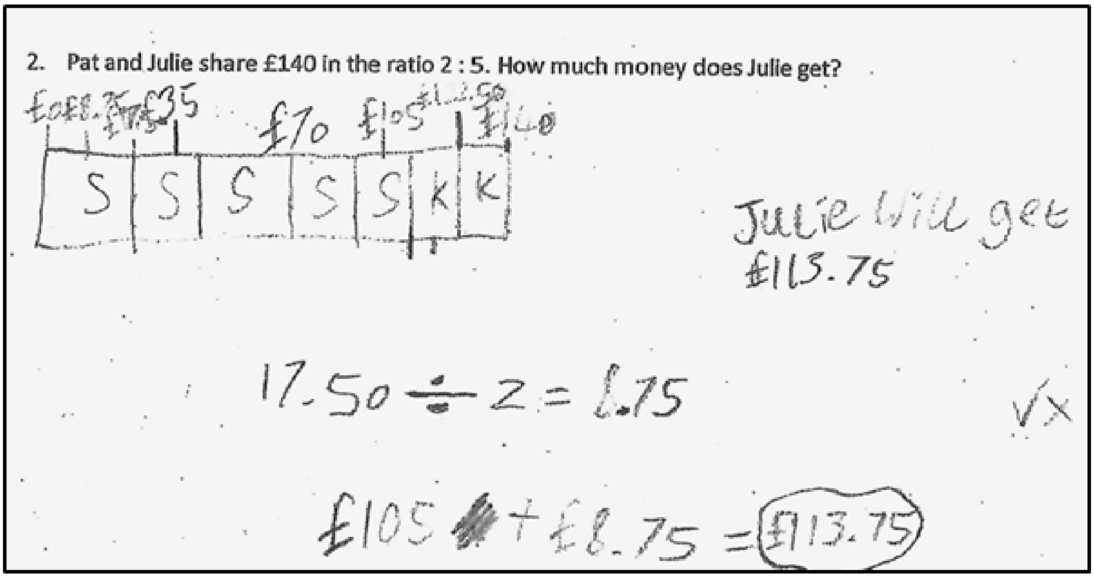

Figure 4-20 Misapplication of the halving strategy

The students spoke very negatively about their ability to carry out division calculations. Several commented how they had never been able to do that 'bus stop thing', but when asked for alternatives they needed to go right back to repeated addition of the divisor in very small chunks. In interview, one student's approach to finding 600 ÷8 was to start listing the 8 times table. He got past 80 before he realised that it might be helpful to count up in 80's rather than in 8's. It was apparent that he was not able to use the standard formal procedure and that the only method he had to fall back on was extremely inefficient and prone to errors. Anghileri, Beishuizen & van Putten (2002) stress the importance of building from students' informal solutions, of enabling them to structure and shorten their informal methods in order to develop greater efficiency, but not at the expense of sense making or loss of ownership. The approach to teaching division in The Netherlands involves staying with the informal for much longer, evolving the method of chunking so that students develop efficiency and speed by removing large chunks at a time. Chunking in this way can become extremely quick but with the advantage that students still have a sense of the size of the numbers involved.

4.1.5 Summary

The script analysis revealed a number of issues associated with learning topics connected to multiplicative reasoning. Students appeared to be relying heavily on memory of rules which led to high incidences of non-attempts and a mis-representation of strategies. Several students talked about not being able to remember how to do a particular question, or a feeling that they had never been able to do it. There was little evidence, particularly in pre-test scripts, of students attempting to make sense of a problem. At times, mis-application of procedures led to unrealistic answers, but students appeared oblivious to the unsuitability of their answers. Their ability to perform basic operations in number is a major barrier to success and a lack of fluency with times tables is still an obstacle for some. The concept of division is particularly challenging with many students showing that they cannot connect with the standard formal algorithm for dividing two numbers and yet they will have met this rule repeatedly over many years of schooling. The need for these students to work with different approaches to learning mathematics is very apparent.

The impact of the RME intervention was significant, not only in terms of usage (36 out of 49 intervention students used an RME model at least once in the post-test), but also in terms of enabling students to make progress. Where students had left blanks or given final answer only solutions in pre-tests, solution spaces became full as students sought to use their bar and ratio table models to find a route to the solution. Drawing a bar or a ratio table prompted the students to think differently about the problems, allowing some to re-engage with informal, sense making strategies, as well as providing a structure within which to organise and record their thinking. This was particularly useful to students struggling with formal division. In addition, the RME models acted as an assessment tool with diagnostic potential for the teacher and the learner. Using a bar or a ratio table encouraged the students to be flexible and creative. Once they had drawn the model they were free to fill in other quantities as they chose, and this placed much less demand on memory and helped to boost confidence.

In terms of marks gained per question, pre- to post-test, the intervention group fared better. Many of the intervention students applied an RME method to several post-test questions, and this is encouraging for two reasons. Firstly, it shows that students were beginning to recognise the unifying potential of the models as a strategy for answering questions across a range of topics in number. Secondly, it suggests that students were beginning to vertically mathematise and see how to apply their context-specific models to a whole range of problems. Given the shortness of the intervention, this is very positive and we would suggest that there is scope for even greater and longer-term impact within the context of a GCSE resit course with longer exposure to a RME approach.

4.2 The impact of the RME approach on classroom interaction

The design of each module in the intervention meant that the same basic structure was followed in all four intervention classes. Within this framework, there were local variations which can be understood as in part dependent on students' responses to an essential element of the RME approach: its departure from a common interaction pattern of closed questions and one-word answers, and a corresponding re-positioning of students as participants in sense-making. This requires the establishment of a different set of 'sociomathematical norms' - "normative aspects of mathematical discussions that are specific to students' mathematical activity" (Yackel and Cobb, 1996, p. 458). From the point of view of many mathematics education researchers (for example Boaler, 2002), this shift is generally desirable in terms of student engagement and understanding, but in an RME approach it is closely linked to its design principles and the need to build on visualisation of contexts and associated moves from 'model of' to 'model for' at a pace which supports students' developing formalisation. This meant that lessons were not necessarily identical across sites: the tutors responded to students' developing understandings and adjusted the pace accordingly. As noted in the independent evaluation (see Appendix 5), this variation was in keeping with the design principles of RME and represents a relatively high level of implementation fidelity. We illustrate these points in this section, contrasting two lessons covering the same material, in the school leavers intervention classes at FEL (FELLP) and at SFS (SFSP).

4.2.1 The survey bar lesson

This lesson appears at an early point in the Number module (approximately 5 hours into the 12), and involves developing ways to compare, add and subtract fractions using a bar (segmented strip). It builds on the ideas introduced in the earlier "Sweet Shop" lesson (see Appendix 2), of using contexts which are 'bar like' when drawn. In this lesson, the context is about displaying the results of various surveys on a segmented bar. As the context develops, students are asked to compare the survey results of a year 7 class of 30 students with those of a year 12 class of 20 students. The aim is to choose a means of representing both groups, thus encouraging students to work on informal ideas associated with the common denominator through the context of choosing (or imagining) a segmented bar which would allow both survey results to be visualised and displayed, and enable meaningful comparisons. The lesson is designed so that students are likely to come up with conflicting views about how to compare a preference for Indian food given by 3 people out of 20 versus 3 people out of 30. The lesson proceeds with numerous survey examples which raise further questions for students about representation and comparison. Towards the end of this group of lessons, students are set traditional bare fractions questions such as $\frac{2}{5}+\frac{1}{3}=$?, but the aim is that many will still be thinking in the context of the segmented bar, either by drawing one, or by thinking along the lines of what size bar would simultaneously represent – in this case - a group of 5 people and a group of 3 people.

Standard methods for teaching addition and subtraction of fractions rely heavily on memory rather than understanding or making sense of what is done through manipulation of the numbers. The approach taken in this lesson encourages students to develop their own ways of thinking about how to add fractions based on meaning and understanding. This lesson also requires learners to interpret and compare data presented in tables and charts and so exposes them to GCSE-type problems traditionally associated with the handling data aspects of the curriculum.

In this analysis, we focus on the following aspects:

1. Teacher-student interaction patterns, specifically turn-taking and wait time;

2. Socio-mathematical norms, specifically (a) teachers' positioning of students and students' self-positioning with regard to authority and control over sense-making; and (b) actions directed towards mathematical explanation and argument.

As a background for our analysis, some findings on the nature of mathematics classes and on standard patterns of teacher-student interaction are helpful. For example, Boaler (2003) compared traditional classes with 'reform' classes – ie classes based on an open-ended problem-solving approach as opposed to the more traditional approach in which mathematical methods are demonstrated by the teacher, and then practiced by students. Her findings are summarised in Table 4-4.

Traditional classes: use of time		'Reform' classes: use of time	
Teacher talks to the students, usually demonstrating methods	21%	Teachers talked to the students in the whole class	16%
Teachers questioned students in a whole class format	15%	Teacher questioned students in whole class format	32%
Students practiced methods in their books, working individually	48%	Students worked on problems in groups	32%
Average time spent on each mathematics problem	2.5 minutes	Average time spent on each mathematics problem	6.8 minutes
Total teaching time used	84%	Total teaching time used	82%

Table 4-4 Teachers' use of time in 'reform' and traditional classes as reported in Boaler (2003)

In addition to these aspects of time use, which are largely to do with teachers' choices, we can also look in more detail at these interactions. One major aspect of the standard IRF exchange is its sequence organisation: the talk involves sequences of turns, not just individual question-answer turns. Schegloff (2007) describes the most basic unit of sequence organisation as the adjacency pair, comprising two ordered turns, in which the second part is paired to the first in the sense that it is expected (eg question-answer). The standard IRF sequence comprises an adjacency pair followed by a sequence-closing third turn which is evaluative in some sense (Schegloff, 2007, p 118). Pairing also entails 'preference' – in an adjacency pair, there will be preferred responses to the first part of the pair. Speakers also aim to maintain intersubjectivity, or to restore it if it is lost, and 'repair' is needed (Schegloff, 2007). Hence 'trouble' (Ingram & Elliot, p. 40) is repaired, preferably by the speaker whose turn it is, and preferably initiated by them rather than another (self-initiated self-repair versus other-initiated self-repair). Repair by another person (other–repair) is least desirable. Silence can be interpreted as trouble, and so when wait time lengthens, both teacher and student feel the need to speak in order to repair it – hence the pressure on teachers to initiate repair through repeating, rephrasing or nominating another student when their initial question is not answered. Pressure to repair in this way can lead to a loss of the dialogic teaching associated with an RME approach.

Turn-taking in classroom interaction tends to follow identifiable rules, including the teacher's right to nominate the next speaker, and that speaker's right/obligation to respond. While this is no different from standard conversation, if the teacher does *not* nominate the next speaker, they have the right to continue – students do not have the right to take the turn of their own volition. If a student is speaking, the teacher is either nominated or has the right to self-select to take the next turn. If the teacher does not take this opportunity, the student can continue. This pattern of turn-taking is illustrated in Figure 4-21.

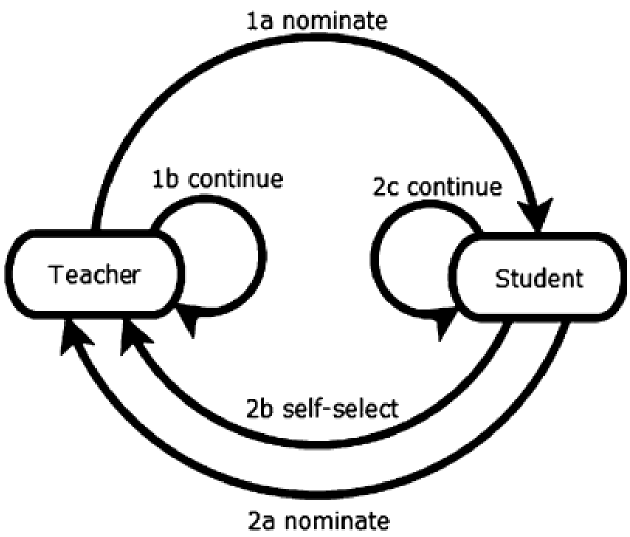

Figure 4-21 Classroom turn-taking structure (From Ingram & Elliot, 2016)

Wait time is a key issue in classroom learning, since it is built into the standard IRF pattern (ie the pause between teacher and student turns). A long-standing finding is that teachers typically leave less than one second between asking a question and repeating, rephrasing or even answering it themselves (Ingram & Elliot, 2016). Following Ingram and Elliot (2016, pp. 42-3), we note four categories of wait time:

- Wait time I (i): pause between teacher finishing and student starting to speak
- Wait time I (ii): pause following teacher finishing and then taking the next turn
- Wait time II(i): pause following student finishing speaking and teacher taking the next turn
- Wait time II(ii): pause following student finishing speaking and then continuing their turn

Extending wait time of all types is generally seen as beneficial in terms of allowing students more time to think and respond, with at least three seconds being a common target for wait time I (ii). Extending wait II times increases the likelihood of students explaining and reasoning, asking questions, speculating and interacting with each other (Rowe, 1986). Rowe also reported that extending wait times affected teachers' contributions, notably increasing the likelihood of questions which encouraged students to elaborate or explain. We take Ingram and Elliot's (2016) approach here, in terms of recognising students' roles in the *joint* construction of pauses. As they point out, turn-taking is an integral part of the IRF pattern, and from this point of view maximises opportunities for pauses in comparison to ordinary conversation, say, but it also relies on both parties, who can 'manipulate lengths of pauses to achieve pedagogical and other social goals' (p. 38). This is highly relevant to the use of an RME approach, which introduces what for many students is a novel way of approaching talk about mathematics, departing from the closed question-one-word answer form of transmission teaching that many have been used to.

4.2.1.1 Lesson A - FELLP

This lesson took place in a class of 9 students from the usual class of 13. This was their second lesson using the RME approach and was 90 minutes long. The PowerPoint slides are illustrated in Appendix 4. In the following analysis, we refer to slide numbers in this PowerPoint.

The general format of the lesson involved an initial period of whole class teacher-student questioning around the context of 'healthy eating' and the interpretation of a school canteen survey of students' food preferences (see SLIDE 1). The topic of fractions is merely mentioned in passing ("What fraction would corn be then?") in the 6th minute, generating 40 seconds of discussion linking the answer (an eighth) to the survey bar. The lesson is supported by the use of 7 survey scenarios which move progressively towards the use of fractions, but fractions are only introduced in explicit form into the survey context in the 53rd minute (SLIDES 4 and 5). Teacher-student questioning on this topic and discussion about fractions of a million in response to a student's volunteered contribution ("She could have asked a million people and you could still get those fractions") follows until the end of 60 minutes. The teacher introduces two new surveys (SLIDES 6 and 7) focusing more on fractions, what they represent and how they can be compared on a bar. Only at 69 minutes are fractions presented as bare fractions questions, although they are still closely linked to use of the bar, and the explicit focus of discussion is how the bar can be used to work out $\frac{1}{3}+$

$\frac{1}{8}$ and similar questions (SLIDE 8). At 79 minutes the students are set to work on exam-type bare fractions calculations (SLIDE 10) until the lesson ends.

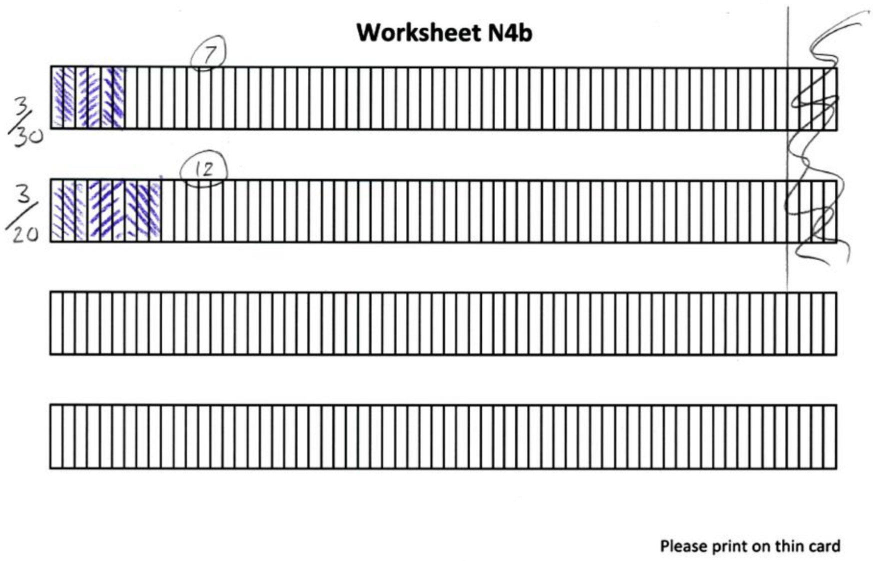

Figure 4-22 Sarah's solution

Alongside this pacing of the role of fractions, the lesson involves the introduction of a complexity which leads to paying attention to common dominator, as explained above. At 21.45, the teacher presents the key question of comparing 3 out of 20 and 3 out of 30, which generates 1 further minute of discussion which is not wholly conclusive. A further task working on this issue is presented in the 23rd minute, lasting for 9.5 minutes. In the 34th minute, the teacher returns to the comparison question and the idea of fractions: “I was just asking a moment ago about Indian, which is more popular, and you’ve got three on each one so it’s quite hard to tell which is more popular isn’t it and what we always do is try get it to a fraction, so what did you say the fraction was?”. The students offer estimates in a 30-second exchange, and the teacher then moves at 34.02 to discussion of one students’ solution (see Figure 4-22). This is prolonged and finally ends at 44.56, with general student agreement that Indian food is more popular in one class ($\frac{3}{20}$) than the other ($\frac{3}{30}$), and an agreed explanation as to why this is the case. The discussion ranges across representation of different numbers of people on equivalent bars, the concept of fairness, and opinion polls. At 44.56, the teacher introduces a new survey (the sandwich survey, slide 3) and at 49.16 asks another ‘which is more popular?’ question followed at 51.44 with another (“where is egg more popular?”). This generates some disagreement, with students initially saying both groups are the same but then changing their minds to argue against the teacher’s ‘devil’s advocate’ “Why do you think it’s year 8? It’s not it’s the same look!”. The lesson moves explicitly towards using the bar for comparing fractions in the final 20 minutes.

Table 4-5 summarises the overall use of time, while Table 4-6 tracks topic and activity through the lesson in numbered sequences. Table 4-2 is similar to Table 4-1 but breaks "Teacher questions students in whole class format" down into two categories: teacher questioning in whole class and teacher-led discussion of students' solutions. It also distinguishes between students working on tasks such as using the bar to represent the survey results, and students solving problems such as 'how much bigger is $\frac{1}{2}$ than $\frac{2}{5}$?'. It is notable that only 15% of class time is spent working on conventional fractions problems.

Lesson A: overall use of time	
Teacher talks to the students in the whole class	3%
Teacher questions students in whole class format	39%
Students work on tasks	22%
Whole class discussion of task solutions	17%
Students work on problems	15%
Whole class discussion of problem solutions	0.3%
Total teaching time used	96.3%

Table 4-5 Teacher use of time in Lesson A

Time (mins/secs)	Topic	Activity	Notable features T=Teacher, S(s)=Student(s)
1. 0-1 [slide#1]	Canteen survey	T introduces the canteen scenario	
2. 1-6.31	Interpreting what the survey bar represents	T-S Q & A	T's first question (how many people were surveyed?) is pre-empted by S volunteering an observation about the bar. Students all join in, sometimes talking at the same time. T's emphasis is on asking Ss to explain answers, and responses are sometimes several seconds long; there are few one-word answers.
3. 6.31-16.20	Class survey on favourite fruit	Student task 1 – draw survey results on pre-printed bars	Students volunteer comments on how they have represented the task and there is a lot of discussion with teacher explaining and comparing strategies.
4. 16.20-19.10	Sharing class survey representations	T-led discussion of solutions	Focus on how students have done the task. Student contributions are up to 20 seconds long.
5. 19.13-20.27 [slide#2]	Canteen survey	T introduces the two groups scenario	Some 'pure context' talk about food preferences accompanies this introduction.
6. 20.30-22.45	Comparing 2 groups	T-S Q & A	Key question at 21.45 "So […] if you compare the year 7 and year 12 class, do

			those 2 classes like Indian the same? Yes or no?" T funnels more than elsewhere in this sequence eg "Because there's only 20 in the class and...?" [rising intonation]. The question appears to remain unresolved.
7. 22.45-32.16	Comparing 2 groups	Student task 2 – draw survey results on pre-printed bars	T moves to colouring bars, one for year 7 and one for year 12. As students work, T prompts re whether their drawings allow comparison between year 7 and year 12.
8. 32.16-44.56	Comparing 2 groups	T-led discussion of solutions	T first discusses one student's solution which uses one strip per person for both year 7 (30 people) and year 12 (20 people) – "it's quite hard to tell which is more popular isn't it and what we always do is try get it to a fraction". T then invites another student to explain her solution, which uses 2 and 3 strips per person respectively (Figure 4-22). Discussion focuses first on her strategy but then students begin to argue about 'fairness'. Finally there is agreement that comparison depends on both bars being the same size.
9. 44.56-52.40 [slide#3]	Sandwich survey interpretation	T-S Q & A on how to represent survey data for comparison	Students volunteer the votes analogy used by T in the previous discussion. Later, the 'where is egg most popular?' elicits the wrong answer from one student (they are the same) but immediate disagreement from many others (year 8)
10. 52.40-60 [slide#4] [slide#5]	Teacher survey interpretation	T-S Q & A on 'how many' questions	Numerous voluntary contributions in this sequence. Extended discussion about fractions of a million and which numbers could be represented in the survey.
11. 60-62 [slide#6]	Background music survey	T-S Q & A on 'how many' questions	T is speeding up here, students call out answers, and a break is called.
12. 62-65.18	Break – students get up from seats to 'exercise'		
13. 65.18-68 [slide#7]	Hot drinks survey	T-S Q & A on how to represent survey data for comparison	Slide presents survey data in the form of fractions only
14. 68-75 [slide#8]	Drawing bars to add fractions slide 1	T-S Q & A as T models answers on board	Students discuss how to use the bar to add $\frac{1}{3}$ and $\frac{1}{8}$ and arrive at 24 strips.
15. 75-78.42 [slide#9]	Drawing bars to add fractions slide 2	Students work on problems	Teacher supports, several students talk out loud while doing the task.
16. 78.42-79.03	Drawing bars to add fractions slide 2	T-led discussion of solutions	A sequence of closed questions and one-word answers as T moves through each question.
17. 79.03-79.20	Bare fractions slide	T introduces bare fractions exam-	T compares these questions to those students are likely to encounter in the exam.

[slide #10]		type questions	"You'll have methods in the past that you can't remember but now you can use the bar to help you".
18. 79.20-79.59	Bare fractions slide	T-S Q & A as T models first answer	T asks what other numbers could be used to solve problems using the bar, and students respond with multiples of 12.
19. 80-90	Bare fractions slide	Students work on problems	Students work silently at first then talk out loud, sometimes to each other, teacher scaffolds and joins discussions.

Table 4-6 Time, topic, activity and notable features across Lesson A

4.2.1.2 Teacher-student interaction

Interaction in this lesson is characterised by the proactive role taken by students in co-constructing the discussion. In contrast to the standard classroom interaction patterns described above, they self-select and initiate new on-topic comments 7 times. Looking at turn-taking and wait times, the teacher finishes a turn and then self-selects as next speaker (type 1b in Figure 4-21) only once, in sequence 4 (Table 4-6), following a Time I (ii) wait time of 4 seconds. Turns are otherwise of type 1a and 2a, with Time 1(i) wait times of less than a second with 4 exceptions. The first entails a 4-second wait in sequence 2 which appears to be an invitation to self-repair a wrong answer:

T: How many people do you think there are in carrot and corn?
S: 100
[T checks others agree]
T: And you said 50 for corn…
S: Yeah. No 25.
T: What FRACTION would corn be then?
S: Quarter.
T: Quarter.
[4 secs Time I(i) wait]
S1: Quarter.
S2: It would be an eighth.
S1: Yeah.
T: An eighth? [looking unsure]
S3: Yeah I'm going with an eighth.
T: How do you get an eighth?
S3: it's 200 and 25 into a 100 is 4.
T: OK … so if these are all 25s.. [turns round to look at group] then it would be an eighth because you'd have 8 of everything in the whole thing.

