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## Movement, memory and mathematics: Henri Bergson and the ontology of learning

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# Henri Bergson and the ontology of learning

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## 1. Introduction

Much of the work on embodiment in mathematics education focuses on how bodily movement is a resource for the learning of mathematics. Whether drawing on physiological, phenomenological or semiotic paradigms, there is a burgeoning literature in the field that attends to how movements of the human body factor into learning mathematics. Research in this tradition opens up the discussion of movement and learning in radical new ways, showing how thinking occurs in the gestures and the bodily movements themselves rather than behind them (Nemirovsky et al., 2012). Indeed studies claim that kinetic bodily movement is in fact a *necessary* condition for the emergence of abstract mathematical knowledge (Bautista, Roth & Thom, 2012), and shown how various sensuous modalities interplay in mathematical cognition (Radford, 2013).

In this paper, we aim to contribute to this new direction of research, by showing how movement is a primary ontological category. Thus the article makes a philosophical argument as well as pursuing the implications of this argument in the study of mathematics classrooms. We use the work of Henri Bergson (1859-1941) to examine the role of time and movement in embodied mathematics. The body, for Bergson, is a "centre of indeterminacy" in a chaotic system of affects and percepts, individuated as a body only provisionally (Bergson, 1896/1988). Bergson uses the term *duration* to describe the way that matter and memory are provisionally entangled in bodies. In rejecting mind/body dualism, Bergson claims that there are no representations or perceptions possessed by the mind. The very idea of an internal representation is absurd, argues Bergson, since there is no evidence to support the claim that we *store* memories as pictures.

Our aim in this paper is to show how Bergson's ideas on bodies, movement and memory can help researchers study mathematics classroom activity for how it *involves* and *evolves* mathematical concepts. We begin by describing Bergson's "intuitive method", a method that he intended for philosophers and scientists alike, as part of his proposal for a new kind of empiricism. We emphasize that for Bergson this method was also intended as a way to live, and a way to be creatively and responsively plugged into the world. In this paper, we focus on how elementary school students use this method in classroom activities, tapping into a collective duration as they grapple with mathematical concepts. We first show how Bergson's method is grounded in his interpretations of the history of mathematics, and we explain his approach to body, movement and memory. We then explain how his method helps us revise our approach to studying the role of the body and movement in learning mathematics.

In particular, we show how classroom events can be examined for how students attend to the provisional nature of both concepts and matter. In other words, we study the extent to which students are able to disrupt the stasis of that which seems to be given and determinate. In the last section of the article we draw on these ideas to analyze two case studies of mathematics classroom video. The video excerpts are from two different Italian research projects. The first occurs in a classroom in which motion capture technology is used to explore students' learning about rates of change, and the second occurs in a classroom in which students are exploring number patterns. In this first case study, we show how dynamic graphing software contributes to students' understanding of movement and time. In the second case study, we show how the intuitive method is at the heart of a robust concept of number.

## 2. Bergson's intuitive method

Bergson was perhaps the most influential French philosopher at the turn of the twentieth century, drawing on both the continental tradition of metaphysics and also on developments at that time in physiology, quantum physics and relativity theory. In such books as *Time and Free Will* (1889/1910), *Matter and Memory* (1896/1988), *Creative Evolution* (1907/2005), and *An Introduction to Metaphysics* (1903/1999), Bergson offers exceptional analyses of the nature of movement, duration, intuition, and life more generally. Bergson is today, however, underappreciated, in part because his focus on ontology was at odds with the growing adherence to positivist paradigms of knowledge in the early twentieth century, and in part because there was little sympathy for him within the increasingly dominant analytic language-centric philosophical tradition. Recent interest in new materialist philosophies has brought Bergson back into the spotlight, in particular the work of Gilles Deleuze (1983/1986, 1985/1989, 1966/1988) and Elizabeth Grosz (2004) has shown how Bergson's theses on difference and duration can be used to examine theories of learning. More recently, discoveries about brain plasticity and amodal perception in the neurosciences support many of Bergson's earlier insights (Malabou, 2008; Protevi, 2009). Bergson's work is strongly linked to kindred projects in the phenomenological tradition of cognitive science, for instance enactivism (Varela, Thomson & Rosch 1991), but it differs in being strongly post-human in its conception of what constitutes memory. In this sense, we see his work as directly supporting attempts to analyze mathematics education through the lens of inclusive materialism (de Freitas & Sinclair, 2012, 2013, 2014).

Bergson was critical of psychological explanations of thought and activity. His approach aligned with pragmatist philosophies—like those of James, Dewey and Whitehead—all of whom were also cast as *process* philosophers. As a pragmatist, he was intent on offering a method of analysis that would bring the best of metaphysics and science together, a method that would identify and diffuse common illusions about mind and matter, while also furnishing a means for invention of new concepts. He named this method the *intuitive method*. It is important to state at the outset that for Bergson, intuition is a *method* and not a faculty of the mind. The intuitive method is a powerful pragmatic and materialist engagement with the world, not a mysterious appeal to a magical internal intuition, not a Kantian intuition all too human, but a new kind of empiricism.

This new empiricism must grapple with the nature of time and movement, states Bergson, for these aspects of life are the most perplexing and the most difficult to grasp. Given the challenges of perceiving the temporal flow of time, and given our physiological tendency to perceive illusions of fixity, we need to develop a method that addresses and perhaps overcomes these limits. Bergson proposes that metaphysics and science must come together in this new method if they are to overcome common habits that dwell on the illusions of fixity and stasis. In other words, this new method will help us understand *becoming rather than being*, and thus help us grasp the fundamental *temporality* by which we live. This is an intellectual method that will attend to the flow of duration rather than the stop and start of actions.

This method will overturn the tendency of the human mind to always see variation as only "the expression and development of what is invariable." (Bergson, 1999, p. 54). This new empiricism will refuse to demote action to an "enfeebled contemplation" and duration to "a deceptive and shifting image of immobile eternity" (Bergson, 1903/1999, p. 54). Bergson critiques theories that always invest more in the immutable than in the moving, demoting the unstable to a diminution of the stable. He argues instead that there is more in the moving than the immutable, and that passing to the unstable (in this case, say, the encounter with a cube) entails expanding that which we attend to rather than reducing or narrowing our attention. Thus For Bergson, concepts are not abstractions in the Aristotelian sense, but rather gradually emerge within the folds of material encounters.

Bergson hoped that his new empiricism would bring science and philosophy together in powerful ways. He saw in the science of the early twentieth century an attempt to study mobility as an independent reality, but also saw that contemporary approaches continued to turn to “fixed, distinct and immobile concepts.” (Bergson, 1903/1999, p. 55). Modern science would never “lay hold” of true movement, “bound as it was to the cinematographical method.” (Bergson, 1907/2005, p. 372) and thus “real time, regarded as a flux, or, in other words, as the very mobility of being, escapes the hold of scientific knowledge.” (Bergson, 1907/2005, p. 366). And he claimed that philosophers had made the same mistake, unable to grasp “the continuity and mobility of the real” (Bergson, 1903/1999, p. 52). He blamed Kant for treating science as aspiring to be a “universal mathematics” which was assumed to be a disembodied ideal. According to Bergson’s reading, Kant’s *Critique of Pure Reason* rests on the assumption that our intellect is incapable of doing anything but “Platonizing”, as Bergson puts it, “pouring all possible experience into pre-existing molds.” (Bergson, 1903/1999, p. 59).

