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The mathematical event: Mapping the axiomatic and the problematic in school mathematics

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Abstract:	Traditional philosophy of mathematics has been concerned with the nature of mathematical objects - or concepts - rather than events. This traditional focus on reified objects is reflected in dominant theories of learning mathematics whereby the learner is meant to acquire familiarity with ideal mathematical objects, such as number, polygon, or tangent. I argue that the concept of event - rather than object - better captures the vitality of mathematics, and offers new ways of thinking about mathematics education. In this paper I draw on two different but related theories of event articulated in the philosophies of Alain Badiou and Gilles Deleuze to argue that the central activity of 'problem solving' in mathematics education should be recast in terms of a problematic of events.
Response to Reviewers:	After submitting the original manuscript in May to SPED, I couldn't let it go, and continued to revise it throughout the summer months and into the Fall. I was particularly motivated to make sure that the use of Deleuze and badiou was appropriate (I sent it out to about 6 people I know who work in this area, received comments, and made some minor changes throughout), and I found a thread in the paper that needed elaboration - that being the tension between axiomatics and problematics (as explored by Deleuze, in his study of the history of mathematics). This thread is at the heart of his work on events. The request by the reviewers that a more detailed link to education be made also fueled my interest in this tension, as it became more and more obvious that this tension operates at the level of curriculum policy. So, as requested by reviewers, I added a new section which took up this thread and developed it. I also added material to the introduction so that readers would have an immediate idea of how the axiomatic/problematic tension in the history of mathematics was related to the idea of an event. In the new section - called "Into the classoom" - I related this tension another more familiar tension in educational research - that between encounter/recognition. I reference a few researchers in mathematics education who work on problem-posing, and I reference some policy documents, but my main sources here are Popkewitz and Cutler - in an attempt to try and mobilize mathematical concepts. I also altered the images - and added one new image - so that the reader would be

	better able to follow my investigation of the 1989 problem.

THE MATHEMATICAL EVENT: MAPPING THE AXIOMATIC AND THE PROBLEMATIC IN SCHOOL MATHEMATICS

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Abstract:

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Keywords:

Event, mathematics, Deleuze, concept, multiplicity, indeterminacy, undecidability

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Introduction

The concept of a mathematical event seems odd in a field dominated by symbolic notation and rules of inference. Mathematics, according to Whitehead (1911), attends to universal laws that transcend the "events of this ever-shifting world" (p. 4). Traditional philosophy of mathematics has concerned itself with the existence of mathematical objects, not the occurrence of mathematical events. The event is said to *occur* and the object to *exist.* Whereas existence of a mathematical object might be material or immaterial, depending on one's philosophical approach, events are always ephemeral incidents, contingent on material circumstances, and full of action and motion and temporality. Unlike objects, events seem to gobble up space and time, taking up the flow of temporality and spilling out into surrounding space. Some have argued that events and objects share in the same ontology and simply differ by degree - both are material inhabitants of space-time, the object simply being a particular kind of event, one that is "firm and internally coherent" over time (Casati & Varzi, 2010). The objects of mathematics, however, are rarely considered a part of this spectrum, since they are often taken to be entirely free of spatio-temporal conditions.

The philosophical question of how mathematics participates in the spatio-temporal, or how mathematics comes to matter, is rarely addressed within education, despite it often being the source of student confusion. Students regularly struggle with attempts to mathematize temporal processes and events, whether they be age-old word problems about journeys between towns, the measure of rates of change, or exploring the probability of an event. Consider, for instance, the notorious 'difficulty' that students have in properly answering the question: "Compare the probability of getting the pair 5-6 with the pair 6-6 when rolling two dice." The vast majority of high school students state that the probabilities are the same, and that it doesn't matter whether it's the pair 5-6 or the pair 6-6, since the result of rolling each dice is independent from the other (Lecoutre & Fischbein, 1998). Educational researchers who study how we might correct this 'misconception' suggest that we show students a 6x6 array of all possible outcomes of the two dice tossing, and offer a justification of the sort "look, there are two ways of getting 5-6 and only one way of getting 6-6" (see figure #1).

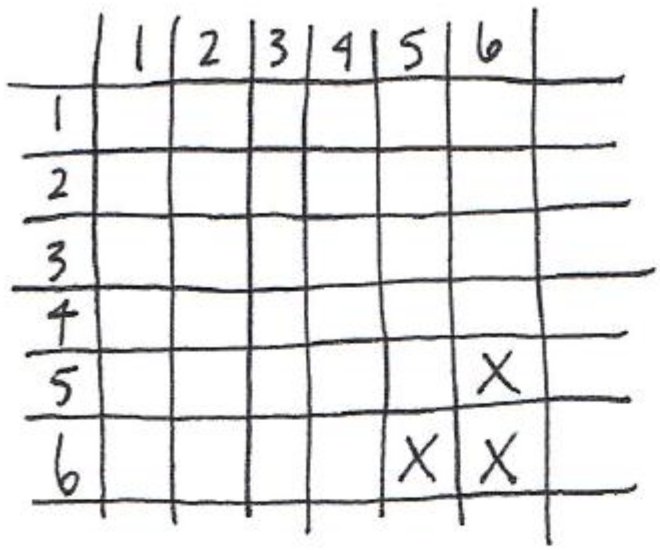

Figure #1: Array representing throwing of two dice

When students see the array they are forced to recognize, through counting, that the there are two outcomes for the pair 5-6 and only one for 6-6. The act of counting 5-6 as different from 6-5 strikes most students as wrong, but the array shows them how one might assess the event as though composed of two possible outcomes. The array functions very importantly here – it spatializes the possible outcomes in a two-dimensional framework, decomposing the event into two directions, and problematizing the assumption that the two dice are thrown together and perceived *all at once.* This form of justification detemporalizes the act of tossing the dice, and decodes the event in terms of a logical ordering rather than a temporal ordering. One can imagine how students might feel the tension between their sensation of the event and the assertion that there *is* a difference in order (a 5-6 pair *counted* as different from a 6-5 pair). I draw attention to this example as it points to how student perplexity is related to the awkwardness of mathematics in thinking the event. Logical and axiomatic relations tend to erase the temporal and ontological, even in this case, when the task attends to a spatio-temporal action. One can trace this tension between temporal and logical facets of mathematical activity throughout the history of mathematics, comparing specific developments that grapple with the temporal, like the infinitesimals of Leibniz, with others that deny it altogether, like the axiomatization program of Peano (Châtelet, 2000; Deleuze, 1994).

Deleuze (1993, 1994) and Deleuze & Guattari (1987) examined the history of mathematics, mapping the tensions between state-sanctioned or “royal” or “major” mathematics and alternative or counter historical lineages of mathematics, which are named “nomadic” or “minor”. According to this account, state mathematicians quelled the dynamic and nomadic strategy of flow associated with nomadic mathematics, and imposed “civil, static, and ordinal rules” on notions such as “becoming, heterogeneity, infinitesimal, passage to the limit, [and] continuous variation” (Deleuze & Guattari, 1987, p. 363). Nomadic mathematics, according to Deleuze and Guattari, disrupted the regime of axiomatics through its emphasis on the event-nature of mathematics. In particular, nomadic mathematics attended to the accidents that condition the mathematical event or encounter, while the axiomatic attended to the deduction of properties from an essence or fundamental origin. In pragmatic terms, this means that nomadic mathematics dwells in the mathematical figure *as an event* rather than an essence or representation. The square is nothing but the process of quadrature. The curve is set free from an absolute metric and embraces and follows relational movement. Deleuze demands that we re-animate the figures of mathematics and set them in motion, so that we might resist the tendency to imagine transcendent references for mathematical figures and instead embrace their event-structure and immanence.