The second and third entail 4- and 5- second waits for what turn out to be correct answers in sequence 4 and sequence 8.

The fourth involves a wait time of 5 seconds in sequence 9:

> *T: Without doing any calculations at all – egg – where is it more popular?*
> *[5 second wait]*
> *S: It's not it's the same.*
> *T: It's the same?*
> *S: Yeah.*
> *S: Yeah it is yeah.*
> *T: It's 6 in both, 6 people in both.*
> *Ss: Oh no.*
> *S: No it's year 8 again.*
> *T: You tell me why.*
> *Ss: [All talking at once] 8 times 6 is 40.*

There is, then, little evidence of 'trouble' in the lesson, in Ingram and Elliot's sense of silences which must be somehow filled or responses which are not preferred and require repair. Students' contributions are often long and spontaneous, reflecting non-standard socio-mathematical norms, as illustrated in the next section.

4.2.1.3 Socio-mathematical norms

Remembering that the standard pattern of interaction places the teacher as an authority and students as passive receivers of knowledge, there are numerous points in this lesson which indicate that the class is not following traditional socio-mathematical norms in terms of the teacher's and the students' positioning as participants in sense-making. In addition to the instances when students initiate new topics, or explain their solutions, there is an episode of disagreement and discussion in Sequence 8, in which students join with the teacher in establishing what is mathematically correct:

> *[This sequence picks up the unresolved question from sequence 6 – "…if you compare the year 7 and year 12 class, do those 2 classes like Indian the same? Yes or no?". T has been talking through one student's (S4's) solution (Fig. 4-22). The sequence has been edited to focus on the discussion.]*

> *T: So the top bar is for year 7 and the lower bar is for year 12. How many bars has she coloured in for year 7? Altogether.*
> *Ss: 60*
> *T: And year 12.*
> *Ss: 60*
> *T: So 60 for year 12 and 60 for year 7. Now erm for Indian, it's that bit for year 12 isn't it and that bit for year 7, that's right isn't it. … Erm so Indian goes up to there*

and Indian goes up to there on the second one. Does that tell you much about which one is more popular?
S1: No cos she's used two bars per person on the first one and three on the second one so it's the same amount of people, she's just used more lines or bars on that one so it's not – effectively it's three people and three people so it's the same.
T: OK. What d'you think S3?
S3: [clarifies which strip represents Indian on each bar].
T: Yeah, that's the Indian for year 12. And that's the Indian for year 7.
S2: It's the year 12 cos they prefer it more…
T: What do you think about – S1's argument was that she tripled the year 12 and doubled that [year7] and that's why it's bigger. Is it fair to do that?
S1: Sir it has to be the same it has to be fair.
T: What do you think S4? Do you think it's fair to do what you've done?
S4: No I just thought it was easier for me.
T: Yeah, on the bar.
S1: If you'd done [inaudible] give the year 7s 60 and the year 12s 40 it would balance out like.
T: OK. But then we'd have 2 bars of different length that would take us back to the same situation of hard to compare them.
Ss: Yeah.
T: I like S4's method cos she's got the same length. Are we convinced that – well we're not convinced that it's fair.
Ss: [inaudible]
[T works through the columns for year 7 and 12 multiplying entries by 2 and 3 respectively]
T: OK and what do those numbers [year 12 column] come to?
Ss: 60
T: And what do these [year 7] numbers come to?
Ss: 60
T: 60 OK. So if you had 2 year 7 classes of 30 with the same opinions this is what you'd get.
Ss: Yes.
T: And if you had?
S: Two year 12 classes.
T: How many?
S4: [correcting] 3 year 12 classes .
T: 3 year 12 classes with the same opinions that's what you'd get.
Ss: Yes.
T: So is that a fair way of doing it?
Ss: Yes.

S: Everybody's got a different opinion so there's more [inaudible] to do it… the answers are going to be the same cos it all depends on what people…
T: It's almost like what we do – you know in general elections they do opinion polls and they ask people. When they do a poll how many people do you think they ask?
S: Thousands.
T: They do ask thousands. yeah erm. How many thousand do you reckon?
S: Ten thousand.
T: But do you know how many people can vote?
S: Anyone over the age of 18.
T: How many might that be in the UK?
S: 30, 40 million.
T: So they don't ask 30 or 40 million people, they ask ten thousand people. And then they generalise on that. And that's kind of what you've done here, you've asked 20 people here and then you've times'd by three and you've got 30 people here and you've times'd by two so we've now got something we can compare. […]
Ss: [inaudible]
T: So let's go back to our question about Indian. So you have 3 people from each class.
S1: It's more popular in the year 12 class now. Cos if you're giving like … one person's got 3 votes, if you times that by 3 then effectively 9 people or 9 votes have gone towards Indian. But whereas for the year 7 you've only given them 2 votes so effectively they've got 6 votes.
T: And this is another copy of S4's, er and this grey bit is the Indian bit, yeah, so for Italian, which age group prefer Italian?
S4: Year 7?
S1: It's…no year 12s, year 12s
T: Yep
[…]
T: And how come you can do it so quickly?
S: It's just like we're looking at the general size of how big the bar is.
T: Yeah. Can you do it quickly on S1's?
S: Not as quickly.
T: Because?
[…]
S: The bar's not the same size.
T: The bar's not the same size. The bars help us compare. Brilliant.

This type of discussion is both required and generated by the RME materials. The absence of algorithms encourages students to interpret the contexts they are presented with and engage with their mathematisation. It is significant that bare fractions calculations are not presented until the very end, and in keeping with the underlying principle of RME, students are reminded that they can link this back to visualisable contexts.

4.2.1.4 Lesson B - SFSP

Lesson A illustrates how students can respond positively to the RME approach in terms of high levels of engagement, enabling a smooth progression towards meaningful manipulation of fractions. Lesson B illustrates some of the challenges presented by GCSE re-sit students, and how these can be accommodated within the RME approach. In this lesson we can point to instances where invitations to participate in ways which differ radically from students' past experience are readily accepted but there are also passages where the joint sense-making that RME makes possible required greater effort on the part of the teacher to generate.

This lesson took place in a class of 19 students. This was their 3rd lesson using the RME approach and was 103 minutes long. The same PowerPoint was used as for Lesson A.

As in Lesson A, the general format begins with the school canteen survey scenario (SLIDE 1) and progresses through to the teacher use of the canteen survey (SLIDE 5), following a pattern of discussion about the slide contexts, followed by work on how to represent them and discussion of solutions. The topic of fractions is raised by a student in the 4th minute, and this is picked up by the teacher, but both references are in the context of the difficulty of explaining why it is difficult to estimate numbers of responses in the SLIDE 1 data. The teacher returns to fractions in the 56th minute with reference to the "is Indian more popular in Year 7 or Year 12" question and 'comparing chunks'. The first explicit reference to fractions appears in 86th minute, during the Q&A session accompanying SLIDE 4: the data are presented as fractions and the teacher demonstrates $\frac{1}{5}$ of 20 using the bar. Fractions become even more of a focus in the final 10 minutes of the lesson, where the teacher spends time on how to represent fractions on a bar so that they will be comparable. The pace of the lesson is such that the later slides which focus on bare fractions are held over to the next lesson.

The overall pace is slower than in Lesson A, and we explore this with respect to teacher-student interaction and socio-mathematical norms below. The complexity introduced by the canteen survey (SLIDE 2) begins in the 27th minute (just 6 minutes later than in Lesson A in fact) with a question inviting a comparison between $\frac{15}{30}$ and $\frac{10}{20}$. This generates over a minute of discussion, followed by the Indian dinners question in the 29th minute, leading to a further 2 minutes of disagreement and discussion. Disagreement arises again in discussion of students' representations of the data on SLIDE 2, beginning in the 43rd minute and extending for the next 20 minutes. The next 2 slides take up the rest of the lesson (40 minutes); whereas in Lesson A the teacher moves through these with Q&A sessions totalling

10 minutes, in Lesson B the teacher sets the students to work on tasks for both slides, adding 15.5 minutes of discussion and 17.5 minutes of student work on representing the survey data. These different teacher decisions appear to reflect the large number of student voluntary contributions in Lesson A, in comparison to student difficulties and slow responses in Lesson B.

Table 4-7 summarises the overall use of time, while Table 4-8 tracks topic and activity through the lesson in numbered sequences.

Lesson B: overall use of time	
Teacher talks to the students in the whole class	5%
Teacher questions students in whole class format	17%
Students work on tasks	41%
Whole class discussion of task solutions	37%
Students work on problems	0%
Whole class discussion of problem solutions	0%
Total teaching time used	100%

Table 4-7 Teacher use of time in Lesson B

Time (mins/secs)	Topic	Activity	Notable features T=Teacher, S(s)=Student(s)
1. 0 [slide#1]	Canteen survey	T introduces the canteen scenario	
2. 1-8.05	Interpreting what the survey bar represents	T-S Q & A	This sequence is marked by a prolonged section with long wait times, and students appear reluctant to hazard a guess on how many people chose melon; they are unable to explain why they are finding this difficult. Eventually there is a breakthrough at 2.40 when one student notes that not all the sections are equal but wait times are still long. A key contribution comes at 4.50 when another student returns to the issue of inequality, prompting T to draw over the slide at 5.45 to show what it would look like if all votes were equal. Responses become spontaneous and immediate.
3. 8.05-21.00	Class survey on favourite fruit	Student task 1 – draw survey	T stresses that there is no right or wrong way to display the information on the class' fruit

		results on pre-printed bars	preferences and that students' solutions are likely to differ.
4. 21-24.15	Sharing class survey representations	T-led discussion of solutions	T focuses on how students have used the pre-printed bar, particularly those using more than one block per person and how they have calculated in order to use the whole bar.
5. 24.15—24.45 [slide#2]	Canteen survey	T introduces the two groups scenario	
6. 24.45-30.38	Comparing 2 groups	T-S Q & A	T asks a year 7/year 12 comparison question ("out of the year 7 and year 12 who preferred American more") ($\frac{15}{30}$ versus $\frac{10}{20}$), at 26.37 with a wait time of 4 seconds before nominating S to give an opinion. S gives year 7 as the answer leading to several Ss responding with argument, including that preferences are equal. T repeats the various arguments without evaluation and then moves to the Indian dinners preference comparison of $\frac{3}{20}$ versus $\frac{3}{30}$ (28.30). Many students contribute and there is disagreement again. T repeats the arguments, finally arriving at "you're saying there's less people so having 3 out of less people you think is more. We'll come back to that."
7. 30.38-42.59	Comparing 2 groups	Student task 2 – draw survey results on pre-printed bars	T tells students to use 2 bars to compare the 2 groups. T circulates and encourages Ss to think about the question and not to guess.
8. 42.59-62	Comparing 2 groups	T-led discussion of solutions	T focuses on one S's solution strategy: "So how did she make her bars come to the same length? Even though there's not the same number of people. Cos that's not what most of you have done." T moves on to repeat the comparison question (51.56) while demonstrating on S's solution that the answer is now clear visually "I think some people gave an argument to say if you've got three out of twenty that's a bigger chunk than three out of thirty". Some Ss argue voluntarily that 'it's exactly the same' (53.55)

			T remarks "So we're still hearing the argument both ways and it's not been resolved for some people – go on" (54.20). In response to a further voluntary contribution T offers an argument concerning fractions and percentages: "So if I was to do fractions to me one out of more is a smaller thing than them [pointing to board drawing]. I know what you're saying they look the same but when you're comparing things it's like when you do percentage you have to put them out of a 100 you have to put them out of the same amount and here they aren't out of the same amount. So fractions-wise I think this is a smaller chunk of this". (55.20)
9. 62-65 [slide#3]	Sandwich survey	T introduces the sandwich topic	T uses this space as a rest in addition to topic introduction
10. 65-75.00	Comparing 2 groups	Student task 3 – draw survey results on pre-printed bars	T sets task: "If you wanted to draw bars to compare these and you wanted to do what S did and you wanted to use the same length bar how many strips would you give to each person?" S's talk to each other, T does some explaining at the board to individuals, and circulates around class.
11. 75.00-85.09	Comparing 2 groups	T-led discussion of task 3	T asks 2 students to put their solutions on the board and asks Ss to explain what they think the thinking behind the solutions is. Students are slow to answer but at 81.10 one makes a lengthy contribution about how to represent data on bar strips. T focuses on how to use the bar to represent comparisons using different numbers.
12. 85.09-89.58 [slide#4]	Teacher sandwich survey interpretation	T-S Q & A on 'how many' questions	This is the first explicit use of fractions in the lesson. T demonstrates $\frac{1}{5}$ of 20 using the bar and works through the question "How many said tuna OR egg".
13. 89.58-90.23 [slide #5]	Teacher canteen use survey	T introduces	

14. 90.23-97.55	Drawing bars to add fractions	Student task 4	T: "how many people do you think Jan would have asked?" – asks students to think alone and then on tables. T prompts: "Think of a number that you can have a third of a quarter of and a tenth of".
15. 97.55-103.32	How to represent the three fractions	T-led discussion of task 4	T spends this time on students' difficulties with Task 4. "If you had to show a third [on the strip diagram], on one of these, how many, how long could you make the bar and be able to show me a third?" Students offer that 200 would not work, that three strips would work, T asks for more numbers and asks "could you make it 5 long?". One student says yes but others offer 6, 8 (rejected by T), 9, 12, 15. An offer of 10 is rejected by other students. S's join in generating multiples of 3 up to 1500.

Table 4-8 Time, topic, activity and notable features across Lesson B

4.2.1.5 Teacher-student interaction

Students are less proactive overall in this lesson, although there are passages where they are quite animated, particularly towards the end of Sequence 8, where they are eager to make contributions to the discussion about $\frac{3}{20}$ versus $\frac{3}{30}$. A striking feature of the lesson is the length of both Time 1(i) and Time 1(ii) wait times. Time 1(ii) wait times are particularly evident early in the lesson, in Sequence 1, when the teacher is inviting students to suggest how many people preferred the fruits. The teacher self-selects (type 1b turns in Figure 4-21) on 5 occasions, with wait times of 12, 11, 10, 4 and 4 seconds. These all come before a key contribution at 4.50. There are four wait 1(i) times in this sequence, of 4 seconds (eliciting a key response at 2.40 about inequality), 7 seconds, 14 seconds and 2 seconds. After the last of these, the teacher appropriates the student's suggestion and the interaction continues with 1a/2a turn-taking to the end of the sequence.

> *T: And how many people do you think preferred melon, that was their favourite fruit? [12 second wait 1(ii)] Melon. S1, what do you think for melon [11 second wait 1(ii)] no idea? Ok what's stopping you having a guess [10 second wait 1(ii] S2 any idea, melon – no? Can you see where melon is on the chart and what did you tell us S3?*
> *S3: There were 200 people.*
> *T: Yeah there were 200 people and this is zero and this is 200 so I'm almost asking how many people were in this bit. So you've got the 200 people lined up there and I'm asking how many people were in this bit. How many people gave that vote. Have you got an idea S5? [4 second wait 1(i)]*

S5: If you split, if you try and split there's 200 in one section so there are 4 sections but they're not all equal so you could do… oh you mean the top one…

T: Yeah cos I asked about melon …

S5: It's the smallest one out of all of them so you could guess that it's - 10? Or.. I don't know.

T: Well I think that's helped what she said, hasn't it. So it's split into 4 but the 4 aren't equal. And presumably when you add all these four up what should you get?

S: 200

T: Yeah. 200 cos there's 200 people altogether. S6, any ideas? [4 second wait 1(ii)] could anybody tell me about any of the other strips what they think how many people they represent. Just raise your hand [4 second wait 1(ii)] could anybody tell me anything about this? You said about splitting it up, S5 [7 second wait 1(i)]

S5: Yes. I thought that … but I'm not sure.

T: You think what?

S5: I think ... fractions … but I'm not sure.

T: You have fractions coming in between did you say? What d'you mean by that?

S5: Erm… it's hard to explain.

T: Try, try, because somebody will help you I'm sure [14 second wait 1(i)]

S5: Erm [laughs; other Ss laugh and some talk off topic]

T: S6 any ideas?

S6: No.

T: What's stopping you having ideas then?

S6: Because a lot of different sizes.

T: Right OK. If they were all the same size how many do you think would be in each one?

S6: 50 each.

T: 50. Yeah because. OK and if they were all the same size, how would they be blocked up, where would the lines be? Do you know? [2 second wait 1(i)]

S6: [Shakes head]

T: Can anybody help with that question? What S6 is saying is if these were all the same size there'd be 50 in each, cos 50 and 50 is a hundred and another 50 is 150 yeah.

S: So that gives you 4 sections.

T: Yeah there'd be 4 sections and they'd be equal. So – and they're not equal – which was your thing wasn't it [to other S] so you'd have to make an adjustment on that. [S6 is nodding all through this]

T: Right do you think this one's going to be more than 50 people or less.

S: Less.

T: Yeah because it's smaller than – can I draw on this? [asking about whiteboard, T draws on slide] Right if it was 4 equal sections, where would the lines be? That was the question I asked.
S: You'd have a point in the middle.
T: Right in the middle yeah because that would be 2 equal sections.
S: And one on each side in the middle.
T: And one on each side in the middle. Right and if the whole thing was 200, as I've said there'd be 50 in each. So it's almost like if I was to fit this one in here.
Ss: It'd be 25.
T: Ah so it's coming close to 25. Ok. That suddenly made – yeah, you fit this in here, it looks like it's got half, that's 25 people. Ok erm tell me about oranges. How many people do you think said oranges?
S: Think it'd be 50.
T: S7 what do you think, where do you think they got 50 from?
S7: Cos this is like, the thing that you've drawn is like the same size.
T: Because the orange looks very similar to …. Ok .. good.

Elsewhere in the lesson, wait 1(ii) times occur 5 times, with 4, 7, 2, 2, 4 second waits. Three of these occur in the last 30 minutes of the lesson, and the first 2 in Sequences 6 and 7, where the Slide 2 dinner choice survey and its representation are debated. Two further wait 1(i) times of 2 and 6 seconds occur in the last 30 minutes, and another of 10 seconds in Sequence 7.

4.2.1.6 Socio-mathematical norms

The patterning of Sequence 1 appears to indicate that the students are initially reluctant to take responsibility for meaning-making, but that this may largely be due to an assumption on their part that an exact one-word answer is required by the teacher – which of course is not possible with the information they are given on the slide. Once this is clear, they appear more willing to speculate. Sequences 6, 7 and 8 emerge as key points in the lesson where the students engage in discussion. This extract from Sequence 6 illustrates the argument:

T: So out of the year 7 and year 12 who preferred American more [4 second wait 1(ii)] do you see what I'm asking S8 there, have you got an opinion.
S8: The year 7s.
T: The year 7 preferred it more than the year 12s, go on tell us why.
S8: [inaudible]
T: So initially you're saying to me you thought the year 7s presumably because 15 is more than 10.
[Ss all talk at once]
It's equal miss.

No it's not.

Because…

T: OK let's hear a few arguments about this. S9 what were you going to say?

S9: [lengthy contribution, inaudible]

T: So your argument is along the lines of if you do 30 and you take off the 15 you're left with 15 and if you do the 20 students and you take off the 10 you're left with 10 so what's that saying.

[Ss all talk at once]

S: That's not equal

T: Go on S10 what were you going to say?

S10: Miss I think it's … because the total of the students so it's going to be a half of them that chose it.

T: So you're saying it's the same because half of them in year 7 said American and half in year 12 said American

S10: Yeah

T: OK. Ermm- Was Indian more popular with the year 7s or with the year 12

[Ss all answer at once, some say year 12 some say year 7]

T: OK so some people are offering year 12, let's have some opinions over here. Erm S11 do you think it was the year 7 or year 12 where Indian was more popular?

S11: Year 12.

T: Because?

S: Because it's out of 20 whereas year 7 is out of 30.

Multiple Ss talking, one voice heard clearly: That doesn't make sense.

T: Let's hear another argument then, somebody said it doesn't make sense you must be thinking of something else. S12 what are you thinking?

S12: Out of 30 is more than out of 20 because there's more students.

T: So you're arguing it the other way round, you're saying there's more students so 3 out of more students is more than 3 out of… whereas you're saying it's out of less students.

S: Miss they're the same I think.

T: Do you I don't think that's what you said.

S: That's what I meant though.

T: Go on then.

S: I think it year 12 because there are less than in year 7 so basically more people want it in year 12 than in year 7.

T: So you're saying there's less people so having 3 out of less people you think is more.

S: Yes compared to those…

T: Well we'll come back to that.

Although progress is slower in comparison with Lesson A, the students in this lesson are eventually willing to participate in argument. One reason for this might be due to the extended wait times earlier in the lesson, which establish these norms.

4.2.2 Summary

The close analysis of these two lessons illustrates a number of features of the RME approach, including the use of context, the shift from 'model of' to 'model for', and a slow move to formalisation maintaining a continued connection to context. It also illustrates some of the challenges of introducing the RME approach to the resit context, in terms of students' responses to the need to engage in ways which are new to them, as in Lesson B – that the difference for SFSP students was quite substantial is indicated in the student and teacher interviews in sections 4.3 and 4.4. However, the analysis here also illustrates how it is possible to maintain adherence to RME design principles through the use of extended wait times and an adjustment of the overall pace in order to support students' developing mathematisation of the context. We return to the implications of this in Section 6.

4.3 The student experience: GCSE resit classes and RME

GCSE resit students are a vulnerable group in terms of the impact of their prior experience of learning mathematics on their confidence and motivation. CDARE's analysis of the attitude test and 'before' and 'after' differences did not show any change (see Appendix 5). However, as the literature review in Section 1 and the analysis in Section 4.2 suggest, negative perceptions of mathematics are likely to be quite entrenched in this group of students. This makes it all the more important to understand students' experience of resit classes in general, and their perceptions of mathematics. Given the impact of their prior experience and the nature of mathematics teaching, we were also interested to hear what intervention students thought about the experience. Hence in this section we focus on the interview data. We interviewed students in both control and intervention classes to find out about their experience of failing GCSE and doing resit classes, their perceptions of mathematics, and – for the intervention students – their experience of RME classes.

4.3.1 GCSE resit stories – why are they here?

The students explained their previous failure to gain a grade C with reference to external factors relating to schooling but also to qualities in themselves. Some had made a number of attempts.