A new empiricism would instead achieve an inversion of thought, practiced in a methodical manner. All that has been revolutionary in science, claims Bergson, has come from this kind of methodical inversion. He looks to the revolutionary developments of the “infinitesimal calculus” as inspiration, as these reflect a radical overturning of conventional thought so as to reconsider the paradoxes of the continuum. As he refers to it below, “modern mathematics” revolutionized our thinking by studying the becoming of concepts (“being made”) over the being of concepts (“ready-made”):

> Modern mathematics is precisely an effort to substitute the *being made* for the *ready-made*, to follow the generation of magnitudes, to grasp motion no longer from without and in its displayed result, but

> from within and in its tendency to change; in short, to adopt the mobile continuity of the outlines of things. (Bergson, 1903/1999, p. 52)

In the infinitesimal calculus, he sees a new "generative" approach to mathematics that finally banishes the image of universal mathematics that Kant assumed—"that chimera of modern philosophy". Instead, these new mathematical processes have shown us how to get in contact with the "continuity and mobility of the real". Just as the calculus pertains to quantitative differentiation and integration, the goal of this new empiricism will be "*to perform qualitative differentiations and integrations*." (Bergson, 1903/1999, p. 53). Duffy (2013) suggests that Bergson was at the same time critical of the calculus when simply used as "as an impetus of mechanistic explanation." (p. 93). Thus he celebrated those developments in mathematics that introduced time and motion into the concepts, embracing modern geometry because it studied "the continuous movement by which the figure is generated" (Bergson, 1907/2005, p. 37), and saw in this a potentially creative and non-reductive aspect of mathematics. On the other hand, he was wary of a calculus that reduced the flow of duration to the instantaneous present. According to Bergson, "such systems are never in that real, concrete duration in which the past remains bound up with the present." (Bergson, 1907/2005, p. 26), whereas a new empiricism must aim to be so.

In this paper, we take up this call for a new intuitive method that taps into the movement and duration of mathematics. But before discussing how such a focus might operate in the analysis of mathematics classroom video, we need to explicate two key aspects of our approach: Body and movement.

## 3. Theoretical Framework

### 3.1 Bodies

Bergson's theses on the body are meant to offer an alternative to dualisms of mind and matter that fail to explain the relationship between these two substances. Whether it be rationalists who privilege the human mind or empiricists who privilege perception, any philosophy that invests in immaterial thought and material action, says Bergson, will fail to overcome the paradoxes of dualism. These paradoxes result from a flawed understanding of the nature of space and time – and reflect a tendency to underestimate the power of matter, and the movement of thought. The most pressing issue for dualism, says Bergson, has always been explaining the relationship between two substances that are ontologically distinct. In Bergson's words, these approaches must "invent an incomprehensible action of this formless matter upon this matterless thought." (Bergson, 1896/1988, p. 23). In *Matter and Memory* (1896/1988), Bergson's argument begins by first carefully examining the distinctive nature of perception and recollection. Perception, he argues, is first of all impersonal and diffused. If we bracket for a moment the way that memory infuses perception with personal meaning, we will begin to grasp how perception is a material encounter between bodies rather than an act of coming to know. The problem with rationalists and conventional empiricists, says Bergson, is that they assume that "to perceive means above all to know." (p. 28). Rather than reduce perception to the hand-servant of knowledge, Bergson asks that we consider perception as a material encountering rather than only a source of knowing. The implications of such a thought experiment are significant for theories of learning, especially those that posit some correlation between perception and the existence of internal representations.

Bergson offers a radical alternative, arguing that perception is not operating through representation, that perception is *in* the object itself. Perceptions are not pictures or photographs taken of the world and stored in the brain, but are rather constituted in the material relations that sustain encounters between bodies. In an attempt to invert the common sense notion that perception is our mental picture of the world, Bergson claims "the image of that object is given within it and in it." (Bergson, 1988, p. 44). Perception is not in the motor centers or the sensory centers, it "is, in fact, where it appears to be." (Bergson, 1988, p. 46). When perceptions are combined with affections (which are simply perceptions caught in loops), a body becomes an animate center of action. For Bergson, the body is a "center of indeterminacy" in a vast meshwork of threaded entanglements:

> Matter thus resolves itself into numberless vibrations, all linked together in uninterrupted continuity, all bound up with each other, and traveling in every direction like shivers through an immense body. (Bergson, 1988, p. 208)

Every body is materially linked to every other body, with more or less tension due to varying proximity. Perception is less about the creation of representations in the mind, and more about this material tension that couples or assembles bodies with other bodies. Bergson is adamant on this point – there are no representations in the mind or in the brain:

> My perception, in its pure state, isolated from memory, does not go on from my body to other bodies; it is, to begin with, in the aggregate of bodies, then gradually limits itself and adopts my body as a center. (Bergson, 1988, p. 61)

> *My body, an object destined to move other objects, is, then, a center of action; it cannot give birth to representation*. (Bergson, 1988, p. 20, italics in original)

The body becomes individuated in this way through the combination of action and affection, two processes that together constitute the "sensori-motor power" of an animate body (Bergson, 1988, p. 61). Action entails reflection and diffraction of forces and pulses, while affection entails absorption. It is with the addition of affection that our sensed body becomes "privileged" over another. This may seem a rather meager privilege to some, since it fails to grant special status to human will or intentionality. Indeed, Bergson will take the concepts of choice and discernment and make them unmistakably post-human:

> … whatever be the inner nature of perception, we can affirm that its amplitude gives the exact measure of the indetermination of the act which is to follow. So that we can formulate this law: *perception is master of space in the exact measure that action is master of time*. (Bergson, 1988, p. 32, italics in original)

In denying the immateriality of thought or sensation, his argument is deeply materialist. But for Bergson matter is rich with both actual and virtual action, and thus we need to reconsider the way we conceive *extension*, which we discuss in the next section. Thus his aim is to revisit the relationship between ideas and images after having broken with conventional dualisms. As he suggests, it seems absurd to claim that there exist "perceptive cells communicating with cells where memories are stored." (Bergson, 1988, p. 91). His main concern is to critique a philosophy that posits "on the one hand, homogeneous movements in space and, on the other hand, unextended sensations in consciousness." (Bergson, 1988, p. 51). He suggests instead that there must be between images and ideas (the extended and the unextended) "a series of intermediate states, more or less vaguely localized, which are the *affective* states." (Bergson, 1988 p. 53, italics in original). It is crucial to make clear that these are not memories of diluted perceptions. His argument turns on our ability to resist treating memory as a faded perception, as though we possessed or stored faded mental pictures by degrees diminished in reference to the material source. For Bergson, matter is simply "contracted" memory, and memory is "expanded" matter. This is a monist philosophy that resists the dualism of mind and matter.

### 3.2 Movement

Bergson offers three theses on movement. The first thesis states that movement is distinct from the space covered (extension). While movement is a present action, the space covered is past and given through extension. Bergson explains the significance of this distinction in relation to how we think about time. On the one hand, extension is treated as homogeneous and uniform, and is thereby divisible—indeed infinitely divisible—into units, but on the other hand, movement cannot be sub-divided in the same way unless one borrows the metric from extension and imposes it on movement. The issue concerns the continuous nature of movement, and the fact that we must impose a unit of measure that breaks such continuity up into parts. Given a mathematical line, for instance, we are able to break up the fixed or given line into parts, but the mobility of movement defies our ability to perform the cut. For instance, you can cut up the video recording of a moving student into discrete images, but you will always lose the movement—it will, as Deleuze (1983/1986) suggests, always happen behind your back, in the interstices between the immobile units you selected. And no matter how many of these units you select for recombination, you will never constitute the entirety of the movement.

