# Evaluating Real Mortgage Mitigation Options of US Subprime Mortgage Homeowners

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## Abstract

*We provide a comprehensive model using a bivariate stochastic DTI (Debt to Income) and LTV (Loan to Value) formulation to assess the likely contribution of the current US government programs(HAMP) to alleviate homeownership financial distress. In a simulation of five available mortgage distress termination options plus continuation and repayment, the HAMP program appears to foster mortgage mitigation, given our parameter value assumptions. HAMP reduces foreclosures, but also incentivises householders to voluntary sell their property at a higher LTV than without a HAMP program. It might also reduce a homeowner's propensity to strategically default. However, conclusive policy appraisal awaits satisfactory empirical work regarding DTI and LTV drifts and volatilities.*

## Introduction

Due to exogenous shocks, a significant number of US homeowners are unable to make the required monthly mortgage payment (Holden et al. 2012, Quercia, Pennington-Cross and Tian 2012). Federal mitigation programs such as the Home Affordable Mortgage Program (HAMP) are designed to mitigate this inability of homeowners to make the required payment rather than their unwillingness to make the payment. How does the homeowner's inability to pay affect the probability of the "ruthless" exercise of strategic default or other options?

To gain insight and develop initial answers to the research objective, we base our analysis on exogenous program criteria for entry to the HAMP program (SIGTARP 2010). The program aims to make mortgage payments of homeowners with a high debt servicing burden more affordable by encouraging renegotiation between lender and homeowner, mitigating the effects of other (undesirable) options such as foreclosure or default.

We simulate the relative importance (defined by the likelihood to be exercised) of the principal alternative options based on program acceptance criteria to HAMP (SIGTARP 2010, Holden et al. 2012). The main benefit of entry to the HAMP program for the homeowner is that they negotiate and receive a one off reduction in their mortgage payment servicing for a period of 5 years resulting in a DTI close to a recommended value of 0.31 (Holden et al. 2012). In a methodological sense, we are simulating and examining a complex mixture of discrete (time) compound, barrier and reset type options.

We attempt to answer the question as to whether HAMP might mitigate US homeowner's motivation to "strategically" default. The question has not been addressed (see Holden et al.

2012 and the growing literature on the topic) in any published academic research but would appear a relevant question when discussing the efficacy of any mitigation program.

This paper develops a model around a problem formulation based on exogenous HAMP entry criteria, incorporating the stochastic factors (LTV and DTI), to provide an insight into the relative likelihood of different mitigation options being exercised. The DTI ratio is a measure that compares a homeowner's debt servicing payments to the income they receive. The LTV ratio expresses the amount of a first mortgage lien as a percentage of the total appraised value of real property.

We compare the effect of income and home equity change of a prime homeowner and their eligibility to enter HAMP on their option exercise under three scenarios: (i) DTI and LTV are decreasing and would not satisfy criteria to enter HAMP; (ii) DTI and LTV are increasing and would not satisfy criteria to enter HAMP, (iii) DTI and LTV are increasing and would satisfy criteria to enter HAMP.

We comment on the expected effect of a HAMP type program on the more typical mitigation options such as a "forced foreclosure" or "strategic default" as well as additional real options introduced by the relaxation of the perpetual ability to pay constraint such as a "voluntary sale". We demonstrate that a possible effect of the HAMP program is that it makes it less likely that a typical US homeowner will default as a result of "unwillingness to pay" but rather willingly sell their home as a result of an inability to pay and the desire to recover some home equity. The next section discusses the model and methodology. Then we provide some provisional results based on assumed exogenous trigger levels and parameter values. Finally, the last section concludes and points to the empirical work required for policy evaluation.

## Model and Methodology

The purpose of the analysis is to consider and estimate the effect that entry to the HAMP program might have on the probability of how often common real mortgage default mitigation options available to US subprime homeowners might be exercised.

We formulate the problem by defining five real competing mortgage mitigation options (Appendix) to occur with a total probability of 1 within the term of the mortgage. We consider two default options, one instead-of-default option and finally two non-default options. We follow up by stating the path dependent (on LTV and DTI) assumptions as to when each option is exercised. Finally, we describe the mitigation effect of the HAMP program within the problem formulation, make assumptions as to how the path dependency of the five competing options are modified (or mitigated) and compare the relative probability of a mitigating option being exercised to a state of the world without a HAMP program.

### The Probability of a Terminal Mortgage Option Occurring

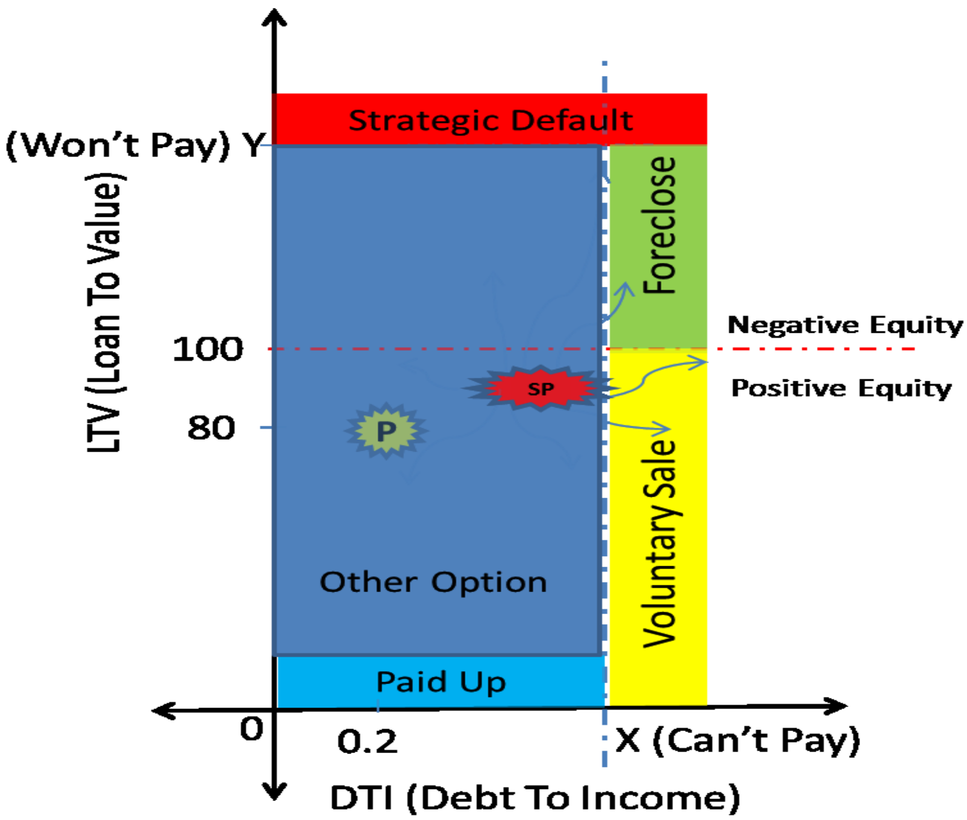
The probability a competing event terminating the term of a mortgage is 1:

**P (Strategic Default) + P (Forced Foreclosure) + P (Paid Up) + P (Voluntary Sale) = 1 - P (Other Option)** [1]

We present this concept in a diagrammatic manner in Figure 1 which shows that a US homeowner who experiences stochastic LTV and DTI over the term of their mortgage must hit one of the termination boundaries or remain within the boundaries (= Other Option).

