Designing with Gradients: Bio-Inspired Computation for Digital Fabrication

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Abstract

Digital fabrication technologies greatly enhance and extend manufacturing possibilities. However, we are still relatively limited in our ability to fully exploit these new methods and create complex architectural structures with *performance-driven* properties. We argue that entirely new computational approaches are needed, using scalable *generative* encodings and advanced bio-inspired *form finding* processes. This paper presents a novel generative model that can create functional and expressive geometries by evolving volumetric *gradient patterns*. Using three case studies, we demonstrate the key advantages of our approach. We demonstrate, using simulation followed by physical fabrication, that our approach is useful for exploring complex, yet buildable geometries in early stage design. Our new method is therefore suitable for performance-driven form finding tasks such as structural optimisation, and holds vast potential for designing exotic multi-material and functionally graded materials in future applications.

1 Introduction

Advances in fabrication technology make it possible to build increasingly complex designs. For example, additive manufacturing technologies (i.e. 3D printing) afford exciting opportunities to precisely control *how* and *where* material is distributed within geometrically complex objects (Oxman, Keating and Tsai 2011). However, the *computational* tools required to fully exploit these technologies and aid the discovery of geometrically diverse, yet high-performance structures, remain relatively underdeveloped (Shea and Gourtovaia 2005, Lipson and Kurman 2013).

To address this challenge, emerging architectural research takes inspiration from form generation in natural systems. The main idea is to integrate simulation, analysis and physical testing into the early stages of design, in order to create performance-driven forms that can be realised with digital fabrication methods (Menges 2012, Magna, et al. 2013, Søndergaard, Amir and Knauss 2013). Notably, this approach does not aim to reduce complex design opportunities to one-dimensional optimisation problems, but, in contrast, enables *new forms* of design instrumentality whereby designers work with materials, fabrication constraints and multiple performance considerations, early on in the design process, in order to *coax-out* desirable architectural characteristics in ways that were previously difficult to achieve.

We believe that this type of bio-inspired approach opens up radically new forms of architectural design and practice. However, we also believe that it is, in its current form, limited to creating relatively small scale and simple designs. We suggest that to exploit the vast formal possibilities offered by advanced fabrication technologies, a key challenge is to develop new ways of *digitally representing* physical designs in ways that move beyond fixed geometric descriptions and/or linear associative models. Specifically, we think that the ability to control and explore complex material structures will be greatly improved with better *generative* design techniques, and that this demands experimental work at the intersection of architecture and a specific area of computer science that is concerned with generative and developmental systems (Stanley and Miikkulainen 2003).

For clarity, we use the term "generative" to explicitly refer to any design process that uses an *indirect* (non-linear) way of mapping between parameters and the final solution. For example, Nature uses truly extraordinary generative mappings to build complex forms. Consider that the human genome contains around thirty thousand genes, yet the human body contains about thirty *trillion* individual cells! (Bianconi, et al. 2013) In simple terms, this means that Nature uses indirect ways of describing form that re-use information, and in doing so creates truly awe-inspiring physical designs that are vastly more complex than the encoded instructions used to build them. Critically, it is this ability to *re-use* information during construction and describe complex physical designs with highly compressed encodings that we believe can improve existing computational design methods. To justify this claim, consider using a traditional *associative modelling* approach (i.e. direct mapping between parameter and geometric transformation) to create a complex structure that comprises heterogeneous materials and exhibits specific performance-driven properties (such as Fig 1). This task is impossible using traditional methods, due to two major problems.

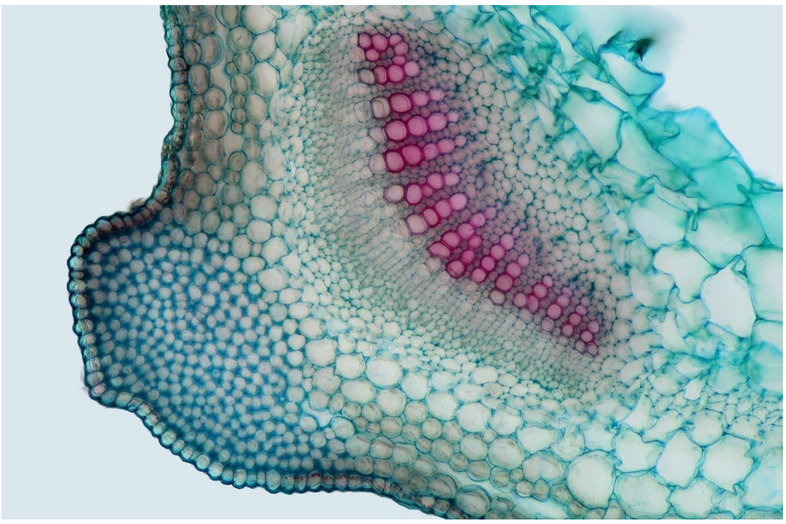

Figure 1. Cross-section through the stalk of a "dead nettle" plant (Lamium maculatum) showing the complex organisation of functional matter, which is difficult to model using traditional methods. Photo by Micropix via Wikimedia Commons / CC-BY-SA 3.0