The second thesis claims that perception of movement entails illusion. Bergson, writing when cinema was just beginning, refers to the illusory "false movement" of the cinema, whereby we see movement based on a sequence of discrete immobile images. Indeed, this cinematic perception pertains to Bergson's second thesis on movement that asserts we have a tendency, given our current sensory capacities and bodily arrangements, to perceive false movement. But it isn't just cinema that indulges in this false movement, for it is a habit found everywhere. For instance, suggests Deleuze, Platonic philosophies of mathematics subscribe to false movement, because they treat ideal forms as eternal and immobile. According to Plato, the instantiation of these forms in the matter-flux occurs as a sequence of discrete inaccurate images or copies. Thus the transformation of a triangle, entailing a movement of dilation or translation for instance, would be "a regulated transition from one form to another, that is, an order of *poses* or privileged instants, as in a dance" (Deleuze, 1983/1986, p. 4, italics in original). Thus Platonic philosophies of mathematics make sense of mathematical movement in terms of discrete instances between quintessential instantiations. Bergson links this tendency to successful traditions in the history of science, citing the innovations of Kepler, Galileo, Descartes, and Newton as examples of how movement has been studied through an abstract quantified time. And although these scientific developments reflect, as Deleuze suggests, a modern practice of examining "any-instant-whatever" rather than the ancient habit of attending to a particular quintessential moment, they continue in a long tradition of treating time as an independent quantitative variable.

The third thesis on movement states that movement is a mobile section of duration. For Bergson, duration is continuous change. This thesis states that movement is not merely translation in space, but also a qualitative alteration of the entire relational ontology at any given moment. This third claim pertains is an ontological claim about the fundamental nature of mind and matter. He posits duration as the undividable continuous flow of time beneath all our counting. The famous example that Bergson supplies in *Creative Evolution* is the event of putting some sugar in a glass of water. There is a radical change in the whole that cannot be summed up by a simple approach to movement. He states “The wholly superficial displacements of masses and molecules studied in physics and chemistry would become, by relation to that inner vital movement (which is transformation and not translation) what the position of a moving object is to the movement of that object in space.” (Bergson, 1907/2005, p. 37). Thus the cup of water is like an open whole, always changing, never closed or complete, but relational. Time or duration is this open whole of relations. The glass of water can be considered a *closed set* of molecules, but this is not the *open* whole of its *continuous alteration*. It is impossible to break the link to the whole, but one can stretch it out, make the thread finer and finer. The distinction between the concept of set and whole relates to their different relation to space and time or duration. As Deleuze (1983/1986) states “the sets are in space, and the whole, the wholes are in duration, are duration itself, in so far as it does not stop changing.” (p. 11). Thus as bodies gain depth they lose their contour and are united in duration. This is a new kind of movement that expresses the whole of duration.

We can summarize Bergson’s insights into these three statements: (1) bodies become individuated and discernible as partially closed sets of relations, (2) movement of these bodies is translation through space and modifies their position, and (3) another movement of constant alteration characterizes the open whole of duration by which the bodies endure. This second kind of movement is not a spatial alteration that occurs across extension, but an alteration that enfolds in the flow of duration. These latter movements are genuine mobile sections of duration, and they are to some extent beyond conventional conceptions of movement. The third statement is the most challenging to grasp, because of its metaphysical claim.

The aim for Bergson—more generally—is to show how extended substance and immaterial thought actually share the same *élan vital,* and that there is a fundamental continuity to life that cannot be divided into two substances. Everything partakes of this continuity or *élan vital* to varying degrees. Thought and perception differ only in degree rather than kind. This *élan vital* or "extended continuum" or "extensity" re-animates extension, and by the same gesture, materializes thought. For Bergson (1896/1988), this term *extensity,* which is a *port-manteau* word that brings *extension* together with *intensity,* describes that which is prior to the homogeneous space that we "stretch beneath material continuity in order to render ourselves masters of it, to decompose it according to the plan of our activities and our needs." (Bergson, 1988, p. 231). If we are able to feel or sense or perhaps even know this "concrete extensity" or "extended continuum", we would then be able to overcome dualism and invest in a truly relational ontology. Bergson argues that "That which is given, that which is real, is something intermediate between divided extension and pure inextension" and "Extensity is the most salient quality of perception." (p. 245). Bergson's ideas resonate with those of the philosopher of mathematics, Gilles Châtelet. De Freitas and Sinclair (2013, 2014) show how Châtelet's ideas support an inclusive materialist philosophy of mathematics education, akin to what we are proposing here:

> Châtelet contrasts the lateral stretch of extension with the cutting and folding of new virtual dimensions through 'intension'. In this, he follows the scholastic tradition of contrasting extension – the interval actually travelled and its duration in time – with intension – its quickness, slowness or 'lateness' (Châtelet, 1993/2000, p. 38). As odd as this distinction might seem to modern readers, it is used by Châtelet to disrupt the privileging of position over motion and to try to imagine motion as the ontogenetic force by which position (or extension) comes into being. These new virtual dimensions are a kind of intensity that deforms the linearity of extension. The intensity of the new

> vertical dimension is what makes extension plastic and elastic. Châtelet uses the 'virtual' to describe the 'indeterminate dimension' in matter that literally destabilizes the rigidity of extension (p. 20). He describes these elastic folds in terms of transverse and vertical impulses that push through the apparent rigidity of extension. (de Freitas & Sinclair, 2014, p. 54)

One of the principle claims shared by Châtelet and Bergson is that there are no mental representations. Of the brain, Bergson (1896/1988) states that it is neither the cause nor the effect of perception, "nor in any sense its duplicate: it merely continues it, the perception being our virtual action and the cerebral state our action already begun." (pp. 232-233). Rather than rank perception over memory, the second a faint or less *intense* copy of the first, Bergson proposes this new term *extensity*. Thus the difference between quantity and quality—or extended and unextended—can be reconsidered in light of their bond or mixture in this term. We attend to something qualitatively or quantitatively, and the difference is "only a difference in rhythm of duration, a difference of internal tension" (p. 247).

> In reality there is no one rhythm of duration; it is possible to imagine many different rhythms which, slower or faster, measure the degree of tension or relaxation of different kinds of consciousness and thereby fix their respective places in the scale of being. (Bergson, 1988, p. 207)

In the next two sections, we turn to two case studies for discussing how Bergson's approach to body and movement sheds significant light on how children engage with mathematics. In particular, we examine children's classroom contributions for how they enact Bergson's intuitive method of grasping the indivisibility of duration and the extensity of extension.

**4. Case study one: Stopping time**

The episode described in this section was part of a larger research project aimed at studying the potential for a graphical approach to functions through the aid of motion detectors in grades two through five (see

Ferrara & Savioli, 2009).[1] The particular lesson considered here was conducted in a regular grade four classroom and focused on different models of motion. It begins with the teacher asking the children to recollect previous explorations they had done in grade three with the software *Motion Visualizer DV* (MV). The software works through the aid of a web camera linked to the computer. Based on live input, it captures and tracks, in real time, the motion of a coloured object in a plane (in this case, the object is an orange glove that enables the tracking of hand movements). As a student moves the object in front of the web camera, the software displays three graphs and the live video of the student moving the object (see Fig. 1).

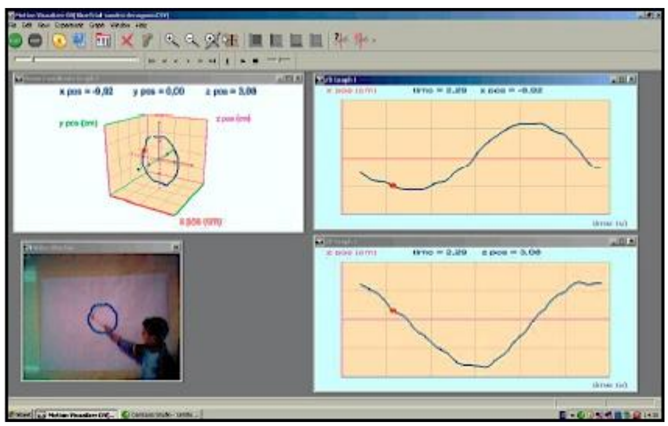

Figure 1. The video and the graphs

Each of three graphs shows the movement of the object in relation to a particular set of dimensions (in the upper left, the motion of the glove is graphed in relation to three dimensions, while the two graphs on the right show the motion in relation to horizontal and vertical dimensions separately. These last two have time as the *x*-coordinate). In grade three, the children had moved the glove in 'Movilandia' along straight trajectories—horizontal, vertical and oblique—and had studied the corresponding motion graphs that were generated on the screen, as well as the graphs that were generated when the glove was kept still, in each case investigating the associated relationships between position and time.