To the right of some arbitrary DTI value X (Can't Pay) a homeowner will either, as a result, of an inability to pay, be foreclosed upon if they have negative equity or voluntary sell their home if they have positive equity. Above some arbitrary LTV value Y (Won't Pay), a homeowner will strategically default no matter their DTI. Otherwise the homeowner, who still has the ability to pay, will either have a mortgage that is paid up or still current (= other option). A typical US homeowner with a subprime mortgage starts with a DTI of 0.31 and LTV of 95% (= P, red star) in the diagram. Prime mortgages start further to the left (= P, green star) and will have a lower probability of other termination events occurring.

Figure 1.Schematic of Alternative US Mitigation Options without HAMP




Therefore if DTI is an independent random variable in the set $0 < \text{DTI} < \infty$ and LTV is another uncorrelated independent variable in the set $0 < \text{LTV} < \infty$, X and Y are arbitrarily chosen boundaries and a homeowner has either positive or negative equity (defined as ≤ or > 100%). Equity is loosely defined as the house resale value less the mortgage value.

We suggest that:

$$P(\text{Strategic Default}) \equiv P(LTV \geq Y \mid 0 < DTI < \infty) \quad +$$

$$P(\text{Voluntary Sale}) \equiv P(LTV \leq 100\% \mid DTI \geq X) \quad +$$

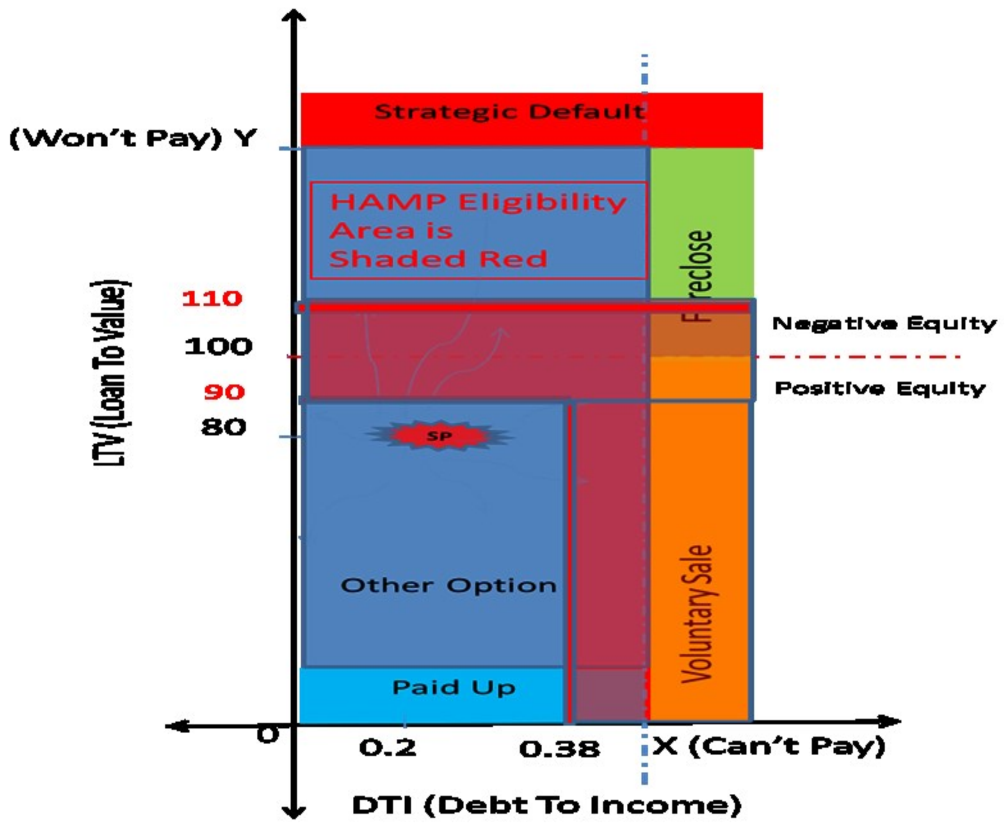
$$P(\text{Paid Up}) \equiv P(LTV < 1\% \mid 0 < DTI < \infty) \quad +$$

$$P(\text{Forced Foreclosure}) \equiv P(LTV > 100\% \mid DTI \geq X) \quad =$$

$$1 - P(\text{Other Option}) \equiv 1 - P(1\% < LTV < X \mid 0 < DTI < X) \qquad [2]$$

The effect of the HAMP program is represented pictorially in Figure 2 whereby US homeowners who have a DTI > 0.38 or within the range 90% < LTV ≤ 110% can negotiate with their lender, enter HAMP and have their DTI reduced to 0.31. HAMP is a temporary mitigating option and does not offer a permanent reduction in DTI as homeowners DTI and LTV will then change subsequent to entering the program. However, it is obvious that entering the HAMP will affect the relative probabilities of any of the terminal options expressed in [2] being exercised.

**Figure 2. Schematic of Alternative US Mitigation Options with HAMP**



An appropriate approach for this particular path dependent barrier type problem formulation is to simulate, using a Monte Carlo approach, the effect of increasing stochastic and DTI on the exercise likelihood of a US homeowner's mortgage options. Firstly, the ability of the homeowner to pay must be checked ($1^{st}$ DTI Trigger) and only subsequently the willingness of the homeowner to pay ($2^{nd}$ LTV Trigger) – a so called double trigger. If the first trigger shows that the homeowner has the ability to pay, then the available real options reduce to those of strategic default, paid up and other option. If, however, the homeowner is unable to pay, then two other real options are introduced, namely the lender forecloses or the homeowner voluntarily sells the property.

We introduce an additional feature into our simulation model whereby a mortgage event is not immediately triggered if one of the stochastic factors e.g. DTI (ability to pay) hits a threshold value in one period but only if it consistently exceeds the threshold for a number of periods. This is not a continuous but rather a discrete time simulation with discrete large monthly time periods. This feature simply reflects the observation that lenders and homeowners do not generally exercise an option immediately in the period following the first missed payment or reduction in home equity. At this juncture, we briefly review the path dependent barrier option literature in the context of the problem as formulated above.

Barrier options are one of the oldest types of exotic options trading since 1967 on the Chicago Board of Options Exchange (Zhang 1998). As a result, literature on exotic barrier options is abundant (Zhang 1998, Wilmott 2006). Snyder (1969) outlined the general approach with a single stochastic variable and a lower barrier which was later extended to multiple stochastic variables and barriers of different forms and durations (Heynen and Kat 1994, 1996).