In *Difference and Repetition* (1994), Deleuze opposes two kinds of deduction associated with these two entwined paths in the history of mathematics. In axiomatics (or theorematics), deduction entails the derivation of a set of theorems from a set of axioms. In problematics, deduction “moves from the problem to the ideal accidents and events that condition the problem and form the cases that resolve it.” (Smith, 2006, p.145). A problematics embraces the “ideal accident” of the square and the prime number, and any other mathematical concept, as an adventurous temporally unfolding process. Axiomatics, in contrast, treats the figure and the number as static and staid - in Platonic fashion - in terms of its essence and its derived properties. As Deleuze (1994) explains, “the reason is that theorems seem to express and to develop properties of simple essences, whereas problems concern only events and affections … as a result, the genetic point of view is forcibly relegated to an inferior rank” (p.160). He cites Proclus’ delineation between the theorematic and problematic conceptions of geometry, where the latter is defined with reference to the events (sections, ablations, adjunctions, etc) that come to “affect” the subject matter (circle, ellipse, line, etc), “whereas the theorem deals with the properties which are deduced from an essence”(p. 54). Smith (2006) offers the example of the line, suggesting that "Euclidean geometry defines the essence of the line in purely static terms that eliminate any reference to the curvilinear ('a line which lies evenly with the points on itself). Problematics, by contrast, found its classical expression in the 'operative' geometry of Archimedes, in which the straight line is characterized dynamically as 'the shortest distance between two points'" (Smith, 2006, p. 148). We can see that the Archimedean definition allows for a multiplicity of curvilinear paths between the points, and defines straightness, rather than line, in terms of distance travelled or movement (de Freitas, 2012a). Smith suggests that such a definition marks the line as a continuous operation and a "process of alignment". Similarly, the circle is a process of rounding, the square a process of quadrature, and so on. "In problematics, a figure is defined dynamically by its capacity to be affected – that is, by the ideal accidents and events that can befall the figure (sectioning, cutting, projecting, folding, bending, stretching, reflecting, rotating, and so)." (Smith, 2006, P.149). Axiomatics, however, prevailed over problematics (through Euclid) as a "triumph of the rectilinear over the curvilinear", and a triumph of essence over event, the latter being seen as the deterioration of the essential form. Desargues' attempt to develop a mathematics of the problem-event, for instance, in his *Draft Project of an Attempt to Treat the Events of the Encounters of a Cone and a Plane*, was opposed and marginalized by the algebraic or analytic geometry of Fermat and Descartes (Smith, 2006). The projective geometry of Desargues was judged "dangerous and unsound" because of its reliance on the diagrammatic and its operative approach to mathematical figures (Smith, 2006, p. 150). Despite that fact, this kind of physico-mathematical approach would find favor in the nineteenth century with Monge and Poncelet, but always as a threat to the dominant analytic tradition and epsilon-delta police whose aim was to rescue axiomatics from the threat of this destabilizing mathematics of the animate or vibrant problem.

My aim in this paper is to rethink the nature of the event in mathematics so that educators can attend more carefully to the ways that our engagement with mathematics entails a problematics. Focus on the materiality of this engagement allows us to better study the mutual entailment of mathematics and subjectivity. Rethinking mathematics in terms of the event allows educators to bracket the dominant logic of necessity and orient their attention towards the question of how the new is introduced into mathematics classrooms. This shift in attention requires a new philosophy of mathematics education, one that troubles the traditional assumptions about the status of 'universals'. Mathematical universals - concepts like squareness or primeness or straightness - are typically contrasted to particulars or individual instantiations. One often hears the claim that a particular drawing of a square is an instantiation of the universal property of squareness. This claim, however, forever demotes the drawing or act of drawing as a mere copying of a more perfect and disembodied square that determines or prescribes the nature of any individual square. In lieu of a discussion of properties like squareness or primeness that are said to be universals, this paper argues that we should attend to the incident of squaring (or becoming prime) *as an event*. In other words, I argue that such an event is not an instantiation of a universal, because the process of instantiation fails to capture the creative and material act of individuation that is entailed when we do mathematics. My aim is to rethink mathematics as a place of events where creativity and contingency prevail, and where the indeterminacy and undecidability at the source of the event is embraced rather than banished.

I draw on two different approaches– that of Alain Badiou and Gilles Deleuze – and argue for an image of mathematics where the concept of event is central. By looking to the history of mathematics, and identifying moments that might qualify as *eventful,* these two philosophers develop very different but related approaches to the very idea of event. I borrow some of their insights, and cobble together a proposal for the study of problem solving as a mathematical event. In order to flesh out this proposal, and show how the philosophical issues are at work in our everyday encounters with mathematics, I use the "first-person" research methods developed by Roth (2012) in his study of the phenomenological aspects of learning mathematics and science. Roth notes that first-person introspective reflection was precisely the kind of method that Merleau-Ponty (1968) and Nancy (2006) used to shed considerable light on the nature of spatio-temporal aspects of experience. In social science research, however, empirical research dominates the field and has largely shunned these other kinds of methods. Like Roth, my first-person method strives to analyze my own encounters with mathematics "in such a manner that more general conditions of knowing and learning are exhibited" (p.4). My approach, however, shifts away from traditional phenomenology, drawing extensively on Deleuze so as to treat the non-human (mathematical concepts) as *animate* folds of the event. As a means of reflecting on the nature of mathematical activity, I focus on one mathematical problem and describe my engagement with it and show how this engagement relates to the concept of event. In the last section, I relate these ideas to work in mathematics education research, and describe how the axiomatics/problematics tension operates in curriculum policy.

Eventhood: Naming and annulling

Theorizing the event has a long history in philosophy. Recently Badiou (2011) – who describes himself as a kind of Platonic materialist[1] - leverages contemporary set theory as a basis for theorizing ontological questions about being and events, arguing that space, time and society can be theorized through the concepts of set membership, the power set, complementarity, etc (Badiou, 2006, 2009, 2011). One has to keep in mind these set theoretic relations as one explores his development of the concept of an event. It seems difficult to imagine an event in a landscape of sets constructed so as to avoid any inkling of motion or action, but Badiou suggests that events are composed of elements that do not belong to the encompassing situation and cannot, for that reason, *take place* across the entire situation. An event interrupts a situation, disrupts the functioning of a discourse, and insists on its own opacity to analysis. It is this latter insistence on its status as dysfunction that makes it difficult to name. We aim to *represent* the event, but it insists on *presenting* itself. And yet what saves the event from oblivion is a semiotic intervention that names the event as *belonging* to the situation. As Feltham (2008) suggests, intervention for Badiou is any naming procedure that marks the event as belonging to the situation "without annulling its eventhood" (p. 103). The naming brings the event into the linguistic realm, but the name itself must be radically new to language or there would be no change in the situation. A new element must be introduced into the recombination of signs, something initially awkward that bumps up against the rules of signification, so that we are constantly forced to consider the extent to which this event belongs to this situation.

