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TONY BROWN, FRANK EADE AND DAVE WILSON

SEMANTIC INNOVATION: ARITHMETICAL AND ALGEBRAIC METAPHORS WITHIN NARRATIVES OF LEARNING

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ABSTRACT: This paper examines a theoretical perspective on the ways in which children progress in learning mathematics. It suggests that there is a difficulty in associating teaching discourses with the mathematics they locate. This can result in an incommensurability between alternative perspectives being offered. The paper resists attempts to privilege any particular account but rather demands an analysis of these discourses and their presuppositions. In developing these themes the paper invokes Ricoeur's analysis of time and narrative as an analytical approach to treating notions such as transition, development and progression in mathematical learning. His notion of *semantic innovation* is introduced. This embraces both the introduction of a new metaphor into a sentence or the creation of a new narrative which reorganises events into a new "plot". The notion is utilised in arguing that the shift in the student's mathematical development from arithmetic to first order linear equations with unknowns reconfigures the contextual parameters governing the understanding of these mathematical forms. It is also utilised in showing how alternative approaches to accounting for such transitions suit different and perhaps conflicting outcomes. For example, demonstrating awareness of generality or performing well in a diagnostic test featuring the solution of linear equations.

INTRODUCTION

This paper is about development in student mathematical performance but not so much how to achieve it. Rather the paper is about how it is seen and how it functions as a notion in guiding our actions as researchers and teachers focusing on students progressing through successive stages in a developmentally formulated curriculum. We shall refer to some work describing the transition children make from work in arithmetic towards simple algebra. This will provide examples of how student mathematical development is treated in the mathematics education research literature. Meanwhile, in literary theory Ricoeur (e.g. 1984) has carried out detailed work on conceptions of time and how these depend on the narratives we create. He argues that we cannot rely on simplistic notions of time understood as the passing of phenomenologically defined events. Rather descriptions of time can always be revisited and revised. We suggest that this opens up an analytical frame within which alternative accounts of transition or development, as understood within educational discourses, can reside. This work falls against a backdrop of an increasingly post-modern attitude prevalent in contemporary theoretical writings, where notions of progression or advancement are treated with a certain amount of caution. Such approaches do not see human actions specified in terms of intention motivated towards some pre-understood ideal. Instead these approaches describe a perpetual state of becoming, governed through the social discourses enacted through the individual. This we suggest prevents easy association between educational strategies and the mathematics they seek to locate since neither can be pinned down in an essentialist manner.

We begin with a brief look at how the essential ambiguity of mathematical phenomena resists clear depiction of transition in mathematical learning in linguistic form. We then review some examples of research in mathematics education that discuss the transition from arithmetic to algebra. We consider how alternative accounts of this transition might be generated and point to the suppositions and assumptions inherent in these. We follow this with an introduction to Ricoeur’s work on metaphor, time and narrative. It is then shown how this might lead us to an alternative approach to describing mathematical understanding. This is illustrated by developing the analysis offered within the mathematical studies. Here we discuss how the introduction of unknowns into arithmetical equations can be seen as metaphorical disturbances which have the effect of adjusting the sense of all of the terms within any given equation. Similarly, we discuss how the introduction of certain teaching models can be seen as impacting on the narrative structure within which “transitions” in mathematical learning are effected. In this way we demonstrate how the intermediate role of language intervenes as a powerful arbiter of the reality to which we attend. Although as authors we are aware that this applies to us and the language we use as well.