4.3.1.1 Teaching at school

School factors included multiple teachers or poor teaching, mis-match between teaching and exam content, and the impact of setting. They were often associated not just with failing but with causing students who had liked mathematics in the past to dislike it. Harriet and Lucy (FELLC) criticise teachers for over-reliance on text books and failing to explain:

> *… the teacher I had in Year 10 was good, but then he left, so we got another teacher in Year 11 and she just like didn't really teach us. She put like a text book on our thing and we kind of had to learn through that. [What makes a good teacher?] Someone who'll explain things properly in a class and not just like throw a text book. (Harriet)*
>
> *… if we like asked her to explain it like a certain amount of times she just couldn't do it. She'd only explain it like two times. … That's why I didn't like it so ... (Lucy)*

Andrew (SFCC) had a similar complaint:

> *… I don't get shown how to do the questions and answers properly and I forget them easy and then they never come back to them and then when it comes to the exam I'll just forget them and don't know them…. Like ratio I get shown once and when I ask for help there's not much help.*

Sienna (SFCP) described the reverse situation, explaining that, having done badly at primary school, she had liked mathematics since entering Year 11, because her teacher did not rely on a textbook:

> *I never used to like it until I got into Year 11 because I got like a teacher that I actually understood, because I used to have teachers who didn't really teach you anything, like you never learnt. … He just, he actually wanted to help and like taught you properly how to do it and went through it if you need help and the other teachers just give you a text book.*

Levi (SFCP) explains that he was a top set student and doing well from years 7 to 10. Mathematics had always been his favourite subject but he became bored of it towards the end of year 10 because it was too repetitive in the run-up to GCSE:

> *It was just like because from Year 7 to Year 10 no one really put pressure on us. … Like we didn't have to like stay behind after school. … but then in Year 11 we had to do it all the time and that like, that was like every day and every lunchtimes and every after school and it just got boring doing the same thing over and over again that I weren't really into doing.*

He blames his failure to gain a C on the fact that teaching had focused on certain areas of the syllabus only:

> *… they weren't teaching us what we needed to know, because like in the test half of the things that was in it I've never seen before. Like I felt like they should be teaching me my weaknesses not my strengths. … but there was questions that … I would skip, but I couldn't do that in the test because I had to answer all the questions, so I have a chance of getting more marks, so I prefer like the teacher should've taught me things that I wasn't good at. Instead I was given the work that I was good at, that I would just finish like that.*

Ruqya (SFSC) told a similar story, demonstrating how aware of marks the students are:

> *… the questions they weren't like what I expected to see in the paper. I don't think the lessons prepared me that much, cos we used to like to focus more on the questions that were like 5 marks. But those were the sort of things that I didn't really need help on. … But I was like quite good at them, it was more like questions got to do with like equations and percentages. [And how many marks go with those questions?] For equations there's about 10 marks.*

4.3.1.2 Setting

The 6th form students raised a different issue: the demotivating effects of setting, and its related teaching. Imogen (SFSP) was in 'a pretty bad set. I was in set 8':

I've got a long experience of people telling me that I'm not good at maths. … Like even my parents say it. They're like you're not that good at maths. It's not really your strong point. … So you kind of, you kind of like when you get told something enough times you're kind of like "okay I'm not good at maths". … I didn't really like it [in set 8] no and I think that's partly the reason why I failed as well, because I thought I must be stupid. My teacher doesn't really care, so…

Her main criticism is of the teaching she received:

I didn't like the way the lessons were being taught. I don't think that's how I learn. Like how the teacher used to learn me, she just used to give us past papers and sit us down and do it and then we'd leave the lesson and she wouldn't even mark it half the time, so we'd kind of go away wondering is this the right answer, is it not the right answer. … I think like because they've got so many people to teach I think they can't really spend time in the lesson writing down exactly how to do every single equation. I think it's more everybody has to just pick it up by themselves.

Pania (SFSP) was also demotivated by failure, feeling 'dumb', and not being able to get more than a C in the Foundation paper:

[Do people think that people in the lower set are dumb then?] Yeah. Cos like when I looked back … said I wasn't doing the Higher, they think like oh yeah I couldn't really do the Higher because I wasn't capable of doing it. So yeah it kind of put me down. [Do you think everybody in the lower set feels like that then?] I think so because they know that you can't get more than a C. So you know that's the only thing you're capable of doing. And mostly like … when you think about getting a C – you think that we're not really getting a lot.

The issue of whether they were entered for Foundation or Higher Tier mathematics was important to many students, because of a general claim that it was easier to gain a C in Higher Tier exams. This was supported explicitly by one of our sites, but rejected by another, as the teacher interviews show.

4.3.1.3 Forgetting and failing to understand

Other students tended to blame themselves rather than their previous schools and teachers for their failure to gain a C. A major theme was simply an inability to remember or to even understand in the first place. Shelby (SFCC) was exercised by the fact that what she learned in September would be easily forgotten by the time the exam came:

... in September what you learn, you don't really learn at the end, you learn this stuff every like term, don't you? … So you've already forgotten it. ... you could come into lesson and you could be like 'Right we're doing ...' and then it re-jogs my memory

> *from last year I think 'Oh I remember doing that'. But if you just wrote the question down in September and went 'Do that' from like July I'd be like 'I can't do that'.*

Abbie (FELLP) said she had never liked mathematics and always struggled with it. She sees this as a personal trait which others don't have:

> *I didn't enjoy it and it takes a while for it like to process in my head. I don't pick up as easy as other people ... I used to like cry before I used to go to my lessons in school, just hated it so much.*

In classes she just aims to get by without causing trouble, pretending to understand:

> *… when everyone else is getting it, I sort of pretend to get it, cos I don't like to disrupt the class or anything. … I don't want to like drag everyone down like, having the teacher explain to me for ages. …. I mean I know there's probably other people in the class who don't get it, but we're all just like being quiet.*

Zoe (SFCP) also describes herself as someone who has never been good at mathematics and isn't interested in it; she can understand what to do in class but 'it just doesn't stick'.

> *I want to like it. Like I want to do good in it, but it just doesn't, when I try it just doesn't stick.*

Others felt that there was a ceiling on their understanding. Lucy (FELLC) says she was good at mathematics at primary school and was also in the top set until her GCSE years, but then couldn't improve, and couldn't understand:

> *I think I was like comfortable there... up until the last two years really. Then the last two years I felt like I couldn't get better… I felt like I was stuck at the same level for the like both the two years, like I didn't get any better. … I think I got to that point as well like, because I wasn't understanding I didn't like it and then I just didn't try.*

4.3.2 Experiencing resit classes

4.3.2.1 Motivation and confidence

Most students framed their accounts of resit classes in the context of needing a grade C in order to apply for further training in a variety of careers - nursing, cabin crew, physiotherapy, apprenticeships – in or order to continue study - access to university courses, health and social care, IT, psychology, and sports science. A handful said they were simply on the course because their college/school required them to be.

For some students, this situation was a source of motivation, as for Lucy (FELLC):

> *I've had a job and everything but it's just been a job. Whereas now I'm thinking of like a career. … I want to do either cabin crew, do you know like on planes ... and you need like an A to C in English and Maths to do that.*

The adult class members described themselves as highly motivated – some had come from functional mathematics classes in the previous year, and were targeting GCSE grade C in order to make major changes to their lives. Nancy (FELAC) was a typical case of returning to study in order to gain a place in nurse training: after having worked for ten years in retail before having a child, she wanted 'to better myself'.

Some students said they were working harder now, and some even had tutors at home. Ruqya (SFSC) described herself as different this year:

> *I think like last year I was the sort of person that like ... I did have an interest in studying, but not as much. … But now I'm like really focussed and like concentrated. … So I'm like 'I want to do this now'.*

The majority of the FE College students talked about other changes in themselves in re-sit classes. They described being more focused and having more confidence, usually because they felt able to ask when they didn't understand in smaller classes with others like them. Shelby (SFCC) is in her second re-sit class and has finally gained confidence to say she doesn't understand:

> *And last year there was more like confident people so you don't want to say 'Oh I don't understand' do you know what I mean like? So I didn't really like say when I didn't understand stuff. So then I did an exam and I got like a D in school and then I got an E in college. And then like this year now I'm doing it again – I just tell him when I don't understand stuff. Like he says to me like 'How do you feel to be one of the smartest ones in the class?' Because I actually ask him now and I actually understand stuff, whereas before I didn't ....*

Lucy (FELLC) was confident that she would be able to improve in the smaller FE class:

> *So yeah I was like more like ... [more] chance to speak to you if you don't understand, cos there's like not as many people.*

Abbie (FELLP) felt more able to speak in a class of people in the same situation:

> *I do feel more like able to speak in this class because we're all in it for the same reason … But in school I couldn't cos everybody was probably at different levels.*

4.3.2.2 Relationships in resit classes

College students explicitly connected their new confidence to the different relationships with tutors at FE college in comparison to school. For Harriet (FELLC) this was about being treated as an adult:

> *… you kind of feel more mature. You know like you do more things independently and things…. I think the kind of the tutors just like put more trust in you and they*

believe in you more … I feel like I'm more confident in college. Like in school I was never that confident.

Joel (SFCP) tells a now familiar story of never being very good at or interested in mathematics and not really trying until Year 10 when it was too late. Consequently he lacked 'the basic knowledge' and only managed a D in GCSE. In his current class he feels 'average', and he appreciates the more personal touch of FE teaching compared to school.

… he's a different person and it's like it makes it a whole lot better because you can actually speak to him. Like in high school and that I was never like, I was never really one to put my hand up and say I don't understand it … In high school it felt like if you put your hand up everyone was just like staring at you …

Relationships with other students are better too:

No one, obviously no one was like mature in high school, whereas like here everyone's mature, so … I don't feel like I'm under pressure like when I put my hand up here. Whereas in high school like with everyone staring at you and stuff it's like …

Like Joel, Sienna (SFCP) appreciates her FE teacher because of his good relationships with students and his willingness to explain. She says she is now enjoying mathematics so much that she would like to carry on to AS level.

The 6^{th} form students had less positive stories. Imogen (SFSP) returned to the setting issue:

… in the high school it's a lot about labelling, like not only do we have the different sets, we have the X band and the Y band and the Y band are meant to be stupid and the X band are meant to be a bit smarter. … I'm going to really apply myself, but if you're in set 4 for Maths, you're going to think I'm not that good at this subject, so I don't see the point of trying. [… is it different in sixth form?] I think it can, it is and it isn't as well. Like I'm still 16 and basically I've only like left high school, so I still feel stupid when I don't understand something and I still feel like I can't put my hand up more than once because other people in the class are going to get aggravated.

Pania (SFSP) is trying to work harder and be more determined despite demotivation:

[Does it feel different being in 6^{th} Form … does that make any difference to you doing the class in 6^{th} Form?] Not really, no.

4.3.2.3 Resit teaching

Some FE students talked about better teaching at College compared to a tendency to teach from text books in school. Zoe (SFCP) prefers college because 'in school we just worked out of text books and like we always swapped teachers and things, so it was never like constant'. Levi (SFCP) also enjoys the teaching at FE college:

… the method of teaching is different from high school. Like in high school they don't really have fun with you or anything on the questions. They just set out the questions and you've got to do it, but sometimes Mike just like goes through it and like he doesn't put too much pressure on us. He puts pressure on us, like challenges us to do work, but it's not too much pressure where we think ah, I don't want to do this. …He teaches us different methods to be able to do the questions. … So we can pick which one we want.

However, despite the obvious motivation of working at mathematics in order to access their chosen career, some students found this difficult to sustain in practice in the face of continuing failure to understand or plain boredom. Although many FE college students described their resit teaching as good, appreciating the smaller classes and better atmosphere, others described it as too infrequent to really help them – Andrew (SFCC) complained that 2 hours a week was too little - or only helpful when it was simply a reminder. It was not able to deliver new learning of material they had never understood, as Caterina (SFCC) explains:

… certain things that I was very good at in the past and I'm doing again, once it is reminded or recapped I caught up very quickly. If it's things that in the past I didn't like it, like ratios or big divisions I'm there like oh I don't know if I'm going to be able, I'm always unsure all the time.

The adults at FEL described a specific issue with teaching at FE, concerning 'new methods'. Nancy (FELAC) explained:

I do it slightly different, but I get to the answer sometimes quicker than others and it's just because I've been taught a certain way ... I find that easier even though sometimes they teach you it in a different way. And I'm like I'm sticking to the way I know, because I know that I get it right sort of thing. … it's purely that I find it a lot more complicated certain ways that she goes round, because on a lot of things you can do it in totally different ways. … And I was going to say some things work for me and some things don't, so I'm sticking to the one that I know that works for me instead of … I'm willing to try new things, but if I don't grasp it straight away I'll go back to the old formula.

This issue came up for the FELAP students too, as discussed in 4.3.4.2.

4.3.3 Perceptions of mathematics

The students did not necessarily dislike mathematics, although many of them did; of these latter, some said they had never liked or been good at mathematics, while others said they had stopped liking it during secondary school. As we have seen, the dominant story was one of being taught a mathematics which they then forgot or did not understand in the first place, and this was coupled with a perception of mathematics as a collection of fairly arbitrary rules which must be learned but are easily forgotten.

4.3.3.1 Learning rules versus understanding

An emphasis on mathematics as learning rules, with right and wrong dependent on the authority of the teacher, emerged when we asked the students how they knew if they understood and were right. Andrew (SFCC) replied:

> *Because I know I've got it right and then the teacher says I've got it right and the book says I've got it right and I remember the method.*

For Ruqya (SFSC), it all depends on the teacher:

> *I will do a question and then I will just ask the teacher if I've got it right. And if I've not got it right and he tells me I've done wrong, I'll go home and I'll practise the questions so I make sure I know what to do. [Do you ever know yourself that you've understood?] No.*

Akia (SFSC) was totally focused on marks:

> *Because I used to do all the homeworks on time, and then my teacher used to mark them. Oh yeah and the exam papers – at the start of Year 10 I was like really bad with them – at the start of Year 11 there was a huge progress. Like in class everyone used to get like … like they used to get half way, but I used to get 80s out of 100, so it was quite a big difference. [So … so how you know you understand … have I got this right? … it's because you get it right?] Yeah.*

Imogen (SFSP) talks explicitly about mathematics as rules to be learned, and understanding as knowing how to apply them. She clearly thinks that there is no common sense involved:

> *I've always believed that Maths is like a game, like if you understand the rules you can understand the game, so if I know how to do something I'll understand all of it. … I think it's like if you learn how to do the equations, you know how to do the … You can forget the equations really easily. It's not like with other lessons with common knowledge involved in it. …. It's not common sense that you have to know the specific things and then …*

Some students were more explicit about understanding and wanting to understand; they weren't happy to just get things right, as Martha (FELLC):

> *I would prefer to actually know how to do it, because then I know whether I've got it right or not rather than before where I just hope that it was right.*

Lucy (SMC) agrees:

Like understanding why it's done that, why you're like ... how it gets to the actual answer like.

Some adult students expressed strong views about the need to understand. Maria (FELAP) was critical of teaching 'these days':

> *I did the function skills 1 and 2 in preparation for this. [And did you like that?] Yes and no. I found it very 'This is what you've got to do to pass the exam' kind of thing. … Which is I think how it is these days isn't it really? Not why you do it or how you do it – this is the answer you need to get to get your exam marks sort of thing. … I need to know why and how. … Need to be able to understand how. … It's very much calculator ... and I need to know if my calculator's not working how I can do it.*

Levi (SFCP) is unusual in his view of mathematics and how he knows he is right: unlike other students he does not think that the teacher is always right:

> *… but I just, sometimes I just, if it's wrong and the teacher is arguing that it's right, I'll just say, I'll just tell her it was wrong. Like I'll just show her, just check over it and show her. …. I just know it was right because if I'm working out like I just know it's not wrong or like I don't know. I don't know. It's like saying 2 times 2 is 4. No one's going to challenge owt like that, but I just, when I finish I just read it again just to, because sometimes I do make some silly, little mistakes.*

The overall effect was that mathematics just didn't make sense. Its arbitrariness meant that it was eminently forgettable, and teaching was confusing, as described by Caterina (SFCC):

> *… if I do it in class I do it right and if I come the next day or two days after it's just oh I'm not sure, because I just really don't know, but it's that feeling that oh I don't think I did understand it or the next day he explains in different ways and you're there like now I'm lost because you're teaching that way and then you're teaching another way.*

Matthew (FELLC) tells a story about not understanding, ever. He doesn't think that mathematics will ever help him in life, and that a C would simply signify 'all the work I put in for Maths and all the hard work I've done trying to get the C' . He sees mathematicians as simply 'Trying to create new sums for us. … Trying to make it harder'.

Some students held on to fixed ability beliefs and the idea that mathematics is hard and only for the clever. Shelby (SFCC) said that 'Some people are just really clever aren't they?', while Ruqya (SFSC) thought 'It's a really hard subject though. … Cos I've been finding it hard since I was small'. She thinks her older sister is good at mathematics because she has taken after their father, and that she's not clever like her siblings. Zoe (SFCP) also thinks that "you've got to be clever to do good in it". She sees herself as a creative person, and so incompatible with 'boring' mathematics:

I think it's like if it was more creative and more fun and like I enjoyed it more then I'd like it better.

4.3.3.2 The value of mathematics

As a consequence of these perceptions, perhaps, most students saw mathematics as having only limited value in life. Shelby (SFCC) thought that no one could like mathematics since it is so useless:

> *No one likes maths do they […] why would you need half the stuff you do in maths in like life? … Like I get some stuff that's like relevant, like percentage and stuff like that like. If you're trying to like work out like the percentage of your tax or like how much is going out this month – you would need it for stuff like that, but I don't think you need like algebra and stuff like ... in day to day life. … why you would need like that.*

Ruqya (SFSC) agrees:

> *If I had a choice I wouldn't do it. … Some of the things are useful but like … do you know when they teach you like the circle like radius and those equations – to me that doesn't apply to me in my daily life. Cos I think you should just know the basic things and not like the hard things.*

As does Pania (SFSP):

> *somehow when it comes to algebra and stuff I don't see how it comes to our lives. … I don't really see the point of learning it.*

Imogen (SFSP) thinks that 'some parts of maths connect to real life', but:

> *I've never seen my mum using Pythagoras' Theorem in everyday life or finding out the circumference of something. … I want to teach ICT in universities and I don't think I'm going to use a lot of maths, so I think, I don't know. It's like it doesn't really click with me.*

Even Yalina (SFSC), who says mathematics is her favourite subject and aspires to be a mathematics teacher, can't see the point:

> *it is very very useful for every lesson. [Can you use it in your everyday life?] Yes I do use it … we can use it in mobile phones, we can use it in shopping, buying things and giving money. Measuring something – it's useful for that. [And what about things like geometry..] Um … I don't think so, I don't find it useful that thing. When I'm doing these kind of questions I think it's not useful, why teachers wants us to do that?*

Where students could connect mathematics to their career aspirations, they could see usefulness, but for the most part this was limited to particular areas of mathematics, as in the case of Martha (FELLC):

> *I think it depends on the situation. I mean I need let's say for Psychology I need, I'll need to be able to do statistics and things like that, so I really, so that's going to be quite important for me, so I do think it depends on the situation that you're in.*

Zoe (SFCP) feels that while she will need to use mathematics as a nurse ("that's why I want to start liking it"), that 'angles and algebra' and 'stem and leaf' aren't useful in everyday life.

4.3.4 Experiencing RME classes

Students' experiences of the RME classes tended to be largely positive, although we were aware that they might be reluctant to make negative comments to us. However, we attempted to obtain detailed comments on what students thought the aim of RME was, and how they interacted with its methods, in an attempt to offset a general desire to please. That this was not in fact the case is suggested by the presence of some negative responses which are indicative of a resistance to the slowing-down effect of RME and its emphasis on modelling and explanation. As we have seen, students' perceptions of mathematics were dominated by their experience of learning what to them were arbitrary rules and a sense of its irrelevance for their lives. The RME intervention challenged this perception and made a difference for some, but not all, students.

4.3.4.1 A different mathematics

The RME approach aims at a general shift in students' engagement with mathematics, enabling then to make connections and build understanding across different contexts and problem types. While we could not expect students to articulate it in this way, particularly after a short intervention, some students appeared to recognise that there was an opportunity for more thinking and understanding in the lessons.

Pania (SFSP) saw RME as 'not spoon-feeding':

> *the thing I like about it cos she explains a lot more. And like she doesn't like spoon-feed us. … So that we can try better and stuff like that, but that's the thing I like about it. [Tell me more about what you mean by not spoon-feeding] Like as in she doesn't like give you the answer but she wants you to try it yourself and then find out yourself … instead of her giving you all the information. […] it kind of helps us more understand it, and then probably I remember a lot more.*

Imogen (SFSP) said that the lessons 'didn't feel like maths' because of the connections to context:

… how she's teaching it's not so much feeling like maths. Like a normal teacher would probably give you a big circle and say here and make a fraction out of it. Whereas she's using actual everyday life things like sweets and it kind of makes you feel like you're not doing maths, but you are doing maths. … I understand the fractions a bit more.

Ruby (FELLP) seemed to realise the way in which RME could capture a variety of problems, with the added bonus of being 'easier':

[What's his [RME teacher] idea as a teacher, what's his plan, what's he trying to do?] Help us find ... how to find an easier way … So with the ratio tables he's shown it that it doesn't matter what question you're being given, that ratio table can help you, because it's used for any calculations.

Joel (SFCP) sees the RME approach as fitting in with his own way of thinking:

Yeah, she's doing it differently from how all the teachers teach maths, but that's kind of like how she does it is kind of like the way that I've always thought about it anyway if you know what I mean. … Just like with I don't know just how she teaches, that's kind of how I've always like thought about it. … Like the way that she like explains how to work it out, like that's how I've thought about how to work out the question. Like if I was doing a test then that's how I'd think about how to work the questions out.

Ryan (FELLP) also seemed to pick up on connections in the sense of RME as 'mixing it up a bit' and making mathematics more fun:

Steve's classes were good, yeah. I learnt like quite a lot from it, yeah. … He's trying to like make it a bit more like … sometimes maths is just straightforward, he's trying to like mix it up a bit, trying to make you understand it more. Trying to make it a bit more fun. … And like trying to break down the questions so you understand them.

Like many others, he saw RME as supporting understanding by breaking questions down:

Breaking it down, yes. It's obviously going to make it like a lot more efficient isn't it? Because if you're just doing it all in a lump sum but if you're doing it just bit by bit, plus you can get more marks for that. … But like he does it like in a way where you can understand it a lot better than …

Maria (FELAP) particularly noted the use of context: 'That's what I like, it's put into a context you can understand'.

Other students picked up on the issue of interest. John (FELLP) volunteered near the beginning of his interview that "I'm finding these maths lessons really interesting", and he saw this as RME's main contribution:

Maths can be quite a boring subject if the teacher makes it boring, but to be honest [the tutor has] made it really interesting, it's been really good. … I think there's only one way you can teach it, it's either … you know like algebra, you can't do it 150 ways can you really, it's one way. So … to be honest I don't think he's changed anything, but he's made it interesting. … He's made it interesting so it's made it easier for us to learn.

Ruby (FELLP) related it all to confidence; she had found the RME ratio tables 'extremely helpful', using them on every question in the post-test:

It's a different way of teaching, but it's also if you find something interesting and you find you can do it you pick up on it better, you feel confident.

Abbie (FELLP) has a lot of difficulties with anxiety about mathematics but said she felt more comfortable in her re-sit class. Commenting on the RME intervention, she says that it has made things easier and had even made her like mathematics:

[What's it been like?] Um ... a bit different, cos there's different methods and stuff, but um ... I do find some of the ways he's taught it was easier to work out things. …It just sort of clicks a bit more. [So what makes it easier?] Um just like the methods, like the bar method and everything. … Like there are questions that can be like put down to small chunks.

She comments that it feels good to understand what she is doing because the bar approach 'breaks it down': "I can't stand maths, but when I get it I actually like it. "

4.3.4.2 Time pressure and "learning a new method"

However, even though they perceived the benefits, some students said that they were under too much time pressure to use the RME strategies in tests. When quizzed about the bar and ratio tables, John (FELLP) said that these were new to him, and that they made mathematics easier, but he had not used them in the post-test. He explained this in terms of the time pressure in the test and his general tendency to 'crumble to the floor' in tests:

Yeah the bar was easier in the lesson when we had more time to practise it. … Whereas in the exam we just had now, we only had like 15 minutes to do it. [so do you mean that you're anxious about using the time for the bar?] Yeah it's like it takes me like a little while…

Abbie (FELLP) was still not quite secure with using the bar; although she appreciates the bar in class, she finds it hard to get started on her own and set up the bar model:

It's easy when I've got it set up, because I know it ... then it all just all comes back.

As these responses suggest, some students tended to see RME as just introducing a new method, a perception confirmed by Tanviha, the teacher at SFS, in the next section.