However, examination of the literature has not yielded specific research or papers which lend themselves to a closed form solution of the formulated problem. On the other hand, Wilmott (2006) suggests that a Monte Carlo methodology is often the best approach with regard to analysing exotic path dependent options, as it is simple to code with a likelihood of fewer mistakes and whose only disadvantages are the difficulty of obtaining the Greeks and its slowness, both of which are minor issues in our formulation. This view is also supported by Vandell (1995) when discussing specific problem formulations associated with an extension of the standard option theoretic problem from a simple bivariate stochastic formulation.

To illustrate the methodological complexity and appropriateness of the chosen approach we make the following observations about our specific formulation. It is of a discrete Asian nature (monthly) and significant errors would be introduced by treating it as a continuous time option (Zhang 1998). Our formulation might appear at first sight to be a simple bivariate option with LTV and DTI as independent variables but this is not so, as DTI and LTV combine in a sequential or compound manner (double trigger action) to knock out the mortgage and trigger a terminal payoff option. Our problem is also less straight forward than many barrier options treated in the literature as in our specific case the homeowner's DTI is reset to 0.31 but the LTV is not reset on hitting the HAMP reset barrier of 0.38.

We make no assumption about the amount of the monthly payment beyond that a repayment formula or schedule is contractually agreed beforehand and that failure by the homeowner to make the agreed payment on time constitutes a trigger event changing their mortgage status. Let $Delay_d = Delay_l = 3$ be the number of (monthly) periods over which the homeowner discovers their "true" DTI and "true" LTV respectively before exercising a mortgage option.

Let $N_i$= a unique US residential owner occupied mortgage homeowner where $1 \leq i \leq N_p$ and $N_p$= number of homeowners in the simulation. We let $T = 30$ years be the term of the mortgage and $per = 360$ the number of payment periods implying monthly payments.

Implicit in this model formulation is that both the homeowner's DTI and LTV processes are significant, independent and non-correlated. Mortgage literature (Campbell and Dietrich 1983, Vandell and Thibodeau 1985) does not contradict this view as empirical research has demonstrated that these two factors are both highly significant but essentially independent.

One might argue that these two factors should be significantly correlated as both might depend in some fashion on interest rates. However, it should be clear from the model formulation that we only model ***short term*** monthly shocks and although interest rates may effect long term property and income trends, current evidence from mortgage literature would suggest that it is short term shocks which trigger default and mitigation, and these shocks are independent of spot interest rate changes.

**The DTI Factor**

This measure is of prime importance and significance within the US mortgage industry as its initial value at origination and consequent development gives lenders an idea of how likely it is that the homeowner will be able to repay the loan over its full term. We do not make any distinction within our model as to why DTI is changing as a result of variations in both income and/or debt servicing requirements and housing expenses. The change in housing expenses could be due to any number of reasons ranging from interest rate changes to property taxes or mortgage insurance. During normal times, most US lender underwriting standards tended to adopt a value of 0.28 as a maximum upper limit for DTI at mortgage initiation. We adopt 0.18 for DTI at mortgage origination, a value considered more typical for prime mortgages. Our exogenous triggers are motivated by the following considerations.

We assume (Holden et al. 2012, Quercia, Pennington-Cross and Tian 2012) that a homeowner will have significant ability to pay constraints if their DTI is above 0.5 for at least three consecutive payment periods and will most likely be foreclosed upon. The HAMP program applies a maximum limit of 0.38 to a homeowner's DTI that is reduced to 0.31 using a waterfall method (Holden et al. 2012) on successful entry to the program.

We assume that the DTI ratio of any homeowner $N_i$, denoted by *D*, follows the simple geometric Brownian motion process given by

$$dD = \mu_d D dt + \sigma_d D dW \qquad [3]$$

$\mu_d$ is the instantaneous expected rate of change of the DTI ratio

$\sigma_d$ is the instantaneous variance of the DTI ratio

$dW$ is a standard Brownian motion.

We make the simplifying assumption that all homeowners have the same $\mu_d$, $\sigma_d$ and $dW$.

**The LTV Factor**

We again take a simple approach and assume that the LTV is a result of many factors. With no change in property price, for the majority of homeowners, the LTV will gradually decrease from month to month as they pay down the loan principal. For others, due to any number of reasons varying from non-payment of principal to a reduction in property prices the LTV may be static or increase. The LTV is only appraised (precisely) once at mortgage initiation and again by the lender or servicing agent in the event of default. Otherwise, the LTV is calculated by the homeowner and lender based on the outstanding principal and arrears as well as a very rough estimation of the property price based on local (if known) property prices or indexes. In the US, conforming loans that meet Fannie Mae and Freddie Mac underwriting guidelines are limited to an LTV ratio that is less than or equal to 80% at origination, a value which we assume in the simulation model. Our exogenous triggers are motivated by the following considerations.

We define a higher LTV trigger of 150% in our model as the boundary where a homeowner will decide to "strategically" default because of the negative payoff received by subtracting the mortgage from the value of the property collateral. The 150 % LTV has been chosen somewhat arbitrarily but is consistent with empirical papers by Gerardi, Foote and Willen (2008) or Guiso, Sapienza and Zingales (2009). We also include a lower boundary at LTV < 1% where the mortgage is effectively paid down and terminated.

A relatively wide range of homeowners are eligible to participate in the HAMP program based on equity or LTV criteria ranging from those with a small percentage of positive equity (LTV > 90%) up to those with negative equity (LTV < 110%). We note that the HAMP programs LTV criteria were recently increased again in June 2012 to 120% (Holden et al.

2012). Currently the 120% limit is one that has been adopted within the HAMP program as a "typical" LTV break point (Holden et al 2012). We have adopted the lower 110% barrier, basing our analysis on the original HAMP criteria.

We assume that if the homeowner has the ability to pay (< 0.5 DTI for three consecutive periods), that where the homeowner's LTV is above 110% for the same three consecutive periods, they will enter and participate in the HAMP program as an alternative to strategically defaulting.

We assume that the LTV ratio of any homeowner $N_i$, denoted by $L$, follows the geometric Brownian motion process given by

$$dL = \mu_l L dt + \sigma_l L dZ \qquad [4]$$

Where

$\mu_l$ is the instantaneous expected rate of change of the LTV ratio

$\sigma_l$ is the instantaneous variance of the LTV ratio

$dZ$ is a standard Brownian motion.

We make the simplifying assumption that all homeowners have the same $\mu_l$, $\sigma_l$ and $dZ$.

**The Interaction of LTV and DTI – the Double Trigger Effect**

We summarise the alternative mitigation options available to the homeowner as well as the program logic of the exercise barriers in Table 1. We assume for ease of explanation that a homeowner is not initially in a HAMP program. At the end of each monthly period, the 1st DTI trigger is simulated and compared against the exogenous triggers and subsequently the 2nd LTV trigger.