Firstly, modelling this type of structure would be incredibly difficult because of the large number of parameters required to describe the geometry, topology and materiality of the design. Yet beyond the initial challenge of considering how and what to parameterise, and then laboriously modelling the structure, the resulting description would be extremely inflexible. Indeed, many of the *necessary* design decisions made to create a workable model will have constrained the geometry and topology of the design to a small number of possible permutations that may not enable suitable performance-oriented properties to be discovered (Hanna 2012). Additionally, making major retrospective changes to the model in order to accommodate new formal possibilities (following initial testing) will often be too difficult and require the designer to remodel the entire design (Davis 2013). Secondly, to create performance-oriented formal attributes, various simulation and optimisation processes are needed. However, designs that contain large numbers of parameters are *not* easy to optimise and quickly become completely intractable. Our central argument is that to improve computational design processes and control significantly more complex designs (such as Fig 1) we need new types of active computational models that embrace notions of *self-organisation* and material agency.

Previous work demonstrates the usefulness of generative design methods for creating complex geometries and performance-driven designs (Hornby 2004, Ayres 2012, Menges 2012). However, generative approaches can also present significant challenges when applied in practice. For example, multi-agent systems use impressively simple rule sets to create complex geometries, which exhibit emergent formal properties. However, the forms they create are generally not functional, and can even be completely unbuildable. Additionally, whilst the rule sets themselves are compact, and therefore potentially useful for scalable optimisation, the algorithmic methods needed to *steer* these types of designs towards useful high-performance solutions are still severely lacking (Ayres 2012). Consequently, most studies have been limited to creating provocative ornamental geometries (Snooks 2012, Ramirez-Figeroa and Dade-Robertson 2013).

This paper presents a generative design process which facilitates speculative early-stage exploration of complex forms (yet also ensures buildable structures), and can aid the discovery of high-performance designs. First, we contextualise our approach and review related work. Second, we describe our model. Third, we present three experimental case studies that highlight the various advantages of our approach using 3D printed prototypes and finally, we conclude with a discussion that summarises our findings, and highlights exciting possibilities for further research.

2 Background

Michalatos and Payne (2013) recently demonstrated an interesting way of digitally *representing* geometry that uses volumetric gradient patterns to control a voxel-based model. This process allows them to vary the material properties of 3D printed structures, throughout their volume, by defining the property of each voxel by a function of its Cartesian (x,y,z) coordinates. The significance is that the method provides a viable way of controlling multi-material physical objects with complex internal architectures. This idea of digitally representing each part of an objects geometry as a mathematical *function* of Cartesian coordinates is also referred to as "functional representation" (Pasko, Sourin and Savchenko 1995), has been widely used in computer graphics, and was recently applied to the design of 3D printed microstructures (Pasko, et al. 2011). We suggest that *functional representations* could offer vast potential for architectural design by providing a compact method of describing complex material structures, while maintaining the ability to explore expressive formal designs. However, a major challenge for this area is how to manipulate these types of volumetric gradient patterns so that the physical designs they encode can develop specific mechanical properties. We believe that recent work in the field of *evolutionary computation* may offer a solution.

Evolutionary algorithms simulate the process of Darwinian evolution and have been widely used in design and engineering to generate physical structures that have specific properties (Holland 1975, Frazer 1994, Kumar and Bentley 2003). However, recent work on *neural evolution* and *evolutionary robotics* has demonstrated that gradient-based patterns (similar to Michalatos and Payne's model (2013)) can be evolved by combining a generative representation called *CPPN* (Stanley 2007) with a state-of-the-art neuroevolution technique called *NEAT* (Stanley and Miikkulainen 2002). CPPN stands for *Compositional Pattern Producing Network,* and NEAT refers to an evolutionary algorithm called *Neuroevolution of Augmented Topologies*. CPPN-NEAT has been described in detail (Stanley and Miikkulainen 2002, Stanley, D'Ambrosio and Gauci 2007, K. O. Stanley 2007), so here we provide a brief non-technical overview, and focus on describing the new design opportunities that this approach affords.

Simple 2D gradient patterns can be easily generated by defining the colour of each pixel on a canvas as a function of its x and y coordinates. For example, a simple gradient pattern can be created by summing the x and y coordinates of each pixel and using the result to define that pixels RGB value, as shown in Figure 2A. More complex 2D patterns can be generated with a similar approach, but using slightly more complicated *pattern producing networks*, or CPPNs. These may use a variety of different mathematical functions, such as Sigmoid, Gaussian and Cosine, and are connected with weighted links (Fig 2B). Critically, Stanley and Miikkulainen's NEAT algorithm can *evolve* CPPNs using an evolutionary approach, whereby nodes and connections are added and manipulated over time (i.e. where networks are *augmented* by evolution). Combining CPPNs with NEAT, Clune et al (2011) use interactive evolution to create small 3D printed sculptures; Cherney et al (2013) evolve soft robot designs with multiple materials to achieve interesting locomotion; Hiller and Lipson (2009) use CPPNs to evolve 3D solutions that meet high-level goals, such as specific deflection of beam structures; and Auerbach and Bongard (2010) evolve virtual creatures with diverse behaviours.