Consequently, some were less keen on learning a new method, particularly when they felt they had a perfectly useful one already, or simply needed reminding of a method they had forgotten.

Although he didn't necessarily use the RME bar himself ("the method is good, but like I don't really do the method properly because I don't like doing the lines"), Levi (SFCP) thinks that the RME approach is "helping a lot of people in the class":

> *Because I was sitting next to Joel and Sienna and then when they did it first they didn't get it right, but then when [tutor] went over the method on the board then they did the next question then they got it, so … Yeah, so they know how to do questions like that now. … because I think Joel likes when things are broken down to him like.*

He sees the RME approach as being about a new method, but he is unlikely to use it, he says:

> *… because her method works, but mine does too. … And I find it easier to do my method than anything. … Like because some people when they go into the test they just know what method they're going to use like for each question. I wouldn't like want to do her method and not get it, like struggle and waste time. … I'd rather do my method and do it fast and know that I'm right. … So I wouldn't want to do it as a gamble like. [How do you know they were right?] Just check over it.*

Sienna (SFCP) sees positives in the RME lessons, but is not sure about the bar; like Levi, she sees that it is helpful for other people but not for her. It appears that she prefers not to seek explanations:

> *I think she is [good], like the way she brings in different things and like the things that we've been doing lately I could do some things in my head, but I think the pictures thing, but I don't think the bar helps me with the fractions. [Okay, can you say why?] I just think like it's wrong to do it and like the other people in the class like they try and explain it. It just confuses me, because I did it. I think my way is an easier way, because I just go straight to it. … Just half it and then quarter it and then find an extra one. … I think it's because I can do the division and … they can't. … It's just when, I only find it confusing when the rest explain it and like they try and get to the answer and then they'll be like finding half and they have to add another one when they could just do a division and then it would give their answer.*

Like other students, she sees the aim of RME as providing an alternative method:

> *I think she's trying to give people different ways or like look at it with a different perspective and if like they struggle one way they'll have other ways to do it, so when you get in the exam if you forget one you've got another.*

The adults at FEL presented a particular case of this issue, which appeared to be based on their frequent experience of finding that mathematics teaching was different from when they were at school. For example, Maria (FELAP) echoed Nancy's (FELAC) preference for 'going back to the old formula':

> *When they first came in ... I think it's cos I had a system in place already that I used and we were being taught a new system, I struggled, and I kept sort of brushing the new system aside cause I had a way of doing ... yeah, cos I already had a way of doing it, I couldn't get my head round their way of doing it. The way I was doing it was very similar, but without drawing a bar and stuff like that.*

She acknowledged that the RME tutor had said this was OK:

> *But he did keep saying to us if you've already got a system in place use it. This is just a system if you can't do it.*

However, she was more than happy to change when she no longer felt confident about the mathematics:

> *Because I don't understand how to solve these problems, I'm enjoying it and benefitting from it a bit more, if that makes sense.*

4.3.5 Summary

Many of the experiences of mathematics teaching related by the students were familiar, echoing the literature on the impact of traditional teaching, setting, and perceptions of mathematics as a set of rules to be learned. Students who had entered FE College from school reported on more positive experiences in terms of relationships and confidence, although they recognised the impact of having too little teaching time. For the intervention students, RME presented something new, which was welcomed by some as giving them an opportunity to understand and to break problems down. However, some students saw RME as just about learning new methods, which some resisted because they did not want to replace existing working methods. This was a particular issue for the adult students, although they could also see the benefits of RME, particularly when they lacked a current strategy or did not understand.

4.4 The teachers' view

We interviewed teachers in all four sites about the challenges of teaching GCSE resit classes, their approach to teaching this particular group of students and how they would like to develop their teaching in order to meet their needs. Four teachers were host intervention teachers, and we also asked them for their opinions of the RME approach.

4.4.1 The challenges of teaching GCSE resit

4.4.1.1 Confidence and motivation

All the teachers identified issues of confidence and motivation as the primary challenges in resit classes. Some noted that previous teaching had been patchy. For those in the FE colleges, attendance was an additional issue. Mike (SFCP) sums it up - students arrive 'with a multitude of problems':

> *It's not uncommon for them to be saying they've had half a dozen teachers, they've had supply teachers. All those issues affect their enjoyment of the subject, their confidence in the subject, so our issue is … two classes of people who don't want to be there, have a lack of confidence in the subject, often don't like it. Having said that the vast majority do have the maturity to recognise they need the qualification, so we don't particularly have issues with behaviour. We will have issues with apathy to a degree, lack of confidence and willingness to work particularly hard, attendance, punctuality sneaks in as the year goes on. Those are sort of the issues we're up against.*

David (SFCC) noted that motivation was the key issue in the classroom, keeping students going who feel they ought not to be in the class because they have narrowly missed a C, but also those who feel demotivated because of persistent failure:

> *… keeping them concentrating, keeping them focussed because at the other end of the spectrum it's just a trial for them every week and they're trying to do things that for the last four years at school they didn't understand. Why should they understand it again in the next year?*

He sees confidence as the major challenge for resit students:

> *Because when we're teaching the subjects, if we do something like Pythagoras, if we do something like something like bar charts, but pie charts for example, "we did this at junior school … we you know we've done this". … but they don't know it. … And it's that, that's the barrier. That's the confidence barrier because they know it and they sort of they won't listen, because I think they're afraid a little bit of listening into bits that they don't understand, so they put up a front of "I know this already" …*

Tanviha (SFSP) sees the problem as one of needing to move on:

Do you know what it's motivation. I think the key thing is trying to get them to not think about the grade that they had back in the summer, but start afresh and then try and work towards the C that they need … I think once we're over that, then it's okay. I mean it's trying to get them remember, reconsolidate, but the past things that they remember and how they apply it again, the hardest part is "oh we've just done this again and we're doing this again and I'm never going to get it".

Kate (FELLC) thinks that lack of confidence leads to non-attendance:

The main challenge has been attendance. … they are reluctant to address I guess the difficulties they've had with it and it lowers their confidence, at which point they decide not to return because they're not enjoying it. When they're in the classroom the difficulties with them tend to be that they are down on themselves about it, so they tend to feel that no matter what you know they're never going to get it.

Like Kate, Carol (FELLC) sees attendance and confidence as part of an overall challenge, particularly within the college framework of three hours once a week:

Well bearing in mind you only see them once a week, so there's no, you've got to keep them very positive, very encouraged and motivated and also tight, so they come, they have friends that come every week, so for me my classes have to have a certain element of enjoyment in it as well, social and you know interaction and enjoyment. Otherwise these people just won't come.

Peter (FELLP) makes the same point:

Yeah, and they may have got less than a D the last time, so they'll have a few 'go's and they will achieve a D, so confidence in using Maths, attendance, normally just their tiredness towards the end of the three hour session as well. It's a challenge.

4.4.1.2 Basic skills

Aside from these issues, the main challenge was addressing the very low base that some students are starting from. Kate (FELLC) noted that "you're talking even looking back to entry level 1, 2 and 3". Lucy (FELLC) is also concerned about students' study techniques, and their lack of experience in taking notes for themselves, echoed by Asad (SFSC), who sees students' lack of independent learning skills as a major obstacle:

.. they've got to be, learn to be independent learners and these, most of the resit girls are developing that independent learning. They've not mastered it. If they would've mastered it, they would've been in set 1 or set 2 and things like that, whatever, because they're capable of working individually for several hours at a time, because they feel comfortable about it. Where we've got girls who are not individual learners, who don't have the acquired skills to actually revise, then they still get

flustered, they still don't know how to revise, then we've got a repeated pattern of failure in revision ...

Mike (SFCP) pinpoints a lack of basic skills and failure to retain as a major problem, which is exacerbated by the time pressure of the course:

The positive is they have seen everything before. … They do have an idea. Your problem comes when you get students who can't do multiplication, who can't do division, because we don't even put that on the scheme of work. We haven't got time ... to go back. We haven't got time to teach them their tables and the number of students who are, I've got a times table chart on the wall, who are looking at the wall or you see them with their fingers. … Because we've condensed everything that they do in Year 10 and 11 into about 30 weeks as it is. I think virtually everything we do or everything we do, the majority of them will sit there and can do it on the day. … The trouble is if I reintroduce it a fortnight later, the vast majority have forgotten.

Peter (FELLP) sees slightly older students as more motivated and willing to work and take notes, but he notes that there may be pressure in the future when all students arriving with a D must do the course. He believes that motivation will dip, and he feels that there may be a particular problem with girls.

But the 17, 16, 17 year olds …Their group thing together, they bring each other down. They talk each other down ... in the classroom and you can see it happening. They out compete each other to be stupid in Maths

Tanviha (SFSP) is the only teacher to point to language issues, which are very evident in her class because some students are not first language English:

I mean if the girls can't access the question and don't understand what's going on, if it's too wordy they'll just give up. … They won't even try to attempt to even read the question and then have a go, so it's kind of like getting them to understand that why don't you highlight the key points? And then take it from there, but with the resit group I think it is just the motivation.

4.4.1.3 Mixed abilities

A related issue was mixed ability. David (SFCC) sees this as the main challenge:

I think the main challenge seems to be confidence, because we have people who just missed a C and we've got people who just got a D and it's a big range and so that's a range of arrivals, but also there's a whole range of there's the people who were lucky just to have missed a, you know got a top end D and people who were unlucky not to have got a C, so even at the top there's sort of people who did it by cramming and who did it by ability and then I think our challenge here is trying to, although they're all D grade, which is a great advantage, I think the disadvantage is sorting, well the *difficulty we have is sorting out who needs to be just improve their practice, so they're confident they'll pass, and others just telling them yeah you don't have to check it at every step of the way.*

Asad (SFSC), also picks out the variability in the class, and relates this to the school decision to teach Higher Tier, when nearly half of his students have been doing Foundation.

Now this is a departmental thing and what the department has decided is that because the results, 85% of our GCSE passes came from Higher, okay and they've decided what's the point of doing Foundation? And they're going to fail, because the majority of the pass mark for grade C as you're aware it could be as high as 75% on the Foundation. …. So we're talking about the very low Foundation and suddenly they've got to do Higher.

He has struggled to get to know his class of 23 girls and to address their varied levels of attainment:

If it was like a smaller group it would've been manageable, but that is one dilemma. The other dilemma is we've got girls who just missed out on a grade C, so we've got this whole differentiation from girls who don't know how to half 64 to some girls who want to do in terms of I don't know some topic in Maths and things about whatever you know, and they can do plotting a quadratic graph quite comfortably.

4.4.1.4 Time pressures

Consequently, time was a major issue for them as teachers. Kate thinks that although theoretically the resit course can fit into a year, time is an issue because of the amount that needs to be covered and the need to consolidate learning and support individual students:

In a year having three and a half hours with students it wasn't a problem, but that was broken over three days in [her former 6th form college]. Here it's one night a week and I'm not sure if that's going to have a big impact …. they're only seeing me once a week and they haven't got the support of me sort of any other time. Like I'm not seeing them, sit down with them and say this is how you could do this more at the time. I think that might be a consequential difficulty…. Two and three hour blocks … loads of concentration on so many topics, it can be a little bit too much. … You know it's quite a sizeable group if you're looking at people who need individual attention, so … they're not getting what they need ... to really make a big improvement.

She estimates that just 40% of her class can be certain of achieving a C, largely because she has insufficient time to get to know her students and target their strengths and weaknesses.

Lucy (FELLC) also thinks that the college's class scheduling of one three-hour class a week is problematic:

I don't think you get three hours' worth of work in three hours … I mean the other option would be to give people homework, … I've been giving a little bit, but I think people won't do it, so you need to make sure you cover a lot in the class I think, because they won't do it you know. They'll walk away and they'll just get the materials out the next week sort of thing. I know that, but I do think you know it's very difficult to get that amount of stuff in the three hours.

Asad (SFSC) thinks the major problem is the pressure to achieve a C in a short time:

Teaching it's all about equipping them to, in terms of life and everything, it is a problem solving activity and things like that, whatever, so they will have in terms of the aspirations, big aspirations from those who want to do up to degree level, things like that and whatever and maths comes up in things like that, whatever. Then you've got people who just want to go to work, work environment and everything … but the question is are we preparing these girls to a work life and everything? The answer is maybe and everything. Unfortunately in all school systems it's the same. There is a tick box exercise. You've got to get the grade Cs regardless. Now unfortunately we can't teach them thoroughly in terms of basic sale price, percentages and things like that whatever, numbers and everything. We're cramming all this information into their heads…

He sees current work plans as impossible:

If you look at the .. first week I'm doing decimal places, standard, sorry significant figures, recurring decimals and upper and lower … all in one week to this group. Okay, next week I'm going to do fractions and so that is more or less telling me I've got to do one topic per week, sub topic. And everything, so in the space of two weeks I could've covered six topics. It's ridiculous.

Consequently, the one-year course is untenable, in his view:

The girls, if you're going through so many topics, it's a two year course higher they've never done before, cramming into six months, I will cover 40 topics in six months and everything. Will they remember it? Good for them. That's not going to work and things like that, whatever. Going back to the question, is the system right or wrong? It's not, of course it's wrong …

4.4.2 Approaches to teaching

Teachers tended to talk primarily about changing students' attitudes to mathematics and motivating them to learn.

4.4.2.1 Addressing confidence

David (SFCC) addresses the confidence issue head-on:

> *Well I just have to, well I apologise to them and then I say just bear with me because not everyone gets this and I turn the emphasis on the fact that people have got a D here. You've all done really well. You've all got a D. That's great, but the fact is the reason you didn't get a C is different from the reason that he didn't get a C and there's only little bits. We're not looking at getting, you know if you got, if you fell five marks short of a C, because most of them seem to know what, we're not looking to get six marks. I'm looking to get another 20 marks, so that you've confidently got a C and the things that you need to get those extra marks, there are only a few marks on each subject, and it's because there's just a few little things that you don't understand on each subject. The problem is you've got to listen to everything because otherwise you're going to miss the bit that you didn't understand last time.*

Kate's (FELLC) main priority is confidence building, which she addresses by breaking tasks down and situating them in story contexts and real life examples. She also encourages students to work together:

> *I think it gives them the ability to explain the methods and give each other different methods which they've not got before and it helps them communicate in language that they prefer rather than just hearing it from me.*

Peter (FELLP) also tried to 'mix things up' with group work:

> *I try to make things active for them, so we'll switch what they're doing quite regularly, so I try not to keep to a routine so to speak. … So they may work in small groups and they might discuss things like, later on we're going to start handling data, so they'll be discussing what words mean … They'll be doing exercises and practising things as well. So all those things. I try to mix things up as much as I can.*

Like Peter, Lucy (FELLC) aimed to do group work, partly as a diagnostic strategy but also partly in order to offset the problem of the three-hour class and enable students to engage more with the material. Although she aims to avoid too much lecturing, she does feel the pressure of time:

> *I think that always unfortunately has a, it can have a bad effect in the way that you think "we've not got time to do this, we've not got time to do" … you know because I actually I really like the idea that people can learn by investigation, but a lot of the time it gets stunted that, by the fact, by the you know the time schedule that you've got.*

4.4.2.2 Changing attitudes

They also attempted to change students' perceptions of mathematics and mathematics learning, focusing on relevance and the value of mistakes. David (SFCC) tries to make mathematics relevant:

if they know what they want to do, you can pull out things and say you know ratio you're going to need this because you're going to be mixing medicines or if you've got a ward of, not a ward of, if you've got a set of patients, of 10 patients, it's going to be a bit different from if you've got a set of five or two or whatever. … you can give them a bit of application and if they've looked into things, and also by now when it comes to percentages for example you can always find someone who's been, got a job and is working so you can pull out percentages and the value of their sort of overtime you know and the value of a rise, that sort of thing.

Like David, Lucy (FELLC) also tries to tailor her teaching to the other subjects students are taking in order to put the mathematics into context.

Yeah, I mean I've asked quite a few of them already what subjects they're doing, so I know like someone's doing motor vehicles, someone's doing media. The new girl today she said she's doing hairdressing, so when I do the examples and things I bear in mind that …

Carol (FELLP) and Peter (FELLP) both try to highlight the value of failure; Peter explains:

I will try to emphasise the value of mistakes and the value of self-esteem in the classroom, so I'll promote mistakes and we'll all learn from someone's mistakes, but that's difficult because you know it's like, it's not, I don't want to make it, you know there's a balance between showing somebody that's a really interesting mistake to make, because everyone will make that mistake and then all of a sudden their ego's dropped and "I've made the mistake and oh no, no. I'm never going to get this. I can't do this" and everyone's joining in.

Unlike some other teachers, Tanviha (SFSP) says that her students are asking for something different:

I don't want to be where they've been like five years and teaching something on the board and just teaching them a certain way. I think they learn better if they're telling each other how they've done a certain part of the question and then bringing that question together and we've done that today and they worked really well, so I got girls who are of similar ability to sit together and then work on a question and then they've just put the input in what rules they've used, because they'll all bring in different rules. … But it is hard…. It's draining yeah. It's draining yeah.

Like Asad (SFSC), Tanviha (SFSP) has a wide range of ability in her group, and has to work hard at differentiation:

With the work in this group, because they range, because I've got some Gs in there, I've got Fs in there, I've got Es in there, so it is quite, even though it's the foundation I've still got quite a bit of a range in there, so they're all split up, completely differentiate things. I've got a lower group doing, on the same topic, but different *type styles of questions and another group doing a different type of question and another one trying, tackling more harder, think about the D and C grade questions.*

4.4.2.3 Targeting skills issues

The teachers also aim to address skills and target gaps in knowledge, but there are limits to what they can do, as their comments on the overall time frame show. Kate (FELLC) has learned that she needs to 'just get cracking with number work straight off', while Mike (SFCP) talks about targeted 'fixes'. However, even with a slightly more homogeneous group in comparison to Asad (SFSC) and Tanviha (SFSP), he struggles to do what is needed, and sees this as being more difficult once the college is taking students with grades below D:

> *… we've got a problem in as much as you've got a one year revision course. We don't have the time to take apart what they couldn't do and put it back together. … It's worked relatively well, okay, up to now because again if the people have got a D they do have a decent grounding. What we're trying to do is we're trying to find three or four things over the year that we can fix. Um, again that may not work so well in years to come where you've got weaker people who you can't do the quick fix with and we could do with more teaching time. … Which is why it would be better if they were separated in the future, we could do this over two years. Um, so you know much as we would like to go back and dissect what they've learnt or not learnt at school we don't have time, so it's looking for quick fixes.*

Consequently, the system relies on time for revision and past papers immediately before the exam, since retention is generally poor:

> *And we have them in here from 9 til 3 and they're on a carousel where each teacher teaches the same lesson maybe five times and the students start in one room and then after an hour they get up and go to the next room and we pick five major topics and try and blitz them as a revision the day before. … From what I've said before from a retention point of view they will have forgotten it and I also know that an awful lot of them won't do any revision anyway, so at least they've had four or five hours the day before.*

Retention is a major challenge, so Asad (SFSC) optimises consolidation:

> *Now some of those girls, even though they've done it they still can't do it. … it's just like it doesn't matter how many times you say it, it doesn't click in and everything, so … one thing that works very well is if I do one topic and repeat it three times throughout the year, it does help. … Even though I try and do it as much as possible to the extreme and everything, but when I find though it's getting too much, then I'll stop it. Then I'll repeat it again, perhaps add a little bit more and everything. That*

way they've got this certain oh yeah, they'll always remember a certain percentage of it.

Despite his efforts, there is not enough time to address all the issues:

I have changed my teaching because I am more concerned about getting them through and getting that grade C, so I am targeting topics which occur regularly and I even wrote down a list of ten topics for this November resit. You know target this, this is going to help you. You, some of them perhaps you know that, but this is going to help you to get through and everything, so it's targeted work and unfortunately if I were to do it thoroughly and properly there's not enough weeks.

4.4.2.4 Higher versus Foundation Tiers

As we have seen, there is an issue over whether students are entered for Higher versus Foundation Tiers. Asad (SFSC) describes teaching approaches in terms of the school's strategy for meeting exam targets, including a switch to Higher Tier for all students, together with a decision to enter some for November resits. Teaching had involved going through past papers whether or not students were entered for the November sitting, but he had made the decision to split his class into students who were likely to pass and those who were not. He had begun to work on individual topics with the latter group and had introduced extra lessons after school:

I think nearly the whole group, have done Foundation and suddenly they've got to … …So really it's just selecting the topics and saying right obviously we're going to stay away from the A and the grade A topics, because it's way beyond them. However, some of the Foundation topics come up in the Higher, like the numbers, you know highest common factor, lowest common multiple, things like that, whatever. They come up all the time. Straight line graphs comes all the time in the Foundation and the Higher, so it's important to cover those topics again and everything…*

The issue of Higher versus Foundation Tiers has also come up at Mike's college, with many students believing that they have better chances in the higher paper:

Um, our argument has always been the level to get a C is the level to get a C. There is that crossover on both papers. It is the same or as near as makes a difference the same standard. If you need more than a C fine. We will talk to you and we will do some extra work and let you do the higher paper or see if we can do that. If you're going to go for the C we would rather you did a paper where you can do everything rather than go in and doing a paper where you can only do half of it and you now only need 30%, but if half the paper's gobbledygook to you it's 30% or 50%. It's not so good you know.

4.4.3 Teachers' views on the RME approach

4.4.3.1 The positives

Teachers were very positive about the RME approach as a pedagogic strategy. Mike (SFCP) saw it as radically different from what students were used to, with far more potential for understanding:

> *Right I think the power of it is if you are trying to … you're trying to teach a method where you'll say to them … say if you think of things like the bars, you're trying to just say to them 'Right what information do we know? – put it down in this bar and see what else we can work out from that.' So it is a more open general approach to problem solving, which I would hope people could apt to different environments. If you get used to that 'What do I know? What else do I know? Where am I trying to go with this? Can I put those combinations together?' – as opposed to the standard way that it's taught in this country. We teach it here as well … 'If you get this question you do this step, that step, that step' – and students are drilled to do that – which works for 70, 80% of questions. As soon as you get a slightly different one … or you forget. … I think [RME] potentially would give you a foundation you could fall back on and apply to lots of areas. But that is really hard for us to get over to our kids on a short block. What I hope it does do with them is it opens their eyes to different ways of dealing with things … and in some cases it switches a light on.*

Carol (FELLP) focuses on RME emphasis on drawing diagrams as enabling students to 'give it a go' and gain confidence:

> *… seeing things in pictures and making sense of it … if you're stuck just draw something and it's that idea of visualising it rather than thinking oh my gosh I can't remember the rules here … So that's been really effective. It's also really good for giving yourself, I think what my students like to do is it gives them space to muddle through it rather than oh my gosh, what's the rule? And not knowing where to start and what their next step is, so it's given them a bit of a strategy and a bit of a process even if you don't know where you're going initially, you can by drawing get there. … It takes a lot of the panic away. It takes the anxiety away and it helps you, even if you don't know the answer you know that you're going to be able to find it if you keep drawing things.*

Like Mike, Peter (FELLP) saw a major virtue of something different from school:

> *Um, I think it's very good techniques for them to grasp onto, because it's different from what they've been doing at school … Yeah, so having another one like ratio tables or whatever and using the grids is a good tool to use, so if they have a raft of tools to pick from and they're not just staring at the paper thinking I can't remember*

how to work out the percentage of one or another, it's somewhere to start at least isn't it?

