Mathematics has maintained an enduring image as a field of knowledge that lends its resources to many intellectual pursuits and practical applications. School mathematics, however, has responded to a commonly conceived purpose of supplying the world’s workforce with the resources needed to support economic wellbeing. Research intended to inform the practices of mathematics classrooms has often reflected local interpretations of this fundamentally economic agenda. Since the advent of international comparisons, for example, governments have been jockeying for a better position in the resulting league tables. Good performance in international testing programs such as the OECD’s Programme for International Student Assessment (PISA) or the Trends in International Mathematics and Science Study (TIMSS) has been interpreted as indicating wider economic competitiveness (Brown & Clarke, 2013). Relatively poor performance, however, has often been cited to justify changing educational policies. For example, a recent British government white paper outlining future policy for education in England proposes to expand employment-based models of initial teacher training. It explicitly cites England’s performance in international comparisons such as TIMSS and PISA as a reason for pursuing this approach to training, so that children will compare more favourably with their peers overseas (DfE, 2010) [1]. The (dubious) rationale of the document was that by enabling “a larger proportion of trainees to learn on the job” and by “learning from our best teachers” (p. 23) student teachers would more effectively
encounter the realities of school and be better able to implement centralised curriculum and assessment, which are in turn designed to improve England’s international performance.

In this article, we argue that our conceptions of mathematics and of ourselves as researchers, teacher educators, teachers and students move on as a result of a broad range of pedagogical or practical agendas, such as improved economic competitiveness or performance in international comparisons. In the first part of the article, we depict the mundane reality of the teacher education programme in which we teach, as an example of a site of specific enactments and conceptions of mathematics. This recently introduced model of school-centred teacher education illustrates how changing practices impact the social construction of school mathematics. Changes in the way mathematics is produced and assessed in schools, for example, have progressively altered the demands made on teacher knowledge and practice.

In the second part of the article, we zoom out to offer a more theoretical account of how the evolution of mathematics more generally might be understood in terms of cultural adjustment. That is, we propose that the empirical reality of mathematics today feeds into mathematics itself to change what it is in the future. In this part of the article, we work with the idea that the fields of mathematics and psychology do not describe pre-existing realities. Rather each field depicts realities that are consequential to past human endeavours or conceptualisations of what mathematics is and of what it is to be human.
Mathematics is built to reflect the image we have of ourselves and becomes part of those selves that it reflects.

We conclude by suggesting that curriculum interventions, whether arising from new models of mathematics teacher education, or from the influence of comparative testing, are not distortions of pre-existing conceptions of mathematics. Rather, they reflect new ways in which mathematics is evolving as a discipline. Such interventions also produce revised conceptions of learners, teachers, teacher educators, researchers, and of how policy works.

**Thinking theory and mathematics in new models of teacher education**

The university element of teacher education in England has become much smaller in most models as a result of the government’s policy of specifying that teacher education should take place primarily in schools. Employment-based courses, a recent extension to this policy, have provided a prelude to the wider establishment of “teaching schools”, modelled on teaching hospitals. The government’s ambition is that teaching schools will progressively replace university contributions to teacher education: “The first two cohorts of teaching schools have been designated—a total of 183 teaching school ‘alliances’. The target is to establish 500 teaching school alliances by 2015” (DfE, 2012, p. 1).

We teach in a one-year, employment-based teacher education programme linked to our university but following the practices of a regional teacher education network comprising universities and associated schools. The programme offers two routes. One route is for
primary student teachers planning to teach mathematics as part of the broader primary curriculum. These student teachers would typically have studied mathematics at school until the age of 16 and later completed a university degree in any subject. The other route is for secondary student teachers specialising in mathematics. These students would have completed a mathematically oriented degree. In each of the two routes the student teachers spend a total of forty hours in university (e.g., a five-hour day once a month for eight months). The primary school student teachers spend about six hours of that total on the topic of teaching mathematics. The secondary school student teachers spend about twenty-five hours on the same topic. For the rest of the programme, students work in paid positions full time in schools for the school year. The student teachers involved are typically would have spent some time working in other jobs prior to teaching. They often choose the employment-based model as an alternative to university study. Many have expressed a preference for “wanting to learn on the job and receive a salary as they train” (DfE, 2010, p. 23).