MULTIPLE ACCOUNTS

How do we account for transition in mathematical learning when such transition resists clear depiction in tangible evidence? Mathematics education research is generally predicated on some notion of improvement being sought. Often this is seen in terms of hastening this transition, whether this be focused on children progressing through a curriculum, or teachers or researchers developing improved strategies for facilitating this. Part of our task here is to problematise conceptions of moving from one domain to another. We suggest that any depiction of source and target domains, or of the transition between them, resist phenomenological accountability. That is, targets expressed in terms of desired trajectories seem problematic insofar as we attempt to claim movement from state A to state B. As many recent research reports in mathematics education research remind us, mathematics and the students studying it are socially construed entities susceptible to temporal and compositional shift (e.g. Brown, 1997). We can always revisit and reread an interpretation of how a child proceeded through some mathematical exercises. We thus need to be cautious in defining states of mind, or even mathematical competencies, that position students in one domain or another. Such critical perspectives have surely lost some of their appeal in any case after the work of contemporary writers (e.g. Walkerdine, 1988) who have resisted psychologically oriented accounts centring analysis around singular developing minds, such as that provided by Piaget's stage analysis. For this reason there is also a need to be cautious in introducing models and to be more aware of their presuppositions and limitations. We cannot easily define such an organisation of events which effects a transition to have happened. For within any such event there are multiple plots highlighting or implicating alternative phenomenological features. We shall set the scene with a few examples of how such issues concern us in our understanding of children's early work in algebra.

Sfard and Linchevski (1994 a, p. 191) assert that algebraic symbols do not speak for themselves. Rather, any algebraic expression has a range of possible senses and can be read in a number of ways. For example, the expression $3(x+5)+1$ can be read as a computational process, a certain number, a function, a family of functions or merely as a string of symbols. Indeed, they claim, we can identify "an inherent process-object duality in the majority of mathematical concepts" (*ibid*). Meanwhile Gray and Tall (1994) introduce the term "procept" to identify this duality. They suggest that the "ambiguity of notation allows the successful thinker the flexibility in thought to move between the process to carry out a mathematical task and the concept to be mentally manipulated as part of a wider schema". This schema however can be conceptualised in a variety of different ways. For example, many studies privilege accounts which assume that arithmetic precedes algebra (e.g. Filloy and Rojano, 1989; Herscovics and Linchevski, 1994, both to be discussed shortly). This is the direct opposite to that described in the work of Gattegno (e.g. 1988, pp. 74). He sees algebra preceding arithmetic where algebra is a "way of speaking of the mental dynamics necessary to transform some mental given into another mental form", within which arithmetic resides. He claims however that school education favours verbal description. This he sees as resulting in an overemphasis on algebraic ways of thinking, at the cost of certain areas of mathematics, e.g. geometric intuition, that do not fit so easily into this sort of categorisation. Thus the school curriculum concentrates on topics that are more easily explained in this way (see Brown, 1996 a, p. 61). Similarly, we take from Mason's (1996) work that the reductionism implicit in emphasising issues of transition through progressively more difficult forms of algebraic equation, draws attention away from the underlying principle of algebra being about the noting of generality. He suggests that many programs of instruction stress the particularity of concrete objects which has the effect of drawing attention away from the general, thus favouring behaviour over awareness (*op cit*, p. 70). It seems that the mathematician's idealism is both located but also evaded within the research models we create in our analysis. This in itself will come as no great surprise; models are inevitably simplifications introduced to help us see structure (Linchevski and Williams, 1996, p. 266). It does however bring into question the virtue of any quest to privilege any particular model or any final declaration as to the mathematical content this locates.

Also, how we see the salient features of mathematical learning is a function of where we are positioned in any educative process. For tutors concerned with the initial or in-service training of teachers, clearly it is important to equip one's students with the sorts of