We now outline our evolutionary model, which extends CPPN-NEAT, and allows us to explore complex and performance-driven material structures by manipulating volumetric gradient patterns.

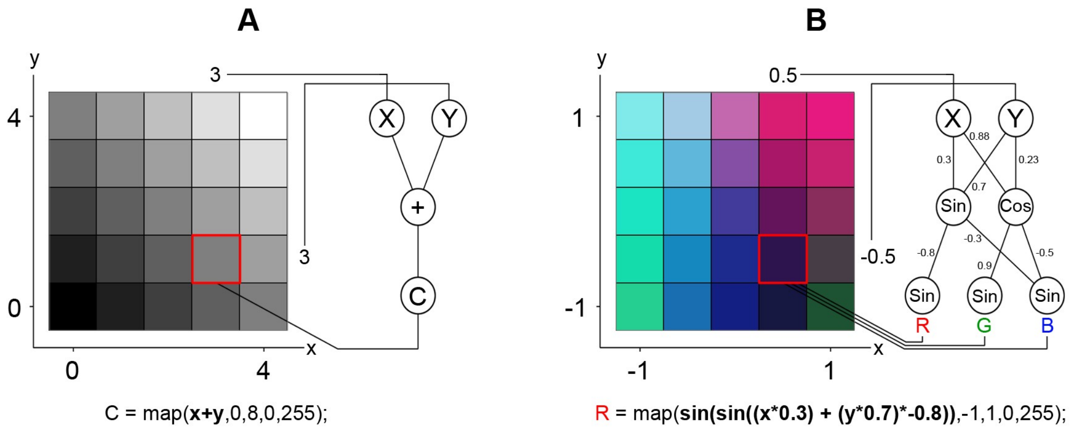

Figure 2. (A) Simple gradient pattern generated by summing the x and y coordinates of each pixel to generate a colour: C. (B) CPPN generated pattern. To generate the colour of the highlighted pixel, the coordinates: x = 0.5, y = -0.5, are fed into the CPPN and a red, green and blue (RGB) value is output. The equation below (B) shows the calculation of the red value.

3 Methods

CPPNs can generate diverse 2D and 3D patterns (Fig 3). These patterns have several exciting properties, which we believe to be useful for computational design; however, they also have one key problem that currently makes them difficult to apply to real-world design and engineering domains. This section will first outline the desirable properties of CPPNs and then describe how our extended method is able to exploit gradient patterns to generate useful (real-world) designs.

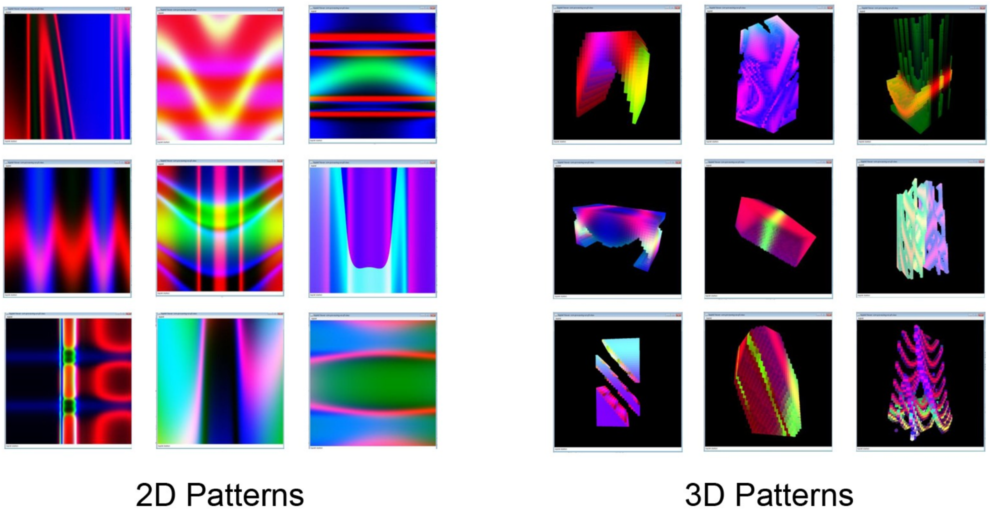

Figure 3. Diverse 2D (pixel) and 3D (voxel) patterns generated by random CPPNs. The 3D patterns are produced using a CPPN with four outputs: the three outputs shown in figure 2, for RGB values, and a fourth output which returns a *threshold* value. If the threshold value is less than zero, the voxel is not created. This process allows us to create different geometric shapes, as well as different colours.