Tanviha (SFSP) has used the bar in her Year 7 class, and is enthusiastic:

> *I used the same method with my Year 7 and I started this last week with them where I introduced percentages and I liked it and it did, they understood it and they could see it better. They realised how to work out 1% without me even telling them how to work out 1%. It was really good. It was really, really good and I can see it, I mean they were a bit of a middle, no actually they were one of the top bands, but I can see how it could work with a lower ability group, because the lower ability are much harder to get them to understand what's going on…. I think it's really, really good in the sense of is it more looking at what's happening instead of just numbers*

4.4.3.2 The challenges

However, Mike (SFCP) notes that some students, particularly stronger ones, resist the RME approach because they are reluctant to learn a new way of doing the easier problems which they can solve and consequently don't see the point. However, these same students can benefit in the long run:

> *Because some of the better ones that we have in these classes when [RME teacher] has done it, and it's all relative obviously, but some of the better ones do sit there and say "well I can do that anyway". … But in terms of an approach that breaks things down and gives you a hopefully a standard way that you can fit lots and lots of mathematical problems into, it's really, really good, because a big issue we have is as soon as a question goes slightly off the basic that they've seen … they're stumped. … So if we have a framework that you can adapt to a whole raft of problems, that would be really, really useful.*

Mike was quite concerned about student resistance, suggesting that the RME approach needs to be introduced early in schooling:

> *I think the problem with dealing with not just the students here, but certainly students of any age in this country is that they have pre-set ways of doing things. By definition the students I'm teaching are not outstanding at maths and there are weaknesses within their thought process. But neither are they massively poor either if they've got a D. That does mean in some areas they've got structures that work. In other areas they've got structures that don't quite work but are deeply embedded – it's a problem. To change that mindset and that thought process with some of them at the age of 16, 17, 18 … is quite awkward. So presenting them with a new way in some cases is met with a 'But I know how to do this' is the issue - even if they actually don't know how to do it. So you've got some structure … which as I say in most cases does have probably some substance to it, but is maybe weak around the edges … So*

sometimes it is legitimate 'I can do these questions, I don't need another way'. The process itself I think, if it as embedded in younger students in this country I think is extremely powerful. I think the visualisation of what's going on and the ability to adapt it to different areas … cos our students struggle like mad to adapt anything - give them a process and a standard question that fits that process and you can drill them to do it. Go just slightly off that path and they're unable to adapt it and to apply it.

Peter (FELLP) raised the same issue:

… so some of them will have this thing where they'll say just tell me what the answer is or tell me what to do, tell me how to do this. … And they don't like that … but I will tend, like Steve tends, not to give answers. He will tend to ask questions and that's, for some of the learners that will frustrate them because they just say "tell me what to do".

4.4.3.3 Pace

As a host intervention teacher, Mike had to give class time up to a slower pace and coverage. Early in the year, in the number module phase, he felt that this was not a problem, because there was some slack in the timetable:

And touch wood I'm on target. I don't need that last week at the moment. Um, you know you can rattle through things. I can afford to lose a small number of lessons ...

However, later in the year he was feeling pressurised:

With every other group I am three or four weeks ahead of [the RME one] and where am I going to squeeze in this and this and this? But you're right about the underlying understanding being really really important, so I'm pulled two ways. … I really like what you do and buy into it, and the other side of me is saying 'damn, with this group I've still got to cover this this and this, and when am I going to do it?', because when I start teaching again I've still got things on the scheme of work to do…

Mike's view is that the time pressure is a major problem, when weaker students probably need a two-year course combining an RME approach with recovery of the basics – "if we had two years where a good chunk of the first year was going back to basics and just dismantling what she knows or doesn't know, or thinks she knows, and putting in some structure … some way of dismantling all the misconceptions and rebuilding the basics".

In terms of the impact of the intervention on her as host teacher, Carol (FELLP) echoes Mike:

Well it is a challenge definitely, because what we have done is spent a lot of time on the number part and I think that's really useful, but it just means the rest of the course is quite squeezed, so the pace is good. I mean it would be nicer if we had *double the amount of time really, because we have to get them, or Paul and Sue have to get them to a point of making this sort of workable quite quickly. Whereas in a sort of three classes a week at school they have a lot more time to develop it you know.*

Despite these pressures, she thinks that the intervention will benefit the students:

… the good thing is when we do this, the making sense stuff, you cover it very thoroughly, so your students are definitely going to get marks in those questions, so you know and it pervades into so many other topics that I mean it's genuinely worth the investment.

Peter (FELLP) is comfortable with the slower pace of RME because of the longer-term benefits:

Yeah, I think once people have got it they can move through it quite quickly. You can apply things across the board can't you? …So like ratio tables, once people have learnt that, if they can see it works elsewhere then they'll use it elsewhere.

Tanviha (SFSP) sees similar problems for resit groups as other teachers in terms of introducing RME in a time-pressured frame, when students are anxious to work more obviously on exam questions:

… at first they understood what was going on and then after a while they felt like why are we drawing bars for? Why do we keep drawing bars? They didn't understand what the bar represented. … They just felt like it's another method. …They didn't realise that it was a skill that they were picking up. It was just a bar, because some of them just drew a bar in one of the questions and then they didn't know how to apply the question to the bar. … So that's where they were. They just drew it and they're like okay, what do I do next? So they didn't realise it's not a method. It's the understanding of what the whole bar would represent and what would you do next?

Tanviha feels that the RME approach is valuable, but comments that the resit students need to understand more about what it is doing in order to accept it:

So you want to try and cover or get what they're really weak at. I mean number is an important part and they said "oh it's like we've been doing bar for the past four weeks". "In fact actually no you haven't been doing a bar. You've been doing fractions, amount of fractions, you've been adding fractions, you've done ratios and you've done percentages. That's five different topics you've done in that space of time and it would take me a lot longer to do, so they didn't understand that concept". They just thought "we're just looking at a bar all the time", but they didn't realise. I think next time what it has to be is them to realise what topic is being related to. Maybe then they would understand "oh I can use this in fractions, I can *use this in percentages, I can use this in ratios, I can use it"… because they just, as they would walk in they'd see a table or a bar they didn't realise what they were using it against.*

4.4.4 Summary

The legacy of students' previous experience of learning mathematics presents particular problems in terms of (1) their tendency to understand the RME approach as 'just another (algorithmic) method', and (2) their resistance to, or lack of belief in, sense-making in mathematics. However, while the RME approach increases the potential for success, it changes the nature and pace of the work, leading to the possibility of resistance from students (especially those on the C/D border). Teachers' views were positive, but they expressed concerns about the time needed for RME in the form of short interventions in an otherwise very different programme of work, and also the extent to which students are likely to engage with it. All see it as an approach with great potential for weak students, but as presenting problems for stronger students who might reject it. Teachers were strongly in favour of introducing RME earlier in school careers.

5 What have we learned?

5.1 The post-16 landscape revisited

Realistic Mathematics Education prioritises use of context and model-building to engage and motivate students, enabling them to visualise mathematical processes and make sense of what they are doing without resorting to mis-remembered rules and procedures which have no meaning, a problem noted in much 14-19 mathematics provision. This problem is exacerbated in short-course GCSE resits, and targeted, meaningful content is needed to break cycles of failure. RME has the potential to raise the self-efficacy of students who have experienced long-standing failure in mathematics, and consequently enhance their engagement, understanding and attainment. However, this is a difficult – but not impossible - context in which to introduce a new approach, presenting challenges for students, teachers and researchers.

5.1.1 Students

Resit students have a history of repeated failure in mathematics, and many have limited basic number skills; they are particularly wedded to remembering a variety of formal procedures, which they nevertheless struggle to retain and replicate. Their confidence levels tend to be low, and they have little sense of authority over a mathematics which they generally do not perceive as having anything to do with common sense. Many are desperate to achieve a pass in GCSE mathematics which will enable them to meet their career goals, and this is a source of motivation for most, although some find this difficult to sustain in the face of not understanding. Some of the 17 year olds believe that if they can 'brush up' on a few topics then they will achieve success, and this can make them very reluctant to engage with a new approach, particularly when it presents a mathematics which is very different from the formal algorithms they wish to revisit. Used to being taught a quick route to formal procedures, they are challenged by being asked for their opinions and strategies, and to engage with other peoples' solutions. They need time to adjust to this approach and to shift their expectations of what a mathematics lesson looks like. However, as the findings indicate, they did employ RME strategies to good effect, and they were able to engage in a different kind of classroom interaction. Some students clearly appreciated the RME approach as a way of helping them to understand more and make connections. The adult students also tried to lean on old remembered formulae, but these were less likely to be available due to the passage of time, and they were open to new methods. They also expressed a desire to understand, and the RME approach was met with enthusiasm by many of these students, some of whom recognised the power of the models to unify topic areas and significantly reduce the reliance on memory.

5.1.2 Teachers

Teachers of GCSE resit feel a great deal of pressure to cover as much of the syllabus as possible in a very short period of time, and this can lead to an approach which involves quickly revisiting the formal aspects of a topic, even though they know that this is going to have a short-term impact and is unlikely to be successful in the long run. They are under pressure to produce results, and are aware that this forces particular choices. They are acutely aware of the difficulties their students have, citing confidence, motivation, attendance, and skills gaps as major problems in learning mathematics and recognise that repeating a traditional transmission and drilling approach is unlikely to be successful. While the host intervention teachers were anxious about pace and student resistance to RME, they were enthusiastic about it as a way of teaching mathematics which could give students a chance to learn. They were keen that students should see the benefits.

5.1.3 Challenges for the intervention

As researchers and tutors in this project, we had an in-depth knowledge of the theoretical framework behind RME and also a working practical knowledge of how to implement RME approaches within the classroom. We were aware of the need to spend time embracing contexts, developing students' informal representations and removing ourselves as the mathematics expert in the room. We had a vision of the classroom cultures necessary to an RME approach, the need for students to think independently, to explain their strategies to the class as a whole and to question each other's' approaches. Post-16 GCSE resit students are more reluctant than most to expose their thinking and to engage with the mathematical ideas of their peers. Finding ways to engage them in a shift of classroom norms proved challenging and caused a particular tension for us in relation to time. Had the intervention been longer we would have spent more time developing the classroom culture, but the confines of a limited number of hours teaching, combined with the concerns expressed by teachers about covering their schemes of work, necessarily led to compromises on our part.

5.2 Suitability of the RME approach for GCSE resit

5.2.1 Effects on student outcomes

The independent evaluation (see Appendix 5) concludes that the RME approach had some benefits despite the limited nature of the intervention. It recognises the challenges for implementation in this context but suggests that this is an approach which is worth pursuing in further trials. Our qualitative analyses show that there is potential for impact, and suggest that this would be enhanced by different time frames for GCSE resit classes, in particular extension to a two-year course.

5.2.2 The materials

The emphasis on use of context in the materials interested and motivated the students. At times group discussion became lively, and spontaneous, as various students offered their opinions and life experiences. Contexts such as the ribbon and the computer download bar encouraged students to make sense of the strategies they used.

The RME models associated with the number module - i.e., the bar model and the ratio table - were shown to be particularly useful for GCSE resit students. They provided them with a new way of representing and thinking about a variety of familiar problems; they promoted informal strategies which can be very helpful to students, particularly those around grade E and F level; and they provided a visual representation through which students already competent in formal procedures can gain insights as to why their procedure works. The RME models are 'new' methods but they are not necessarily trying to replace the 'old' ones. They unify many elements of the number curriculum, which is particularly important within the confines of a short course, and they have the potential for use within other strands of the curriculum.

The algebra module offered less obvious cohesion than number in that the models associated with algebra were more varied. One of the algebra topics focused on how to use the bar model for solving a variety of word problems. Unfortunately, due to time constraints, this session was only trialled with one of the four groups, but we have since developed this material as part of a CPD package and it has been well received by teachers, who have found it empowering to realise that the bar model has potential to develop approaches in algebra as well as in number. The pressure to move from contexts which develop pre-formal algebraic thinking to formal symbolic algebra was more apparent in the algebra module, not least because this was a shorter module.

The materials are made up of a variety of genuine problems and as such they make a case for enabling students to develop their problem solving abilities as an integral, rather than 'bolt on' experience. This is particularly valuable in preparing students to sit the new 9-1 specification GCSE with an increased emphasis on problem solving and contextually based questions.

5.2.2.1 A note on algebra

The intervention group performed at a lower level than the control group in both pre- and post-tests in algebra, although at a higher level on the delayed test relative to the control group. Here we consider the issues and the successes relating to the algebra materials in more detail:

- The Algebra module was relatively short (9 hours) and in reality even shorter: two classes received 7.5 hours, another class 4.5 hours. This meant that tutors had to attempt to make short cuts through the materials or miss out sessions altogether. Such a short intervention is not conducive to an RME approach in terms of allowing students to genuinely embrace contexts and to gradually develop their informal representations.

- GCSE requires that students reach some demanding formal levels of symbolisation and algebraic manipulation, and in some cases the gradient of moving from the RME contexts written into the materials through to answering GCSE-type questions was too steep. This issue was exacerbated by the reduced amount of lesson time given to the intervention and created tensions for tutors, teachers and students.
- The algebra module uses a range of contextual situations and their associated models. The use of models is more subtle than in number and more wide ranging, and students were not always able to see how to relate the formal demands of a question to the informal strategies developed during the sessions.
- Some of the contexts used were very well received, including the ‘chip shop’ situation where students were able to develop reasoning and representations relating to simplifying, expanding brackets and factorising. The context of the seesaw and traditional weighing scales helped students to become successful in solving equations with ‘x’ on both sides. By representing a formal algebraic equation with an informal weighing scales picture, students unable to answer this question in their pre-test were able to deduce a solution. An example is shown in Figure 5-1.

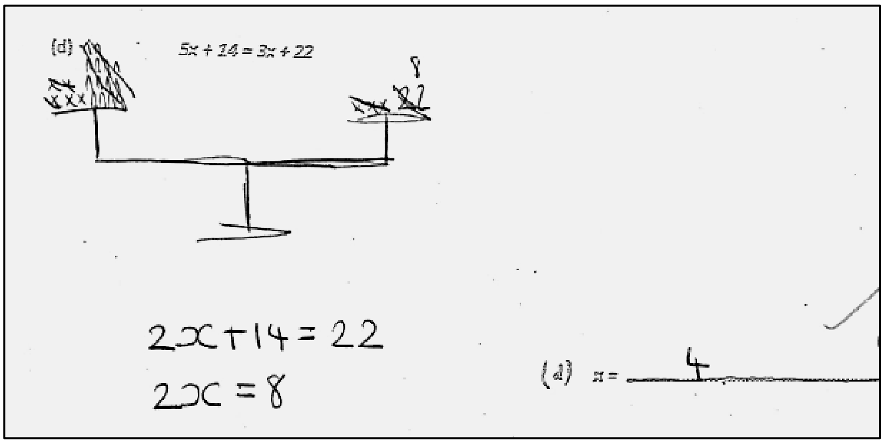

Figure 5-1 Algebra post-test example of a student using an informal strategy to solve an equation

- The Algebra module delivery may have benefitted from prioritising the order of sessions in order to build from the success of the Number module and in particular to focus initially on developing use of the bar model for algebraic thinking.

5.2.3 Timing and student engagement

The modules for this intervention were designed to take into account the relatively short amount of time available within a GCSE resit course, while balancing this with the deliberately slow pace of an RME approach. In reality, the contracted version of the algebra module and the length of time taken for students to open themselves to these approaches would suggest that a longer intervention could have more impact. At times researchers leading the lessons were forced to compromise RME principles in order to keep up to date with the coverage contract established with the host institutions. Teachers working with RME approaches for, say, the first term of a GCSE resit course would be able to devote more time to following student thought processes and promoting group interactions.

The positioning of the modules within the GCSE resit courses may have led to other issues. The number module was situated part way through the first term after host teachers had already established their classroom norms, meaning that classes viewed the intervention teachers as 'different to usual'. For some students this caused a reluctance to engage, and this seemed to be more evident when classes had been working with their own teachers for longer. In an earlier pilot phase, the intervention had commenced at the start of the year to good effect. The Algebra module fell towards the end of the second term, when the pressures relating to speed of coverage of topics were most pronounced.

Students' expectations of classroom norms relating to mathematics teaching and learning meant that some found it difficult to engage with how they were being asked to think and work. Setting up an RME culture where the student, not the teacher, makes decisions about what strategies to use and whether a solution strategy is correct or not, takes time; with two of the classes this was difficult to achieve within the confines of the short modules. This investment of time has the potential to hugely benefit students and would be easier to achieve within a longer intervention.

Once a formal procedure is taught to a class, there is often a shift of authority away from informal strategies, and a pressure to use the standard procedure and related reluctance on the part of the learner to use a method that they may now perceive to be inferior. This attitude was certainly present amongst some of the students we worked with, who were initially resistant to working with informal methods, perceiving them to be a backward step. This raises the question of how to adapt the intervention to help convince students to value the informal more. It also raises a much bigger question about the way mathematics is taught to students who find themselves in GCSE resit classes and in particular whether, how and when lower attaining students should be taught formal procedures.

6 Implications and recommendations

Overall, our curriculum design objectives were to produce materials that were true to the principles of RME but covered a significant amount of the Foundation Tier GCSE curriculum. However, while we have seen some success, particularly in terms of the qualitative analysis of the Number module tests, the speed of introduction of these models and the accelerated process of 'progressive formalisation' of the models may have impacted on the success of the intervention in terms of impacting on students' performance in examination conditions. In the Netherlands, RME is the adopted pedagogy from the moment that children begin their formal education and the approach is sustained throughout their schooling. In the Post-16 sector, RME will be 'competing' with other known but often misremembered and misunderstood strategies. Within this context, a number of implications arise from our findings with respect to mathematics teaching and CPD.

6.1.1 Recommendations for mathematics teaching at GCSE resit

6.1.1.1 The benefits of a two year course

We recommend that a two-year course would be beneficial, particularly for students who have achieved well below a standard pass grade. There are many students who repeat GCSE twice and achieve the same grade, or even worse, both times. Very few students are likely to move securely to a standard pass in what is at most a 9 month course. As we have seen, many lack basic skills. A two-year course would give teachers the time to not only address basic skills gaps, but to interest students in mathematics, explore the often deep-seated misconceptions that students have at this level, promote genuine understanding, and produce students who not only have a better grade in mathematics, but also a greater facility to use the subject in the future. The importance of such a shift is underlined by the changes in the new 9-1 specification towards greater depth, more focus on problem-solving, and the provision of clear mathematical arguments.

6.1.1.2 Incorporating a full term 1 RME approach in one year courses

We recognise the pressures on providers to offer resit courses as one year courses only. In this context, we recommend that the benefits of the short intervention in this project are extended and consolidated by employing RME-based materials and pedagogy throughout term 1. This would enable coverage of more content and work on unifying models which would provide a robust base for a more effective traditional focus on revision and past papers in terms 2 and 3.

6.1.1.3 Building student learning skills

Students not only need to improve their basic skills: they need to develop learning skills. Many students did not believe that mathematics made sense, and consequently never reflected on their answers. They needed to develop skills of questioning their own and others' strategies, and of sharing and evaluating ideas. Interaction is a key principle of RME and needs to be fostered and to some degree learnt by students used to traditional teaching. We recommend a 2-week introductory module focussing on learning, with regular re-visiting throughout the course. Given the current pressure on time, this would ideally be part of a two-year course, although this could be time well spent even in a one-year course.

6.1.1.4 Building a classroom culture that supports learning

Post-16 resit students understandably feel that they have 'failed' in mathematics. They have little confidence in their own ability, and just want 'to be told 'what to do. The analyses in Section 4 show students who are reluctant to engage in genuine discussion about mathematics or to make any real effort to 'make sense' of the subject. Yet these are crucial elements if they are to improve their understanding and ultimately their attainment. Hence, teachers need to effect a shift in classroom culture and student attitude. Section 4 clearly shows that this is possible, but it requires skill and intent on the part of the teacher and also, again, it requires time. It is also important that teachers have materials, classroom activities, and ways of working which support this shift, and RME provides this. Part of the problem with the notion of a 'resit' class is that it hints at the fact that only the exam counts and the only success criteria is a 'pass'. In this context, neither teachers nor students are really encouraged to explore the subject, create interest, or deepen understanding. This project has shown the potential for RME to do these things while still improving student performance.

6.1.2 Implications for Continued Professional Development (CPD)

The independent evaluation report recognises the need for "effective and achievable CPD for teachers" and asks how this might be constituted. The number of teachers who have already signed up for the CPD courses which we have developed as a result of this project is an indication of how much they feel the need for support at this level. There is recognition that 'doing the same thing over again' is not just unproductive, but also demoralising for both students and teachers. We sense a genuine desire for change. To effect change, however, teachers need both materials that support the change, and also regular Professional Development. A simplistic 'front-loaded' CPD model will not be sufficient: the pressures on teachers, well documented throughout this report, will see them reverting to previous practice. Section 4 clearly shows that using RME materials can bring about a change in classroom culture in which students will engage in discussion and sense-making. It also shows, however, that this is a difficult process and that teachers need support if they are to effect this change.

6.1.2.1 Lesson study as support for substantial and sustainable CPD

Many GCSE resit teachers' knowledge of mathematics is likely to be procedurally dominated, so that even if they want to operate in a different way, they may not have the expertise to do so. RME can support the development of teachers' pedagogic and subject knowledge: models bridge the gap between informal and formal understanding, and their use enables teachers to feel less pressure to move to early formalisation. A well-designed RME-based CPD has the potential to develop teachers' support for students working at differing levels of abstraction, so that those who find more formal notions difficult can continue to make progress and develop strategies for solving problems. We recommend that CPD provides opportunities for teachers to meet regularly with curriculum designers and other teachers in order to understand the relationship between the design principles and the materials.

One way of providing a CPD framework to support this development is Lesson Study (see Huang & Shimizu (2016); also http://tdtrust.org/what-is-lesson-study). This model involves teachers working with RME materials and planning lessons in pairs, teaching lessons and observing each other, and feeding back on course days where they have the opportunity to analyse and reflect on their implementation of the material with other teachers and RME tutors. We recommend that this model is followed through the course of the academic year, with up to 5 course meetings, enabling teachers to develop their pedagogic and subject knowledge in ways that will support a sustained RME approach in the challenging environment of the GCSE resit classroom.

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8 Appendices

8.1 Appendix 1 – The pilot study, 2012-2013

The majority of the pilot took place in a large further education college, working with one teacher and two groups of students with approximately twenty in each group. There were two interventions, in October (Number) and January (Algebra), both consisting of around 10 hours teaching together. Some lessons were delivered by the usual class teacher, some by the research team, and some by both. The students varied in age from 17 to 50, and were studying mathematics GCSE for a wide variety of reasons. We focus here on student performance in Number.

Students were tested on standard GCSE Number questions prior to the intervention. One week after the intervention finished, students were given the same GCSE questions as a Post-test. Two months later they were given different GCSE questions of a comparable level of difficulty to the original questions, on the same topic, for questions 1 and 2 only. Table 8-1 shows the percentage of students who answered each question correctly.

Number: GCSE type questions	Percentage of students answering correctly (n = 39)		
	Pre-test	Post-test	Delayed post-test
Q1 Rates	35%	82%	65%
Q2 Percentages	28%	72%	69%
Q3 Comparison	11%	68%	N/A

Table 8-1 Pilot results

The data show improvement in the performance of the students on all three questions. Of particular note are the delayed Post-test results for questions 1 and 2, which suggest that for most students the improved performance extended beyond the short term.

<u>Qualitative analysis of pre-test questions</u>: many students left questions blank, or wrote ‘can’t do this’. Sometimes they wrote down an answer with no working or explanation. Evidence of students attempting to ‘make sense’ of the problem was rare. Interviews revealed that their strategy often was to attempt to remember previously taught procedures which had not been understood, or had been committed to memory but then forgotten.

<u>Qualitative analysis of post-test questions</u>: many students filled the page with their own individual versions of the RME models experienced in the intervention phase. They were not only successful at adopting these strategies, but also showed confidence and flexibility in making their own choices about how to apply them to a range of topics in Number.

Interview data: We interviewed a sample of students after they had completed the post-test. The focus was on the strategies they had employed on both pre- and post-tests. For example, one student had written very little prior to the intervention when answering the GCSE questions. For two of the questions she simply wrote down answers (one correct, one incorrect), for others she wrote nothing. By contrast, in the post-test, her solutions included diagrams using models from the interventions, which showed clarity and structure. At interview she was able to explain her reasoning and commented with confidence on how she was now able to engage with a problem using the models as a basis.