[1] This data was also examined for how the technology furnished opportunities for studying creativity in mathematics classroom (see Sinclair, de Freitas & Ferrara, 2013).

We see in this case study how the original continuity of the child's movement of the glove is captured and calibrated in four moving images. The first is the video that re-presents the 'concrete extensity' of the original action, and the others are the graphs of 'Cartesiolandia' where the software decodes the motion in terms of position and time. On the far right of the screen, time is treated as extension, literally spatialized and counted along the *x*-axis. In the upper left graph, time is the invisible fourth dimension, calibrated by the software, but not mapped onto one of the visible parameters. Indeed the software displays a moving point that traces out a curve in each of these graphs, against the back-drop of the grid, as though the original moving gesture (with glove) was the cause or determinant of the trace. This causal link is crucial for the pedagogy, but also relates to how Bergson describes our relationship to the sensory-motor, which is often cast as a mechanistic power. This classroom episode is a perfect example of how we "shut up motion in space" (Bergson, 1896/1988, p. 218) and imagine that space is the "ground on which real motion is posited" (p. 217). It also shows how movement is usually considered derivative or secondary to the more primitive givens of position and fixed grid. This is precisely what Bergson hoped to interrogate. Thus even with this exciting technology, we are still training our students to demote motion to a series of positions, thinking of it as a secondary linkage between states or positions.

But does this technology also help our students attend to the varying rhythm of perception? Does it help them with the count, the tally or the measure as a rhythm of duration rather than a fixity of space? How might it help students connect with the *qualitative* multiplicity that is within or beneath the quantitative extension of Cartesiolandia? Could we develop a pedagogy that helps our students grasp this truth of relativity? As Bergson claims, "That which usually hinders this mutual approach of motion and quality is the acquired habit of attaching movement to elements – atoms or what not – which interpose their solidity between the movement itself and the quality into which it contracts." (Bergson, 1986/1988, p. 203). Thus we fail to see how movement is quality itself—"beating time for its own existence through an often incalculable number of moments" (p. 202).

We see in the technology an opportunity for the students to engage with two of Bergson's theses on movement. The first is evident in how the screen combines four moving images, all unfolding together, and yet all spatially distinct. To see all of these occur simultaneously as distinct motions *and yet* the same motion, is to begin to grapple with how motion is *not* the space covered. The aesthetic incommensurability between these four moving images allows the students to begin to grasp the more fundamental incommensurability within duration. This simultaneity shows the heterogeneous multiplicity implicit within the original activity with the glove.

Bergson's second thesis on movement concerns our propensity for perceiving "false movement". The false movement of video (an apparently continuous movement composed of discrete images) operates centrally in the MV technology. Indeed, we are so acculturated to video and other moving image technology that we are all experts at perceiving this false movement.[2] But in the context of mathematics education, this propensity for our perceiving false movement is extremely effective in actually helping our students attend to the 'any moment whatever' of mathematics. As we explained above, this entails shifting from a Platonic philosophy where movement is the movement between quintessential instantiations of an ideal form, and embracing a more modern scientific approach—perhaps the start of a more materialist approach—whereby we attend to the diverse arbitrary moments. In DGS this fact is commonly used in pedagogy to help students inductively—through a series of arbitrary instants—identify a pattern or rule. Although 'false movement' allows for this development, it still reduces movement to a series of discrete instances, and thus the continuousness of the movement is not fully realized.

The third thesis on movement is harder to study, and yet we believe that it is exactly in this metaphysical claim that we can begin to study the movement of thought and mathematical concepts. The third thesis states that the original activity occurs within the "extensity" or qualitative alteration of the entire

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[2] See Doane (2002) for a historical discussion of time and the moving image.

relational 'flat' ontology of duration. Time or duration is this *open whole of relations*, expanding and contracting in an ever flowing process that propels the present into the future. The body plays a pivotal role in this process, as that which is partially individuated and affecting other bodies in relation to it. As we said earlier, it is impossible to break the link to the whole, but one can stretch it out, make the thread finer and finer, as is suggested by the diagram of the cone. Thus bodies are in duration, and their sensory-motor movements are also united in duration. This new kind of movement of contraction/expansion expresses the whole of duration and it is *the actualization of the virtual*, where the virtual is that dimension of memory that fuels the creation of the new.

In order to study movement as the actualization of the virtual, we need to unpack Bergson's concept of duration in the context of the mathematics classroom, and see how well it helps us study the movement of mathematical thought and concept. We do so by turning to a discussion of a video excerpt in which the students were asked to remember and reflect on the experiments with the MV from the previous year.

The video begins with the children who are asked to remember about past activities with the MV, starting with motion trajectories of the glove. The discussion occurs without the presence of graphs or screens. In talking about how many "shapes" had been seen in the movement of the glove, several children recall three main, straight trajectories. After one child summarizes them as "oblique, vertical and horizontal", the student Marco claims "a vertical line never appeared in Cartesiolandia," and the teacher invites the children to discuss this claim. In the following excerpt, three children—Gaia, Elisa and Beniamino—give different explanations, shifting back and forth from talking about what had happened to what could happen.

Teacher: Why could not a vertical line appear?

Gaia: Because the glove, um, when it moves, it moves (*right hand raised in the air miming a short movement*) from bottom (*right hand closed in a fist indicating a specific position*), since, um, when it appears in Cartesiolandia, the glove is always at the bottom (*indicating a specific position*) and then it makes the line in this way (*right hand shifting horizontally from left to right, Fig. 2a & 2b)* as they moved, and it does not start in this way (*right hand moving twice along a vertical direction, from top to bottom, Fig. 2c & 2d)* to make, um, the vertical line.

Teacher: What do you want to add, Elisa?

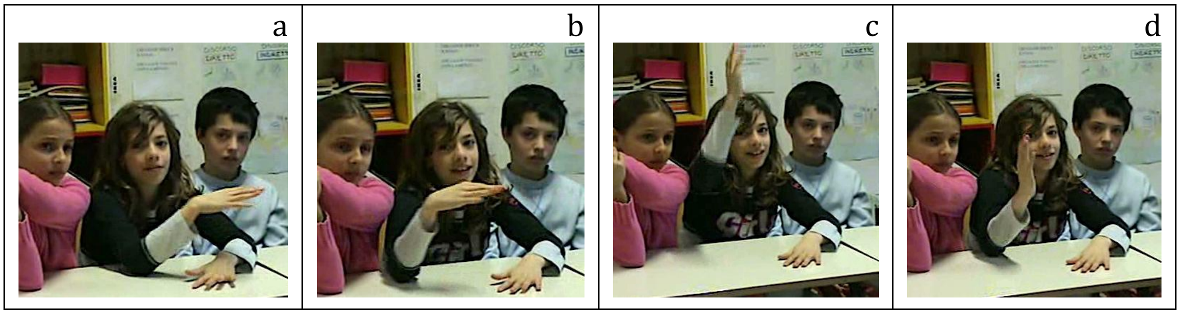


Figure 2. Gaia's explanation

Elisa: To me, because in the table (*left hand kept still in the air, right hand miming the axes*) that is in Cartesiolandia, it appears, um, to come vertical, it does not arrive at the end of the table (*open right hand moving horizontally, from left to right, Fig. 3a & 3b*), but it should arrive at the end (*right hand repeating previous gesture, Fig. 3c & 3d*).