Badiou offers the example of the introduction of the imaginary root $\sqrt{-1}$ by the Italian mathematician Cardano and others in the 16th century. Complex roots for polynomials were controversial at the time, and the very idea of naming such a thing made many mathematicians uncomfortable. Chuquet (1445-1500) showed that some equations lead to imaginary solutions, but dismissed them, stating "tel nombre est ineperible" (such a number is nonsense). Cardano (1501-1576) used solutions to polynomials yielding square roots of negative quantities, but called them "sophistic" and said they were "as subtle as they are useless" (Brown, 1998). Complex roots were introduced to enhance the study of equations, but their emergence is a good example of an event that enters the discourse of mathematics through a naming procedure and is then repeatedly considered and encircled as a recalcitrant obstacle upon which an entire theory of complex numbers emerges. The naming doesn't "quell the undecidability" (Feltham, 2008, p. 103) of the event once and for all,

[1] He offers a counter-reading of Plato that reclaims the idea and the realm of ideas as a site of eternity and multiplicity (Badiou, 2011).

but puts the event into motion across the situation, allowing it to grow and saturate the situation. Indeed, Arkady (2012) points out how even current planar representations of complex numbers (the Argand plane), don't really capture or adequately visualize the structural relations between these numbers. The complex plane breaks with the habits of what he calls "Euclidean mathematics" in that it can't map nicely onto a geometry of phenomena, despite it being put forward, by Gauss and others, as a way of legitimizing these kinds of numbers. In Badiou's historical account, the imaginary root remained a "conceptual inexistent" until a mutation in habits of calculation (occurring over a long period of time) granted it *being-there* status in this world (Badiou, 2011, p. 62). The event, for Badiou, is thus a site from which a "truth" can emerge. Events of this caliber are rather rare in life and in mathematics, as they must be adequately intense and strongly obstinate in that which they bring forth. These events are incorporated and subjectivized in the work of individuals like Cantor, Cardano and Galois. Such mathematicians troubled the very foundations of their contemporary's mathematical practice by "raising up" an "inexistent" mathematical entity that had no prior existence nor name. As Badiou states, an event is "a perturbation of the world's order (since it disrupts the logical organization – the transcendental – of this world), as the raising up of the inexistent attests" (p. 91).

Setting aside the appeal to Platonism, and the fact that a set theoretic rendering seems absent of action,[2] this depiction of the mathematical event is a powerful way of describing the rupturing nature of invention in the history of mathematics. I want to leverage this image in rethinking the learning of mathematics and suggest that one can study mathematical activity by pointing to how a "conceptual inexistent" rises up in a situation and, once named, it begins to saturate the given discourse. But what exactly is a "conceptual inexistent"? How can I leverage this idea without it being tied to some ideal Platonic form hovering over us? How can we speak of a "conceptual inexistent" without falling back on the concept of a transcendent form? Moreover, in the case of $\sqrt{-1}$, the inexistent cannot be the "imaginary root", since that, by necessity, has already been named when it is announced. If we aim to speak of the event prior to naming, but without Platonic transcendence, then we have to shift our attention to the problems (not solutions nor naming) that constitute the mobility of the event. The problem *as problem* carries with it the undecidability and dysfunctionality that are the engines of the event. It is the pursuit of a solution to the given equations that constitutes the event for Cardano. The problem affirms its opacity to analysis by refusing determination and

[2] Hence the move by Badiou towards the mathematics of Category theory where there are both objects and arrows.

pursuing its flight and movement. Problematizing the act of finding roots of an equation inserts a movement into the situation that causes the mathematical discourse to vibrate and stammer. A problem, therefore, is at the source of a mathematical event. This is true at the historical level, but also at the level of the classroom where mathematical problems are taken as common place.

Take a simple problem: show that a square can be dissected into 1989 squares.[3] I found this problem in a book, and it travelled with me for weeks. It refused me, kept me out, perturbed me. I set it down to attend to other things, and would be drawn back into it later. At first, I tumbled into an easy and incorrect interpretation, thinking that I merely had to write 1989 as the sum of squares.

Since 1989 (not being a perfect square) was a little bigger than 40^2, I wrote,

$1989 = 40^2 + 12^2 + 15^2 + 4^2 + 2^2$. There was, of course, an immediate disconcerting feeling, a nagging sensation, that my doing so hadn't attended to the (physical) constraints of the problem, and that I had entirely overlooked the spatial constraint that these squares must *dissect* a square. What would that look like? It was evident that my solution, a solution that came too easily and was suspiciously convenient, and also wrong, could not dissect the square in a clean way. The problem affirmed its opacity to analysis – my analysis – and thereby sustained a sort of intrinsic undecidability, destabilizing whatever mathematical discourse on which I was able to draw. My arithmetic was made to tremble and stammer as I realized my blind spot in not considering the need to dissect a square. The problem became a recalcitrant obstacle (with indeterminate borders) and pushed back, reshaping the terrain of my mathematics, functioning like an attractor for all my thinking, speaking, doodling. I quickly found a square factor of 1989, so that I might write 1989 = 9 x 221, which allowed me to divide the original square into nine squares, although the structure of each of these remained indeterminate. Still, this felt like progress. I now had to decompose each of the 9 squares into 221 smaller squares. Again, 221 was not a perfect square, so the dissection would involve squares of different sizes – introducing an asymmetry into a perfectly symmetric container, that being the square in all its obstinate squareness. I began to draw squares (figure #2), and to physically carve up the squares with my pen, looking for some sort of iterative pattern (a kind of dissecting machine) that might be made to generate 1989 squares. I began to feel like I was burrowing into the problem, literally digging up its materiality and moving piles of numbers around the problem space.

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[3] *Fomin, D., Genkin, S., Itenberg, I. Mathematical circles: Russian experience.*

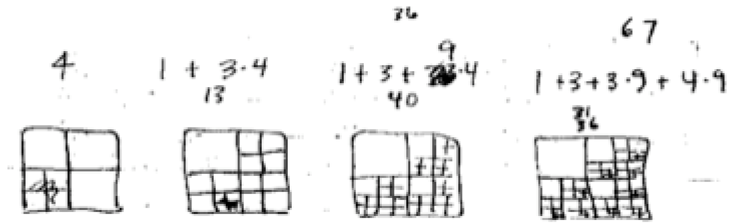

Figure # 2: Excerpt from attempts at engaging the problem

Since my engagement with this problem was not likely to bring about some monumental creation of a new mathematical entity that would revolutionize mathematics, it seems difficult to map Badiou's theory of the event onto my experience. Indeed, focusing closely like this on a more mundane experience with a mathematical problem (like the kinds encountered in classrooms) doesn't seem to suggest the dawning of a conceptual inexistent *as form* but the affirmation of a material indeterminism that lies at the source of all our actions and concepts. This indeterminism is indeed inexistent in precisely the way an event is inexistent – events occur rather than exist. This is an important point - *indeterminism and mobility occur rather than exist*. Thus an event partakes of becoming but not being. In my attempt to attend to the event-nature of mathematical encounters of the everyday sort, I have to wrestle the event away from its association in Badiou with the emergence of both truth and determinate form. I am not, however, denying the emergence of truth and form on all such occasions, as might occur when a student discovers an established or new theorem through investigation. But my aim is to map the event-structure of the everyday problem – the problem that doesn't promise to unveil some fundamental structural entity. In such instances, I believe that the event qua event is my engagement with the problem, in other words, the event entails my coupling, submission, and sensory entanglement with a problem which is itself at play and in the process of becoming. In the case of dividing a square into 1989 squares, my engagement involves the destabilizing of my subjectivity and a concommitant problematizing of the concept of squareness. This problematizing of squareness, the perturbation of the concept, reflects the power of the problem to punch through discourse to the murky and mobile world of indeterminacy. In other words, *the problem is an event of perturbation of a concept*. Thus the problem became a problem only when it shook my cherished assumptions and set my mathematical discourse trembling with indeterminacy.