Overall findings of the pilot study

- Students showed sustained improvement in their performance in Number.
- They were able to adopt the RME models of the bar and the ratio table and by the end of the intervention could successfully apply these to solve a wide variety of problems in Number.
- Students showed evidence of adopting sense-making approaches to solving problems in Algebra. This enabled them to make progress in solving equations, expanding brackets and solving simultaneous equations, although several students appeared to need more time to develop their understanding of the RME-based approach to Algebra.
- The Algebra material needed to be re-designed to take account of the limited time available. In particular the material needed to enable students to build on their success of using the bar model in Number to solve a variety of problems in Algebra.
- The visual nature of the RME models helped many students, and the idea that drawing a picture to represent a problem was a tool for 'unlocking' students who felt they were 'stuck'.
- Teachers need support if they are to adapt an RME approach in the Post-16 context. In particular, they need help to recognise how widely applicable the RME based methods are to solving a large number of GCSE problems. They also need help to recognise that RME methods do not necessarily need to replace the methods that students bring themselves, but instead can be used to enable them to develop an understanding of why their particular method works.

The pilot study demonstrated the potential benefits of RME with this group of students and teachers. It also highlighted the need for further work on understanding the challenges for this group, and the specific materials requirements.

8.2 Appendix 2 – Design principles

Examples from the teaching materials illustrate the use of RME theory within our curriculum design.

Figure 8-1 Introducing the context

Figure 8-1 introduces a context that involves Aidan investing his redundancy money in a sweet shop. Although the context is not real to 16 year old students it is 'realisable'. Van den Heuvel-Panhuizen (2005) and Gravemeijer, Van den Heuvel & Streefland (1990) suggest that a context is valid if it is 'realisable' to the student and that with younger students, fairy tales often provide useful contexts. At this stage, the question posed to the group is non-mathematical and is designed to get them involved in the context.

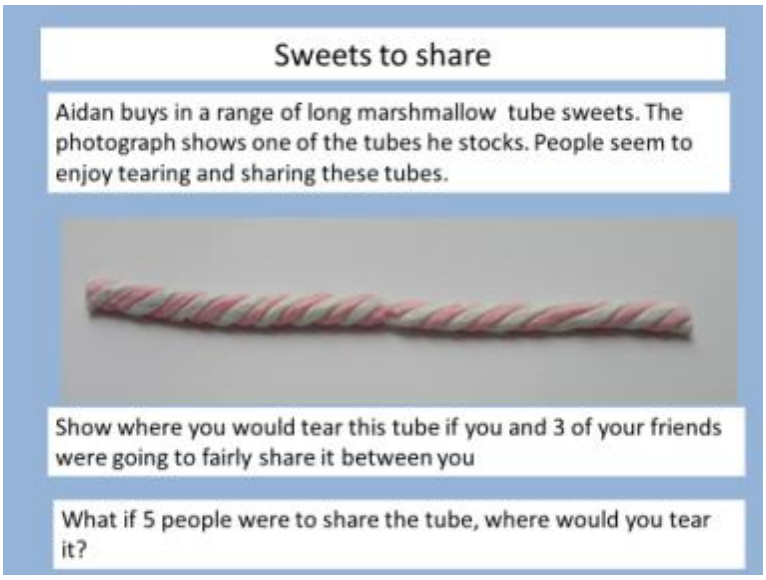

Figure 8-2 Introduction of fair sharing

In Figure 8-2, the concept of fair sharing is introduced with a question which provides purpose to a mathematical task. Providing a sense of purpose is a key element in the materials.

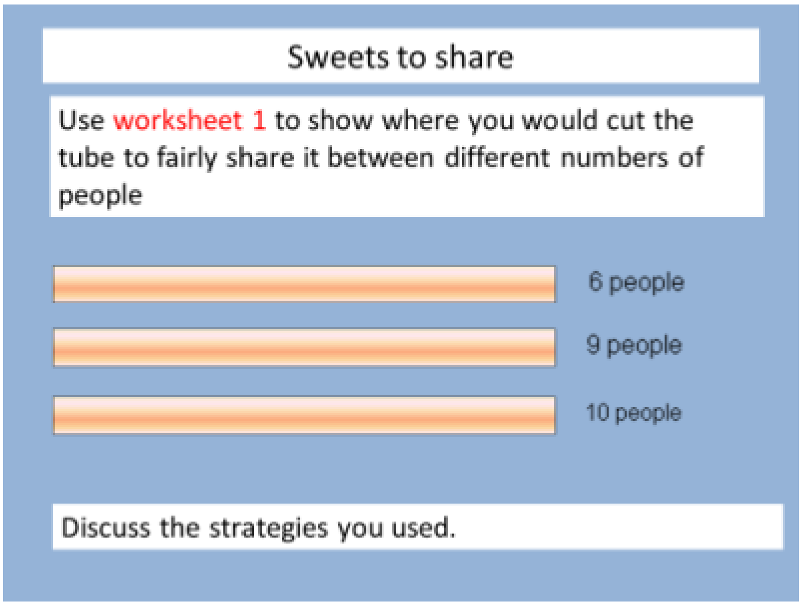

Figure 8-3 Introduction of 'model of'

As we move onto Figure 8-3, the marshmallow is represented as a long rectangle or bar. In other words, the bar has become a 'model of' the marshmallow (Treffers, 1987). This 'model of' provides an informal representation of the mathematics but remains closely connected to reality (in this case, the marshmallow tube).

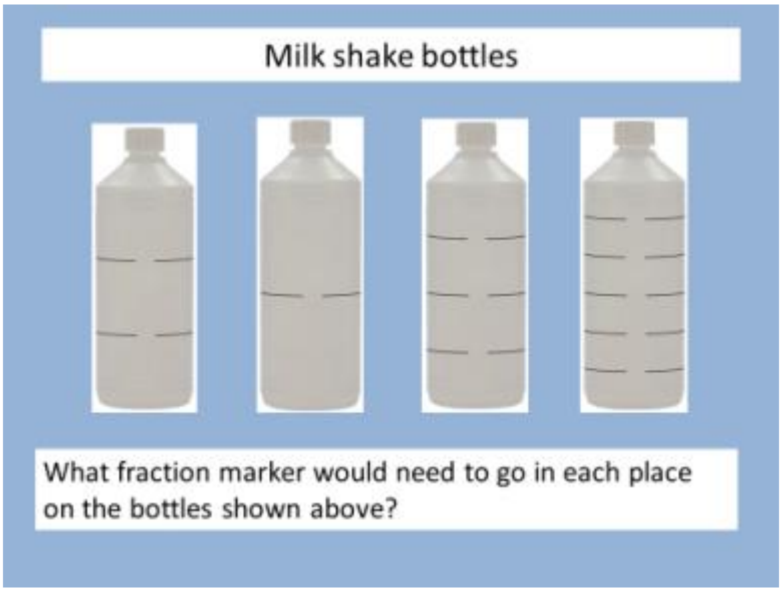

Figure 8-4 Repetition of 'model of'

A second context is introduced in Figure 8-4, again leading to a rectangular representation that is close to reality. This again results in a 'model of', and it is the repeated use of 'models of' that will allow students to appreciate the generalisable use of the bar model and thus begin to use it as a 'model for'. Fraction notation is introduced here, but within a problem that is very close to reality.

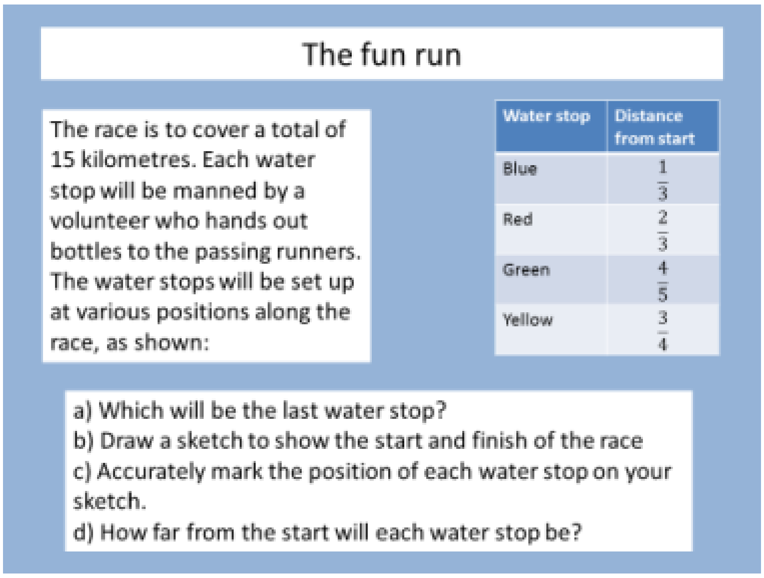

Figure 8-5 Bridging the gap between informal and formal

A third context is considered in Figure 8-5, again encouraging the students to draw a bar. However, here the bar represents a distance of 15km and students are required to find specific fractions of that distance. The bar has become more abstract at this point, and some students may represent it as a double number line. Van Den Heuvel-Panhuizen (2003) describes this as bridging the gap between informal understanding overtly linked to reality, and more formal mathematical systems. This gap is closed further in the next slide.

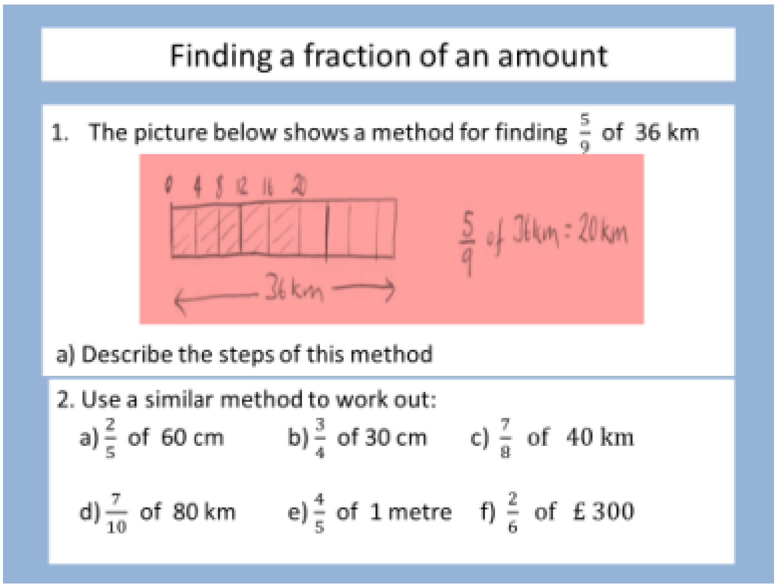

Figure 8-6 Closing the gap and the top of the 'iceberg'

Question 2 of the slide depicted in Figure 8-6 presents problems which are approaching the abstract world of mathematics and would be at the top of Webb's 'iceberg' (Webb et al., 2008).

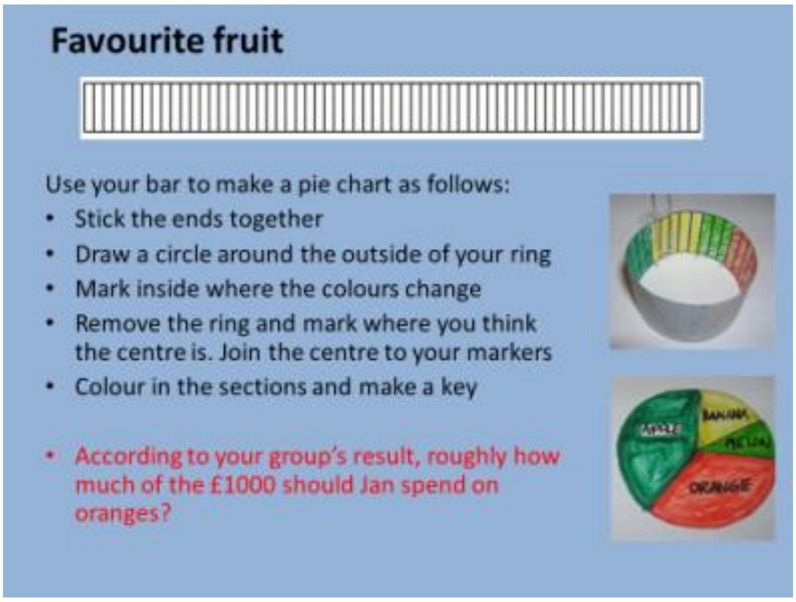

Figure 8-7 Use of models in multiple areas in mathematics

Figure 8-7 shows an activity from later in the series of lessons. Now the bar has become a stacked bar chart, and at this point students will draw pie charts from the bar chart. This exemplifies the repeated use of models in a variety of areas of mathematics thus making effective use of the limited time available in Post-16 resit courses.

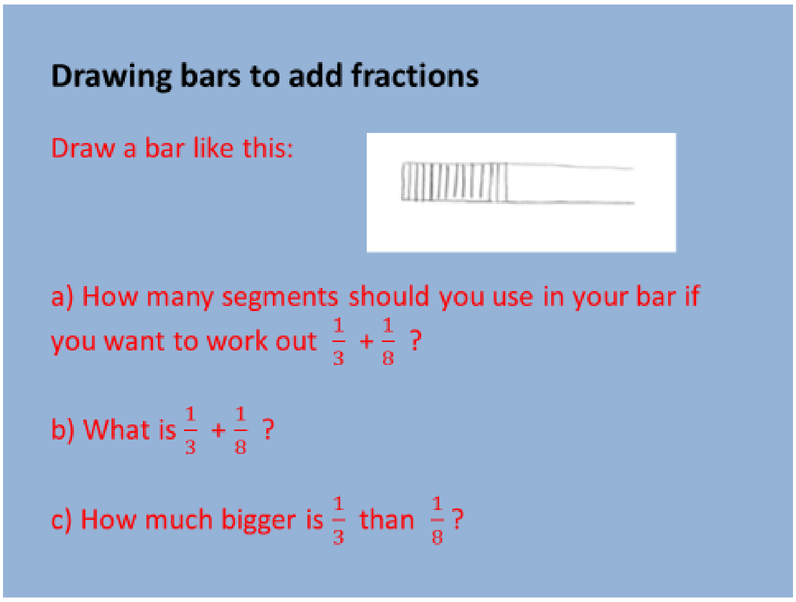

Figure 8-8 Visualising fractions to make sense of common denominator

In Figure 8-8, a segmented bar is being used for adding fractions. The purpose here is to provide a model that will form a visualisation of the fraction and allow students to make sense of the use of common denominators. Students with a recollection of adding fractions, however strong or weak, map their solutions onto the bar to allow them to gain further insights into the mathematics.

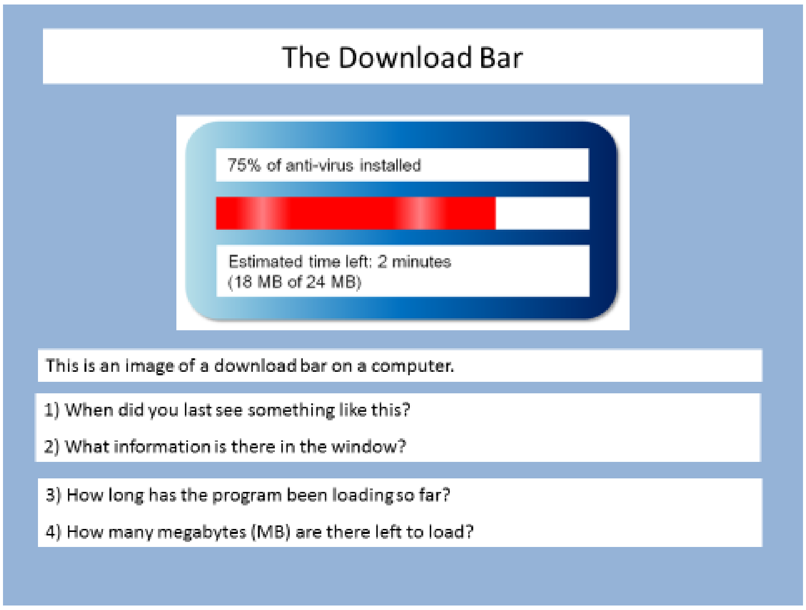

Figure 8-9 Using the bar in a familiar context

The problem in Figure 8-9 is introduced in the early stages of a lesson on percentage. The bar is again used in a context familiar to the students and they will be encouraged to discuss the context.

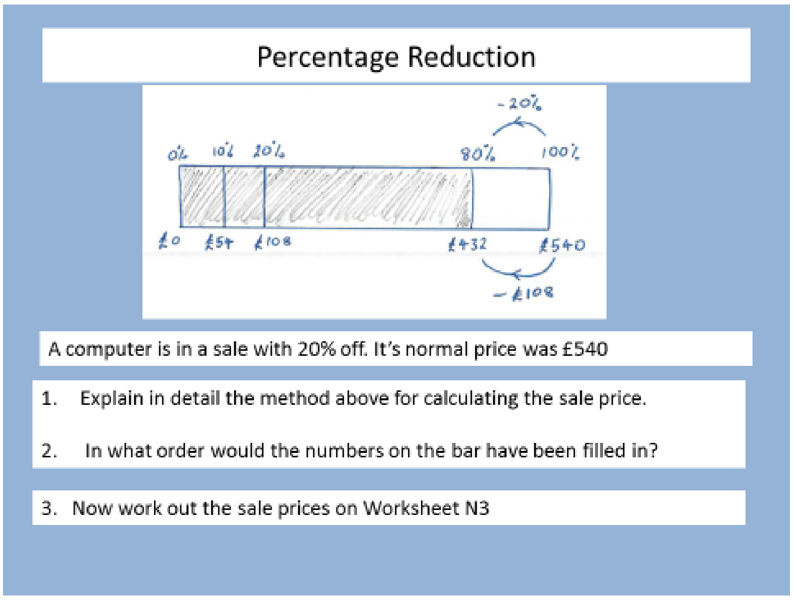

Figure 8-10 The bar as a 'model of'

As we progress through this unit of work, the students increase and reduce amounts by a given percentage (see Figure 8-10). At this point, the bar has become a 'model of', with the students using it for a variety of questions involving proportional reasoning (Treffers, 1987).

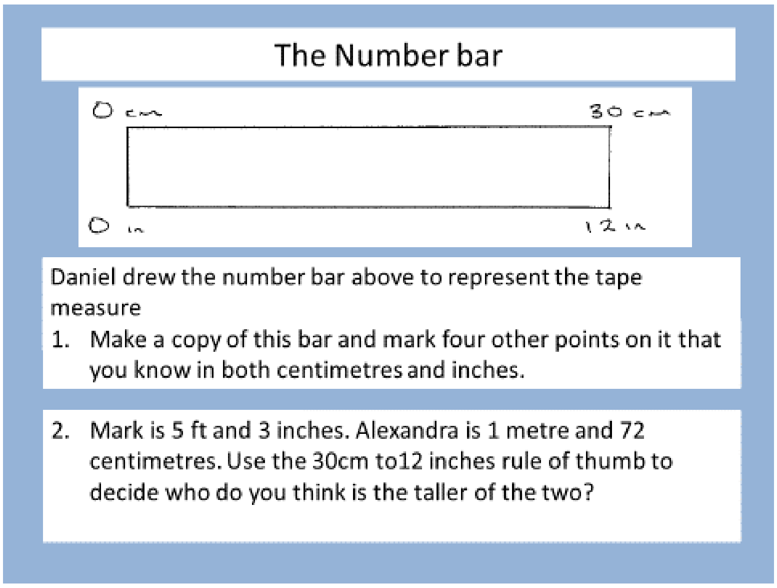

Figure 8-11 Recognising the limitations of the bar

The bar model in Figure 8-11 is close to reality but here students are encouraged to become aware of a limitation. They will understand that in certain situations there will be a need to make conversions between much larger measurements but that the bar will not allow this as it is a representation which is drawn to scale. This creates the need for a new representation and a purpose for the introduction of the ratio table.

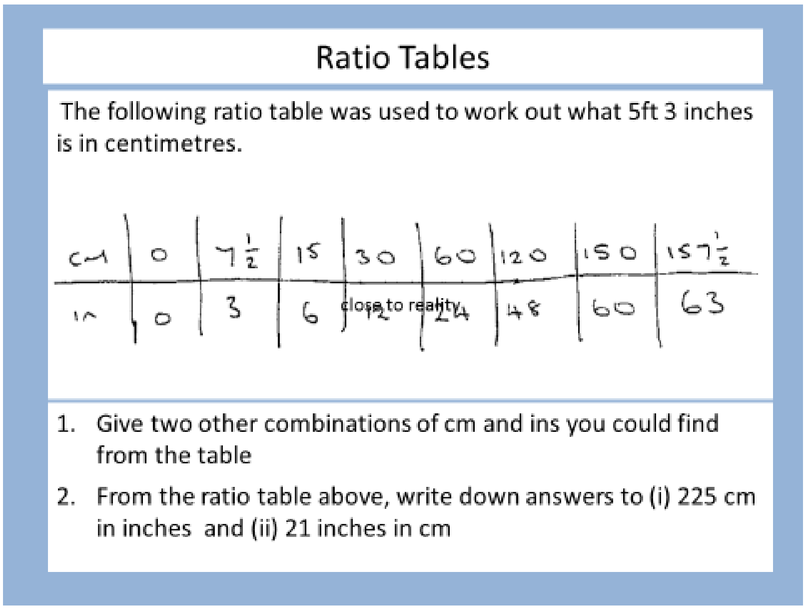

Figure 8-12 Ratio tables as a more flexible model – a 'model for'

The ratio table in Figure 8-12 is a more flexible model as it allows proportional pairs to be calculated and recorded without needing to be to scale. Soon after this, students will begin to use the ratio table to record in non-ascending order to answer a variety of problems. The ratio table is a 'model for', and could be considered to be a 'pre-formal' model. It does not resemble the context that it represents, and yet the use of real life units on the left hand side allows the user to refer to the context. For example, doubling the numbers in a column would make sense (twice the inches would equal twice the centimetres). However, adding two to the top and bottom would not make sense (two more inches would not mean two more centimetres). Middleton and Van den Heuvel-Panhuizen (1995) concluded that after being exposed to ratio tables,

> *.. students who traditionally had trouble with computation began to perceive the underlying structure of the mathematics and became more proficient at computation of rational numbers*

This would suggest that the ratio table would be a useful tool to develop for Post-16 resit students who typically have poor computational skills and number sense. Along with the bar model, the ratio table is an example of a pre-formal model that van den Heuvel-Panhuizen describes as a means to bridge the gap between real life problems and formal mathematics (Van den Heuvel-Panhuizen, 2003).