As shown in Figure 3, CPPNs can create regular geometric patterns with repeating motifs, which is a consequence of the periodic mathematical functions that they use (such as *cosine*). This means that designs encoded with CPPNs can exhibit symmetries, and perhaps, more interestingly, imperfect symmetries that will likely be useful for complex designs. A second interesting property of CPPN encodings is that the designs they encode effectively obtain infinite resolution. That is, as shown in Figure 4, 2D patterns generated with CPPNs can never pixelate, because as the resolution is increased each pixel simply re-queries the CPPN to generate a new pixel colour. The implication of this is that structures described with CPPNs are extremely flexible and easy to manipulate during the early stages of design. Indeed, not only can the dimensions and resolution of designs be varied whilst retaining their evolved logics (in a similar manner to NURBS surfaces), but the connection weights of evolved CPPNs can actually be extracted and manipulated in real-time, providing similar interactivity as designs described by popular associative modelling platforms, such as McNeel's *Grasshopper*. Yet, unlike existing associative modelling platforms, CPPNs can be easily (and automatically) adapted with NEAT. Note this behaviour follows for 3D gradients, and highlights the most significant feature of CPPNs for optimising complex designs - *scalability*. Generative encodings such as CPPNs are useful for evolutionary design problems because they do not become more difficult to optimise as designs scale up (Hornby 2004). In simple terms, the *mathematical search space* (that contains all possible solutions) does not become larger and more difficult to search as designs increase in resolution and/or more elements are added. Critically, we believe that this ability to create smaller search spaces, which contain diverse solutions, will provide a significant advantage when aiming to design truly complex and performance-oriented structures.

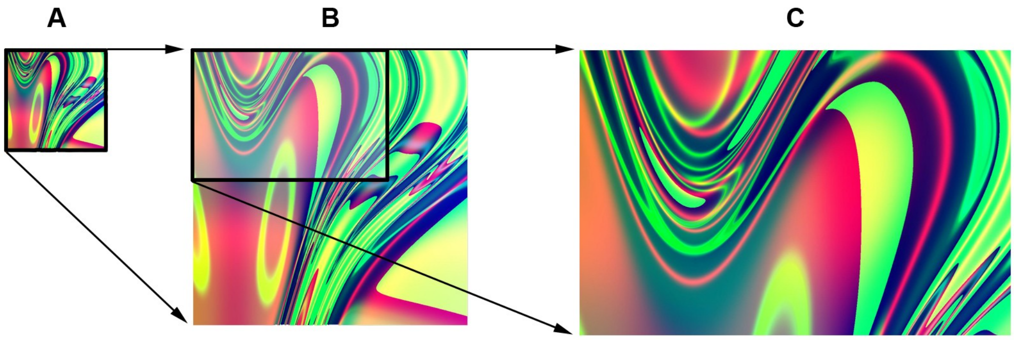

Figure 4. Infinite resolution of CPPN generated pictures. (A) Shows the original resolution of the image. (B) The resolution of the image is increased, and the pixels re-generate new colours based on the CPPN. (C) One corner of the image is isolated and increased in resolution. Notice that the image can never pixelate and has, in theory, infinite resolution.

As we have discussed, CPPNs have many desirable qualities for describing complex 3D designs. However, as with other generative design techniques, they also have problems ensuring *buildable* solutions. For example, when creating voxel designs with CPPNs, it is easy to generate designs that have disconnected (Fig 5) or non-manifold elements, which render them un-buildable. To exploit the beneficial properties of CPPNs and create functional 3D designs, our model incorporates an additional *growth* process, which uses local *agent-based* interactions to make *all* designs buildable.

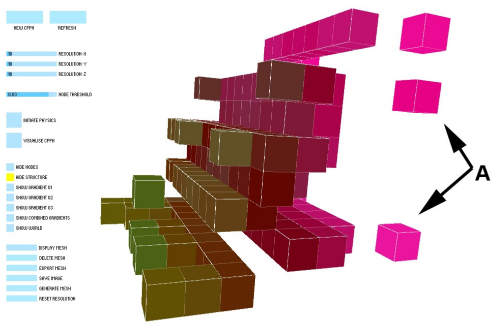

Figure 5. CPPNs can generate unbuildable designs. (A) highlights voxels which are completely disconnected and unsuitable for fabrication.

We have previously described our growth process in detail (Richards and Amos, 2014), so here we provide only a high-level description of the procedure (see Fig 6). The key insight for our model is that, instead of using CPPNs to describe the absolute position and properties of each voxel, we use CPPN generated gradient patterns to define connected paths for *painting* through voxel space.