A perturbation of a mathematical concept introduces temporality into what was taken as eternal. How might we then recast our mathematics so that it attends to the event-structure of these entities and concepts? We would need to re-write the dense noun phrases that saturate our math textbooks using verbal phrases instead. But would that be sufficient? In the next section I turn to Deleuze who suggests that verbs destabilize the static ontology of objects, and asks that we consider what happens when we undo the adjective and return to the activity of the verb. What if we were to speak of the green tree as the occurrence "the tree greens"? What if we lived in a world where numbers were always becoming individuated in some manner (both actualized as prime, odd or square and counter-actualized against the canon)? Why not perceive the ebb and flow of becoming prime rather than the quality of being prime? And why not then invite the counter-actualizations that are too often banished from the mathematics classrooms, those being the untimely becomings that refuse the fixity of the canon?

The temporality of the event

Here I turn to Deleuze's approach to the event as it seems to be better suited to the discussion of everyday mathematical events of the kind discussed above.[4] For Deleuze, events are quite common occasions where creative and inventive actions occur, actions (both human and non-human) that then organize and structure the behavior around them, whereas for Badiou events are rare historical moments when a Truth rises up (Bowden, 2010). We have seen above how the event for Badiou has no formal place in a given situation and therefore resists recognition. Deleuze shares this commitment to the dislocating nature of events, a commitment that denies the event any well-defined spatio-temporal location. Deleuze's approach in *Logic of Sense*, however, sets the event travelling in time away from the present, moving infinitely back into the past and infinitely forward into the future. For Deleuze, the event is a change in intensity where the virtual and the actual re-combine and the effects of this change multiply and proliferate the many futures of the situation. This approach lends itself to the study of smaller actions at the micro-level of activity, tracing the way these fold into significant changes in a situation. From the Stoics, Deleuze takes a philosophy of change that decenters substance and privileges flow, indeterminacy, alteration and mutation. He argues that events are ontologically primitive in that substances are derived from them. The event is free and without determination – its ontological reality *is* the virtual. The virtual is the movement or mobility or dynamism that is at the heart of all our actions. The event escapes the trappings of the present in its potential movement (back and forth) along an uncountable and indeterminate set of temporal

[4] In *Logic of worlds* (2009), Badiou briefly discusses Deleuze's theory of the event, as articulated in the book *The logic of sense* (Deleuze, 1990), summarizing what he considers to be Deleuze's "axioms" of a theory of the event, and then contrasts these with his own axioms. Many scholars (Bowden, 2011; Smith, 2005; Williams, 2009) have identified the ways in which Badiou entirely misrepresents Deleuze's concept of the event.

directions. It is this potentiality that determines the change that has occurred or will occur. The event is thus never fixed in a present moment, but rather a moving and unraveling thread trailing off behind and ahead. Events are the manifestations of singularities or disruptions in the norms of behavior in a situation, and they function as unconditioned magnets or attractors that re-organize the spatial practices and actions of those around it. In other words, events put indeterminism to work like an engine, re-arranging and re-entangling the threads of spatio-temporal unfolding. Like exhalation, they operate through break-down, but like inhalation, they affirm a directionality.

Consider the same problem of dissecting a square into 1989 squares. My path through this problem reveals how this break-down and affirmation unfolds. As I sustained my engagement with the problem, I began to map a lineage of related problems along one thread, identifying problems that might have come prior to it, often deemed “simpler” or “subsidiary”, problems that participate in the same kind of indeterminacy, but in lesser degree. Was it possible to dissect a square into any given number, even or odd? I drew a series of squares (Figure #3) and tried to divide them into five, or six or seven squares, and so on, repeating the request, and studying the way the problem evolved. In so doing, I split and splice the various threads of the event that were woven through each of these “sub-problems”. The original problem participates in the spatio-temporal flow of material actions I had taken or would take. I reached back into the history of this event, drawing 4x4 and 3x3 squares, and I reached ahead beyond the current focus on 1989 towards the case of any number. I found myself stretched along one such extended event, repeatedly dividing squares into an even number of squares by partitioning off a corner with an L-border around it. This sequence of actions congealed into a solution to a problem that was never formally posed (“can you always divide a square into an even number of squares?”), and yet it was a problem that somehow participated in the emergence of my 1989 problem. I spliced this trailing thread of events into another, pursuing the potential of generating an odd number of squares. I drew another square and partitioned off a central square with a border to get 13 squares, and I repeated the action to generate 17 squares (Figure #4), but after doing so I realized this method didn’t furnish me with the case for 15 squares.

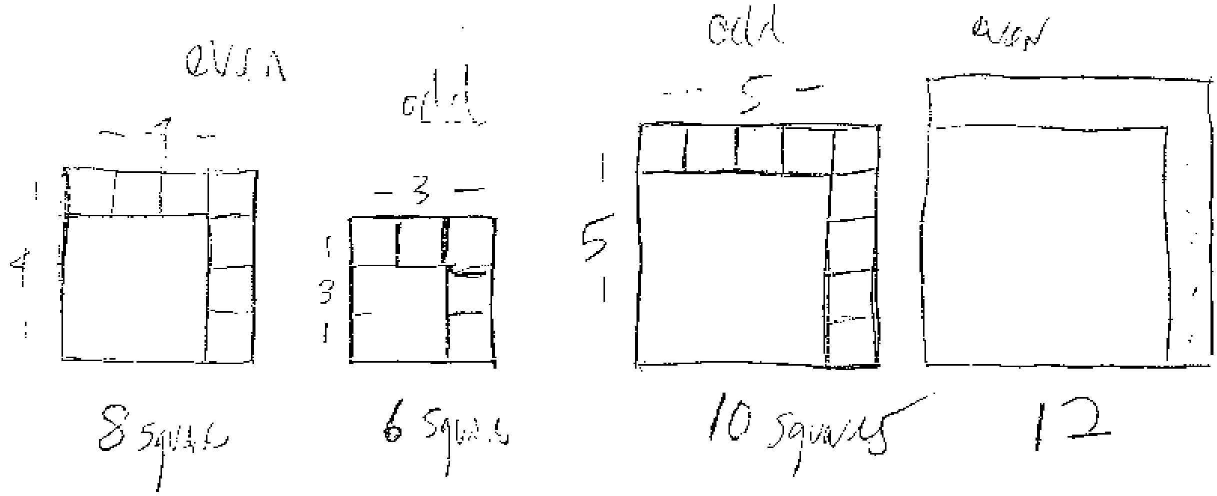

Figure #3 The L pattern.

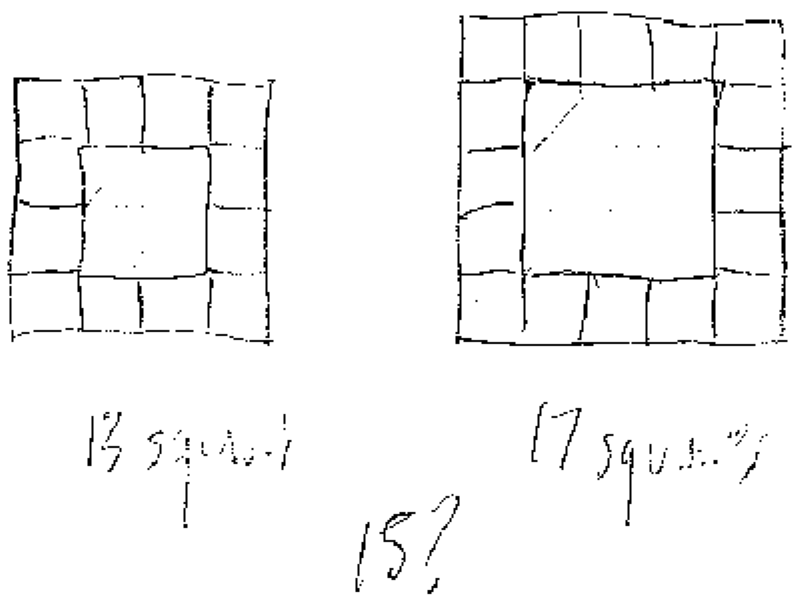

Figure # 4: A central square.