8.3 Appendix 3 – The number and algebra module contents

Content of Number module at a glance:

Unit	Content
The Sweet shop (4 hours)	• Using contexts which lead to bar model representations for fractions • Developing the idea of what a fraction is • Comparing fractions using various strategies • Drawing and using a bar model to **find a fraction of an amount** • Drawing and using the bar model to **divide in a given ratio**
The Canteen Survey (4 hours)	• Making connections between the bar model (segmented strip) and the circle model (pie chart) • Developing ways to **compare, add and subtract fractions** using a bar (segmented strip) • **Division of fractions** using a bar
Solving Problems using the Bar Model (3-4 hours)	• Using contexts that lead to bar model representations for percentages • Drawing and using a bar model to **find a percentage of an amount and percentage increases and decreases** • **Finding the original amount after a percentage change** • Problems involving **depreciation and repeated percentage change** • Problems developing an understanding of proportional reasoning
Ratio tables (4 hours)	Using ratio tables to solve problems associated with proportional reasoning; • Direct proportion • Recipes • Conversions • 'Best buy' Unitary method

Content of Algebra module at a glance:

Unit	Content
I Think of a Number (1.5 hours)	• Introducing algebraic notation. • Solving equations where the unknown appears once. • Rearranging equations where the unknown appears once.
Graphs (1 hour)	• Reading expressions • Creating tables of values and graphs of linear and quadratic functions
Fish and Chips (1.5 hours)	• Simplifying algebraic expressions • Expanding and factorising algebraic expressions
Easy to See (1 hour)	• Encouraging students to make sense of algebraic equations. • Solving a variety of equations, including quadratic, using common sense strategies.
Word problems using a bar model (2 hours)	• Enabling students to represent a variety of word problems using one or more bars • Developing strategies for comparing bars in order to solve a variety of word problems • Recapping and developing the use of the bar as introduced in the Number module
Balance (1.5 hours)	• Using the context of weighing scales to develop a conceptual understanding of balance • Solving equations with ‘x’ on both sides and brackets, using the context of balance

8.4 Appendix 4 - The survey bar lesson

Slides used in Lesson A and Lesson B

SLIDE 1 Healthy eating

City college has appointed a new cook, Jan. In her last college Jan made a big impact on getting students to eat more healthily. One of her strategies was to survey her students about their food preferences. She started by asking about fruit and vegetables.

Jan made these pictures from the results she got:

SLIDE 2 Favourite type of food

Another way Jan thought might increase the number of students eating in the canteen was to have themed lunches. So one day it would be Italian, another day Indian and so on. Again she surveyed students to find out what themes might be popular. Students were asked to choose their favourite theme.

Here are the results from a year 7 class and from a year 12 class:

Type of food	Year 7 class (30 students)	Year 12 class (20 students)
Indian	3	3
Italian	6	5
American	15	10
Spanish	1	2
British	5	0

Is Indian food more popular with the year 7s or the year 12s?

Is Italian food more popular with the year 7s or the year 12s?

Favourite type of sandwich SLIDE 3

Students in year 8 and year 10 were asked to choose their favourite type of sandwich.

Here are the results:

Flavour of sandwich	Year 8 class (25 students)	Year 10 class (40 students)
Ham	8	16
Cheese	4	7
Tuna	7	11
Egg	6	6

a) What size bar would be good to use if you wanted each bar to have the same number of pieces?

b) Are tuna sandwiches more popular with the Year 8 or the Year 10 students?

Favourite type of sandwich SLIDE 4

Jan asked 20 teachers at City college about their favourite flavour of sandwich.

$\frac{1}{5}$ said ham, $\frac{1}{2}$ said cheese, $\frac{1}{4}$ said tuna, $\frac{1}{20}$ said egg.

a) How many of the 20 teachers said ham?

b) How many said tuna or egg?

Canteen use SLIDE 5

At a staff meeting, Jan asked people how often they used the canteen.

$\frac{1}{3}$ said every day,

$\frac{1}{4}$ said three or four times a week,

$\frac{1}{10}$ said once or twice a week.

a) How many people do you think Jan asked?

b) How many people said they never used the canteen?

Background music SLIDE 6

Some Year 12 students suggested that it might be good to have background music playing in the canteen. Jan asked some students what they thought.

$\frac{1}{4}$ said no to music, $\frac{1}{6}$ said they didn't mind either way

a) Jan seems to think she asked 40 people for their opinion. Is this possible?

b) Suggest some numbers of people she might have asked.

Favourite type of hot drink SLIDE 7

Jan wondered what types of hot drinks might be popular with students. She thought that younger students might have different preferences to older students, so she surveyed Year 7 and Year 12 students for their favourite hot drink.

	Tea	Coffee	Hot Chocolate
Year 7	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{2}$
Year 12	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{8}$

a) What sizes of segmented bar could you use to represent both $\frac{1}{5}$ and $\frac{1}{3}$

b) What is the difference between the fractions of students in year 7 and those in year 12 whose favourite hot drink is tea?

Drawing bars to add fractions SLIDE 8

Draw a bar like this:

a) How many segments should you use in your bar if you want to work out $\frac{1}{3}+\frac{1}{8}$?

b) What is $\frac{1}{3}+\frac{1}{8}$?

c) How much bigger is $\frac{1}{3}$ than $\frac{1}{8}$?

Drawing bars to add fractions SLIDE 9

a) Draw a segmented bar to help you show that $\frac{2}{5}+\frac{1}{2}=\frac{9}{10}$

b) How much bigger is $\frac{1}{2}$ than $\frac{2}{5}$?

Drawing bars to add and subtract fractions SLIDE 10

Use the method of drawing a segmented bar to work out the following:

a) $\frac{1}{4}+\frac{5}{12}$

b) $\frac{2}{7}+\frac{1}{3}$

c) $\frac{1}{2}+\frac{3}{5}$

d) $\frac{1}{4}+\frac{3}{8}$

e) $\frac{6}{15}+\frac{3}{10}$

8.5 Appendix 5 – The independent evaluation report

Independent evaluation of 'Investigating the impact of Realistic Mathematics Education approach on achievement and attitudes in Post-16 GCSE mathematics resit classes'

Mark Boylan and Tim Jay

Sheffield Hallam University

8.5.1 Introduction

Sheffield Hallam University (SHU) undertook an independent evaluation of the Nuffield Foundation-funded Manchester Metropolitan University (hereafter abbreviated to MMU) investigation of the impact of Realistic Mathematics Education approach on achievement and attitudes in Post-16 GCSE mathematics resit classes GCSE resit intervention. For brevity the intervention is described in the report as the RME GCSE resit project.

This report should be read alongside MMU's description of their intervention in Sections 1-3 of the main report, which provide further detail of the intervention, its theoretical background, materials and activities. A summary is provided here as part of the implementation evaluation.

In the main report, previous implementation and research on the Realistic Mathematics Education (RME) intervention is described. In the post-16 context a recent review of interventions that have the potential to support attainment at a GCSE equivalent has been undertaken[1] l. This considered both previous use of RME and also an intervention that ha some similar features - for example, the use of mathematical models and an emphasis on conceptual understanding[2]. This review suggests that there is some evidence that interventions similar to RME could have a positive effect on attainment. Given that, the Post- 16 GCSE resit intervention is a newer context for RME. Therefore, MMU undertook a relatively limited intervention in terms of number of students involved and the number of hours of intervention across the academic year, to test RME in the GCSE resit context and to research the use of an RME approach in this context in terms of students' and teachers' experience, and practical issues of implementation.

In addition, the intervention focused on two areas of mathematics - aspects of number (proportional reasoning) and elements of algebra.

In the intervention, the researchers worked directly with the students, and class teachers had the opportunity to observe. Whilst MMU had undertaken development work in GCSE resit contexts and so materials had been piloted, in terms of a trial methodology this could be considered a pilot[3].

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[1] Maughan et al. (2016). Improving Level 2 English and maths outcomes for 16 to 18 year olds Literature review. London: EEF.

[2] Swan, M. (2006). Learning GCSE mathematics through discussion: what are the effects on students? *Journal of Further and Higher Education*, vol. 30, no. 3, 229—241.

[3] See https://v1.educationendowmentfoundation.org.uk/uploads/pdf/Classifying_the_security_of_EEF_findings_FINAL.pdf

The aims of the evaluation were:

(i) To provide an independent evaluation of the impact of the RME approach as operationalised in the project, on Post-16 GCSE resit students' achievements and attitudes in mathematics.

(ii) To provide independent advice to the MMU team on issues of fidelity and teacher CPD.

(iii) To provide independent advice on the scalability of the intervention for a larger efficacy trial using a randomised controlled trial methodology.

8.5.2 Evaluation methodology and methods

8.5.2.1 Evaluation approach and rationale

A quasi-experimental design was appropriate for this pilot in which 4 classes in three different locations received the RME intervention and 4 classes in the same locations did not. The quasi-experimental design allowed:

- comparison of outcomes for students in relation to attainment and attitudes
- comparison of student learning outcomes as a result of a limited experience of alternative RME teaching as a supplement to prevailing teaching in post-16 and usual approaches
- identification of professional learning implications

However, there are limits to the reliability of the comparison of impact given the size of the samples and the fact that this was necessarily a clustered trial. Further, constraints on recruitment of the intervention and comparison groups meant that randomisation at class level was not possible[4]. MMU were able to recruit teachers who were willing to have their classes participate in the project and so these were assigned to the intervention condition. Given this, there is a threat to the security of the trial due to possible imbalance between the intervention and comparator samples on both observable and likely non-observable relevant characteristics.

We compared impact between intervention and comparison classes on areas of mathematics targeted by the intervention - aspects of number and algebra. There were three phases to the evaluation of impact, corresponding to the two phases of the intervention plus a final delayed post-test.

[4] For this reason, the trial did not meet the standards for CONSORT registration.

The testing approach was designed to minimise the disruption for students and classes involved, an important ethical consideration and one designed to help to reduce attrition. The delayed post-test included a mixture of number algebra and other GCSE mathematics questions. The delayed post-test aimed to assess whether there was evidence a sustained impact of the intervention. However, it is important to note that the length of the delay varied between number and algebra module teaching and the delayed post-test.

8.5.2.2 Measures of impact

8.5.2.2.1 Attainment measures

MMU and CDARE agreed the overall test design, with questions/sections populated by MMU. Questions were based on or adapted from GCSE questions thus leading to a degree of external validity of the assessment tool.

The Phase 1 test, focused on number, used the same questions in the pre-and post- tests. This was to support planned diagnostic interviewing. The test had 7 questions, of which 2 asked for a self-assessment of whether the answer was correct and for an explanation of this. This generated qualitative data about learners' relationships to mathematics and their strategies.

The Phase 2 test contained 7 questions focused on algebra and used the same questions in the pre-and post- tests.

The delayed post-test contained 15 questions, 5 on number, 5 on algebra and 5 on other topics.

Tests were marked by MMU researchers using agreed mark schemes, based on a GCSE approach giving marks for methods as well as final answer.

In addition, papers were analysed using the following criteria:

- Attempted/not attempted (1-0) – to provide a data source on possible impact on resilience.
- Applies a relevant RME model appropriately (1-0) – to assess the effectiveness of RME teaching regardless of effect of impact. This also provided indirect evidence of whether the RME intervention had increased the students' range of appropriate methods – for example a student might have answered a question correctly in the pre-test and in the post-test answered the question correctly again, but using a different method.
- If using an RME approach, makes sense of the problem (1-0) – whether an RME approach was used in a way that was appropriate and/or potentially productive to provide data on impact on sense making.

8.5.2.2.2 Attitude measures

The attitude scale was developed by MMU and was adapted from the Understanding Post-16 Participation in Mathematics and Physics Project[5].

Additional items were designed to identify epistemologies of mathematics in the target group in the RME GCSE resit project.

8.5.2.3 Analysis

Statistics were generated regarding attrition and balance across conditions.

The attitude scale was analysed by Principle Component Analysis by SHU. Reliability analysis (Cronbach's alpha) was used on the resulting factor in order to test for internal reliability. It was outside of the scope of this evaluation to assess test-retest reliability.

Analysis was undertaken to consider if attitudes to mathematics are affected by participation in the RME intervention. This analysis was an ANCOVA, with 'attitude to mathematics' as the dependent variable, 'experimental group: intervention/control' as independent variable, and controlling for attitudes to mathematics at pre-test.

The analysis also considered if attainment in mathematics was affected by participation in the RME intervention. Alongside the analysis of attainment, we carried out a similar analysis for the dependent variable 'use of RME principles' as described above in order to determine whether students in the RME intervention group respond to questions in the target and non-target domains using RME approaches (e.g. visualisation and modelling). The measure of difference in attainment could be used to calculate a Minimum Determinable Effect size (MDES) for future studies.

The analysis considered if there are relationships between attitudes and attainment. This analysis helped to assess associations among the variables measured. Part of the rationale for the intervention is that an RME approach can lead to different perceptions of mathematics and of how to go about doing mathematics, which will in turn lead to higher levels of attainment. Multiple regression analysis was used to assess relationships among attitudinal factors, use of RME principles, and effects on attainment in both target and non-target domains.

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[5] Reiss, M, (2012) Understanding Participation in Post-16 Mathematics and Physics (UPMAP) ESRC End of Award Report, RES-179-25-0013. Swindon: ESRC. http://research.ioe.ac.uk/portal/en/projects/understanding-participation-rates-in-post16-mathematics-and-physics-upmap%2845a02dd7-1141-4e70-a97b-bcb65adc79f4%29.html

8.5.2.4 Process evaluation

SHU undertook a light touch process evaluation, consisting of the following activities.

- Visit to 1 college during phase 2 and write up. The visit included a joint observation with an MMU researcher to evaluate the use of the observation protocols, and to consider the RME approach in the classroom in relation to a possible efficacy trial.
- Telephone interviews with the 4 intervention teachers (originally it had been planned that the college visit would include a face to face interview with the host teacher, but this was not practical).
- Review of MMU research processes in relation to both intervention and control teacher practices.

8.5.3 Test conduct and marking

8.5.3.1 Conduct of the tests

There were no issues reported with the conduct of the number test.

There were a number of issues that the MMU team identified in relation to the algebra test.

- Test conditions for the post-test were not consistent. Due to the distance involved and time constraints, at one site intervention and control students were supervised by their class teachers for the post-test. During post-test interviews two students from the control groups at the same site revealed that their teacher had helped them to answer questions during the test. Closer examination of scripts from this site revealed that some control students produced very similar responses to particular questions. At the other two sites supervision was by one of the intervention staff.
- One control group is known to contain more able students than the corresponding intervention group due to the way the students are grouped.
- RME marking. It is much harder to see overt use of RME methods in the case of Algebra than number. In the case of the Algebra test, it was only possible to see use of RME in some of the questions. It is possible that students were using RME models to answer questions (but this could only be revealed through in depth post-test interview).

There were also issues reported by MMU with the delayed post-test:

- Test conditions for the delayed Post- test were not consistent. At one site students did half the questions as part of a GCSE mock paper. They did the other questions later. This led to a loss of data as several students were absent (in particular, those in the intervention group were not present for the second half of the test).
- Researchers supervised the tests for three pairs of intervention and control classes. At the other site, the same one where issues were identified for the algebra test, class teachers supervised. At this site, there was evidence that some students were allowed

access to calculators and that in some cases students had been given help by their teacher.

8.5.3.2 Test attendance

MMU undertook an analysis of patterns in test attendance. The results show a pattern of increasing absence from courses. Although this information is based on just 5 data points, it is worth pointing out that students did not always know that they were going to take a test, and so each point represents a normal day as far as they were concerned. Absence rates are very similar for intervention and control groups, with better attendance by intervention students with one exception – the algebra post-test. However, this greater absence was not statistically significant. There was one incident of a significant difference between the groups: absence from the algebra pre-test was far lower for the intervention students (chi square = 7.600' df = 1, p = 0.006). Percentages are listed in the table below, in order of occurrence through the year.

Test (* p=0.006)	Intervention group absence	Control group absence
Number pre-test	12%	15.3%
Number post-test	20.3%	23.6%
Algebra pre-test	12%	30.6%*
Algebra post-test	37.8%	30.6%
Delayed post-test	31.1%	41.7%

Table 8-2 Test absence

8.5.3.3 Test marking and moderation

MMU blind marked the tests. Tests were mixed for intervention and comparison students clustered in each school and then double marked by the researchers who were not directly researching/teaching in that school. Following moderation MMU derived an agreed mark. MMU provided a data file that included details of student demographics and other key data (gender, age, prior GCSE grade, when GCSE last taken, college, class teacher).

Due to the reasons identified above concerning the conduct of the algebra test and initial analysis of outcomes (see below) for efficiency, SHU undertook moderation of number tests only.

SHU undertook a process of randomisation to identify a sample of the number test that was then requested from MMU. The stratification matrix sampled by groups and between intervention and control. The sample was moderated to evaluate consistency, and following blind marking by SHU a 100% measure of inter-rater agreement was found between SHU's marking and MMUs.

8.5.4 Data analysis

This section reports findings from the analysis of data collected by the MMU project team. The first part of the section focuses on addressing the question of whether the intervention had an effect on students' attainment in number or algebra. The second part addresses some further questions about outcomes of the intervention, including those focusing on students' use of RME approaches in their solution strategies and on potential group differences in response to the intervention. The third and final part of this section focuses on questions relating to the attitude survey completed by students before and after the intervention.

8.5.4.1 Balance and attrition

Data were collected from 147 students, in eight groups. 75 students, in four groups, were part of the intervention group, while the remaining 72 students, in another four groups, were part of the control group.

Missing data were an issue for all analyses in this section. Complete data (pre-, post- and delayed-post-test scores for both number and algebra) were received from only 52 students (29 intervention, 23 control). Numbers will be reported for each analysis, and students are included in each analysis for which complete data were collected; i.e. a student will be included in analyses of number data even where there may be algebra data missing. The high level of attrition has implications for interpretation of findings, although the post-16 mathematics retake context is often subject to similar rates of attrition.

8.5.4.2 Did the intervention raise levels of attainment?

This section essentially aims to answer four questions. Did the intervention lead to higher levels of attainment in number and/or algebra for those in the intervention group, relative to the control group, at either post-test, or delayed post-test?

- Was there an effect on attainment in number at post-test?
- Was there an effect on attainment in algebra at post-test?
- Was there an effect on attainment in number at delayed post-test?
- Was there an effect on attainment in algebra at delayed post-test?

Figure 8-13 shows pre-, post-, and delayed test scores for number. This shows that scores for both groups increased. The intervention group started at a lower level, but at post-test and delayed test performed at close to the same level as the control group. Note that delayed test scores reflect results from a test comprising number and algebra items which were testing the same concepts as the pre- and post-tests, but with different questions; the delayed post-test also had a higher maximum score, and so these scores are not directly comparable with pre-and post-test scores.

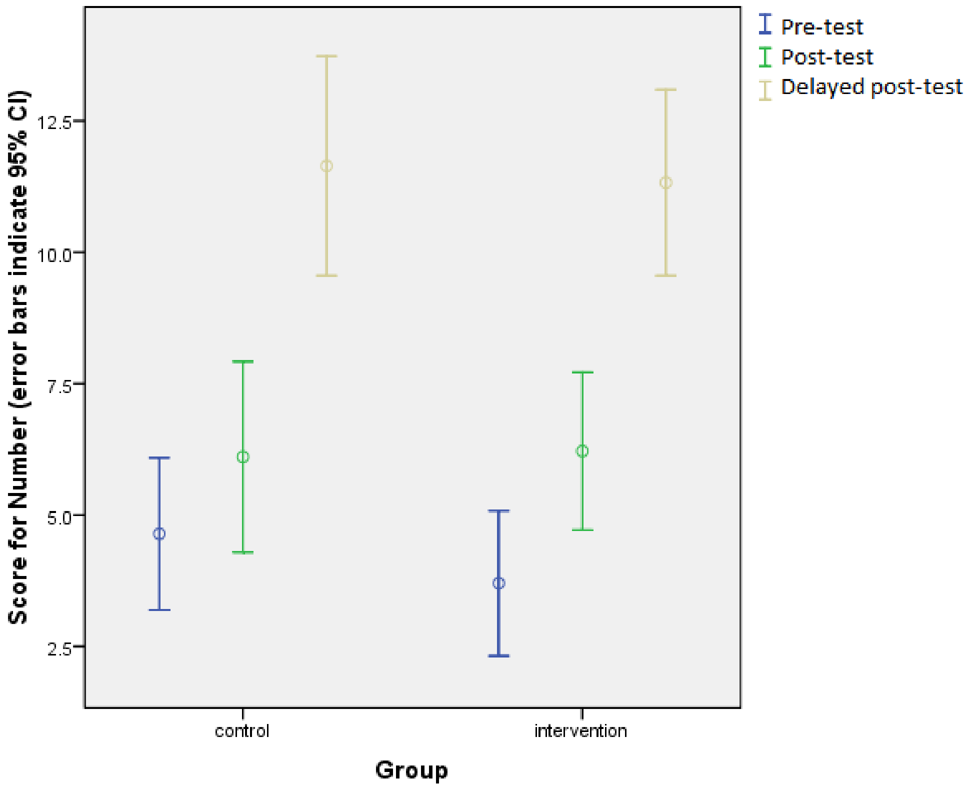

Figure 8-13 Effect of intervention on attainment in number

An ANCOVA with group as independent variable (44 students in the intervention group, 52 in the control group), post-test number score as dependent variable, and pre-test number score as covariate shows a significant effect of group ($F_{1,93}$=4.55, p=0.035). We can interpret this as evidence that the intervention has been effective, although the effect size is small: partial eta squared =.047, meaning that 4.7% of the variance in post-test scores can be accounted for by participation in the intervention (Cohen's d = 0.26, based on adjusted means). Much of the effect appears to consist of the intervention group 'catching up' with the control group. The groups were not balanced in terms of prior attainment, and without further investigation it is not possible to tell whether the observed effect may be due to regression to the mean, different test conditions for the two groups, or some other factor. An independent t-test looking at the difference in score on the number post-test between the two groups did not reveal a significant effect (t=1.51, df=109, p=.134). However, we can conclude that, for the Number assessment, the intervention group improved to a greater extent than did the control group, between pre-test and post-test.

A second ANCOVA was carried out, with delayed test score as the dependent variable. This shows no effect of group ($F_{1,71}$=0.229, p=.634).

Figure 8-14 shows pre-, post-, and delayed test scores for algebra. It appears to show that the intervention group performed at a lower level at both pre- and post-test, but at a higher level in the delayed test, relative to the control group. Again, delayed test scores are not directly comparable with pre- and post-test scores as they derive from a different test, with a higher maximum score.

An ANCOVA with group as the independent variable, post-test algebra score as dependent variable and pre-test algebra score as covariate showed no significant effect of group (F1,75=1.08, p=.302). A similar ANCOVA with delayed test for algebra as the dependent variable showed no effect of group ($F_{1,67}$=2.61, p=0.11).

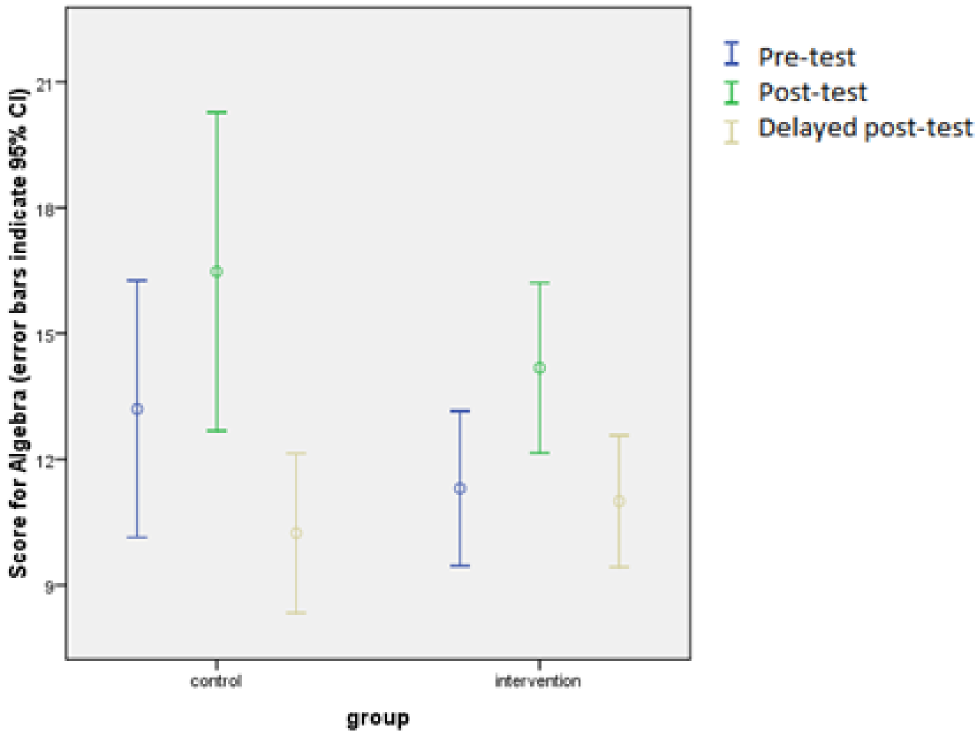

Figure 8-14 Effect of intervention on attainment in algebra

The only significant effect in this section is the greater improvement between pre- and post-test for Number, by the intervention group compared to the control group.