Our practitioner research on the programme started some four years ago with an interest in the role of educational theory in this model of training. Theory had been part of university-based training in the past. Did it still exist as part of employment-based training? If so, what did it look like? Where was it located? In the early stages of the course, “theory” was often seen by student teachers as the stuff that was written in books and thus a bit distant from the immediacy of practice. Their priority was to get on with the job of teaching. Early experience in the first placement school was often very
positive, but a number of the student teachers started to find working with just one mentor rather restrictive (see Jones & Straker, 2006).

All student teachers moved to a second school three months into the course where they found that expectations and practices could be rather different. A new role began to emerge for the university component as the course progressed. Rather than focusing so much on what worked in a specific placement school, the issue was what worked for students more generally in schools. That is, the university sessions became redefined as venues where more generic teacher knowledge was created. Theory became the creation of analytical writing by the student teachers themselves, to support their practice across different schools. The university sessions initiated and responded to the student teachers’ own classroom-based research as part of their getting to know how they might successfully work within a school classroom. They became a place in which their classroom practice could be critically evaluated against broader educational concerns. [2] Our empirical findings of how theory was conceptualised are described in other papers (Hodson, Smith, & Brown, 2010; Smith, Hodson, & Brown, in press a).

We then looked, in the same way, at what had happened to mathematics (Smith, Hodson and Brown, in press b). That is, rather than supposing that there is a real mathematics to which teacher education seeks to provide access, we asked what, empirically, mathematics had become for the student teachers as a result of following the employment-based model of training. The detailed government-produced “non-statutory” assessment framework for how the curriculum was to be covered had now been
abandoned. Yet many schools still had schemes of work closely tailored to this framework. The schemes were typically staged according to the levels in the main curriculum. The student teachers, therefore, found themselves in schools where the curriculum structure was ever present in the shaping of classroom activity and of mathematics.

Our findings revealed, for example, how many student teachers felt coerced into teaching to the textbook or scheme of work. One secondary school mathematics student teacher described what she perceived as the relentless overseeing of the content and methodology of her teaching by her head of department: “The other day I was doing something a bit different and then he’s going, ‘You can’t do the end of chapter tests on that because you haven’t done exercise 5b!’ I feel as though he wants me to do every single question in the textbook.” Another extract from a discussion held with secondary school student teachers suggests that some freedom to apply the teacher's own ideas could be derived from following the school’s scheme of work. However, this had to be assessed using the government Assessing Pupils’ Progress (APP) framework, which the school followed: “I will plan my lesson, I use the scheme of work and I do this by myself. I don’t have anyone to tell me what to do—no one checks that. There’s no textbook to follow. I just teach my lessons so that they can do that, can use these words. At the end of topic, they have to do the APP.”

Findings from the primary teachers demonstrated a similar exertion of school influence on what counted in mathematics. One student teacher, for example, in reflecting on a
question posed about how he would decide to teach mathematics had this to say: “We have a policy, certainly for the four rules ... I was doing ratio ... and they were coming up with methods and I was looking at the class teacher asking, ‘Shall we go down this route?’”

In English primary schools, mathematics is most usually taught with much whole class input, where the teacher must react to children’s responses. This can be a risky business for student teachers when under the watchful eye of their mentors. In these situations, it was most important for student teachers to be seen to use the “correct” method. One student teacher described how moving from whole class teaching to individual activity, where children could experiment on their own ways of reaching solutions to mathematical problems, enabled him to “really see what the children could do”. Ironically, he still needed to check the validity of the method used by a particular child with his mentor. She confirmed, with some hesitation due to the apparent deviation of the method from the more typical school approach, that if the children “got there, we’d probably support that [method]”.

In short, we found that many student teachers learn to teach mathematics by participating in current school practices that closely follow the curriculum and the demands of national tests. Furthermore, schools and government agencies set criteria as to how this engagement was validated. Periodic national tests influenced the forms through which mathematical ideas were encountered. The consequence of these framings is that
mathematics encountered in schools has a tendency to focus on those areas relating to the tests.

