Supplemental Material

For a quasi-steady jet flame, the total air entrainment upstream of lift-off length relative the fuel injected is estimated as [13],

$$\bar{\phi} = \frac{2 \cdot (A/F)_{st}}{\sqrt{1 + 16 \cdot (H/x^{+})^{2}} - 1}$$
(S1)

The cross-sectional average equivalence ratio $\overline{\phi}$ at the position of lift-off length (*H*) is expressed in Equation (1). x^+ is the characteristic length of fuel jet. It is defined as,

$$x^{+} = \frac{\sqrt{C_a} \cdot d \cdot \sqrt{\rho_f / \rho_a}}{a \cdot t \operatorname{an} \theta/2}$$
(S2)

where C_a is orifice contraction coefficient, d is injector orifice diameter, a is a constant with the value of 0.75 [11], In this work, the area-contraction coefficient C_a is assumed to be 0.95 [22, 28]. θ is obtained from non-evaporating spray measurement [28].

The oxygen ratio was introduced by Mueller et al. [29] to account for the difference in mixture stoichiometry of oxygenated fuels. The oxygen ratio is defined as the amount of oxygen available in the fuel-air mixtures divided by the amount needed for stoichiometric combustion. The oxygen can be obtained by,

$$\Omega = \Omega_f + \frac{1 - \Omega_f}{\phi} \tag{S3}$$

where Ω_f is fuel oxygen ratio. It is defined as the amount of oxygen in the fuel divided by the amount of oxygen required for stoichiometric combustion.