As shown in Figure 6, we begin with a 3D gradient pattern, and instead of using outputs to describe properties of voxels (such as RGB values as Fig 6A), we use this information to seed the properties of 3D grid of nodes. Nodes have three different properties: (1) a *range*, which describes which other nodes they can see and communicate with (Fig 6B), (2) a *brush size*, which describes how thick paths between nodes will be, and (3) a *concentration* variable that is useful during evolution. Using this information, each node constructs connections with surrounding nodes that are within their range (Fig 6C) – note this is done using a "data-tagging" method which we have previously described (Richards et al, 2012). Following growth of connections, we are left with a network (Fig 6D) that is always suitably connected (i.e. no disconnected elements). This is achieved by ensuring minimum connectivity of nodes. Finally, we *paint* through a voxel space, with a specified resolution, and create a voxel model (Fig 6E). Again, node growth ensures a minimum "brush" thickness while painting through voxels, in order to avoid non-manifold elements.

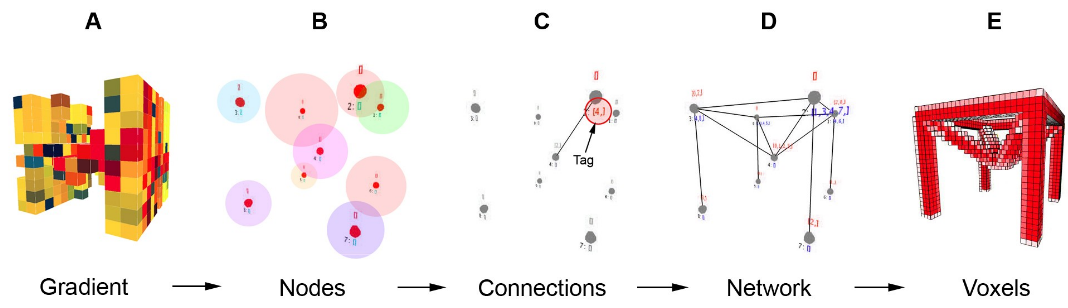

Figure 6. Growth process to generate buildable CPPN generated designs. (A) CPPN gradient, (B) nodes query the CPPN and obtain growth properties, (C) connections are grown between nodes. (D) Network paths are generated by growth, (E) Network paths are used as guides to paint through a volumetric space of voxels.

The benefit of this additional *agent-based* growth process is twofold. Firstly, we can embed important *buildability constraints* and ensure that all designs generated are fully buildable and can be subjected to performance simulations without causing errors. The second advantage is that we can perform optimisation and analysis procedures at various levels of abstraction. For example, during early stage design exploration it may be desirable to perform quick structural analysis at the level of connected bars (Fig 6D) and later fine-tune the design at the level of voxels (Fig 6E) or mesh, thereby creating significant CPU savings.

Our model is implemented with *Java* and *Processing*, and utilises Karsten Schmidt's *toxiclibs* library to paint through voxel space and control meshing procedures. The following section presents the results of our initial case studies, and demonstrates that we can exploit the various formal benefits of CPPN encodings and create functional designs.

4 Design Experiments

4.1 Geometric Complexity

This case study demonstrates that desirable properties of the CPPN encoding persist when we introduce our growth process, and that this allows us to generate complex, yet buildable, geometries.

Figure 7 shows two structures created with our CPPN-based method. To generate these designs we first situate a grid of nodes within a 3D physics simulation. We then grow network structures with CPPN instructions, and treat nodes as particles and connections as springs (using Verlet Integration). Finally, we create mesh-based structures by painting through voxel space (as Fig 6E) and applying an isosurface to the solution. During this process, if nodes move (due to physics), new connections are constructed and old connections are destroyed as nodes move in and out of "range" of their neighbouring nodes.

Using this model, we can explore a diverse range of forms in the early stages of design. Interestingly, the forms generated exhibit visually regular features with repeating motifs, as well as symmetries and even imperfect symmetries (Fig 7A and 7B). Yet due to the physics-based growth process, the geometries are also *always* buildable (Fig 7C and 7D).