A significant aspect of the change in student teachers’ understanding of mathematics related to how university mathematics teacher educators conceptualised their roles. They had been accustomed to spending a significant amount of time with student teachers in the university. Later, as increasing responsibility for training was relocated to schools, the content that had been previously covered in the university was condensed. The number of topics being covered was reduced and those that remained were dealt with at a brisker pace. At first we found this new arrangement quite stressful, compressed as our previous role now was into an increasingly small amount of time with the student teachers. Ironically, however, student teachers, thrust as they were into the hurly burly of school classroom activity, found the university sessions altogether more relaxing. Close pursuit of the curriculum in school framed their conceptions of mathematics, whilst university sessions provided reflective space. For primary school student teachers, the six hours at university early on in the course that provided a guide to the curriculum that they would be following were soon forgotten. Later in the course, mathematics was discussed as just one of the subjects that they were responsible for teaching. For secondary school student teachers, the twenty-five hours largely tackled issues relating to their teaching in schools.

The orientation of the university component of the programme had shifted from one of input to one of response. Its role in supporting the student teachers had relatively little to do with introducing broader issues in mathematics education, research in the field,
subject knowledge being rethought as pedagogical content knowledge, and so forth. More experienced university-based mathematics teacher educators found themselves subject to a very different conception of their practice to the one to which they had become accustomed earlier in their careers. The traditional content of mathematics teacher education, insofar as it was still addressed, was distributed across school and university settings. Many of the ambitions advocated for teacher education (e.g., Askew, 2008; Rowland, 2012) or for subject knowledge (e.g., Ball, Hill & Bass, 2005; Davis & Simmt, 2006) had been deleted from the list of training priorities. If mathematics education research still influenced the practices of the student teachers, then the route through which this influence was achieved is not entirely apparent. It is also unclear how, within this model, one would seek to influence practice through mathematics education research. To whom would research about classroom practice be addressed and how would knowledge derived from this research filter into teacher knowledge?

So how might we understand this model of practice impacting wider understandings of mathematics, mathematics teaching, mathematics teacher education and mathematics education research? In describing changes to teacher education and its impact on conceptions of mathematics our opinions are inevitably referenced to our own experience. We are responsible for overseeing the mathematics teacher education of a set of students. In our local world, teachers get to be teachers by following the route that we have briefly described. Our impact on the experience of the student teachers is limited, as a consequence of spending so little time with them. Whatever our own mathematical credentials, or fantasies of what might be achieved in other circumstances, our everyday
challenge is to attend to what might be achievable within the model that governs our practice, and to work according to the outcomes of that model. Insofar as we are part of the community leading attempts to improve mathematics education, we may adopt a critical or resistant attitude in our efforts, but we must face up to the net effect of our actions, even though so many decisions have been taken out of our hands [3]. Student teachers, meanwhile, develop a specific, pedagogically-oriented conception of mathematics. We argue that they conceptualise their teaching within a rather restricted model of education.

The evolution of mathematics

We have argued that the specific administration of school mathematics and associated teacher education described above changed conceptions of mathematics in our locale. From a wider perspective, we see the conceptions of school mathematics that influence our actions as a function of the discursive environment and the way that that environment formats mathematical activity. That is, the way in which mathematics is administered and conceptualised in the specific pedagogical environment determines what mathematics is. In England, successive governments have each followed rather authoritarian modes of curriculum definition and teacher education in the name of collective success (Brown & McNamara, 2011). This approach feeds into public and individual perceptions of what mathematics is. We are part and parcel of a population where the majority of people understand the scope and purpose of mathematics through the filter of their own school education [4]. Moreover, our sense of who we are is built through our own practice and
the linguistic categories available to us, which are conditioned by particular, culturally preferred ways of making sense.