mathematical insight described above. However, an alternative style of objective might be assumed by a policy maker seeking to promote effective performance in tests designed to facilitate international comparisons in mathematical achievement. Here understanding is often reduced to being assessed against more quantifiable performance criteria.
It seems inescapable that pedagogical discourses are a function of the enterprise they support. And that these discourses govern the choice of teaching devices, which in turn condition the mathematics learnt through them. For example, in his analysis of a teaching scheme, Dowling (1996) found mathematics which was designed for less able students to be of a very different nature to that given to their more able peers. For any given topic the emphasis in the instruction varied between the texts designed for the two sorts of students. It seemed to result in exclusion for the less able from the real business of mathematics as understood in more abstract terms. Instead they were caught in the discourse of “less able” mathematics. The analysis identified at least two levels of mathematics, each characterised by a discourse with associated styles of illustration, questioning, perspectives assumed, etc. But clearly there are many such discourses operating in mathematics education. And as with the distinctions between mathematics designed in the schemes for less or more able students, differences between these discourses are swept over. In many situations this is a consequence of outcomes being seen primarily on a register of mathematical content, independent of the processes that lead to these. Nevertheless each of these discourses misses the mathematics it seeks to locate. Each is characterised by some sort of illustrative approach which simultaneously serves as a teaching device, but this arguably draws us away from the mathematics. (The educational strategy of getting analogies to fit the ideas being taught is also well trodden in science education. See for example Heywood and Parker, 1997). This, of course, is also true of the mathematics designed for the more able students following the scheme where situations are couched in more overtly mathematical form. But each of these discourses is predicated on some sort of mathematical objective. This might be tied down to performance in a specific discursive frame, such as the solving of a linear equation, or more transcendental mathematical claims such as abstraction, the noting of generality or intuition. Ricoeur’s analysis will assist us in distinguishing between these alternative narrative layers. It will also help us in showing how mathematical development in children might be seen as being conditioned by and dependent on narratives embedded in the teaching and learning process. But for now we consider how such narratives are generated.

THE TRANSITION FROM ARITHMETICAL TO ALGEBRAIC THINKING: TWO ALTERNATIVE ACCOUNTS

We shall focus on certain aspects of two studies taken from the field of research in mathematics education. These will provide examples of papers which address the student’s transition from arithmetic to algebra, and in particular, the introduction of unknowns into arithmetic equations. Filloy and Rojano’s (1989) work sees this transition as mathematically defined. Meanwhile, Herscovics and Linchevski (1994) work sees it as cognitive. These will show different ways in which this transition is described within the research literature. We will also be referring to another paper by Sfard and Linchevski (1994 b) which suggests an interesting disruption to any dichotomising of these two perspectives. These examples will then be utilised in support of our introduction of Ricoeur's theoretical perspective.

Didactic cut

How might we characterise the shift from arithmetic to algebra? Filloy and Rojano (1989) introduce the notion of "didactic cut" between arithmetic and algebra. They see this as arising when the child's arithmetical resources break down in tackling linear equations. They suggest that a sharp delineation between arithmetic and algebra can be identified when in a first degree equation the student encounters for the first time the unknown on both sides, e.g. Ax+B=Cx+D. When the unknown only appears on one side they suggest the solution can be found intuitively through purely arithmetical means and hence with existing skills, such as counting procedures or inverse operation. Thereafter they claim additional resources are needed and to overcome this barrier. The students then require assistance from the teacher who needs to provide some sort of device which enables the student to negotiate access to the new domain. They go yet further by suggesting that this introduction of teaching strategies results in an inevitable diversion in reaching mathematical objectives. This is because the teaching devices create obstacles through their introduction of intermediate codes, ie between functioning at the concrete level and the fully syntactic algebraic level. They talk in terms of taking students in the "the direction of what algebra is intended to achieve". However, this "direction" cannot be specified directly but needs to be alluded to through teaching devices. Whilst these devices assist in broaching new territory, they inevitably draw attention away a little from the conceptual understanding being sought. These

"hinder the abstraction of the operations performed at the concrete level and are due to a lack, in the transition period, of adequate means of representing to which the various operations lead. The obstacles arise from a sort of "essential insufficiency' in the sense that modelling ... tends to hide what it is meant to teach (Filloy and Rojano, 1989, p. 25).

Such modelling, they suggest, is characterised by two components, namely transition and separation. They argue:

When either of these two components is strengthened at the expense of the other, the new objects and operations become harder to see (ibid).