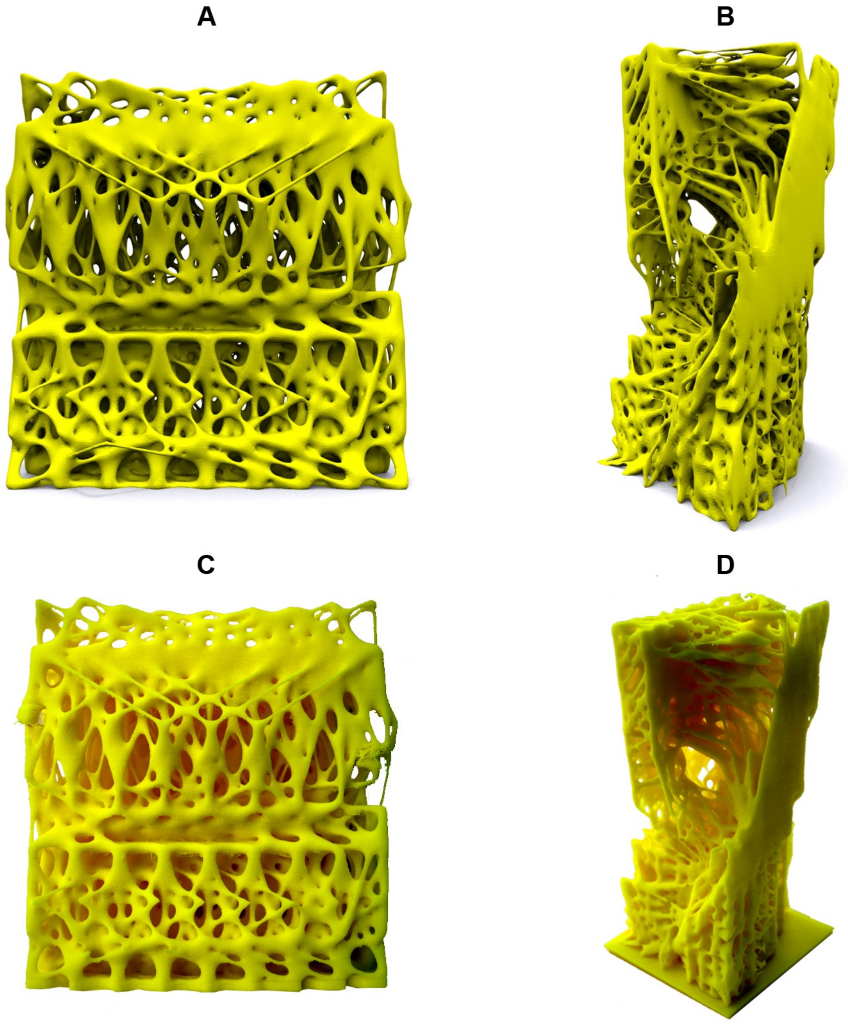

Figure 7. Complex geometries generated with model. Geometries exhibit repeating features, symmetries and imperfect symmetries, and have been

fabricated with a desktop 3D printer. (A-B) Computer renders, (C-D) Actual 3D printed models.

4.2 Evolving Functional Performance

Generative encodings often cause problems when creating functional designs. However, because our CPPN-based model is built on top of the powerful evolutionary algorithm: NEAT (Stanley and Miikkulainen 2002), we can easily use our method to discover functional morphologies. We now show this using some simple 2D and 3D topology optimisation problems.

Topology optimisation is widely used in structural engineering to increase the overall stiffness of designs whilst minimising weight (Søndergaard, Amir and Knauss 2013). The goal is to create physical structures that resist deflection for a given loading, using a restricted amount of material. To validate that our model can discover functional, performance-driven designs, we evolved solutions to four well-known 2D truss optimisation problems (Achtziger 2007). Figure 8 illustrates the four benchmark problems, our best evolved solutions and the known global optimum. For all benchmark problems, the objective is to create a structural form that can hold the imposed loads, *F*, with minimum deflection, using a limited amount of material, *V*.

We evaluate structures using the linear direct stiffness method (Felippa 2013), which is a common finite element method (FEM). This process involves modelling the stiffness properties of each element in the structure and using this information to assemble a larger global stiffness matrix, which describes the mechanical properties of the entire structure. Using the global stiffness matrix, it is relatively straightforward to obtain mechanical properties of the designs, such as node displacements, and use this information to evolve designs with desirable functional properties. As shown in Figure 8, our model is able to generate 2D designs that have good structural performance, and which share many commonalities with the known global optima (for further details see Richards and Amos (2014)). However, our approach also works for 3D problems and can be therefore be used to explore *physical* performance-driven designs (Fig 9).