In this second part of the article, we consider how formulations of pedagogical objects in mathematics such as the reconfiguring of mathematical tasks to meet new curriculum demands, comprise qualitative adjustments to mathematical objects more generally. Specifically, we consider how the impact of changes to school mathematics on mathematics itself can be understood. “Mathematics”, of course, requires some qualification, to ensure that it has any meaning at all. Mathematics could mean the vast field of mathematics that is beyond the scope of any one individual. Alternatively, mathematics could be seen as the majority view of what mathematics is and how it manifests itself in everyday practices across given populations. There cannot be final agreement on this. A choice needs to be made as to what counts as mathematics and this choice always depends on circumstances that transcend most conceptions of mathematics. Nevertheless, it seems clear that the field of mathematics itself has been transformed as a consequence of certain areas being explored more than others, for example, statistics rather than geometry. Of course, most people do not explore many of these areas and so enjoy a relatively restricted view of mathematics.

In the example discussed in the previous section, school mathematics is susceptible to regular makeovers based on curriculum changes that redefine its content and preferred points of reference. New mathematical priorities, such as the need to meet the demands of international comparative testing, have come into prominence whilst others have faded,
such as problem solving based activity. A casual inspection of school textbooks or exam papers through successive decades would evidence a substantial shift. Mathematical objects are converted into pedagogical objects or standardised test items, such as those evident in TIMSS and PISA, which then influence the form of school mathematics (Brown & Clarke, 2013; Morgan, Tang, & Sfard, 2011). On the one hand, school testing regimes have increasingly partitioned mathematical activity so that children and teachers are better able to successfully meet the priorities of international comparison (Askew, Hodgen, Hossain, & Bretscher, 2010; Brown, 2011). On the other hand, teacher education models have been modified to secure greater compliance with those curriculum demands (Brown & McNamara, 2011). Further, key examinations for 16 years olds have been shaped to meet new demands, but have reduced students’ capabilities and dispositions towards further study in mathematics (Pampaka, Williams, Hutcheson, Wake, Black, Davis, & Hernandez-Martinez, 2012). Test scores, enjoyment and functionality can pull in different directions.

To represent mathematics as universal, spanning nations and generations, comes at a price. TIMSS and PISA measure and compare school mathematics in different countries on a singular scale. Yet the resultant conceptions of school mathematics now define and regulate the boundaries of school mathematics. At a recent conference, a Mexican delegate spoke of how TIMSS criteria made her country answerable to American priorities for school mathematics (Garcia, Saiz & Rivera, 2011). An Ethiopian educator spoke of how teachers and students were obliged to engage with pedagogical forms largely unrecognizable in their country situation (Gebremichael, 2011). England
sacrificed its earlier facility with problem-solving approaches in order to meet newly understood TIMSS objectives [1]. Meanwhile, a delegate from Finland revealed that her country’s strong showing in the exercises did not release them from having to discuss their achievements in terms of the international priorities (Krzywacki, Koistinen & Lavonen 2011). School mathematical knowledge derives from this newly described world backed up by governments using these conceptions of mathematics to set their policies and to materialise these new understandings of mathematics. These policy priorities may often exclude some powerful and interesting areas of mathematics. The revised priorities may then grow up to police the practices now developed in the name of mathematics (Kanes, Morgan, & Tsatsaroni, 2010). Teachers are subject to skills criteria referenced to the curriculum success of their pupils. Pupils are understood through the grades they receive. The social parameters that govern our actions move on, as do the mathematical activities that are subject to these parameters.

Mathematics itself is reconstituted according to evolving social priorities and criteria. For authors such as Bachelard, Lakatos, Althusser and Badiou science is a practice marked by the production of new objects of knowledge (Feltham, 2008, pp. 20-21) [5]. Mathematics, for example, evolves through successive attempts to algebraicize its objects, such as through reaching new generalisations in newly encountered conditions. The advance of mathematics is defined by the production of such objects, often in response to newly defined utilitarian objectives, pedagogical circumstances or, as we are arguing here, as a result of new formal assessment demands. For example, an earlier article in this journal sought to question the history of circles, where “circle” was being taken as an example of
a mathematical object (Bradford & Brown, 2005). The question being asked related to how much a circle as an object was a function of the cultural layers that had been brought to it. Circles are common entities and they have featured in many of the stories that we have told about our world. We may feel that we have got to know circles from many perspectives, which results in them acquiring a broad set of qualitative features. Stellated octahedra in contrast have been denied that level of intimacy and familiarity with humans. But, geometrically speaking, is there any reason as to why one might be privileged over the other? Attention to circles has indeed impacted their perceived importance, so that circles are far more prevalent in peoples’ lives and work than stellated octahedra, perhaps simply because people are primed by everyday life and by the school curriculum to find and use them. This cycles back into the now accepted “reality” that circles are clearly more important than, say, stellated octahedra.