For example, the noting of generality might become obscured if the student becomes locked within the domain of a particular model or teaching device, having separated herself from the task of seeking the abstraction implied by the more concrete domain.

Cognitive gap

Herscovics and Linchevski (1994, pp. 59-61, see also Linchevski and Herscovics, 1996) also seek a "clear-cut demarcation between arithmetic and algebra". However, they question Filloy and Rojano's notion of didactical cut on the grounds, they claim, that it focuses on mathematical form rather than process. They introduce the notion of a "cognitive gap" which "is characterized by *the students inability to operate with or on the unknown*" (p. 75, their emphasis). This they see as moving the boundary being considered from one between two mathematically defined domains to one separating developmental stages in the learner's conceptions. Their findings with seventh grade students suggested that in equations where the variable appeared just once (e.g. ax+b=c or 37-n=19), nearly all students solved the equations arithmetically by inverse operations. However, a fundamental shift was noted when the variable appeared twice, either on just one side (e.g ax+bx=c) or on both sides (e.g. ax+b=cx+d). They found

> the majority of students were able to solve them only by reverting to a process of systematic approximations based on numerical substitution. Although students managed to spontaneously group terms that were purely numeric, at no time did we witness any systematic attempt to group the terms in the unknown. We came to the conclusion that the students could not operate spontaneously with or on the unknown. The literal symbol was being viewed as a static position, and an operational aspect entered only when the letter was replaced by a number. This inability to spontaneously operate on or with the unknown constitutes a cognitive obstacle that could be considered a gap between arithmetic and algebra. (Linchevski and Herscovics, 1996, p. 41)

Having identified this gap the authors then carried out empirical research to examine ways in which it might be crossed. Their findings include some suggestions of specific teaching techniques designed to overcome this particular gap, such as exercises in grouping like terms, developments of the balance model and decomposing into a difference to facilitate cancelling subtracted terms (Linchevski and Herscovics, 1996). They do however stress that "it is only when they (the students) achieve a more general perspective on equations, solutions and solution procedures that they can appreciate the value of a more general solution process" (op. cit., p. 63)

STORYING TRANSITION

It seems to us that the difference between these two examples from the research literature highlights the potential for creating alternative accounts (or stories) of how this boundary is broached. And the need for care in privileging particular accounts. We continue with some discussion of the student's perspective on this transition. This is followed by a consideration of some of the research issues involved in depicting the transition.

Student preferences

Quite apart from accounts offered by researchers we can observe *storying* carried out by the students themselves within their own learning. That is the way in which they describe their own transition, and the ways in which they situate any developing understanding of particular mathematical ideas within their broader conceptions of what constitutes mathematics. (See for example, Ruthven and Coe, 1994, Rodd, 1993.) Any addition to the student's mathematical repertoire is understood within a broader narrative frame within which narrower conceptions of mathematics reside. That is, students utilise a broad range of metaphorical apparatus in supporting their own mathematical thinking. Yet these are situated within their broader narrative accounts of why things are as they are and how they connect with other bits of mathematics and life outside. The student's experience we conjecture is not of a straightforward switch from arithmetic to algebra. Their *storying backdrop*, the contextualisation that receives and conditions any unfamiliar statement, needs to be extended at the same time. Although as Sfard and Linchevski (1994 b) indicate children do this in different ways and have different needs as regards providing supporting rationale for their implementation of procedures. They show us that we cannot assume consistency between children as to their apparent readiness to occupy a new domain and that this readiness is not straightforwardly associated with broader mathematical ability. They discuss a child whose preference for considered interpretive assessment of meaning slows him down against a peer more amenable to unreflective implementation of techniques. It is this example which gives rise to their distinction between "interpreter" and "doer". They characterise the students' respective motives as follows:

"the meaningfulness ... of the learning is, to a great extent, a function of student's expectations and aims: true interpreters will struggle for meaning whether we help them or not, whereas the doers will always rush to do things rather than think about them. The problem with the doers stems not so much from the fact that they are not able to find meaning as from their lack of urge to look for it. In a sense they do not even bother about what it means to understand mathematics. (Sfard and Linchevski, 1994 b, p. 264)

This muddies the water in any attempt to draw clear distinctions between mathematical and cognitive domains. On the one hand we have unreflective performance of mathematical procedures, on the other a more sustained attempt to understand which seems to work against performance at least in the short term. This seems to disrupt any straightforward attempt to correlate cognitive ability with mathematical performance. The preferences of interpreter and doer seem to conflict. Their mathematical progression is in different ways dependent on, among other things, chosen teaching strategies, the assessment instruments applied etc. These in varying degrees impact on the student through the way in which they perceive their work being evaluated. Also learning theories used in explaining this progression might be seen as partisan, prejudicing against particular learners or against certain capacities or potentialities present within all learners.

Research concerns in depicting transition

Insofar as mathematical learning supports both intrinsically mathematical concerns as well as more utilitarian enterprises, facility with both abstractions and concretisations seems crucial. Mathematical agendas perhaps privilege the former, while more utilitarian agendas (including those frequently assumed within school mathematics) privilege the latter. It was these concerns that led us to question the ways in which differences of results between the two studies cited have been put down to sampling differences, different experimental conditions etc. (Herscovics and Linchevski, 1994, p. 75). Rather than to incommensurability of the research models being applied. The very quest for some “clear-cut demarcation” between arithmetic and algebra seems fraught with difficulty from the outset. We cannot breach the inevitable divide that separates mathematical and cognitive domains within such models. Between the two models there appears to be a dichotomous choice between seeing the transition as separating, in the former, two distinctive mathematical forms and, in the latter, two developmental cognitive stages. It is this disjunction that a more hermeneutic analysis would seek to dissolve (e.g. Brown, 1997). Here we pursue this style of analysis by focusing on how these various factors might be seen as being embedded within the narrative frame that guides the way we talk about mathematical transition and thus how this transition is marked. This analysis will assume a softer relation between the contextual parameters of mathematical activity and features being identified within these to highlight how the narrative layer can be seen as being instrumental in creating both.

SEMANTIC INNOVATION

Over the last twenty or so years Paul Ricoeur (e.g. 1978, 1981, 1984, 1985, 1988) has carried out extensive work on how both metaphor and narrative can be seen as dual components within the overarching frame of what he calls *semantic innovation* (1978, p. 55; 1984, pp. ix-xi). This analysis initially focused on the introduction of a new metaphor into a sentence. He later extended this to include the emplotment (*muthos* - organisation of events) provided within some new narrative account. That is, the inclusion of a new narrative within our account of the world we experience results in a reconfiguration of the way in which the world is experienced and acted within. Ricoeur argues that such semantic innovation can be understood as an extension of familiar understandings held by an individual of the actions s/he takes towards incorporating figures of speech that allow the capturing of mental experience not readily accommodated within previous versions of his or her linguistic usage. He further argues that the passage of time does not lend itself to being described as a sequence of events, features or stages. Instead time needs to be understood as being mediated by narrative accounts of such transitions. These rely on interpretations which at a very basic level cannot be seen as comprising phenomenological features.

Metaphor

Ricoeur considers how a metaphor functions within a sentence, where metaphor is depicted as follows:

The rhetoric of metaphor takes the word as its unit of reference. Metaphor, therefore, is classed among the single word figures of speech and is defined as a trope of resemblance. As figure, metaphor constitutes a displacement and an extension of the meaning of words; its explanation is grounded in a theory of substitution. (Ricoeur, 1978, p. 3)

He suggests that the use of a metaphor activates strains and stresses throughout the whole sentence in which the other words function in emphasising the "impertinence" of the new metaphor (1984, p. ix). These strains and stresses project us out of the realm of familiar literal meanings towards creating a meaning effect unattainable within the parameters of the previous realm. This also resonates with the seminal work of Jakobson where:

Metaphor (i.e live metaphor as opposed to clichés that have lost their figurative force) is necessarily perceived as incongruous or surprising, at first apparently not compatible semantically with its context. Groden and Kreiswirth (1994, p. 419).