Should any of the conditions described in Table 1 (No HAMP Program) persist for longer than 3 months then that particular terminal option is triggered. If the government then introduces a HAMP program and the homeowner meets the entry criteria as specified in Table 1 (With a HAMP Program) then the homeowner enters HAMP and their status changes from 0 to 1 and DTI is reset to 0.31. They then are subject to the same periodical simulation (as without a HAMP Program (Table 1, Whereupon) and do not qualify for a DTI reset again.

**Table 1. Table of Exogenous Trigger Values for the Option Simulation Program**

| HAMP Status | 1st DTI Trigger | 2nd LTV Trigger | Option Description |
|---|---|---|---|
| No HAMP Program | | | |
| | **IF** | **AND** | **THEN** |
| 0 | DTI < 0.5 | 1% < LTV < 150% | Other Option |
| 0 | DTI < 0.5 | LTV ≤ 1% | Paid Up |
| 0 | DTI < 0.5 | LTV ≥ 150% | Strategic Default |
| 0 | DTI ≥ 0.5 | LTV < 100% | Voluntary Sale |
| 0 | DTI ≥ 0.5 | LTV ≥ 100% | Foreclosure |
| With a HAMP Program | | | |
| | **IF** | **OR** | **THEN** |
| 0 -> 1 | DTI ≥ 0.38 | LTV ≥ 110% | Enter HAMP => DTI Resets to 0.31 |
| Whereupon | | | |
| | **IF** | **AND** | **THEN** |
| 1 | DTI < 0.5 | 1% < LTV < 150% | Other Option |
| 1 | DTI < 0.5 | LTV ≤ 1% | Paid Up |
| 1 | DTI < 0.5 | LTV ≥ 150% | Strategic Default |
| 1 | DTI ≥ 0.5 | LTV < 100% | Voluntary Sale |
| 1 | DTI ≥ 0.5 | LTV ≥ 100% | Foreclosure |

Note: An option is triggered only if DTI or LTV are at a trigger level for at least consecutive three monthly periods

We also consider the appropriateness of treating both DTI and LTV as standard log normal gBm processes. This assumption is relatively uncontroversial for LTV (Vandell 1995) but may be open to discussion in relation to DTI and the underlying income dynamics.

Income distribution and dynamics is difficult to simulate and the measurement and interpretation of US homeowner's income is even more fraught than the measurement of property values and subject to many caveats.

However, recent empirical research by Quercia, Pennington-Cross and Tian (2012) would suggest that DTI for low and moderate income US households is in the first instance symmetrically distributed with a bell shaped distribution.

An examination of literature on US homeowner income dynamics (e.g. Gottschalk and Moffitt 2009) does not provide evidence that other formulations such as mean reverting income is more widely used or provides any better results than the simpler standard geometric Brownian motion. Although a homeowner's income may revert over the longer term, this is of no consequence to the homeowner or lender who are faced with short to medium term payment difficulties. The HAMP program also has a relatively short duration of 5 years leading us to choose a standard geometric Brownian motion stochastic process.

## Simulated Results and Interpretation

Initially we assume that a US homeowner with a subprime mortgage at origination has a DTI = 0.31 and a LTV = 95%. We define a worst case (economic) scenario as where the homeowners' LTV and DTI are increasing at a rate of 2% per year, and a best case scenario as where LTV and DTI are reducing by 2% per year. We assume that no HAMP type program is available in the best-case scenario but is introduced in a worst-case type scenario to homeowners who meet qualifying criteria.

We take the same global approach to analysing each individual terminal option.

1) We compare the percentage homeowners who exercise with a HAMP type program to a state of the world without a HAMP type program for different volatility parameters.

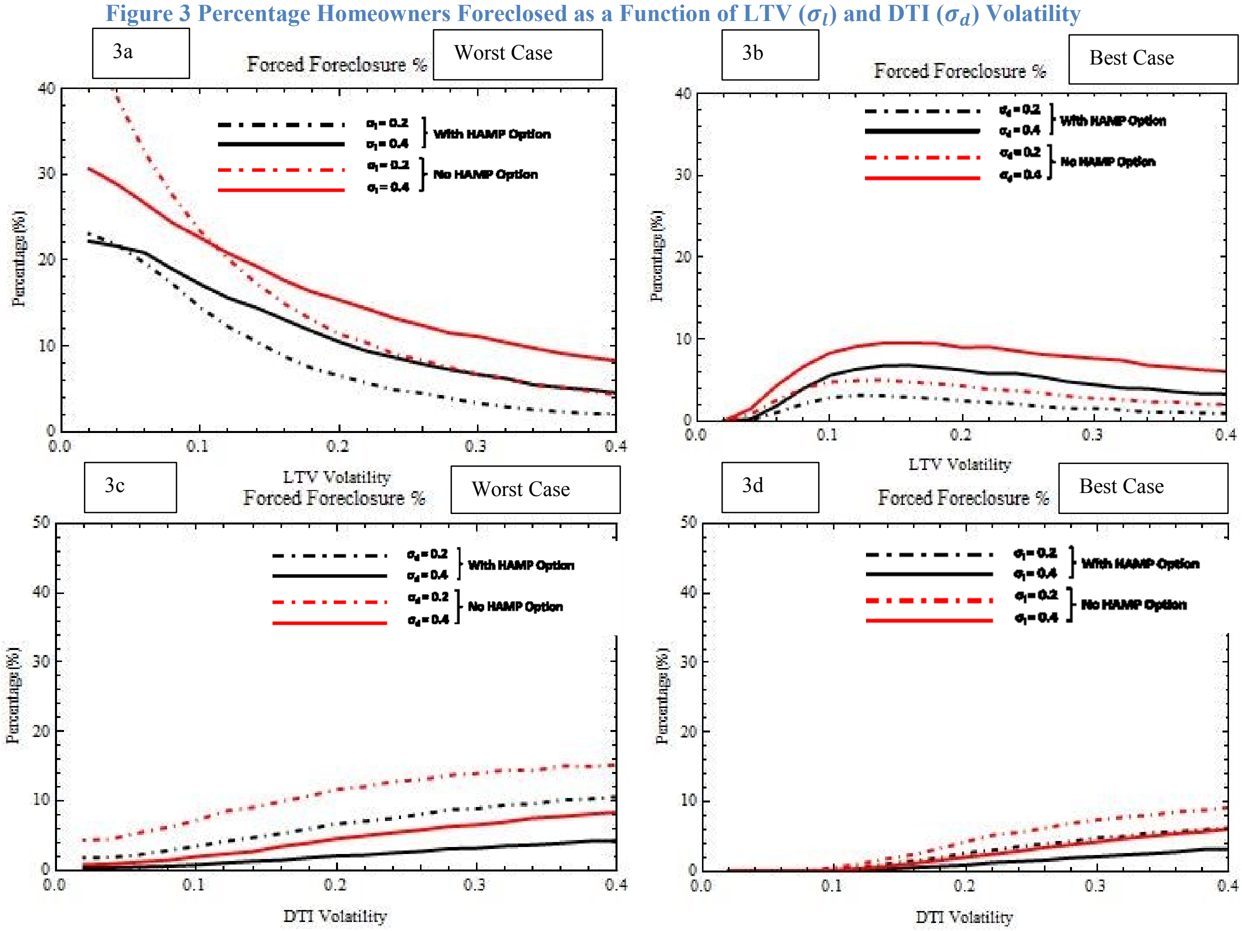
2) We estimate the terminal change in LTV from mortgage origination. This indicates (but does not quantify) whether entry to the HAMP program "preserves" or "destroys" LTV (a measure of equity) for the homeowner.