More generally, certain elements of mathematics have been touched more frequently by pedagogical or practical concerns. The field of mathematics has been marked out according to how it has been seen as supporting practical agendas. Some bits (e.g. statistics) are much more popular than other bits (e.g. topology) and for this reason tend to be more likely to be noticed in schools, used in everyday life, secure research grants [6], etc. Yet, it is actually quite difficult to sort mathematics according to which bits are empirically referenced like circles and which bits are not so common in appearance or utility, such as stellated octahedra. The historical circumstances that generated mathematical objects are often lost. The objects may have become a part of who we are such that we are no longer able to see them. Mathematical models exist as knowledge that
sometimes support empirical enterprises to certain limits but, ultimately, as empirical support, the models always reach their limits.

So far in this article, we have considered what has become of mathematics as a result of it being reshaped to meet pedagogical or practical demands. Is mathematics then merely a social construction linked to our practical ambitions that anchor the existence of mathematics and the existence of its objects? Or does it have some underlying truth that, as it were, holds mathematics in place? This question requires a deeper philosophical analysis of whether there is a mathematics beyond all of our socially motivated encapsulations. A recent public debate addressed the issue of whether science was held in place by rationality or faith. The scientist, Richard Dawkins, “raged against any kind of mystery in the cosmos, preferring instead to settle for a cold universe driven by the machine of pessimistic reason.” Alister McGrath, a professor of theology, in contrast, posited a “pure religious thinker” governed by a faith (Davis, 2009, pp. 13-14). A second debate however, between the theologian John Millbank and the philosopher Slavoj Žižek led to an assertion that faith and reason are not simply opposed (Millbank & Žižek, 2009). They each argued in different ways that the work of Hegel undermined any dichotomy between the mythical and the rational. For Hegel “the object is always-already bound up in the complex mediating process of the subject’s thinking it, and conversely, the subject’s thinking the object is itself bound up in the object’s very existence” (Davis, 2009, p. 14). Or, as Žižek (2011) says, “What we experience as reality is not the thing itself, it is always-already symbolised, constituted, structured by way of symbolic mechanisms” (p. 240). Žižek (2012, p. 144) identifies three positions in Hegel’s
formulation: “In the first, reality is simply perceived as existing out there, and the task of philosophy is to analyse its basic structure. In the second, the philosopher investigates the subjective conditions of the possibility of objective reality” *i.e.*, we ask where are we coming from in seeing it that way. “In the third, subjectivity is re-inscribed into reality” *i.e.* our assumptions of where we are coming from become part of reality.

Similarly, humans are social constructions as a consequence of particular attributes being privileged in our understanding of them. Social constructions can change what humans are. For example, mathematics education research has often been governed by Piagetian conceptions of the mind developing. In the last few decades, however, discursive constructions have become more familiar (for example, Walkerdine, 1988). In these later models the focus is not so much on the mind developing as on changing the story or structure according to which we work. Barad (2007) has shown us that it is never entirely clear where the physical human stops and where the operation of cultural machinery begins. One might take the example of someone keying in some commands to a computer that have a tangible impact on activity elsewhere.

Both mathematics and psychology describe realities that are consequential to past human endeavours or conceptualisations. As fields of enquiry they are enterprises, or objects, built in the human’s own self image (and then built into the human self image) that trap us into thinking that there are universal realities of what it is to be human and of what it is to be mathematical. TIMSS and PISA, for example, are in the business of serving an image of mathematics characterised by particular forms of questioning and, by serving
that image, they make mathematics itself seem more real, or part of a more enduring reality.