Although I am dwelling on my actions and how these actions pursued a thread in time, I want to underscore that Deleuze has a dispersed theory of subjectivity, whereby the situation rather than the subject undergoes events. Rather than focus on the subject, it is the system of linkages (the assemblage) that reacts to stress and increased intensity. Events happen through people, rather than to them (Williams, 2009). Events intersect with the many other series that flow through the subject. In the foregoing activity I emerge as a mathematical subject (no matter how novice) rather than my being the subject who then acts. For both Badiou and Deleuze, events figure crucially in the individuation of subjects. Without events, we lose the significance, that is, the activity or mobility or intensity, by which a subject congeals and amalgamates. Williams (2009) suggests that both Badiou and Deleuze problematize theories of agency altogether, since there seems to be no will, no intention, no centering of agency in an individual act. Following this decentering of human agency, a mathematical subject is not a site for cognition and performance, but rather a meeting or series of encounters. As Williams (2009) explains, change is more primary than the subject who congeals through change; the subject becomes a unified entity when it is part of change. Thus change, difference, and movement are more primary than position. My engagement with the 1989 problem, therefore, must be seen as part of a series of actions that are expressive of various kinds of intensity and flowing energy across the situation rather than willful results selected by an agent. Although Deleuze and Badiou have different ideas as to how (and why) this dispersal of energy occurs, they seem to share a similar commitment to studying the complex multiplicity of forces at work in any situation.

Where I find Deleuze more compelling for a theory of learning particularly relevant to mathematics is in his emphasis on creativity or inventiveness (or the bringing forth of the new) through problems and paradox. The event, for Deleuze, must be seen in terms of the problem or problematic idea rather than in the terms of a truth or a solution. In *The logic of sense,* Deleuze uses the work of Lewis Carrol on logic, language and mathematics to map alternative ways of thinking about sense and non-sense. The book draws on Stoic and Leibnizian philosophies so as to introduce a fourth dimension to language, called the “sense-event” which both grounds and destabilizes the various manners by which meaning is made, those being denotation (causal links), signification (truth conditions), and manifestation (personal stake). The sense-event is precisely that which will sustain the circle that binds these three components together. As such it cannot be subsumed within a theory of possibility (conditioned by logic) but must instead be “unconditioned [and] capable of assuring a real genesis” (Deleuze, 1990, p. 19). The event subsists and is expressed in language but is prior to any logical determination or reference. It is crucial to note that Deleuze is pointing to the inadequacies of language and any theory of language (or representation) in addressing the nature of events. He is reclaiming “sense” from the linguists and troubling the meaning of the term by showing how paradox and bodily event are buried in it.[5] My scribbling on the page as I attempt to grapple with the problem (to make sense) must be seen as that which animates the problem without ever exhausting it or fully determining it.

And yet Deleuze argues that states of affairs, qualities and quantities are a part of substance, and that the incorporeal (the event) is sterile, "impassive", ineffective and *on* the surface of things: "The Stoics discovered surface effects"(p.7). Against depth and height, irreducible to bodies and to ideas, the "incorporeal event" frequents the surface: "There is nothing behind the curtain except unnameable mixtures, nothing above the carpet except empty sky" (p. 133). The event is the "impassive" site (neither active nor passive) where action proliferates into all of its possible unfolding. Events grow like crystals along their edges. The event is thus a site of paradox. Indeed paradox is the "dismissal of depth, a display of events at the surface, and a deployment of language along this limit." (p.9). The möbius strip offers a useful model for this sort of disorientation or "sterile splendour" in that it is only one surface – a surface where things and language meet - and yet one's direction is reversed when one returns to where one started. One must cut the möbius strip in order to *make sense* of its directionality in relation to an external set of axes, but one can also live on the strip and incorporate a moving set of axes on the surface so as to make *intrinsic* sense. "The agonizing aspect of the pure event is that it is always and at the same time something which has just happened and something about to happen; never something which is happening." (Deleuze, 1990, p.63).

Problems: the source of events

In *Difference and Repetition* (1994), Deleuze draws on the philosopher of mathematics Albert Lautman's theory of the problem or problematic idea. According to Lautman, a problem has three aspects:

> Its difference in kind from solutions; its transcendence in relation to the solutions that it engenders on the basis of its own determinant conditions; and its immanence in the solutions which cover it, the problem *being* the better resolved the more *it* is determined (Deleuze, 1994, 178-9).

Solutions to a problem - be they numbers, triangles or relationships - are at source spatio-temporal dynamisms or dramatizations of the indeterminacy that characterizes the problem. The triangle is progressively determined through the actualization of this virtual space of movement and dynamism.

[5] According to Smith (2005, 2006), this issue is poorly grasped by Badiou in his reading of Deleuze on the event.

The problem consists of the indeterminacy prior to complete resolution or actualization. Lautman goes on to suggest that problems or problematic ideas emerge in relation to a set of dialectical opposites (even/odd, whole/part, continuous/discrete) for which resolution is never fully granted. And although these opposing notions transcend mathematics,[6] he argues that mathematics itself can give birth to the new through the mixes and amalgamations of diverse theoretical embodiments of these opposites. Consider, for instance, the way that parity (even/odd) functioned thus far in my engagement with the problem above. Or, to offer a more historically significant example, consider the way parity functions as a creative force in the ancient Greek proof of the irrationality of radical 2. The proof works through contradiction (assuming the opposite of what it aims to prove, deducing a contradiction, and thereby concluding that the initial assumption was wrong). In assuming that radical 2 is rational, one deduces the existence that a number is both even and odd - an impossibility! Thus, it must be the case that radical 2 is not rational. Even-odd parity is the engine by which the proof functions, and it is also the force that engenders the irrational number. Parity is precisely the kind of instrument that functions simultaneously to work on both the logical level and the material or ontological level. It marshals logical constraints *and* creates mathematical entities. Parity is leveraged in this famous proof to unleash and make actual the virtual (the irrational number) *while* operating according to the laws of legitimate inference. The proof is the making of something – quite literally, a sculpting or creative activity that brings the new number into being. But it is also, of course, a process by which the possible is realized. The sculpting tool of parity works in both the realm of the possible and the realm of the material.

Problems have both this ontological and logical element – they bring forth the new while also delineating the possible. Treating them only as obstacles to solutions detracts from this ontological creative aspect. Deleuze follows Lautman in troubling the usual coupling of problems to solutions. The traditional model for mathematics education is to consider the problem as given or ready-made and to consider the solution that which will dissolve the problem (Brown, 1998). From this traditional perspective, the difference between problem and solution is one of degree, and “the problem simply contains less of the solution than the solution does.” (Durie, 2006, p. 171). Such an approach to problems doesn’t honor the problem as a site of creation and genesis, and focuses instead on the skills and psychology of those who may or may not solve it. To see the problem as lacking sufficient solution, suggests that what’s at stake are “failures of recognition” on behalf of the knowing subject,

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[6] The solutions are immanent to mathematics, but the problems transcend mathematics. Both Deleuze and Badiou use this approach to go backwards, through the solutions found in mathematics, to the problems of interest to philosophy.

who fails to recognize the solution in the given problem. Deleuze argues that thinking is not a matter of (mis)recognition, but rather an encounter. The difference between recognition and encounter is gigantic - the former binds us to epistemological concerns while the latter delves into the ontological nature of thinking. If learning is an encounter, then the problem rebuffs my thinking not because I fail to recognize the solution in it, but because I encounter its material otherness through my senses, and I experience the limits of my senses in so doing. In these kinds of encounters we experience the limit of the sensible, and it is this encounter that forces us to declaim the problem as problem. Problems are thus acts of constitution and investment, arising from the encounter with the limits of sensibility. Here we are inverting the usual lineage whereby right/wrong falls out of the solution, and arguing that the contours of right/wrong are in fact constituted by the problem itself.