We demonstrate the outcome of the model parameters on the exercise probability of foreclosure option in detail, presenting and discussing both figures and tables, by way of example and will later dispense with the detailed explanations of the figures for the other options to aid clarity. The reader may obtain the wider range of graphical information on request.

**Foreclosure Option – Volatility Effects**

Foreclosure occurs when a homeowner's income (modelled by DTI) is not sufficient to make the monthly payment and they have not enough wealth or equity (modelled by LTV) in their property whereby a voluntary sale (or downsizing) would pay off the loan. The y axis of the four subplots in Figure 3 represents the percentage homeowners who exercise the foreclosure option. The x axis of the top two subplots (3a,b) respectively worse and best case represents the LTV ($\sigma_l$) volatility while the x axis on the bottom two subplots (3c,d) represent DTI ($\sigma_d$) volatility.

**Figure 3 Percentage Homeowners Foreclosed as a Function of LTV ($\sigma_l$) and DTI ($\sigma_d$) Volatility**




**Simulation Parameters**

$\sigma_l$ , $\mu_l$ , $\mu_l$ , $\sigma_d$ , $\mu_d$ , $\mu_d$ = Variable, Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%

Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000

We show four trend lines in each subplot. In the top two subplots (3a,b), representing the percentage of homeowners exercising foreclosure plotted against LTV ($\sigma_l$) volatility, we show trend lines for DTI ($\sigma_d$) = 20% (dashed trend line) and 40% (continuous trend line). In the bottom two subplots (3c,d) representing the percentage of homeowners exercising foreclosure plotted against DTI ($\sigma_d$) volatility, we show trend lines for LTV ($\sigma_l$) = 20%

(dashed trend line) and 40% (continuous trend line). Red lines represent results without any (NO) HAMP program – the status quo in good economic times. Black lines represent results when a HAMP type program is available.

Comparing the two left hand sub plots to the two right hand sub plots, foreclosures decrease as the homeowner's economic circumstances improve. Foreclosures increase linearly with increasing DTI volatility (3c,d subplots).

With increasing LTV volatility (3a,b subplots) foreclosures decrease at moderate to high levels of LTV volatility. Increasing LTV volatility therefore offers some homeowners the opportunity to benefit from increased positive equity and reduces the likelihood of foreclosure. Lower LTV volatility, on the other hand, reduces the likelihood that a homeowner with negative equity will ever have positive equity.

It should be noted that some other homeowners are more likely as a result to exercise the strategic default option at these higher LTV volatilities. In contrast to other mitigation options, little change in the percentage of homeowners exercising this option occurs at higher LTV volatility but all the "action" takes place at low to moderate levels of LTV volatility.

Table 2, column 2 summarises (for $\boldsymbol{\sigma_l},\boldsymbol{\sigma_d}$ = 20% and 40%) the percentage (of N=50,000) homeowners exercising a particular option during the best and worst cases depicted in Figure 3. The right hand sub table is the best-case scenario i.e. where LTV and DTI are decreasing by 2% a year and the left hand sub table is the worst-case scenario where LTV and DTI are increasing by 2% a year. Each individual sub table summarises the percentage of homeowners who exercise one of 6 options – numbered 1-6. The top half of each sub table shows the results for a HAMP program and the bottom half for NO HAMP program. Finally results are presented for different combinations of LTV and DTI volatility with $\boldsymbol{\sigma_l}$,$\boldsymbol{\sigma_d}$ = 20% and 40%.

**Table 2 The Effect of LTV ($\sigma_l$) and DTI ($\sigma_d$) Volatility on Homeowner Option Exercise Frequency**

| Worst Case Scenario $\mu_d = \mu_l = 0.02$ | | | | | | | | | Best Case Scenario $\mu_d = \mu_l = -0.02$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\sigma l$ | $\sigma d$ | 1 | 2 | 3 | 4 | 5 | 6 | | $\sigma l$ | $\sigma d$ | 1 | 2 | 3 | 4 | 5 | 6 |
| H | 0.2 | 0.2 | 46.1 | 6.6 | 37.0 | 3.8 | 0.0 | 23.8 | H | 0.2 | 0.2 | 38.0 | 2.6 | 22.1 | 19.5 | 0.1 | 26.9 |
| | | 0.4 | 47.4 | 10.6 | 30.5 | 5.1 | 0.0 | 26.0 | | | 0.4 | 47.1 | 6.3 | 17.1 | 16.4 | 0.1 | 26.7 |
| H | 0.4 | 0.2 | 44.2 | 1.9 | 41.6 | 3.7 | 3.9 | 18.6 | H | 0.4 | 0.2 | 31.2 | 0.9 | 36.2 | 8.4 | 14.2 | 16.7 |
| | | 0.4 | 44.7 | 4.5 | 36.2 | 5.1 | 4.3 | 21.1 | | | 0.4 | 41.1 | 3.2 | 30.5 | 7.2 | 11.4 | 19.5 |
| NH | 0.2 | 0.2 | 29.6 | 11.8 | 43.2 | 15.3 | 0.0 | 0.0 | NH | 0.2 | 0.2 | 19.7 | 4.2 | 25.4 | 50.6 | 0.1 | 0.0 |
| | | 0.4 | 29.4 | 15.2 | 37.8 | 17.6 | 0.0 | 0.0 | | | 0.4 | 28.9 | 9.1 | 20.7 | 41.2 | 0.1 | 0.0 |
| NH | 0.4 | 0.2 | 30.7 | 4.5 | 46.4 | 12.7 | 5.8 | 0.0 | NH | 0.4 | 0.2 | 16.7 | 1.9 | 39.6 | 23.0 | 18.8 | 0.0 |
| | | 0.4 | 30.7 | 8.4 | 40.8 | 14.1 | 6.0 | 0.0 | | | 0.4 | 25.8 | 6.2 | 34.3 | 18.5 | 15.2 | 0.0 |

Simulation Parameters

Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%, H =HAMP Program, NH = NO HAMP program, $\sigma_l$ and $\sigma_d$ = 20% or 40%

Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000

Column Option Descriptions : Percentage Homeowners who 1 = Voluntarily Sell, 2 = Forceclose, 3 = Strategically Default, 4 = Otrher Options, 5 = Paid Up, 6 = Enter HAMP

Note that the sum of the columns in the case of a HAMP program will be greater than 100% indicating that some of those who enter HAMP early in the term will eventually terminally exercise another option later such as e.g a Voluntary Sale.