8.5.4.3 Was there increased use of a RME approach in the intervention group?

Analyses from this point on in this section have been carried out only for data from the Number tests. This is partly because of the above finding, that there was an effect of the intervention on attainment in Number, but not Algebra, and partly because of reported irregularities in the way that Algebra tests were carried out.

In this case, 'an RME approach' was observed through use of the bar model or the ratio table.

Table 8-3 shows that, while neither group used an RME approach to answer questions during the pre-test, the intervention group did use an RME approach to answer some questions during the post-test. This difference between groups was significant ($t=8.73$, $df=97$, $p<.0005$). Of the 49 students in the intervention group who took the Number post-test, 36 used an RME approach at least once.

There is also a significant correlation between students' improvement in score between pre- and post-test in Number, and their degree of use of an RME approach ($r=.258$, $n=86$, $p=.016$). However, when including only those students in the intervention group in the analysis, the correlation is not significant ($r=.227$, $n=44$, $p=.139$), so this finding should be interpreted with some caution.

	Intervention	N	Mean	Std. Deviation	Std. Error Mean
Pre-test RME use	control	59	.02	.130	.017
	Intervention	64	.00	.000	.000
Post-test RME use	control	50	.00	.000	.000
	Intervention	49	2.76	2.232	.319

Table 8-3 Descriptive statistics for use of RME approach for number items

To find out whether increased use of RME could account for increases in performance in the number test by the intervention group, a multiple linear regression analysis was carried out, with post-test number scores as the outcome variable. Model 1 has just pre-test number score as a predictor. Model 2 adds post-test RME frequency, and then Model 3 adds the group variable indicating whether participants were in the intervention or the control group.

Table 8-4 shows that, after having taken account of variance due to initial variation in pre-test scores, an additional 2.4% of the variance in post-test score can be accounted for by variance in use of RME approaches ($F_{1,83}=6.083$, $p=.016$). After both pre-test score and RME use have been accounted for, then no additional variance is explained by whether the participants were in the intervention group or the control group ($F_{1,82}=0.219$, $p=.641$).

					Change Statistics				
Predictors	R	R Square	Adjusted R Square	Std. Error of the Estimate	R Square Change	F Change	df1	df2	Sig. F Change
Pre-test only	.807[a]	.651	.647	2.722	.651	156.982	1	84	.000
Pre-test, RME use	.822[b]	.675	.667	2.643	.024	6.083	1	83	.016

Pre-test, RME use, group	.822[c]	.676	.664	2.656	.001	.219	1	82	.641

8.5.4.4 Analysis of attitudes data

KMO (0.791) and Bartlett's test of sphericity ($p<0.0005$) were used with the pre-test attitude data to show that the data were suitable for factor analysis. A Principal Component Analysis (PCA) revealed that a 1-factor solution gave the best account of the data, accounting for 31% of the variance.

A regression method was used to create a single score for 'pre-test maths attitude'. The PCA was repeated for the post-test questionnaire data, which included an additional 4 items. A

Table 8-4 Linear regression statistics to show relationship between use of RME and post-test attainment in number

1-factor solution still explained the data best, so the additional 4 items were not included in the single score for 'post-test maths attitude'. The pre- and post-test attitude scores were analysed for any differences by group or as a result of the intervention. A 2-way ANOVA with group (27 students in the intervention group, 24 in the control group) and time (pre-test, post-test) as independent variables, and attitude as dependent variable revealed no significant main effect of group ($F_{1,49}=0.697$, $p=.408$), no significant main effect of time ($F_{1,49}=1.375$, $p=.247$), and no interaction between group and time ($F_{1,49}=1.920$, $p=.172$).

8.5.5 Implementation evaluation findings

8.5.5.1 The RME GCSE resit intervention

Here the intervention is briefly described. This should be read alongside the research report and other publications by MMU which provide a fuller description.

8.5.5.1.1 Rationale and theoretical background

The intervention aimed to address the needs of 16-18 years old students and others who do not obtain a grade C GCSE during compulsory schooling and for whom there is a low conversion rate when resitting[6].

The intervention is based on the Realistic Mathematics Education approach. Important features of RME are:

[6] Department for Education (2013) *Level 1 and 2 attainment in English and Mathematics by 16-18 students.* Statistical first release accessed at www.gov.uk

- use of carefully chosen contexts that are meaningful to students and models to support student visualisation and understanding of mathematical processes and concepts (a meaningful context is one that can be imagined or visualised)
- unlike other context rich pedagogies, context in RME is used not primarily for application of mathematical learning but to support learning and conceptualisation
- careful choice of models to bridge informal understanding to formal understanding and abstraction and to connect different areas of mathematics and consequently a slower movement towards formalisation
- the encouragement of an explorative and problem solving approach by students

In the RME GCSE resit intervention the most prevalent models used were bar models and ratio tables, but other models – for instance balance scales – were also used.

8.5.5.1.2 Recipients and contexts

The intervention took place in three sites involving four intervention host teachers:

- an 11-18 single gender school (one intervention class, one control)
- two large FE colleges (three intervention classes - two 16-18 years old classes, and one adult/mature student class, three controls)
    - Two of the teachers whose classes were involved in the intervention had previously hosted MMU researchers developing and trialling materials for use with the GCSE resit classes. One of these teachers described having adopted some of the modelling approaches as part of their general practice as a result. A third teacher worked at a college where MMU had previously researched but had not personally been involved. There had not been any previous activity related to RME by MMU on the fourth site.
- As stated above the total numbers of students for whom evaluation data was obtained were 75 in the intervention groups and 73 in the control.
- Mathematics teaching in the settings for GCSE resit classes took place in lessons that ranged from 1-3 hours long.
- In the FE settings in particular teachers reported very challenging circumstances in terms of staffing of GCSE resits, timetabling and student motivation, some of the issues identified were
    - the current practice is for GCSE resit course to last one year (and given the examination timings less than a year's teaching)
    - a prevalent culture in GCSE resit classes of focusing on examination with the attendant danger of 'teaching to the test' and rapid progression through the curriculum
    - lack of resources or time for teacher CPD
    - a culture of low priority or value of GCSE resits and a history of low conversion rates arguably leading to low expectations

- irregular attendance of some students

8.5.5.2 Implementation

8.5.5.2.1 The intervention

MMU tutors taught the material with host teachers observing or taking some non-active role in the lesson (e.g. marking at the back of the class). In some cases teachers attempted to help individual students in lessons, although steps were taken wherever possible to prevent this. For the most part, tutors would be elsewhere or be undertaking other tasks whilst in the room.

MMU tutors used materials designed specifically for the project including presentation materials and student activities on paper.

The number module was designed to last 12 hours of teaching and the algebra module 9 hours. MMU tutors taught RME during usual timetabled teaching hours, thus it replaced usual teaching rather than supplementing it.

From information from telephone interviews and from the MMU team, the teaching time was achieved for the number module, and apparently exceeded at the 11-18 school where students had four hours of mathematics per week in one hour lessons. On one site the timing of the algebra module was later than had been intended and there was disruption to algebra teaching so the intended 9 hours was not fully achieved. Teaching time for the Algebra module was limited partly due to the anxiety of teachers about coverage of the course during the Spring term. While the module was designed to take 9 hours, only one class received this amount, while two classes received 7.5 hours tuition and another 4.5 hours. This led to inconsistencies in which topics were taught with which classes. For example, the bar for word problems was only taught to PT1; the balance scales for equations to PT4.

From telephone interviews it appears the teaching approach was consistent by different MMU tutors, with one significant adaptation/variation at one site being that lessons were shorter but more frequent.

A feature of the RME approach is to work with contexts that are meaningful and to respond to students' developing formulation. Therefore, exact replication of teaching to all classes would be inconsistent within the theoretical framework. It was clear during meetings with the MMU team that all team members had a consistent and shared view of the theoretical framework and of RME. Thus, a relatively high level of implementation fidelity in terms of quality can be inferred.

8.5.5.2.2 Differences between the intervention and usual teaching approaches

During the telephone interviews, teachers commented on differences between the RME intervention and usual teaching approaches.

All four teachers commented on the issue of pace of curriculum coverage, contrasting the amount of time they have to teach a topic when trying to cover the whole examination syllabus in approximately 90 hours of teaching time, with the amount of time taken to cover material in the RME lessons.

Two discussed the usual approaches as being focused on the use of instrumentalist and procedure orientated methods:

> *"here we give a technique to deal with each type of problem, we try drill them with a way of doing a problem" (Teacher 2)*
> *"we focus on a single method" (Teacher 3).*

This was contrasted with the RME explorative approach, the development of understanding and the use of model that is applicable to more than one type of problem.

A third teacher who had previously worked with MMU researchers has adopted some of the approaches, specifically mentioning ratio tables. The fourth teacher, though new to working with MMU described their teaching approach as similar to that of the MMU researcher, though clearly there is no way of verifying this claim.

The teachers contrasted the intervention with their own usual teaching. A limitation of the evaluation is that data was not collected on teaching approaches in the control classes.

8.5.5.2.3 Issues affecting intervention delivery

The MMU tutors found it challenging to negotiate time for teaching. Whilst when interviewed teachers were enthusiastic about the approach and said they would use bar modelling, they also commented on the need to move through the curriculum quickly. The latter need was related to the one year length of the resit course and the limited time for mathematics teaching.

In the FE settings variable attendance was an important issue identified by teachers as affecting outcomes during the telephone interviews. One teacher described attendance as generally being 60% to 80%. In the view of the teachers RME lessons were neither better nor more poorly attended. It is important to note that some of the students in the trial are likely to have experienced less than the 12 hours intended RME mathematics in number or the 6-9 hours available for algebra. The issue of attendance amplified concerns by teachers about curriculum coverage, given that they expected students to miss a number of lessons.

Resit students often have a negative relationship to mathematics, and many have prior attainment significantly lower than the C grade they are aiming for. For many, GCSE 'failure' is only the most recent in a long history of 'failing' in mathematics.

All four teachers commented that some, or in the case of the 11-18 school most, students were resistant, in general, to new approaches to learning mathematics. For some this appeared to be related to a lack of confidence. The two teachers in the FE colleges of the

16-18 classes both noted how students who had attained a D in GCSE often had a view that they only needed to gain a few more marks to convert their grade to a C. This was one reason, it was believed, for absences in some cases. The teachers pointed to the way in which students had been coached and drilled for GCSE at 16 and so often had significant gaps in understanding.

8.5.5.3 Responses

8.5.5.3.1 Student responses

Student attitudes were inferred from teacher reports and the one observation. Students were actively engaged in the RME lesson that was observed. Teachers reported that students have a mix of responses to the RME lessons and materials: some were highly engaged, other students rejected being taught new methods or approaches, focusing instead on being taught 'how to pass the exam' and to use methods they had previously been taught. One teacher stated that the RME approach was challenging for students as they had usually been taught to "go straight for an answer" (Teacher 2).

Three of the four teachers related student disinclination to using new methods to students' prior attainment or student self-perception of their mathematical ability. Students described as relatively stronger were less favourable, initially, at least to the RME approach, although one teacher commented on how this changed when the models were used for harder problems that such students had not been able to previously engage with.

These three teachers contrasted this less favourable response amongst such students with the response of the students with lower previous grades who were more receptive from the outset. One commented "if you keep repeating the same ways then they just freeze up" (Teacher one) and therefore RME was valuable as it offered a fresh approach. One teacher in contrast suggested that it was relatively higher attaining students who could engage more fully with the RME models.

One teacher also linked attendance to student response, as students who missed a session then did not always find it easy to re-engage with the teaching. This contrasted with the usual approach in which a topic or two was covered in a single session before moving on and so lessons were self-contained.

However, from the interview data it is difficult to separate possible responses to RME teaching from the student responses to new and different tutors. All teachers commented on this issue, noting that it took some time for students to be 'won over', even though they also commented positively on the MMU teachers’ capacity to build relationships with students quickly. One of the teachers who had worked with MMU previously contrasted this year with a previous year when the MMU teacher was able to meet the students at the start of term and so relationships were built as part of the induction to the resit course. The teachers pointed to students' lack of confidence as a reason why they had some reluctance to engage from the outset with new and different teachers.

This issue of initial student response may have lessened the effectiveness of the first session of the intervention. Based on this, students in the intervention groups potentially only experienced 9-10.5 hours rather than 12 hours of number RME teaching, and a similar situation for the algebra module, which as described above already had less teaching time than intended.

However, the overall view of the teachers was that students responded positively to the RME lessons. Two teachers commented on seeing some students using the bar model in later work separately and independently from the RME intervention lessons or tests.

8.5.5.3.2 Teacher views on impact on students and on delivery

All the teachers interviewed were positive about the RME approach, one describing it as 'brilliant' (Teacher one). As stated above, one had already tried to integrate some of the models into her own teaching. The teacher in the 11-18 school believed the models were valuable and would be more beneficial if introduced earlier to the students and so the department was considering adopting/exploring RME with Y7.

Three teachers commented on the value of the models in helping students to problem solve. This was both identified as a particular weakness for the resit students but also an important issue given new GCSE specifications.

Two teachers pointed to the importance of embedding the approach in usual teaching. One noted that after the intervention lessons both the students and himself quickly reverted back to 'same old ways' (Teacher 2). He suggested a better approach would be for the intervention to happen at the very start of the year and then for the models to be used consistently.

Two teachers pointed to the need for appropriate professional development to support them to use the RME approach themselves, but also pointed to the difficulties for FE teachers to access professional development of any type. One teacher noted in particular they would not have been able to apply the models to algebra themselves without observing the MMU researcher.

8.5.5.4 Issues potentially affecting the security of the evaluation

It is important to note the possibility that some of the research aspects of the intervention may have also had an effect on outcomes. In particular, one of the research team worked with individual students in a small number of lessons using a data capture pen. Whilst such conversations were not aimed at 'teaching' the students there is a potential effect on attainment. In addition, as stated above, there was some variability in the extent to which teachers observed lessons, or acted as participants in lessons supporting students, though steps were taken to minimise possible effects of this. Above, issues were noted that affect reliability of the algebra test.

8.5.6 The potential for future developments and trials

8.5.6.1 Intervention design

This section draws on the data analysis findings, the implementation evaluation and findings reported by the MMU team related to student engagement and learning and teacher views and responses. These indicate that the intervention would benefit from further development in order to increase the likelihood of having impact. Any further development will need to address the issues identified above that effect GCSE resit classes, in particular designing an intervention that takes into account attendance issues and the time needed for unconfident students to adapt to the new approach. At the same time, the intervention findings also underline the need to address the low GCSE conversion rate.

The outcomes of the test analysis and other data potentially could inform the MMU teams reflection on the posited underlying change mechanisms. We suggest two possible approaches that could be considered in relation to this: extending the RME intervention for whole class teaching or alternatively developing a one to one intervention. Either approach would require further development of the project as a professional and curriculum development programme.

8.5.6.1.1 Whole class intervention

The evaluation findings suggest that one approach to developing the intervention would be for it to extend over two years. This is because of the challenge of implementing the intervention in the context of a single year course as well as the MMU findings and the RME conceptual framework about the need to take time to develop understanding. Such an approach would also address the need that can be inferred by teachers that students may need to 'unlearn' both approaches and mindsets that are barriers to their success.

The number module had a short term small effect. Further, this effect was linked to the use of the MMU RME approaches used in this module. The implementation evaluation suggests that following the intervention there was some reversion back to previous techniques in at least one class and some students. Issues of attendance were identified again potentially lessening the impact of the intervention for some.

The positive outcome might be increased by extending this module and approach over two years to address these issues.

In any case, any extended intervention would require attention to CPD for teachers so that students would experience the intervention pedagogy for substantially longer than the 12 hours aimed at in the evaluated study. One teacher pointed to the value of such a module being introduced early in the curriculum, possibly at the start of the year.

The lack of observed effect of the algebra module may be due to challenges in implementation. However, the analysis above suggests that the RME approach to algebra for GCSE students may need to be further developed, in particular also requiring a more sustained intervention.

If other aspects of the curriculum were to be addressed through an RME approach then the evaluation findings also suggest that a longer and more sustained intervention would be advisable. Notwithstanding that the evaluation provides no evidence of what impact RME might have in other areas of the curriculum as these were not included in the intervention.

8.5.6.1.2 Developing a one to one intervention

Given the challenges of implementation in the current GCSE resit FE context, an alternative is to consider the development of a one to one intervention based on the RME number materials. There is evidence for the effectiveness of one to one tutoring in mathematics in other phases[7] . As a supplement to regular instruction, such an approach would overcome some of the contextual issues particularly around attendance. However, If RME was the basis for one to one tutoring an issue would arise about potential conflicts between RME approaches and regular class teaching. So this would not necessarily reduce the need for a programme of teacher CPD.

8.5.6.1.3 Professional development needs

For an intervention of either design to be scalable, consideration would need to be given to how the intervention could be implemented by others and so what would constitute effective and achievable CPD for teachers.

Whilst the implementation was limited and so strong conclusions cannot be drawn there are some suggestions of issues to consider. Two of the teachers had worked for a number of years with MMU researchers. In one case the teacher described how this had influenced her teaching, particularly using the two models. In the other case, the teacher was honest that whilst he was positive about the models and methods, pressures to focus on the exam meant that the impact on his own teaching had been limited. This underlines the challenges of changing teachers' practice. Further, when asked to describe the important features of RME, all four teachers focused on the bar model with one also mentioning the ratio table. This means that the underlying principles of RME were possibly not that well understood.

[7] Torgerson, C., Wiggins, A., Torgerson, D., Ainsworth, H., & Hewitt, C. (2013). Every Child Counts: testing policy effectiveness using a randomised controlled trial, designed, conducted and reported to CONSORT standards. *Research in Mathematics Education*, *15*(2), 141-153.

However, it is important to note that teachers had not been actively engaged in a professional development project but had only largely observed RME teaching and had informal discussions with researchers. Although evidence from these teachers must be treated with caution, it suggests that observation alone, for example through use of video CPD materials, will not be sufficient for post-16 teachers to adopt RME approaches. RME professional development in the post-16 sector will require considerable investment.

8.5.6.2 The prospects for trialling future interventions

Here we provide indications of the size and scope of trials needed for testing future developments of the intervention. Given the need for further development and the outcomes of analysis presented in this report, it is not appropriate to calculate minimum detectable effect size at this point; rather indications of the size of trial needed are given. Before moving to a full trial, further piloting of redesigned interventions would be required, unless other relevant evidence of potential effect size is available.

8.5.6.2.1 Whole class intervention

A whole class intervention could be evaluated using a clustered randomised controlled trial. A useful reference point is the EEF approach to security rating[8]. To have a clustered trial with a minimum rating of 4/5 in security in terms of detectable effect size, then there might typically be 40-50 sites with two classes per site in both the intervention and control conditions - in the order of 2000 students in each condition. One centre that took part in the RME GCSE resit project had 400 students annually in GCSE resit classes. However, the issue of clustering in a trial means that increasing the number of students on a particular site would be less important in terms of the power of a trial than increasing the number of sites. It would be important to take steps to control attrition, a particular issue in the FE sector and it would be prudent to over recruit in the expectation of attrition.

The number of FE and sixth form colleges combined is approximately 300 centres and these might be the sites which would consider themselves as having the greatest need in relation to GCSE resits. There are approximately a further 1500 state schools that have post-16 provision. However, the size of GCSE resit groups in these centres is probably smaller and recruitment to a trial may be more challenging. Recruiting a total of 100 sites, and for validity a large number of these being FE sites, would be difficult.

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8 https://v1.educationendowmentfoundation.org.uk/uploads/pdf/Classifying_the_security_of_EEF_findings_FINAL.pdf

For a trial of this size, delivery would need at least in part to be by class teachers and so requires the development of an appropriate professional development programme and appropriate piloting of these.

8.5.6.2.2 One to one interventions

Designing and implementing a one to one intervention should be considered if a credible theory of change mechanism warrants it, including potentially evidence of success of such approaches in other contexts. If such an approach was considered then an advantage of one to one interventions is that recruitment and randomisation can happen at individual level. A consequence of this is that the numbers of students in the trial can be much lower, for example an evaluation of Every Child Counts had 600 students in the trial[9]. Detection of an effect size of the order of 0.2 would require a multisite trial involving 15-20 sites with 20- 30 students randomised to the intervention and control condition who then individually received an RME intervention on a one to one basis. Ideally, such a trial would include a placebo - potentially of additional mathematics tuition in classes for an equivalent amount of time to the one to one intervention.

8.5.6.2.3 Implementation issues

The RME GCSE resit project and this evaluation has identified that implementation in the FE context is challenging as is maintaining robust and secure protocols for testing. Independent invigilation would need to be included in any future trial costs. As would clear memoranda of understanding with participants about what is expected for participation.

8.5.7 Conclusion

8.5.7.1 Limitations

Sample size and attrition. There were 147 participants in the study, across intervention and control conditions. However, complete data sets were obtained for only 52 (29 in the intervention group and 23 controls). This is a very high rate of attrition, reflecting the volatility within this age-group and level of study with regard to attendance and engagement. Attrition poses a considerable risk to validity, as it is not possible to dissociate reasons for withdrawal from factors relating to the intervention or to assessment. This is perhaps the most important limitation for future researchers of post-16 mathematics

[9] Torgerson, C., Wiggins, A., Torgerson, D., Ainsworth, H., & Hewitt, C. (2013). Every Child Counts: testing policy effectiveness using a randomised controlled trial, designed, conducted and reported to CONSORT standards. *Research in Mathematics Education*, *15*(2), 141-153.

interventions with this population to consider; researchers must consider ways in which retention of participants could be improved.

Allocation to conditions. Participants were not allocated to conditions at random. This has implications for potential bias, and for balance (e.g. participants in the control condition performed better on the Number pre-test than participants in the intervention condition). Ideally, allocation to conditions should be carried out either at random, or, following pre-test in such a manner that there is balance in key variables between conditions.

Test design. Tests for the assessment of number, algebra and other topics in mathematics were designed by the MMU team responsible for delivering the intervention. This brings risks that the measures are not sufficiently independent from the intervention[10]. Use of independently developed tests could have improved the validity of the evaluation. An alternative approach could have been to carry out a process evaluation with control groups in order to confirm that equivalent content had been covered in control group sessions as had been covered in intervention group sessions.

Test administration. There was inconsistency between sessions and between conditions in the administration of some tests. All tests should be carried out under the same conditions, ideally with independent invigilation in order to ensure that scores for each participant are independent from one another and the class teacher.

8.5.7.2 Conclusions

The RME GCSE Resit intervention led to a short term impact on number but did not lead to sustained impact on number. There were indications that the MMU RME approach - at least in the case of the number module the use of the specific models taught - is potentially beneficial. However, the intervention was relatively brief with only a maximum of 12 hours of alternative teaching for number and 9 for algebra using the RME approach, with some students receiving less than this, particularly in the case of algebra. Therefore, it is not possible to draw conclusions about the potential of RME if it was embedded into a full GCSE resit course or in general about the value of RME for this age group or profile of learners.

The intervention has contributed to knowledge about the context and needs of learners in the post-16 contexts. This can support future intervention design.

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[10] Slavin, R., & Madden, N. A. (2011). Measures inherent to treatments in program effectiveness reviews. *Journal of Research on Educational Effectiveness, 4*(4), 370-380.

The evaluation and study underline the need for developing interventions to address the needs of GCSE resit students. However, they also indicate how challenging it is to address these needs and further to evidence success in this area through an experimental approach.

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