I moved in and out of the problem of 1989 squares. I encountered the limits of sensibility through my reaching out and feeling the contours of the problem – those limits were encountered in its visibility on the grid paper, its spatial partitioning, its movement back and forth along a line of potential patterns, its insistence on irregularity and asymmetry. It wasn't that it withheld its solution – masked it like a secret – but rather it came alive with intensity and pulse, dispersing energy across the lines of engagement (and lines of flight) along which I moved, sometimes haltingly. When I was able to feel the alternating pulse of the problem – the back and forth between the balanced arrangement of a perfect square with the internal irregularity of each square in relation to the others – my pace slowed and I dwelled without haste or anxiety near the limits of the sensible. And although I wanted a solution – indeed the prompt is exactly this, to pursue a solution or "show" how a solution can be obtained – I didn't feel that the solution was hidden or masked, but that the problem was the engine or source (the genetic element) that was bringing forth my actions. Aesthetic considerations caused me to explore the partitioning of 221 into 4 squares, each with its own internal and distinct partitioning. Why partition it into 4? Why not 9 again? Rather than appeal to intuition, the best answer is that I *wanted* the problem to unfold with a partitioning of 4. It was my *desire to produce* a solution that partitioned the first square into 9 and then each of those 9 into 4 squares. The desire and the problem were mutually entailed. I was unaware that my problem was part of a long tradition of similar problems, nor was I aware that there might be other ways of pursuing this problem. The 1989 problem was in fact part of a lofty lineage - In 1770, Lagrange had proven that any natural number can be expressed as the sum of four integer squares. Others had explored variously related problems – the number theorist Emil Grosswald published an entire book in 1985 on the topic of representing integers as sums of squares. I knew none of this. My emotional attachment to the problem and the manner by which I entered it linked less to my knowledge of the field, and more to the *mutual* *entailment* of my (limited) knowledge with the structural conditions of the problem itself. Picking up my pencil once again, I wrote down a sequence of square numbers: 4, 25, 36, 81, 64, 16, 9 … looking for groups of four that when added would furnish a number whose last digit was 1 (to make 221). I found myself writing the numbers 121 and 100 since I knew these were squares. But 100 seemed to resist my attempts at decomposition into an odd number of squares. Distracted, I shifted away from this particular set of numbers, choosing instead 144 (always beginning with a big square, so as to limit the choices for the others), and found these four that worked ($12^2 + 4^2 + 5^2 + 6^2$). My reasoning was far from sophisticated. What stands out is how this encounter with the problem was in large part driven by an aesthetic and sensory energy – the making of a prolonged event. I felt joy at having found one such arrangement of four squares (to be repeated in each of the nine squares), and I drew the image of my solution (figure #5), and only then did I wonder whether this was the only way to dissect the square into 1989 squares.

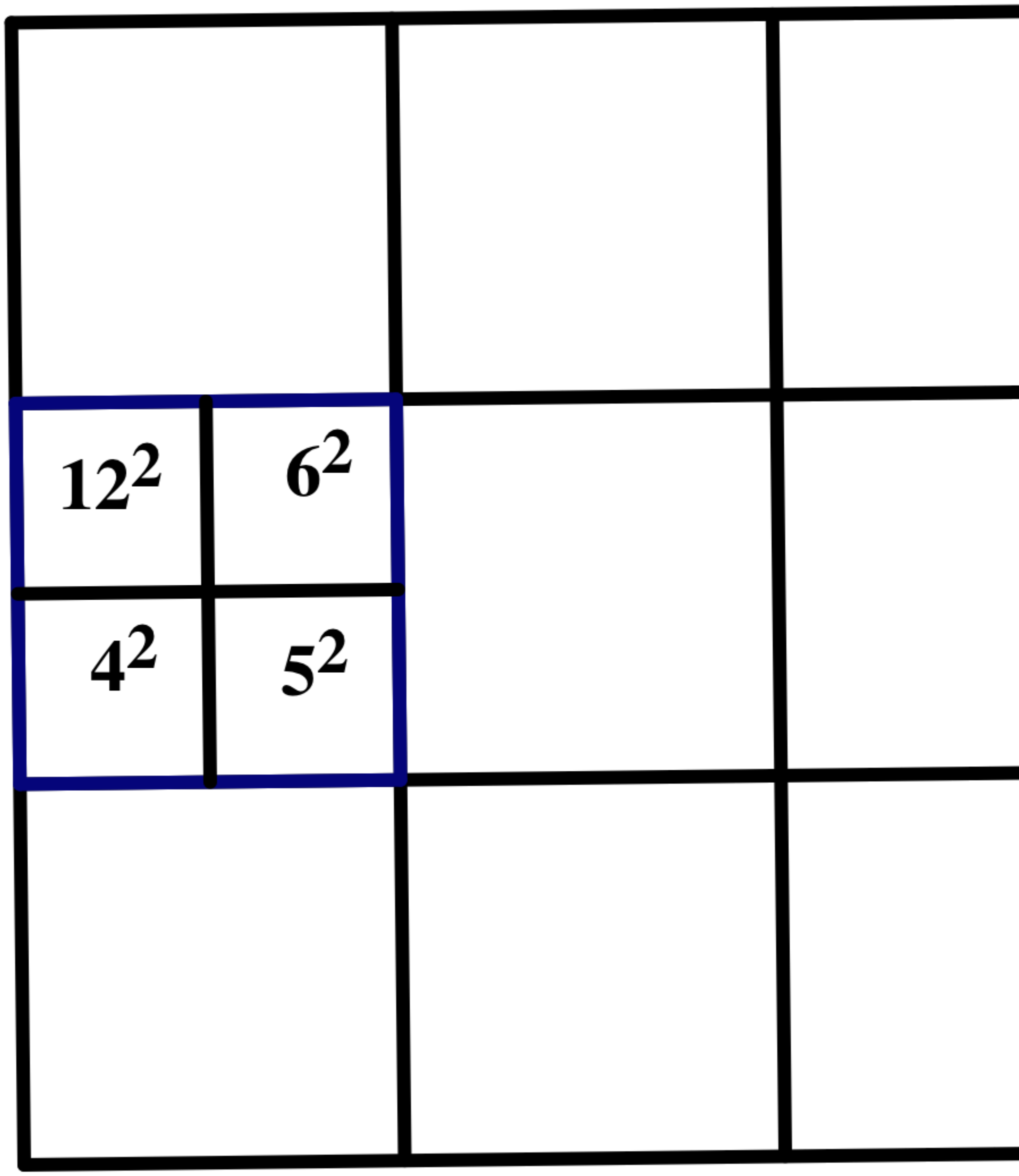

Figure #5

In this act of wondering whether there are other dissections, the problem is further infused with intensity and new lines of flight erupt. After all, I had produced a dissection, but the question asked for a demonstration that such a dissection was possible rather than asking *for* a dissection. I had worked materially through the making of a solution, but the question was about the logical possibility of a solution. The problem was asking me to make my material method more machinic, to abstract from my fumblings and design more of an operative process that might work the freedom of possibility so as to produce or actualize solutions as multiplicities.