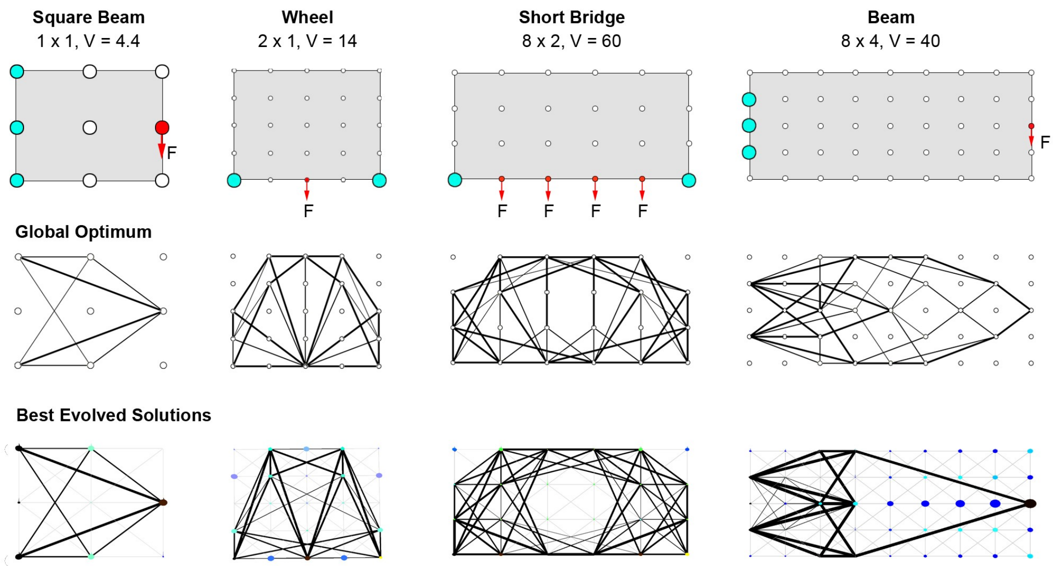

Figure 8. 2D benchmark problems. (Top row) Description of the four problems. Blue nodes represent nodes with restricted degrees of freedom, whereas red nodes are subjected to imposed unit loads in the direction of the associated arrow. (Middle row) Best known solutions. (Bottom row) Best evolved solutions with our CPPN-NEAT model.

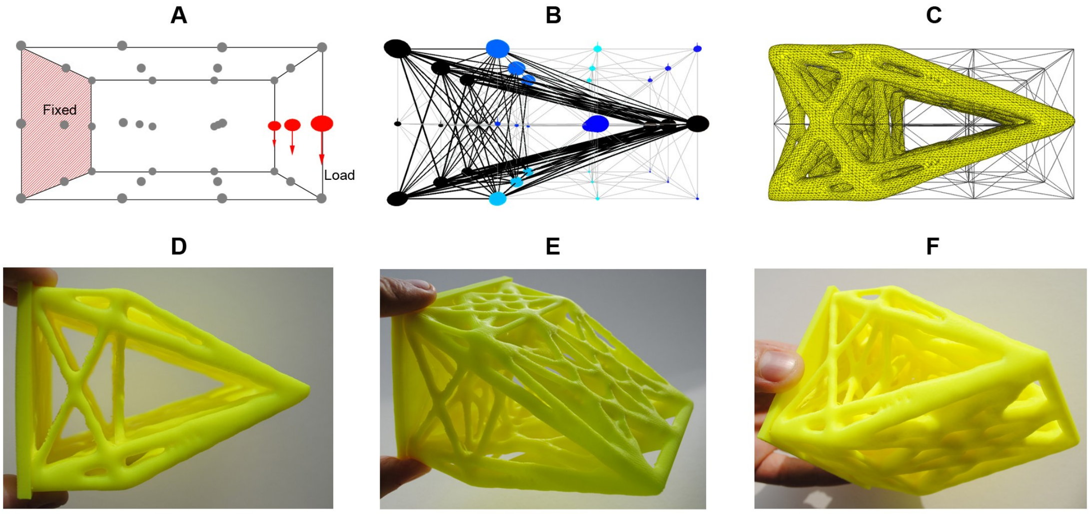

Figure 9. Example truss solution obtained with CPPN-NEAT. (A) Problem definition, (B) Evolved truss design, (C) Meshed truss design for 3-D printing, and (D-F) Photographs of fabricated example truss design.

4.3 Towards Multi-Material and Functionally Graded Structures

The two previous case studies illustrate that our CPPN-based model can successfully create expressive (yet buildable) geometries, and address simple performance-oriented optimisation problems. However, we believe that this could also offer game changing possibilities to design complex multi-material composites and/or functionally graded structures with advanced additive manufacturing technologies. This final case study highlights what we believe to be an exciting trajectory for further work.

We evaluate our previous truss designs as connected bars (i.e. not voxel or mesh designs) with variable cross-sectional area and *homogenous* material, in order to reduce the computational expense of simulation (Fig 9B). However, by adding an extra output to our CPPN encoding, we can also vary the *material properties* across voxel and mesh-based designs, and thereby optimise much more complex structures with *heterogeneous* material composition. Figure 10 illustrates how we can use our CPPN-based model to define variable material properties across non-standard geometry. Additionally, because our approach utilises a scalable evolutionary algorithm, we can feasibly search large mathematical spaces (of possible designs) for material structures that have *any* performance-driven properties, and are thus not limited by traditional homogenisation techniques (Bendsoe and Sigmund 2003).

The ability to simultaneously define the geometry, topology and material composition of efficient large-scale physical structures (as suggested by Fig 10) is a long-term goal of this research trajectory. However, our CPPN-based model may also provide more immediate benefit to design areas that apply shape optimisation. For example, it is entirely possible to vary the surface thickness and/or materials across typical shell structures (as a function of the UV properties) in order to achieve specific mechanical performance, but this remains for further research.

Critically, we believe that future work in this area will facilitate new opportunities to explore complex heterogeneous material, and enable less wasteful designs by exploiting emerging fabrication advances (Oxman, Keating and Tsai 2011) using CPPN-based methods.