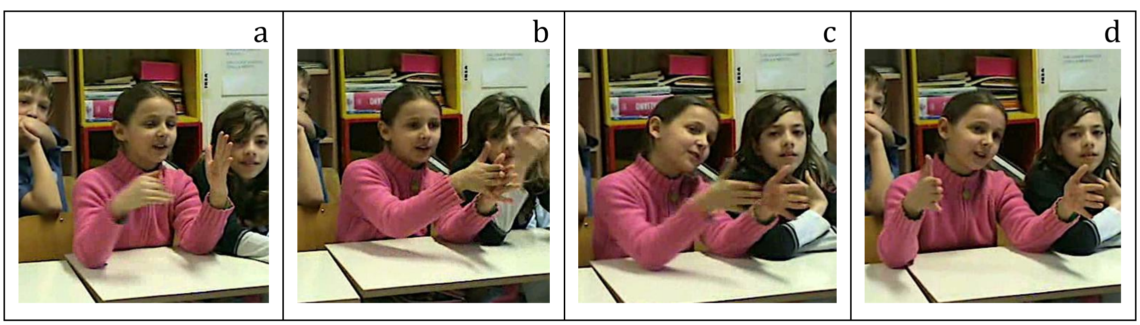


Figure 3. Elisa's explanation

Teacher: What about you, Beniamino?

Beniamino: I wanted to say that, as Elisa said, there is the table (*left arm raised vertically*), where here there is time (*right hand moving twice horizontally, from left to right*) and here (*left hand *shifting twice vertically, from top to bottom*) there is the movement you make, um, but you cannot, for example, in little time, say, 10 seconds, in few seconds make, um, be able to have such a movement (*left hand miming a vertical line*) on a platform, that is in a place making you understand that time passes (*left arm raised vertically, and right hand moving horizontally from left to right, Fig. 4a & 4b)*, since it would be as if you stopped time (*left hand pointing to a specific position, Fig. 4c)* and moved (*left hand jumping twice in the air, Fig. 4d)*.

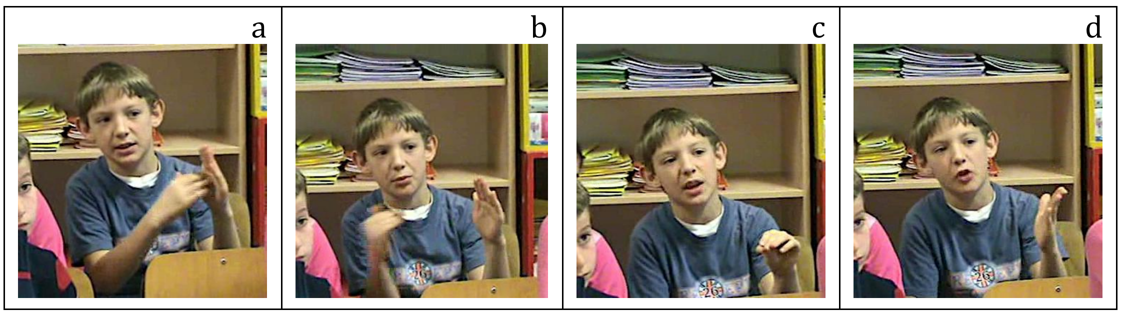


Figure 4. Beniamino's explanation

How are these students thinking about time and movement? Although the first two students speak of the movement of the glove, they are in fact attending to the variable of position to make their argument. In the case of Gaia, she is particularly focused on where the glove starts its movement, or on the position of the glove when the experiment begins. She says "it moves from bottom … is always at the bottom". Although the glove itself does not move in Cartesiolandia, Gaia is describing the way that the virtual aspect of the glove's movement is actualized in Cartesiolandia. Elisa adds to this, in the same vein, and argues that the glove ("it") must arrive at the end position. The two girls are analyzing the movement in terms of position. On the other hand, Beniamino opens up the discussion in new ways, and argues about the continuousness of time. He taps into Elisa's reference to the "table" of Cartesiolandia, and states that you cannot have movement in "little time" or in "a place making you understand that time passes" because "it would be as if you stopped time." His argument points to the entanglement of time and space, and the fact that one can never stop time in "a place making you understand that time passes". We see in Beniamino's contribution an example of Bergson's intuitive method, by which Beniamino attends to the ultimate indivisibility of time or duration.

In what ways are these students' bodies becoming individuated through their interaction? How does Bergson help us study the role of the human body in learning mathematics? As Bergson (1986/1988) states, perception is not *in* the eye or the brain or the touching hand, "it is where it appears to be" (p. 46). This formulation contests the existence of internal images and states that "the image of that object is given within it and in it." (p. 44). It might be easier to see this at work in a case study where the students speak and gesture about an object present to them, but we selected this case study because it lent itself to studying the false movement of Bergson's second thesis.

How does Bergson's theory of memory help us understand the activity of the students as they deliberate and gesture about the impossibility of a vertical line in Cartesiolandia? Obviously there is the cliché memory of recollection, which functions here as the teacher prompts the students to recall what they learned. This kind of memory entails an act of recognition. But in addition to this kind of memory, there is another kind that entails a linkage with perception qua duration. This second kind of memory is the habit or tendency or capacity that Bergson calls the intuitive method. Memory of this kind is *the perception of the movement of thought*. This second kind of memory entails attending to more than simply the movement of bodies across extended space. This other kind of memory has a different relationship with time. Time is no longer indirectly represented through bodily movement—which is how we tend to study it, dwelling on the discrete slices of the sensory-motor image of the moving body. This other kind of memory can engage directly with an indivisible continuous time—what Bergson calls duration. Such a claim might appear to be non-operational, and in principle be immeasurable, but might also help us understand what unfolds in the contribution of Beniamino.

Following this proposal, we study the students' gestures and spoken words as *affective states*, mobile and indeterminate, and 'effective' at communicating precisely because they are not representational—these gestures and spoken words are always mobilizing, as Châtelet (1993/2000) would suggest, a dynasty of future problems and questions. Thus the gesture is crucial in linking the sensory-motor to that other movement—the movement of thought and concept—that allows for the actualization of the new. In the same stroke, so to speak, the gesture opens up onto the potentiality of the body. As allusive devices, gestures have no antecedents and bring something new into the universe. When the students perform vertical and horizontal movements with their hands, using one hand to gesture "movement" and the other "time", their thought is literally contiguous with their past perceptions, and this alloy of duration is what propels their actions into an unscripted and contiguous future. If one links up with the whole of duration—a whole that is heterogeneous and never identical to itself because of its ongoing alteration—then one can begin to grasp the profound mobility of all things, including mathematical concepts. In other words, the intuitive method allows our students to connect with the lack of identity of mathematical concepts—their intrinsic difference—to follow the alteration of thought or concept, and creatively intervene in this alteration. In this case study, the students are literally bringing forth the new of the impossible vertical line, an object that breaks with conventions of space-time representation.

**5. Case study two: Extensity and the concept of number**

The episode we discuss here was part of a wider research project aimed at introducing early algebraic thinking through the use of patterns, rules and rhythms in grades one through five. The specific lesson presented here was conducted in a regular grade two classroom and focused on a sequence of 'buildings' that gives a finite figural model of the sequence of odd numbers. It begins with the children seated on the floor in semicircle in front of four buildings (formed of paper blocks) placed on a piece of toilet paper (extended on the floor from its roll) with a uniform distance between them. We use the word 'building' to designate the combined structure of row and tower (see Fig. 5). Each building is made of a (horizontal) row and a (vertical) tower, where the row always has one block more than the tower. The first building is made of three blocks, two for the row and one for the tower. This particular shape of the buildings could prompt thinking of the total number of blocks ($2n$+1) in each building as a sum of the kind $n$ + ($n$+1), where $n$ is the position of the building on the toilet paper as well as the number of blocks of the tower.