For Deleuze, it is our tendency to forget problems once they are solved, to erase the memory of the problem and idolize the solution as abstract and originary that leads us into the dry world of propositions. In abandoning the problem, the proposition claims an abstract general universality (Deleuze, 1994, p. 162). As Alunni (2006) suggests, Deleuze's complex binding of ideas with problems is meant to decenter the solution and rethink the concepts of transcendence and immanence. All too often we construe the solution as the particular or discontinuous in relation to the continuity of the context within which the problem was articulated. Durie (2006) points out that Deleuze tries to avoid the binary between universal and instantiation (or particular) by characterizing the problem in terms of indeterminate multiplicity. An indeterminate multiplicity is unlike a set in that the latter "maintains a relation to a certain external principle (or essence) which determines the elements of the set (the set of whole numbers, for instance)" (Durie, 2006, p. 174). In contrast, an indeterminate multiplicity submits to fundamental "laws" that govern the kinds of relations between elements, but these laws are "wholly indeterminate" with respect to the form of the elements themselves (Durie, 2006, p. 174). Within a multiplicity there are singular and regular elements – but their singularity and regularity is determined through their relationships rather than their specific form. Durie offers the example of a curve changing at its maximum, where the relationship between the singularity and the regular points in its neighborhood together constitute the curve (p. 175). He also refers to Poincaré's notion of an attractor or singularity to which a dynamic system will tend when it falls within the basin of attraction or neighborhood of pull. The problem is conditioned by an *indeterminate* distribution of singularities, and differs in kind from the *determinate* specification of particulars that constitute the solution of the problem. For instance, the 1989 problem, so simple and yet so taxing, is conditioned by the relations between numbers rather than by the numbers themselves. Instead of universals, Deleuze sees these as material relations. Parity, divisibility, and squareness are indeterminate of the actual entities that crystallize as I work the problem, although they condition and inform the cases under study.

The relational ontology that underpins the problem points to the mobility and dynamism that sustain an event *as* an event. For Deleuze, this mobility is a mark of the virtual or potential expressed through the problem. Deleuze argues that such an approach makes the problem both transcendent and immanent to the solution, that is to say, the problem is an indeterminate system of differential relations (occupying a virtual or potential space) and also incarnate in the actual relations articulated in the solution (Deleuze, 1994, p. 163). Problems are more than solutions cloaked in a fog of confusion. The essence of problems is not their solution. Fetishizing the solution as essence is what leads us every time into transcendentalism. We pay tribute to the solution and erase all memory of its originating problem, and suddenly we claim prior unflinching status for the result. The imaginary roots put forth in the 15th century – that which is named as the solution – abandon the messy problems from which they emerged, and are made to join the pure untainted world of ideal forms. It doesn't seem fair to the problems! Instead, Deleuze (following the philosopher Albert Lautman) demands that we see how solutions are not definitive, but remain shackled to their problems, carrying with them the conditions that set them in motion. “Problems are the differential elements in thought, the genetic elements in the true” (Deleuze, 1994, p. 162). What would be gained (and lost) in mathematics education if we tried to keep in mind the “reciprocal presupposition” that couples solutions and problems? How do we keep at play the internal genetic structuring of the problem itself, the way in which the problem is a site of creation? The vast majority of mathematics classrooms are places of consensus, rather than places of inventiveness and experimentation. Why does mathematics education work so hard to maintain a culture of axiomatics and not problematics?

Into the classroom

During the last few decades, mathematics education researchers have promoted “problem-based lesson plans” as a model that works in mathematics classrooms (citation). Various studies have shown how experiences with “non-routine” problem solving and “problematizing” can improve teacher confidence about mathematics and encourage students to take risks with their ideas (Hiebert et al, 1996; Hiebert et al, 1997; Selden, 1989). Brown and Walter (2005) have argued that cultivating a habit of “problem posing” engenders a more adventurous approach to mathematics problems as alterable situations. Crespo (2003) has shown how this kind of facility with problem-posing can alter teacher practice significantly. The literature on problem-posing in mathematics education consistently produces studies of rich classroom interaction where students are invited to experiment with diagrams and produce conjectures about shapes, numbers and other ‘mathematical entities’ and relationships. The work on problem-posing resonates with the Deleuzian argument of this paper, suggesting that educators should treat problems as sites of creativity and invention, and less as that which must be resolved to make way for the solution. Despite the insights of this research, however, and despite the evidence that mathematicians themselves operate in this way, mathematics classrooms remain sites of compliance where regimes of truth enforce an axiomatic image of learning. In most classrooms, the problem remains a mere curtain covering up the rigid atemporal mathematical concept behind it. Why is it so hard to embrace a problematics of mathematics problem solving?

The answer relates in part to the particular ways in which curriculum is construed in policy documents. The 2011 U.S. Common Core Standards promotes a new emphasis on “conceptual understanding of key ideas” and yet the document supports an image of mathematics as axiomatics. Citing concerns about a curriculum that is “a mile wide and an inch deep”, as well as the “conceptual weakness” found in most U.S. textbooks, the Common Core document pushes for less procedural mastery and more in depth conceptual work. They argue for the need of a “coherent” curriculum that addresses “the deeper structures inherent in the discipline” (Schmidt, Houang & Cogan, 2002, p.4). The Common core claims to present the key ideas in mathematics and to show how the “organizing principles” like place-value and the rules of arithmetic “structure those ideas”. This policy move is meant to address the over-emphasis on skill mastery in mathematics classrooms, where students perform procedures without demonstrating any other understanding of the associated meanings. I am not arguing against such an aim, but I want to draw attention to the tacit assumptions at work in their appeal to the “inherent” structuring of concepts. The authors of the document rely on a theory of learning trajectories or “learning progressions” in order to create a new “sequence of topics and performances”. In so doing they correlate supposed cognitive behavior (learning) with mathematical concepts, stating “the development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time.”(p. 4). This model of learning progressions is teleological in that it assumes that learning is a developmental progression towards a ready-made concept, thereby fixing the goals for the learner and emptying the concepts of any vibrancy and indeterminacy. Moreover, the document makes the usual contrast between procedural mastery and “understanding”, and although this is effective in directing teachers’ attention to how they are overly invested in “rote learning”, it can also be taken up as a manual/mental binary that locates understanding in the *heads* of those sufficiently privileged to participate in what we consider “higher-order” understanding (Walkerdine,1988). In addition, and especially relevant to the ongoing tension between the problematic and the axiomatic, the authors of the document describe “understanding” in terms of “justification and explanation”, revealing a bias towards the logical foundation of certainty in mathematics rather than the aesthetic or creative elements entailed in mathematical practice. Of course explanation and justification are important aspects of mathematical practice, but they operate in tension with other aspects, like inventive diagramming, problem-posing and rule-breaking (de Freitas & Sinclair, 2012). Explanation is also associated with teacher emphasis on verbal communication skills in the classroom and the privileging of the alpha-numeric over other modalities (de Freitas, 2012b). My point here is not to advocate that we abandon all explanation, but to show

how the axiomatic continues to be privileged in curriculum policy through the working of this pedagogical demand. The impact of this on the enacted curriculum can be seen in the fixing of the mathematical concept (squareness, primeness, odd-even parity) as the rigid structural element that determines the inferential unfolding of activity. According to this model, learning is a process of coming to recognize these concepts, to abstract them away from the messiness of their instantiations, and submission to the rules of inference. As a way out of this static treatment of the concept, Cutler and Mackenzie (2011) suggest a "pedagogy of the concept" by which they aim to revise *concept* as a material and ontogenetic device – much like the way Deleuze talks about the event. According to this approach, learning is less about *recognizing* concepts, and more about *encountering* concepts (or becoming concepts). The provisional assemblages of concept-human that emerge in classrooms are indeterminate processes of becoming that cannot be captured in the "learning progression" model. Indeed, concepts operate through mobility and creative processes of bringing forth the new.