The introduction of a HAMP program would lead to reduced foreclosures. From Table 2 (best case, RHS column 2) which summarises key parameters from the simulations for a LTV ($\boldsymbol{\sigma_l}$) and DTI ($\boldsymbol{\sigma_d}$) volatility of 20%, approximately 4.2% of homeowners will be foreclosed upon in a best-case scenario without a HAMP program (NH). When the homeowner's economic situation worsens (worse case, LHS column 2) this increases to 11.8% without a HAMP program (NH) but reduces to 6.6% with a HAMP program (H).

**Voluntary Sales Option – Volatility Effects**

Voluntary sales occur when the homeowner's DTI is not sufficient to make the periodic payment over a three-month period but enough positive equity (as determined by LTV) exists in the property to allow a voluntary sale. We analyse the voluntary sales option in a similar manner to that described for the foreclosure option. In the interest of brevity, we will not repeat the analytical procedure or present the same graphical figures but rather use the summary results in (option) column 1 of Table 2.

Overall homeowners exercise the voluntary sales option more often as the economy deteriorates. From Table 2 (best-case scenario, column 1) for a LTV ($\boldsymbol{\sigma_l}$) and DTI ($\boldsymbol{\sigma_d}$) volatility of 20% approximately 19.7% of homeowners will voluntarily sell in a best-case scenario when NO HAMP program is available. When the homeowner's economic circumstances deteriorate (worst-case scenario, column 1) this increases to 29.6% without the presence of a HAMP program but reaches 46.1% with a HAMP program. HAMP induces a large increase in voluntary sales by homeowners- a desirable outcome from the viewpoint of lenders and regulators, because the forbearance effect created by resetting DTI is such that the increase in some homeowner's LTV leads to more voluntary sales.

**Strategic Default Option – Volatility Effects**

The strategic default option is exercised regardless of the DTI if the negative equity of the homeowner as determined by the LTV reaches 150%.

As the economy deteriorates the number of homeowners exercising the strategic default option increases significantly. However, examining the values in Table 2, (worst case scenario, column 3) the introduction of the HAMP program might see strategic default increasing from 25.4% to 37.0% as compared to an increase from 25.4% to 43.2% with NO HAMP program - a 7% reduction.

A HAMP type program might induce roughly 15% fewer people to strategically default which might be seen as a desirable outcome by lenders and regulators.

**Paid Up Option – Volatility Effects**

This option is exercised if the LTV of the homeowner is below 1% over a three-month period. Effectively, the mortgage is repaid and it is extremely unlikely that any other option will be exercised.

As might be expected, when the economy deteriorates the number of homeowners exercising the paid up option decreases. The option is insensitive to whether or not a HAMP type program is available with the same number of homeowners exercising the option regardless. Thus, one could conclude that the availability of a HAMP type program should have little effect on the rate at which some homeowners pay off their mortgages.

**Other Option – Volatility Effects**

Homeowners who are still current and have never entered a HAMP type program (if available) within the three month period at the end of the computer simulation are deemed to have exercised the Other Option.

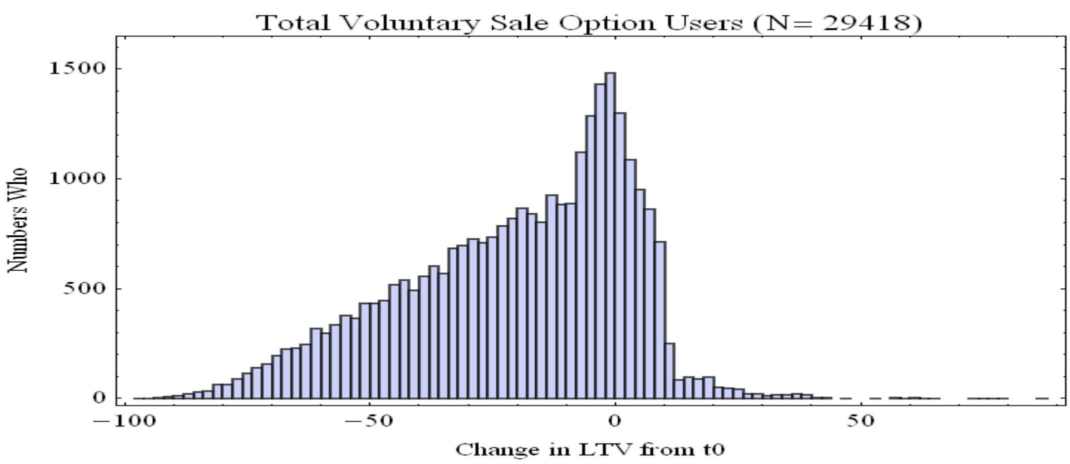
With deteriorating economic conditions, the percentage homeowners exercising the other option decreases from 50.6% to 15.3% with NO HAMP program (table worst case scenario, column 6 and (with a HAMP program) further reduces to 3.8% because of homeowners exercising other options and because around 27% of homeowners remain in the HAMP program at the end of the term (because of entering HAMP after 25 years).

**How Much is the LTV Change Affected by the Option Exercised?**

We now address our second analytical question to discover what effect the introduction of a HAMP program has on the average change in LTV from mortgage origination compared to a NO HAMP scenario. This might help increase our understanding of whether homeowners who enter HAMP experience beneficial changes in LTV from a scenario where NO HAMP program is available? Strictly speaking, rational homeowners should only be motivated by future expectations but it is not unreasonable to assume, that notwithstanding HAMP benefits and lender forbearance, that the average US homeowner may compare terminal LTV to LTV at origination. Similarly, even if rational lenders only consider NPV (Holden et al. 2012) they may also be concerned with their loan losses and the net change in LTV of their mortgage loans and customer's DTI from origination.

We assume that the difference in LTV between mortgage origination and the exercise of a terminal option such as strategic default, forced foreclosure or voluntary sale gives a measure of the change in equity to the homeowner (and therefore the lender risk). The difference, depending on the type of option, can be negative or positive as demonstrated in the histogram in Figure 4 for a simulation of 100,000 US homeowners in a worst case scenario with HAMP.

**Figure 4 Change in LTV from Mortgage Origination for those Exercising the Voluntary Sale Option**




**Simulation Parameters**

**Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%**

**Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000**

**Worse Case = $\mu_d = \mu_l = 0.02$, , H =HAMP Program, $\sigma_l = \sigma_d = 20\%$, $LTV_0 = 81\%$, $DTI_0 = 0.18$**

We construct the histogram by selecting those homeowners who exercise the option and divide these into LTV "buckets" across a (change in LTV) range from -100% to +20%. Finally, we numerically integrate the area to calculate the mean change in LTV from origination for those homeowners. We report the mean of the changes in LTV in Table 3 for the voluntary sale, forced foreclosure and strategic default options.