Figure 5. The sequence of buildings on the toilet paper

The children participate in a classroom discussion led by the teacher about the meaning of the sequence. As soon as the teacher asks: "What do we have here?", Agnese answers: "They seem to be buildings", while others just say: "Cubes". The children are invited to look at the sequence and to say what they think. The discussion develops with some of students sliding forward to the centre, in front of the sequence, in order to talk about the buildings.

In this case study, we use Bergson's theses about movement and duration to study the first five minutes of the video excerpt. During this time, 5 students come up to the row of buildings and discuss the buildings with the teacher. The names of the students are Giorgia, Sophia, Agnese, Lara and Riccardo. There is a great deal of gesture and body movements throughout the video excerpt, as the students slide along on their knees beside the buildings. Rather than interpret this sensory-motor movement in terms of how it might operate as a semiotic resource, we study the activity as the *actualization* of the virtual movement of thought. Following Bergson, the virtual is that dimension of memory that fuels for each child the creation of the new.

When Giorgia comes in front of the sequence, she starts to point and count horizontals from one building to the next, and then returns to count verticals:

Giorgia: Here there isn't the 1. It starts from 2 *([Fig. 6a])*, 3, 4, 5 *(shifting along)*, then it stops *(stopping at the end)*. Instead *(coming back to the beginning)* above it starts from 1: 1, 2, 3, 4 *(shifting along [Fig. 6b])* and stops at the 4 not at the 5 *(indicating the base of the last building [Fig. 6c])*.

Figure 6. Giorgia counting rows and towers from one building to the other

Giorgia points out how each of these distinct sequences of rows ("Here") and towers ("above") is growing, attending to the number of cubes in each row and tower, rather than the entire number of cubes in each building. She attends to the growths of horizontals and verticals as two separate growths, making perceptible a specific quality in the figural sequence. We can begin to grasp this intrinsic mobility of the sequence of buildings as soon as another child, Sophia, creatively intervenes in the discussion to actualize a new quality of odd numbers.

Sophia comes forward, to draw attention to what she deems to be the absent start of the sequence ("a sequence of cubes" in her words, in which "there are 3, it skips the 1 and 2"). In other words, she draws attention to what is not visible or present, that being other smaller buildings that might have come before the one that is visible. She bounces alongside the toilet paper as she talks, gesturing with her hands towards the gaps between buildings, and first highlighting the "two numbers" that, she thinks, "are always skipped". The questions of the teacher: "In which sense?" and "What does the 3 represent here?"

cause Sophia to slide alongside the buildings and name the number that is skipped between each, thereby troubling her initial claim of “always”. She places her hand on the paper between each building, actualizing the pauses as rich with something that is not there, *but could have been there*:

Sophia: (*The teacher saying: “3 cubes. Then?”*) Then, you skipped the 4 (*touching with her right hand the corresponding free space on the toilet paper [Fig. 7a]*) and you put, you skipped one number and you put 5 cubes (*shifting along* and *silently counting the cubes of the new building [Fig. 7b]*). You skipped one number again (*indicating the free space [Fig. 7c]*), and you put 7 cubes. (*The teacher pulls the toilet paper a little towards herself, while Sophia is shifting towards the next building and, looking at it, counting the number of cubes of the fourth building, as she expresses through the movement of the mouth and of the head [Fig. 7d]*)… and you skipped again one number *(indicating the free space on the toilet paper with her left hand [Fig. 7e])* and then you put 9 cubes. (*looking at the teacher and gesturing in the air to indicate the new cubes [Fig. 7f]*).

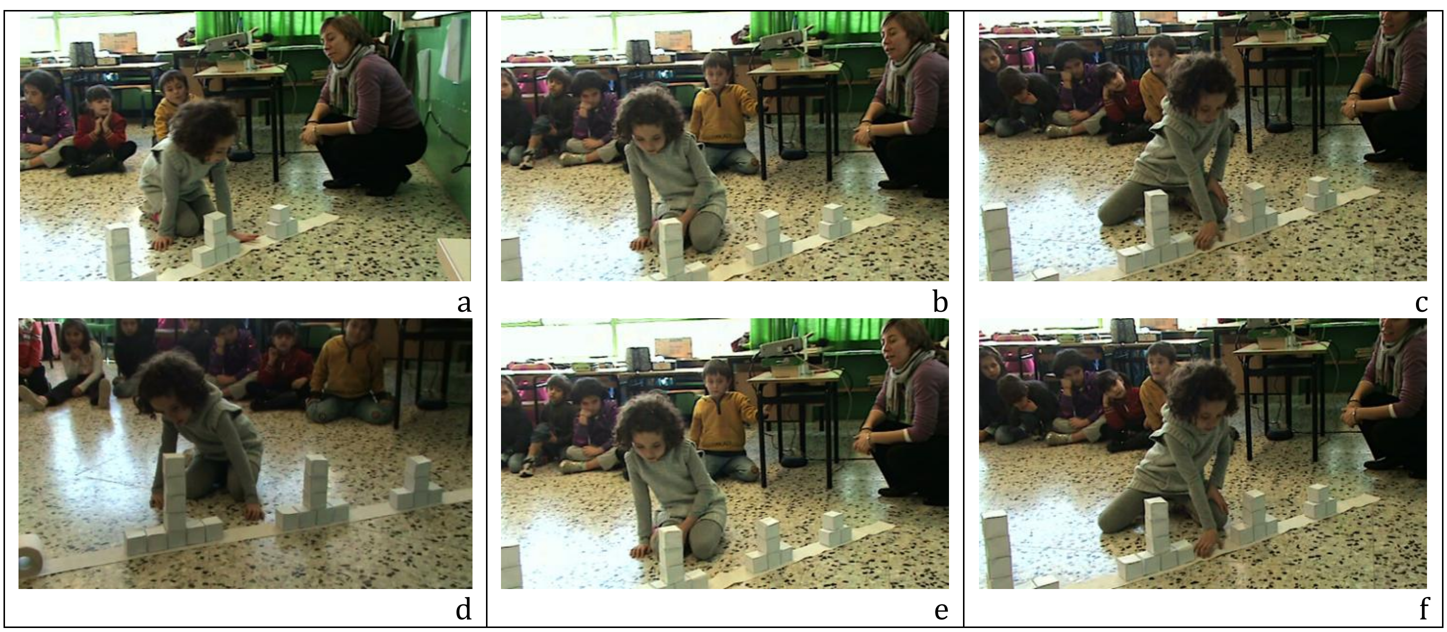


Figure 7. Sophia counting between the absent numbers and the present cubes

As Sophia slides along, she counts the blocks in each building as is evident from the tentative almost imperceptible movements of her mouth, head and eyes. Sophia assembles the building as one object. She attends to the sequence of the buildings, rather than the sequence of the rows and towers, as did Giorgia.

Following Bergson in thinking that everything is an image, we propose that the sequence of buildings is a moving image. Such movement is more or less apparent, depending on how the children engage in the event. This moving image appears in one sense to be similar to the false movement of film or video, as outlined in Bergson's second thesis on movement, insofar as there is a discrete set of buildings and the task is to perceive the continuity of the pattern 'beneath' them. But there is more to this moving image than simply the children's movement alongside the paper and across "the space covered". If we were to dwell only on that movement we would mistake distance covered for movement, as outlined in Bergson's first thesis on movement. Instead, we propose that these excerpts of data present evidence of the *movement of thought*. Rather than interpret the sensory-motor movements of the children as *representations* of this movement, we suggest that the sensory-motor is the *actualization* of this movement. The difference between these two assertions is hugely significant for how we think about learning. One of the implications of our assertion is that the *mathematical sequence* of odd numbers is both physically modelled by the buildings *and* actualized collectively as the event unfolds.