**Conclusion**

The learning of mathematics may be helpfully understood as seeing and experiencing mathematics as coming into being, or participating in the becoming of mathematics, making it come into being (see, for example, Krummheuer, 2009; Roth, 2010). The learner may experience mathematics as part of herself, a self that is also evolving in the process (Brown, 2011). Mathematical objects and the ways in which we relate to them would never finally settle in relation to each other. The building of mathematics then reflects the image we have of ourselves and becomes part of those selves that it reflects. We have taken two perspectives on this evolution in the two parts of the article. In the first part we have considered how conceptions of school mathematics change as a result of rethinking the needs of teacher knowledge. In the second part we took a broader historical perspective. Yet, we may not experience our immersion in mathematical changes in this way. We understand ourselves as operating in a rather more restrictive space decided upon by legislation and by expectations beyond our active control. If the world is built in our own image, our children may encounter that world as an external demand out of line with their own perceived needs. Following Hegel, Malabou (2011) suggests that the individual “does not recognise itself in the community that it is nevertheless supposed to have wanted [...] The individual is ‘alienated from itself’” (p. 24). The “self is already implicated in a social temporality that exceeds it own capacities for narration” (p. 28). These fractures in our self-image can result in adjustments to our
tangible reality and to how we encounter it. Mathematics is a function of how we organise its supposed content at any point in time. Yet it is also a function of the narratives that report on how we experience it through time, and of the hermeneutic working through of those narratives that generate new dimensions of mathematics (Doxiadis & Mazur, 2012). These narratives may be productive, misguided, manipulative, or functions of particular administrative or ideological perspectives (Lundin, 2012; Pais, 2012). For example, ideal accounts of mathematics can readily become policing structures in the service of compliant behaviour transforming how subsequent students experience mathematics. Curriculum innovation and associated testing can activate new, perhaps unexpected, modes of mathematical engagement or educative encounters across a community. People or communities more or less identify with these new conceptions of mathematics and shape their practices accordingly. For example, the model of teacher education described earlier reduces options for student teachers to see beyond compliance with the current curriculum and the associated assessment that seeks to mirror those international ambitions. Similarly, we have suggested that TIMSS and PISA have shaped mathematics through their widespread influence over how mathematics is understood, how it is conducted, how it is reproduced, thus rewriting what it is. As a result they have influenced the demands placed on teachers and students thus shaping who they are, perhaps locking them into unhelpful caricatures that can hinder adjustment to new circumstances. Meanwhile, mathematics education researchers can be cast in terms of supporting this kind of agenda. Many grant applications, for example, are oriented towards improving performance in a given regime, steering school learning back to a correct path more in tune with “what mathematics
really is”. The challenge for researchers in mathematics education is to recognise their political role in understanding how changing circumstances shape both mathematics and humans.

Notes

[1] Curiously, the document mentioned the PISA result where England had moved down from 8th to 25th position, rather than a TIMSS result, previously used as the main point of reference, where England had risen from 18th to 7th. This disparity may be explained by the greater emphasis on word problems in PISA, which relates to England’s earlier relative proficiency with problem solving. There are alternative interpretations: http://fullfact.org/factchecks/school_standards_oecd_pisa_data_media_conservatives_education-2423

[2] Our empirical findings of how theory was conceptualised are described in Hodson, Smith and Brown (2010) and Smith, Hodson and Brown (in press, a).

[3] It may be conceivable that, as suggested by a reviewer of this article, “a broad boycott by university personnel, coupled with a successful public education campaign, could lead to policy changes”. Much mathematics education research appears to be predicated on this sort of possibility. But can we wait for a broader resolution not immediately available in some countries? Decisions are often governed by politics rather than some higher consensual rationality. One of us was recently signatory to a widely reported open letter to the press written by 100 senior academics to the Secretary of State for Education for England criticising the restrictive nature of a new curriculum formulation. The Secretary
of State dismissed the letter as being from “bad”, “Marxist” academics, which he then used to further argue that teacher education be removed from universities.

[4] A recent survey in Britain claimed that only 50% of adults function above the level of an average eleven year old and very often members of the other 50% were quite proud of their limitation (Garner, 2012).

[5] Similarly, Deleuze and Guattari (1996, p. 2) see philosophy as “the art of forming, inventing, and fabricating concepts.”

References


Department for Education (2012) Call for “Expressions of interest” for: Teaching Schools evaluation.