Table 3 Table of Homeowners Option Exercise Frequency and Average Change in LTV from $t_0$.

| | | | | % Homeowners Exercising | | | Change in LTV from $t_0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 1 | 2 | 3 | 1 | 2 | 3 |
| Best Case | NH | $\sigma_d$=0.2 | $\sigma_l$=0.2 | 19.7 | 4.2 | 25.4 | -36.9% | 24.0% | 69.7% |
| Worse Case | NH | $\sigma_d$=0.2 | $\sigma_l$=0.2 | 29.6 | 11.8 | 43.2 | -29.3% | 26.1% | 70.4% |
| Worse Case | H | $\sigma_d$=0.2 | $\sigma_l$=0.2 | 46.1 | 6.6 | 37.0 | -20.3% | 28.2% | 70.5% |

Simulation Parameters
Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%, H =HAMP Program, NH = NO HAMP program
Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000
Worse Case = $\mu_d = \mu_l$ = 0.02, Best Case = $\mu_d = \mu_l$ = -0.02, $\sigma_l = \sigma_d$ = 20% or 40%,$LTV_0$ = 95%, $DTI_0$ = 0.31

Column Option Descriptions : Percentage Homeowners who 1 = Voluntarily Sell, 2 = Forceclose, 3 = Strategically Default.

### Voluntarily Sale Option – LTV Effects

For $\sigma_l = \sigma_d = 20\%$, we calculate (table 3 column 1) a mean change in LTV of -36.9% in the best case No HAMP situation, -29.3% in the worst case NO HAMP situation and -20.3% in the worst case HAMP situation. Homeowners, exercising the voluntary sale option after entering a HAMP type program have a LTV that is on average 9% higher at option exercise compared to when NO HAMP is available.

Although the mean LTV has decreased for both the HAMP and NO HAMP cases in the worst case scenario, because homeowners "hang in" longer by entering the HAMP program and gaining the DTI reset benefit, they also "lose" more equity due to a higher LTV at termination.

**Forced Foreclosure Option – LTV Effects**

For $\sigma_l = \sigma_d = 20\%$, we calculate (table 3 column 2) a change in LTV of 24% in the best case No HAMP situation, 26.1% in the worst case NO HAMP situation and 28.2% in the worst case HAMP situation. Homeowners, being foreclosed upon after entering a HAMP type program, have a LTV that is on average 2.1% higher compared to NO HAMP which appears beneficial to homeowners but (perhaps) at the expense of lenders.

**Strategic Default Option – LTV Effects**

We can conclude (table 3, column 3) that the change in homeowners' LTV from mortgage origination is almost identical where a HAMP program exists than where NO HAMP program exists. We calculate a change in LTV of 69.7% in the best case NO HAMP situation, 70.4% in the worst case NO HAMP and 70.5% in the worst case HAMP situation.

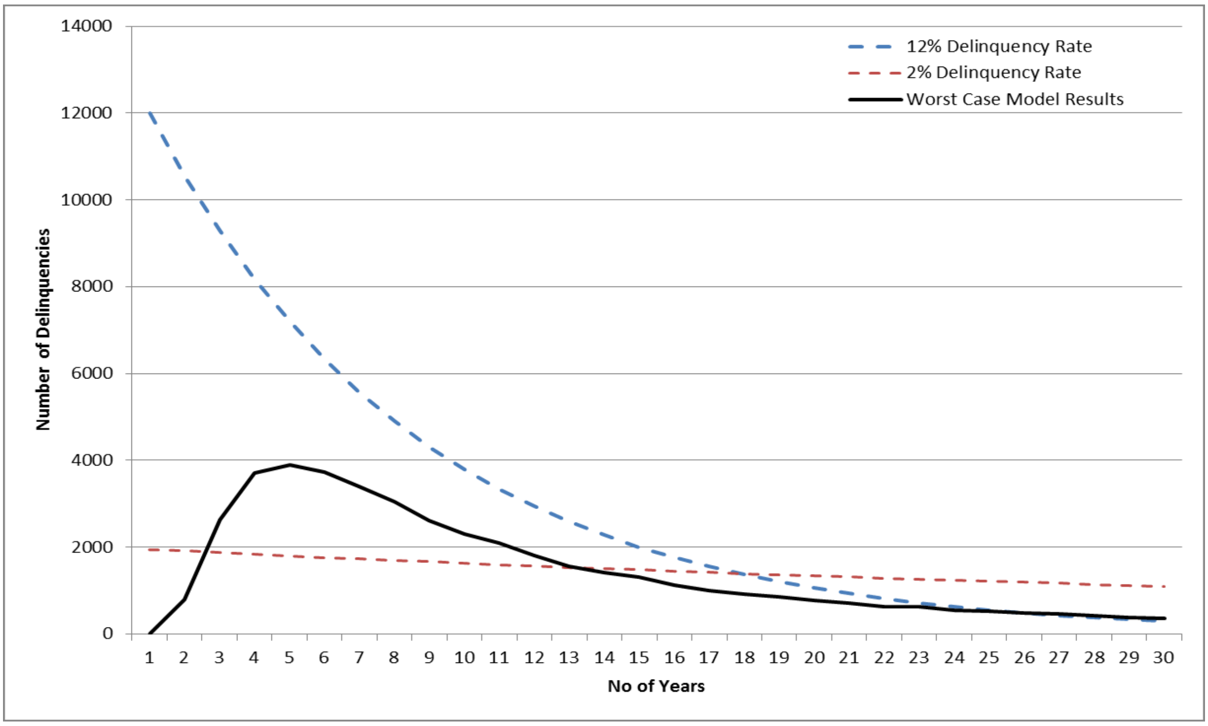
In other words, homeowners, defaulting even with a HAMP type program, have a change in equity that is the same compared to a NO HAMP type situation. Lenders are no worse off with respect to defaults in terms of the losses they might suffer on loans. However, they (might) still gain from the much greater number of homeowners who voluntarily sell. Homeowners who enter HAMP have of course benefitted from the DTI reset.

**Model Validation**

We stress that this paper is purely a theoretical contribution based on commonly accepted HAMP exogenous parameters and the model results have not been validated empirically. However, we believe that the dynamic behaviour and magnitude of the results appear intuitively correct and may bear some resemblance to "reality". Conclusions drawn might indicate at a macro level the possible effect of policy changes in HAMP trigger parameters and subsequent medium term effect on householder's option exercise probabilities.