In Jakobson's formulation metaphor, which is seen as a shift in literal *sense*, is contrasted both with *metonymy* and *synedoche* . *Metonymy* is where something related stands in for the thing being referred to (e.g. "he's taken to the bottle" rather than "he's taken to the drink"). *Synedoche* is where a part of the thing stands in for the whole ("You mustn't show your face around here again"). These are both seen as shifts in *reference* (Groden and Kreiswirth, ibid.). See also Ricoeur's discussion of these distinctions (1978, pp. 55-59). The distinctions between these various terms are discussed in relation to mathematics education by Tahta (1991) and Presmeg (1997). The former offers an important comment by Wilden (1980, p. 58):

Metaphor and metonymy are not entities, they are categories of distinction, not bags to put things in ... this polar distinction itself has a signification only in a context, and since everything has everything else as its context, it is up to the commentator to define the context he is talking about.

We shall discuss this in relation to an example taken from the mathematical studies cited. The studies distinguish between forms such as 7+?=23 and 7+x=23. Both studies see the former as being strictly within the realm of arithmetic. Meanwhile, the latter, in appearance at least, ventures into simple algebra or "pre-algebra" (Filloy and Rojano, p. 20, Herscovics and Linchevski, pp. 60-63). Similarly, in their study Sfard and Linchevski (1994 b, p. 259) consider some children who have met the former but not the latter. Both ? and x appear as surprising to a student familiar with equations such as 3 + 4 = 7, but surprising in different ways. Taking ? and x as words being introduced into sentences, they have different metaphorical effects. Previously these symbols have been met in different circumstances, both in the mathematics classroom and elsewhere, and will hold particular senses for the student. Applying Ricoeur's analysis, our interpretation of the distinction between ? and x would, very crudely, be as follows. The ? is understood as being a simple "missing" number. Meanwhile, the introduction of the x hints at a new realm within which the significance of x might be seen as rather more than this simple substitution of a fixed numerical value. The inclusion of ? maintains affinity to the class of arithmetical equations, whilst the inclusion of x shifts the context being attended to by asserting affinity to the class of pre-algebra equations as yet unknown by the student. The x does not just add a term, it charges all of the terms within the complete mathematical sentence with a new meaning. In one sense the x can be seen as a simple metaphorical substitution of a single digit number, giving a new flavour to the equation. But in another sense its introduction also synedochially signifies a new realm beyond such simple manipulation. The introduction of the variable into the equation thus results in a complete recontextualisation of the equation resulting in a reevaluation of each component term. So at once the x signifies a single digit, but to someone unfamiliar with such usage it also signifies the possibility of a whole new mysterious realm of mathematical activity, with new syntax and new rules of composition. The mysterious nature of this may not only block the student's transition into facility with these new forms but may also through obscuring familiar forms incapacitate students from performing familiar skills. And indeed the child soon discovers that the x cannot be read only as a simple fixed number.