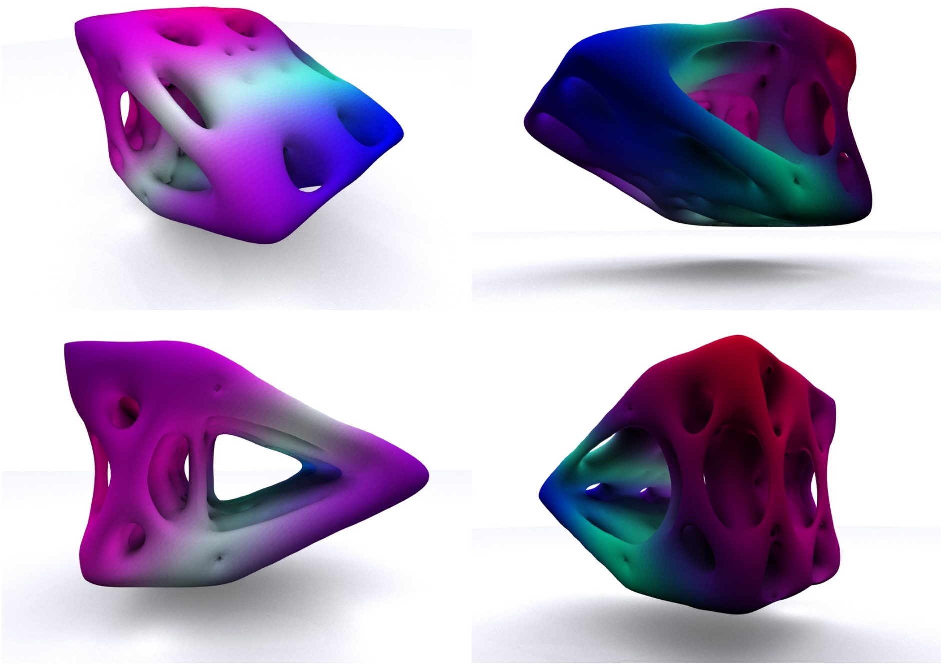

Figure 10. Illustrative truss design with varying (yet non-optimised) material properties.

Discussion

This paper demonstrates a novel method of working with volumetric gradient patterns to control performance-oriented material structures. We argue that this approach offers a valuable way of extending emerging architectural research (Menges 2012, Michalatos and Payne 2013) with a powerful generative encoding. Our case studies show that our CPPN-based model can create expressive, yet buildable, structures, which exhibit desirable geometric regularities and symmetries. Additionally, we show that we can discover performance-driven designs that fulfil 2D and 3D topology optimisation problems. Finally, we highlight exciting new opportunities to describe and control complex heterogeneous material structures to exploit emerging additive manufacturing technologies.

Further work may explore three key areas. Firstly, the computational time needed to evolve voxel-based and mesh-based designs is currently significant. To address this limitation we intend to exploit distributed or cloud-based computing, and significantly reduce the required processing time to facilitate experimentation with more complex performance-oriented designs such as compliant mechanisms. Notably, evolutionary algorithms are particularly well suited to parallel computing due to their structure, as are our matrix calculations for the FEM analysis. Secondly, to explore multi-material and/or functionally graded solutions, we will initially develop and/or integrate suitable file formats to control fabrication machinery (such as Hiller and Lipson (2009)). For example, our current approach requires us to generate cumbersome UV maps (Fig 11A) that specify various colours and/or material properties of 3D structures (Fig 11B), which is undesirable and computationally expensive. Finally, further work will scale-up of our initial studies to consider possibilities for large-scale architectural structures, as well as targeted application as part of larger integrated design processes.

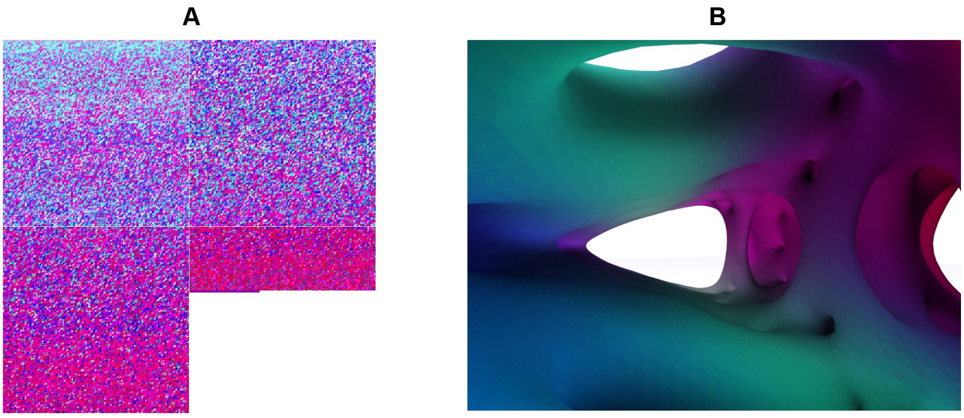

Figure 11. On-going challenges with file formatting. (A) UV map of all mesh faces. (B) CPPN controlled geometry where colours illustrate varying material properties.

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