> Perhaps the real challenge to Cartesianism lies not in reconfiguring how we know what we know but in dethroning the epistemological project itself by considering the body of knowledge on the same material plane of existence as the lived and the physical bodies? Perhaps the challenge is to treat learning as an ontological rather than an epistemological problem? (Cutler & MacKenzie, 2011, p. 63).

The new core curriculum refuses to engage with the indeterminacy and undecidability of mathematical concepts, treating them instead as rigid timeless truths that students will either recognize or misrecognize. This is due in part to the "alchemy" of curriculum policy whereby various material practices that occur outside of schools are packaged into school knowledge (Popkewitz, 2004). Alchemy tends to involve a radical overhaul of the actual material practice, transforming these practices into prescriptions about legitimate forms of thinking. In the case of mathematics, one particular alchemic transmutation entails centering the axiomatics and banishing the problematic. The argument in this paper is meant to show how problematics is at the heart of mathematics, but that it is regularly confined and constrained by the tendency towards axiomatics. By pursuing the mathematical event, my aim is to bracket our ongoing commitment to the verification of truth (through justification and explanation) so that we can imagine how a problematics might figure more prominently in school mathematics. The philosophers of mathematics Corfield (2005) and Baker (2009) have both argued for the notion of the *mathematical accident*, suggesting that such a notion allows us to rethink the nature of mathematical practice. The very idea of an accidental truth runs counter to the usual assumption that mathematics pertains to necessary truths, or in the least, analytic truths, and it breaks with the assumption that explanation is the measure of all mathematical meaning. Many mathematicians will speak of “mathematical coincidences” or “mathematical accidents” when referencing a fact that seems uncanny. For instance, the fact that 1828 occurs twice in the first 10 digits of the decimal expansion of *e* might be deemed a coincidence since its appearance is surprising. An accident, unlike a coincidence, might be an event of some significance that breaks with all causal connections and direct determination. An accident is a kind of event, one that is deemed, in retrospect, to be fully undetermined. Baker (2009) argues that the degree to which an adequate explanation exists determines whether a mathematical fact should be considered an accident. Since many mathematicians still find brute case-crunching computer proofs somewhat unsatisfying, there are today significant mathematical facts that would be considered accidents. Baker (2009) cites Gödel’s incompleteness theorems, pointing out how these results seem to make space for unprovable truths:

> My presumption has been that there may well be mathematical truths such as the Goldbach Conjecture, that are both mathematically interesting and are accidental either because they are unprovable or because their best proof is highly disjunctive. There may, in other words, be interesting mathematical truths that are true for no reason (Baker, 2009, p.153).

While some mathematicians might bracket the coincidence or the accident as simply a sign that we are in need of a better axiomatic or encompassing theoretical approach, I take from Baker a willingness to see accidents as affirmative sites of disruption, as engines for inventive mathematics, as problematic ideas that make the laws of inference tremble and quake. It’s not that we are simply lacking an adequate explanation or unifying superstructure, since that would demote the problem to an unfortunate obstacle to be overcome as soon as a solution can resolve it. The accident *is* the perfect problem, since it draws us down into the muck of perplexity and insists that we reckon with the prospect of the irresolvable. Deleuze (1990) draws a distinction between events and accidents, where the latter is the manifestation of the event in a state of affairs. The event is an “ideational singularity”, but it is realized in a spatio-temporal state of affairs (p.53). It’s not the case that the event is problematic, but that problems are the condition of the event. As Smith suggests (2006), “it is the nature of axiomatics to come up against so-called *undecidable propositions*, to confront *necessarily higher powers* that it cannot master”(p. 158). Problematics is then the form of experimentation by which mathematics brings forth these accidents.

Pursuing a problematics in the classroom entails an encountering of mathematical accidents, an opening up to affective investments, and an inventiveness that embraces the aesthetics of mathematical practice. The *National Council of Teachers of Mathematics* (NCTM) has consistently put forth curriculum recommendations regarding standards for mathematical practice. The process standards, for instance, list "problem solving, reasoning and proof, communication, representation, and connections." Intended to capture the kind of cognitive or intellectual labor entailed in doing mathematics, none of these process standards have captured the deeply material activity of a mathematical problematic. NCTM has also specified particular proficiencies, and while many of these point to the importance of adaptive, strategic and flexible ways of engaging with mathematics, and thus might support a problematics of problem solving, the over-all agenda is to develop in students a "productive disposition" aligned with an axiomatic image of mathematics. Such a disposition is defined in terms of a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy." If taken at it's word, this highly rationalist and utilitarian perspective cannot reckon with the non-sense and paradox that fuels experimental thinking in the field, nor grapple with the indeterminate materiality of mathematical concepts. A more inventive and aesthetic mathematics allows students to engage with the material substance of knowing, by attending to the ways in which the body bumps up against an object of knowledge less as that which is (mis)recognized, and more in terms of coupling, encountering and disrupting space-time relationships. As Deleuze (1994) suggests, "Something in the world forces us to think. This something is not an object of recognition but of a fundamental *encounter"* (p. 139).

Conclusion

Perhaps the relation between mathematics and humanity is best thought in terms of a double problematic – on the one hand, "that of problematizing human events, and on the other, that of developing as various human events the conditions of a problem" (Deleuze, 1990, p.55). Indeed, the recreational mathematics that I pursue here, and that Deleuze studies in the work of Lewis Carroll, offers this double aspect. I entered into the problem of 1989, not only or even primarily to solve it, but to assemble with it, engage in its trailing history and its potentialities, to tap into the genetic element of the problematic; for surely mathematics shouldn't be simply about "solving supposedly given problems, [but] methods of invention appropriate to the constitution of problems" (Deleuze, 1994, p. 161).

I have argued that the work of Badiou and Deleuze help us theorize problem solving in mathematics in terms of the event, and that such an approach makes room for new approaches to teaching and learning problem solving in mathematics. The event is contrasted to the entity or object because of its durational aspect, genetic element, activity, indeterminism and undecidability. Together these present an alternative way of thinking about problem solving that has less to do with uncovering a secret (or acquiring knowledge of an eternal entity) and more to do with creatively bringing forth the new. The problem of dissecting a square into 1989 squares functioned as a site for studying the nature of mathematical problem solving. By focusing on my various forays into this problem, I was able to show how the experience of problem solving was an engagement with the limits of the sensible and a movement along generative lines of flight. My detailed description of the actions I took as I engaged with the problem show how, even in this trivial case, problem solving entailed the perturbation of concepts and the mapping of their event structure. For Roth (2012), first-person methods are rigorous studies of consciousness at the point of its emergence, allowing a researcher to attend to the immanence of embodied engagement. For me, this method allowed for a way of sustaining my attention on the ways that the problem figured prominently in conditioning my activity, and perhaps more importantly, I was able to depict how engagement with a mathematical problem entails an ontological element that doesn't pay lip service to a learning trajectory. Curriculum policy always seems to enforce an axiomatics on mathematics practice, refusing to allow for a problematics of conceptual indeterminacy and creativity. My hope is that this experiment in self-study has troubled taken-for-granted assumptions about mathematics problem solving, and has shown how we might rethink mathematics as event.

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