But how do the children attend to the virtual extensity of the continuum? Do they deploy Bergson's intuitive method? In our approach, everything is always already movement, although some things appear to be more still than others. The buildings for instance, are contracted movement, apparently still, while the gestures have more freedom of movement than the buildings, and thought has the *most* freedom of movement. We propose that all these movements are contiguous or in contact, and that it is simply our own current perceptual limitations that stop us from grasping the underlying indeterminacy and mobility of matter. As Bergson argued, we tend to perceive individuated objects, but this tendency can be overcome through the intuitive method. The intuitive method overcomes the dualism between mind and matter that haunts so many attempts to address the role of the body in learning. The movement of thought is precisely that which partakes of the quivering potentiality of the concept of number, while also animating the quantitative extension of the paper and the discrete and countable rhythm of the buildings. It is through the qualitative indivisible movement of thought that the children link up with the virtual multiplicity of duration "within and in" the sequence of buildings. The children's collective movement of thought attends to a particular rhythm in this case, that being the rhythm of the sequence of odd numbers, which is only one of the diverse potential sequences that might have been actualized in this material encounter. This contiguity of perception and thought (or perception and duration) is what fuels the fluidity and unscripted nature of Giorgia's and Sophia's gestures and physical actions. This relates to Bergson's third thesis on movement insofar as there is a different time or duration that "operates 'alongisde' the time" of the bodily movements of the two girls.

Sophia attends to the gaps between buildings, extending the claims that Giorgia made. In this action of tapping the paper, Sophia perceives other *potential* buildings in the pauses or breaks between the visible buildings. She thinks of the paper as rich with buildings that are not there, counting their absent number of cubes. In this sense, Sophia attends to the elasticity of the physical model, investing the paper with a potentially infinite plasticity. We see in her engagement with the buildings an example of how the students tap into the *virtual extensity* of the extended line. For Bergson, extensity is the fundamental ontology of perception, and yet every finite discretely partitioned model will fail to capture it. Sophia's actions are perhaps an example of what Bergson called the intuitive method, whereby one resists the usual habits of thought, and tries to glimpse the flowing continuity of extensity.

How is matter contracted and memory expanded through the children's intervention? What is the nature of collective memory? Through this intuitive method, the students enter the temporal dimension, expanding and contracting that which came before them. But rather than see this through a developmental model whereby students build towards a unified concept, abstracted from their encounters with the model, we suggest that this method actually affirms the heterogeneity of duration. In other words, these two students are folded into a multifarious duration—forming a complex relationship whereby each contracts and expands the thought of the other. As we discussed in the section on duration, one might imagine the collective movement of the event as a flowing undulating line that reaches into the past and future, occasionally zig-zaging across a curved 'cross-section' of the cone diagram. But the challenge is to consider duration as simultaneous and not extended in a spatial metaphor. Thus the difference between the two girls' contributions is not measured in relation to a pure unsullied *concept* to which they collectively aspire (for this would be differences in degree to which they perform the mathematical concept in question). But instead we want to grasp how their contributions partake in *degrees of difference*, that is, the 'extent' to which they partake of the undulating difference (which we use here to designate mobility/virtuality) that is at the heart of extensity. The term *difference* is helpful here as it points to a fundamental lack of identity in the concept of number, and thus helps us attend to the mobility or potentiality of the concept.

As the event unfolds, the children are immersed in a multiplicity of incommensurable durations. Although we tend to see only the linear unfolding of one sequence of actions, duration is robust and highly differentiated. Sophia, for example, expands Giorgia's actions, adding a new dimension to the discussion, and actualizing a specific "line of differentiation" within the assemblage of building-number-gesture. Thus there is a sense that these actualizations refine or hone the concept under discussion. The challenge is to keep one's eye, so to speak, on the virtual, and not simply attend to the contracted matter of the concept as it is collectively articulated. As Deleuze (1966/1988) points out in *Bergsonism*, "When the virtuality is actualized, is differentiated, is "developed", when it actualizes and develops its parts, it does so according to lines that are divergent, but each of which corresponds to a particular degree in the virtual totality." (p. 100).

We now turn to the next children's explanations, to further develop our interpretation. These students are Agnese, Lara and Riccardo. The next student Agnese diverges from Sophia's assembling of row and tower into a building, and attends to the sequence that links a row in one building to the tower of the next building, saying:

Agnese: Then three, three (*pointing to the row of the second building with two open fingers placed horizontally and pointing to the tower of the third building with the two open fingers placed vertically*). Four, four (*indicating the two lines of four cubes [Fig. 8a]*). And five (*pointing to the five cubes at bottom [Fig. 8b]*), above (*imagining the new building with the five vertical cubes on top [Fig. 8c]*), it should be five (*gazing at the teacher*).

a b c

Figure 8. Agnese relating each horizontal to the next vertical

Agnese expands the previous events, attending to the new sequence that links one horizontal to the next vertical. In so doing, she builds a different pattern, perceiving another *potential* relationship between one building and the next. A new qualitative degree of difference becomes manifest. Moreover, she deploys new kinds of gestures for talking about horizontals and verticals, and thus shows how the sensory-motor movement is part of the ongoing alteration that characterizes the movement of thought and concept. In the flow of the heterogeneous duration, the succession of the two types of gesture soon reveals the novelty of the temporal fold of Agnese's material relation with the buildings, as well as the contiguity of her perception and thought.

The actualization of the new pattern also attends to the first *potential* building that could come after the visible buildings. In her action of continuing the sequence, Agnese perceives the potential relationship of the 'last' visible horizontal with the next absent vertical, which "should be five". Other children tap into her reference to the new building, and extend her claim to consider new qualities of the sequence. Lara perceives the potential new building as a whole:

Lara: We had to put here (*scrolling the roll on the right and indicating the free space on the toilet paper at the end*) six (*miming the row*) and five (*miming the tower*; *Riccardo speaks together with her: "Even if we didn't put 1, it makes 6 and 5", miming the two rows*).

The teacher interjects, asking her then *what we need to do to make the next building*, and she replies:

Lara: Above there have to be five (*miming the vertical*) and below (*miming the horizontal*) six (*shifting the roll back to its original position*).

In the flow of duration, another student Riccardo enters this temporal fold:

Riccardo: Here (*pointing to the first building and positioning himself on one side of it*) there is one cube and two cubes (*pointing with both hand on the floor close to the first building, while looking at the teacher*), first there is the smallest and then the biggest (*shifting towards the second building*). Here two cubes (*indicating the vertical with the left hand open in a vertical interval*) and then three (*pointing to the horizontal with the right hand open in a horizontal interval, while the left hand keeps the previous position*). Here three (*sliding with his body and with the two hands kept open in the previous positions as to maintain reference to the vertical and to the horizontal, being the left hand closer*), then four (*the right hand close to the horizontal*). Here (*shifting to the next building and indicating the vertical*) four and then five (*looking at the building, changing his bodily posture and indicating the horizontal*).

Riccardo perceives a *potential* rule of "the smallest and then the biggest" in each building, meaning two consecutive numbers. He extends Lara's gesture of the pattern to the number pattern, expressing a new degree of difference. In his new action of using the two hands together for talking about the smallest and the biggest, Riccardo attends to the *number-buildings* as wholes and perceives the rhythm of the sequence. His sensory-motor movement is part of this new temporal fold in which one can grasp how his perception and thought are contiguous.

The children first think of the paper as rich with new buildings that are not there but could continue the sequence. Then, Ricardo moves to the number pattern, freeing a new quality of odd numbers from the fixity of the physical model. In this sense, Ricardo expands the virtual extensity that the other children tapped into, investing the paper with a potentially infinite continuity. We see in their engagement with the buildings and the paper an example of how they tap into the intuitive method, entering the flow of duration, expanding and contracting the previous events and adding further qualities to the movement of thought and concept.