To compare our results with “reality”, we present in Figure 6 an extrapolation of two delinquency rates at 2% and 12% on the mortgage survival of 100,000 homeowners over a 30 year period. A pre-2006 delinquency rate of 2% is reported by the OCC Integrated Banking System in Spring 2013, but 12% since 2009. We next compare the extrapolation against our general model results using a DTI = 0.18 and LTV = 85% (combining foreclosure and strategic default probabilities) using the worst case parameters. As already noted, our model result has a characteristic seasoning effect and furthermore the “average” seems to lie around the 2% pre-2006 trend. Therefore, our model underestimates the current high US delinquency rates. This underestimation arises because we overestimate the number of US homeowners who enter HAMP. Introducing modelling rules which reduce the number of homeowners participating in HAMP increases the number choosing other terminal options.

**Figure 5 Comparison of Two Delinquency Rates vs Model Predictions on 100,000 Homeowners**




**Simulation Parameters**

Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%, H =HAMP Program, NH = NO HAMP program, $\sigma_l$ and $\sigma_d$ = 20% or 40%

Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000

## Conclusion

We conclude by summarising the main findings at our reference volatilities of $\boldsymbol{\sigma_l}$,$\boldsymbol{\sigma_d}$ = 20%. We examine the simulated effect of HAMP on real mortgage mitigation options and by extension answer our question as to whether ignoring the ability to pay of homeowners is justified by its assumed "limited" effect on strategic default and foreclosure (Vandell 1995). We repeat that the outcomes of the HAMP program are very dependent on the initial LTV and DTI at origination which in this exposition are assumed to be subprime mortgage homeowners.

1) A HAMP type program leads to a significant increase in subprime homeowners exercising the voluntary sales option from 29.6% to 46.1% - a desirable outcome from the viewpoint of homeowners, lenders and regulators due to the (presumed) reduced associated deadweight costs. This occurs because the temporary mitigation effect created by lowering the DTI of homeowners is such that the positive development of some homeowner's LTV due to higher volatility induces more voluntary sales. This conclusion might, however, be difficult to verify empirically as it is probably not easy to divine homeowners' motivations for selling.

2) Forced foreclosures double during the lifetime of a HAMP program from 11.8% to 6.6% which would seem a desirable outcome to homeowners and regulators alike.

3) A HAMP type program induces roughly 15% fewer subprime homeowners to strategically default which might also be seen as a desirable outcome by lenders and regulators. In better economic circumstances, the most common terminal option chosen is the strategic default option (25.4%, Table 2). When economic conditions

worsen, although the percentage of homeowners that strategically default increases to 37.0% (with HAMP), this increase is much less than the increase in voluntary sales to 43.2% from 19.7%. The HAMP program is certainly not a "free rider" program.

A unique aspect of this simulation model is the ability to compare the homeowner's final equity worth as approximated by their LTV to that at mortgage origination. Although general LTV drift is positive (i.e. value is destroyed) during an economic crisis we simply pose the question whether the HAMP program is more likely to "create" or "destroy" more or less equity (LTV) value for homeowners or lenders compared to NO HAMP program. In this regard, we presume that strategic defaults are "good" for homeowners and "bad" for lenders while voluntary sales and foreclosures can be either "good" or "bad" depending on the terminal LTV with or without a HAMP program.

4) Subprime homeowners who exercise the voluntary sale option from within a HAMP program have a LTV that is on average 9% higher compared to a NO HAMP type situation. Except at very low LTV volatilities, homeowners are worse off in terms of their terminal LTV by waiting longer to sell their property. Homeowners benefit from the reduction in mortgage payments but at the expense of more negative equity when they eventually sell their home. The HAMP program is therefore not a "free ride" charter.

5) Subprime homeowners, being foreclosed upon after entering a HAMP type program, have a LTV that is on average 2.1% higher compared to a NO HAMP type situation. Lenders are consequently slightly worse off with respect to foreclosures in terms of the losses they might suffer on loans. However, this might well be compensated by

their gain or lower loss from the much greater number of homeowners who voluntarily sell.

6) With respect to the strategic default option, a subprime homeowners' average terminal LTV is almost identical after entry to the HAMP program than where NO HAMP program exists. Lenders are no worse off with respect to strategic defaults in terms of the losses they might suffer on loans. However, they still gain from the much greater number of homeowners who voluntarily sell. It is a moot point whether the homeowner will take the presence of a HAMP program into account when considering their strategic default decision. On the other hand, they will gladly make use of any DTI reduction. It remains difficult for regulators or lenders to design a mitigation program where strategic defaulters do not attempt to game to their own advantage.

These form the main conclusions from our study of how we might expect terminal options such as voluntary sales, foreclosure and strategic default to behave in the short to medium term given the presence of a HAMP type program.

The main effect using this model is that a HAMP program steers more US subprime homeowners towards a voluntary sale and slightly reduces the number of defaults and foreclosures. This outcome, depending on deadweight default costs, is most likely to be advantageous to US homeowners and lenders.

Unfortunately, the two main options in percentage terms, affected by the HAMP program, voluntary sales and strategic default, are notoriously difficult to measure empirically. It will remain difficult for opponents and supporters of the HAMP program to conclusively assess its benefit to US society. We have demonstrated that, due to its immediate benefit to distressed US homeowners - the significant increase in voluntary sales and reduction of strategic defaults with the consequent reduction in default deadweight costs, it is probably better to have a HAMP program in place rather than having NO HAMP program.

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## Appendix

**Forced Foreclosure**

Forced foreclosure occurs when a homeowner does not have sufficient income, as determined by DTI, to make the monthly payment to the lender. We assume that the homeowner has no savings or other means of paying down the loan. Furthermore, the LTV or homeowner's equity is negative such that a voluntary sale is of no interest to the homeowner or lender. The option is triggered by the lender if the homeowner is delinquent over a number of monthly periods.

**Voluntary Sale**

Voluntary sale occurs when a homeowner does not have sufficient income, as determined by DTI, to make the monthly payment to the lender. We assume that the homeowner has no savings or other means of paying down the loan. However, in contrast to the forced foreclosure option, enough positive equity exists in the property to make a voluntary sale attractive to the borrower as the least worse option.

**Strategic Default**

Strategic default occurs where a homeowner considers that the amount of negative equity in the property as determined by LTV is such that it makes more sense to permanently default on all future payments and "put" the property to the lender. The homeowner may have enough income (DTI) to make the mortgage payment but their other assets or wealth might be so low that it is of little benefit for the lender to pursue the borrower for the difference between the mortgage amount and the realised house value.

**Paid Up**

We assume that this option is exercised if the LTV of the property falls below 1% through either appreciation of the property value or reduction in the outstanding loan, as most homeowners whose property has a LTV of less than 1% will not consider exercising other options such as default or prepayment due to their expense and risk.

**Other Option**

Homeowners that have never exercised one of the four previous (terminal) options will therefore have an “Other Option” status. In other words, they continue to make due periodic payments to lenders when the model stops computing after the term period of 30 years. They are thus not necessarily “current” as it is possible that they are into (e.g. first month of) payment difficulties.