## 6. Conclusions

Bergson bemoans how we have "shut up motion in space" (Bergson, 1896/1988, p. 218). Our tendency is to treat space as the background, and imagine that space is the "ground on which real motion is posited" (p. 217). This reflects a human tendency to attend to fixity, and to believe that rest is anterior to motion. We tend to focus on the states of rest, and then demote motion as a secondary linkage between states or positions. In contrast, Bergson argues that it is movement that is primary and that it "deposits space beneath itself." (p. 217). In this approach, everything is mobile, and everything is always already moving, even if certain things are freer to move than others. The *most* freedom of movement belongs to *thought* and to the *concept*, the least to the cubes that sit on the classroom floor. Bergson argues that our apparatus for measure or our perceptual organs may be calibrated to sense one rhythm but not another, and we wrongly assume that movement can only be sensed through one calibration. Thus we fail to see how movement is quality itself—"beating time for its own existence through an often incalculable number of moments" (Bergson, 1896/1988, p. 202). The intuitive method is then a means by which we can recognize and alter our perceptual habits and grasp this fundamental movement.

The case study analyses in this paper show how Bergson's ideas are relevant to mathematics learning. First, Bergson helps us look differently at the nature of mathematical behaviour, directing our attention to how students tap into the 'intuitive method'. This method allows them to engage with the fluidity or plasticity of mathematical concepts and objects alike. We believe this method functions as an important component of learning. The children do not collectively aspire to absolute timeless concepts. Instead, they are inserted in the *mobile collective memory* that moves forward, partaking in the fluid alteration of the concepts as much as they partake in the world of sensory-motor activity. Indeed, the mathematical concepts (both the function and the sequence) are physically modelled and actualized collectively as indeterminate and unfinished folds of extensity, opening onto an unscripted future. The case studies show us how the concepts are themselves *mobile* and full of *potentiality*, open to deformation and the remapping of the (im)possible. This argument aligns with the "inclusive materialism" of de Freitas and Sinclair (2014), who argue for the materiality and potentiality of the concept, suggesting that concepts must be sustained as "operative, mobile and creative" in classroom activity (p.218).

Second, the case studies in this paper also show how Bergson's approach can be used to study the role of the *body* in learning. The human body is privileged in our approach, but only insofar as it is a centre of contracting memory and movement. Thus the human body is continuously mobilizing, through sensory-motor and semiotic engagement, new lines of differentiation in the flow of duration. In our analyses, we follow the children's bodies, but not as wilful agents acquiring concepts and communicating ideas. Instead, we analyse how these bodies are individuated in the flow of duration *only provisionally*, as they tap or plug into the vast mobility of matter and memory that saturates the classroom. Thus the human body is always partaking of a relational assemblage that tentatively contracts into the body under study. Moreover, the usual mind/body dualism that we find rampant in mathematics education research is sidestepped by our attention to the *contiguity* between thought and perception, and the *coupling* of memory with matter.

In summary, we offer an alternative to theories of embodiment that interpret the sensory-motor as a representation of cognition. Rather than a representation of thought, sensory-motor gestures and other actions are *actualizations of the virtual movement of thought.* Each action brings forth the new through creatively *reconfiguring* the chaotic movement of thought. Our hope is that this approach better equips researchers to follow the collective ontology of becoming in classrooms. In other words, Bergson helps us think about the collective in new ways. This collective is post-human and includes not just students or student actions, but other material agents that operate at different scales and different rhythms. In the contributions of the children we can see how matter and memory are simply different dimensions of duration, that is, different degrees of contraction and expansion of the flow of duration. Learning becomes the *collective process of actualizing the virtual movement of thought*—where thought is not possessed within one body or another except insofar it saturates the entire learning assemblage.

## References


Bautista, A. Roth, W.-M. & Thom, J.-S. (2012). Knowing, Insight Learning and the Integrity of Kinetic Movement. *Interchange, 42*(4), 363-388.

Bergson, H. (1889/1910)*. Essai sur les données immédiates de la conscience.* (Trans. F.L. Pogson: *Time and Free Will*). Whitefish, MT: Kessinger Publishing Company, 1910.

Bergson, H. (1896/1988). *Matière et mémoire*. (Trans. N.M. Paul & W.S. Palmer: *Matter and Memory*). New York, NY: Zone Books, 1988.

Bergson, H. (1903/1999)*. Introduction à la métaphysique.* (Trans. T.E. Hulme: *An Introduction to Metaphysics*). Indianapolis, IN: Hackett Publishing Company, 1999.

Bergson, H. (1907/2005). *L'Evolution créatrice*. (Trans. A. Mitchell: *Creative Evolution*). New York, NY: Cosimo Books, 2005.

Châtelet, G. (1993/2000). *Les enjeux du mobile*. (Trans. R. Shore & M. Zagha: *Figuring space: Philosophy, mathematics and physics*). Dordrecht, The Netherlands: Kluwer Academy Press, 2000.

de Freitas, E. & Sinclair, N. (2012). Diagram, gesture, agency: Theorizing embodiment in the mathematics classroom. *Educational Studies in Mathematics, 80*(1-2), 133–152.

de Freitas, E. & Sinclair, N. (2013). New materialist ontologies in mathematics education: The body in/of mathematics. *Educational Studies in Mathematics, 83*(3), 453–470.

de Freitas, E. & Sinclair, N. (2014). *Mathematics and the Body. Material Entanglements in the Classroom*. Cambridge, UK: Cambridge University Press. (forthcoming)

Deleuze, G. (1983/1986). *L'Image-mouvement. Cinéma 1.* (Trans. H. Tomlinson & B. Habberjam: *Cinema 1, The movement image*). Minneapolis, MN: University of Minnesota Press.

Deleuze, G. (1966/1988). *Le Bergsonisme*. (Trans. H. Tomlinson & B. Habberjam: *Bergsonism*). New York, NY: Zone Books, 1988.

Deleuze, G. (1985/1989). *L'Image-temps. Cinéma 2.* (Trans. H. Tomlinson & R. Galeta: *Cinema 2, The time image*). Minneapolis, MN: University of Minnesota Press.

Doane, M.A. (2002). *The emergence of cinematic time: Modernity, contingency, the archive*. Cambridge, MA: Harvard University Press.

Duffy, S. (2013). *Deleuze and the history of mathematics: In defense of the new*. New York, NY: Bloomsbury Publishing.

Ferrara, F. & Savioli, K. (2009). Why could not a vertical line appear? Imagining to stop time. In M. Tzekaki, M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of the 33th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, 33–40). Aristotle University of Thessaloniki & University of Macedonia: Thessaloniki, Greece.

Grosz, E. (2004). *The nick of time: Politics, evolution, and the untimely*. Sidney, Australia: Allen & Unwin.

Malabou, C. (2008). *What should we do with our brain?* New York, NY: Fordham University Press.

Nemirovsky, R., Rasmussen, C. Sweeney, G. & Wawro, M. (2012). When the Classroom Floor Becomes the Complex Plane: Addition and Multiplication as Ways of Bodily Navigation. *Journal of the Learning Sciences, 21*(2), 287-323.

Protevi, J. (2009). *Political affect: Connecting the social and the somatic*. Minneapolis, MN: University of Minnesota Press.

Radford, L. (2013). Sensuous Cognition. In: D. Martinovic, V. Freiman & Z. Karadag (Eds.), *Visual Mathematics and Cyberlearning, Mathematics Education in the Digital Era* (Vol. 1, 141–162). New York, NY: Springer.

Sheets-Johnstone, M. (2012). Movement and mirror neurons: A challenging and choice conversation. *Phenomenology and The Cognitive Sciences, 11*(3), 385–401.

Sinclair, N., de Freitas, E., & Ferrara, F. (2013). Virtual encounters: The murky and furtive world of mathematical inventiveness. *ZDM – The International Journal of Mathematics Education, 45*(4), 239–252.

Stevens, R. (2012). The missing bodies of mathematical thinking and learning have been found. *The Journal of the Learning Sciences, 21*(2), 337–346 .

Varela, F. J., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. Cambridge, MA: MIT Press.