Narrative

Ricoeur (1988, p. 241) has argued at length that "temporality cannot be spoken of in the direct discourse of phenomenology, but rather requires the mediation of the indirect discourse of narration". Features of time, progress, development and shift are not constituted through agreeable criteria. They all depend on interpretations reflecting attitudes produced within history, ideology and auto-biography. Any such transition embraces the components of a new domain. This entails meeting new syntactic forms which characterise the new *emplotment* (how events are organised in to accounts of what happened) and comprise new styles of compositions, new styles of questioning, a new way of acting, a new way of feeling and new ways of making sense that begin to become familiar through practice. Any movement to a new way of living can only be spoken "by means of the complex interplay between the metaphorical utterance and the rule-governed transgressions of the usual meanings of our words" (Ricoeur, 1984, p. xi). Ricoeur suggests that this moves beyond mere *seeing as*, but rather becomes "*being as* on the deepest ontological level" (*ibid*, our emphasis). This shift also underlies Ricoeur's (1981, pp. 182-196) use of the term *"appropriation"* in his analysis of how we read text. In this the act of reading extends to incorporating the change in the way of living consequential to absorption of the work. Within our analysis such appropriation is not limited to interpretation at the level of immediate comprehension. It also needs to include reflected upon application within performance where the student's storying backdrop is given time to settle. This is discussed more fully in relation to teachers analysing their own reflective writing produced over a period of time within practitioner research studies by Brown (1996 b, 1997, pp. 172-211). Here the meaning of any given piece of teacher writing can be revisited according to how it is composed alongside other pieces written at other points in time in creating any fuller account. For example, a piece of writing depicting an interaction between teacher and pupil might have been created originally as part of an attempt to better understand a teaching technique. It may then later be interpreted as evidence of where that teacher was at the time in terms of conceptualising his or her practice as part of a broader account of how that teacher's practice has developed. We shall consider shortly how this style of analysis might apply in discussions of children's mathematical work and the sorts of narrative that might accompany this.

Ricoeur's analysis commences in a curious way. He begins with an account of Augustine's twelfth century work *Confessions* which discusses the paradoxes of time, an account built up without any reference to narrative issues. He then follows this with a discussion of Aristotle's *Poetics*, which addresses the notion of Plot, written some 1500 years earlier, an account formulated with no reference to time. He then combines them in a thesis within which time and narrative are mutually constitutive whereby "time becomes human to the extent that it is articulated through narrative mode, and narrative attains its full meaning when it becomes a condition of temporal existence" (1984, p. 52). Following Aristotle he suggests Plot combines *muthos* (emplotment - the organisation of events) and the *mimesis* (the imitation of an action) (Ricoeur, 1984, pp. 31-51). But in his analysis Ricoeur introduces three senses to the term mimesis. Firstly, "a reference back to the familiar pre-understanding we have of the order of action (mimesis 1). Secondly, an entry into the realm of poetic composition (mimesis 2). And thirdly, a new configuration by means of this poetic refiguring of the pre-understood order of action (mimesis 3)" (*op cit*., p. xi).

How might such emplotments feature in analyses of children's mathematics? We shall revisit the paper by Filloy and Rojano to see how Ricoeur's analysis might assist us in developing their account of mathematical transition in the context of early algebra. Filloy and Rojano (1989, p. 21) consider the example of Ax + B = Cx within a "geometric model" where a rectangle of dimensions A and x plus a rectangle of area B is equivalent to a rectangle of dimensions C and x. This organisation of events might be seen as an example of emplotment in Ricoeur's sense:

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They also offer an alternative emplotment, namely, a "balance model" where A objects with equal unknown weight x together with B objects of equal known weight are equivalent to C objects with the same unknown weight x (ibid). We agree with Filloy and Rojano that such emplotments are necessary to facilitate transition. The contrasting senses of mimesis mentioned seem pertinent in developing their analysis of children moving from arithmetic to algebraic functioning by way of such models. Here we shall focus in particular on the shift from "undoing" as learnt in the former to carrying out "the same operation on both sides of an equation" in the latter (cf. Sfard and Linchevski, 1994 b). We now refer briefly to Filloy and Rojano's discussion of children using the geometric model.

Filloy and Rojano (ibid) offer the example of a girl who showed considerable proficiency in solving arithmetical equations, including those with negative solutions. Her attention is then turned to the task of dealing with equations where a variable appeared on both sides. In the first instance the student's grappling with algebra can be seen as an attempt to apply previously held arithmetic understanding into new types of problem (mimesis 1). The girl is then introduced to the geometric model to assist her with equations where the variable appears on both sides. There is now an attempt to grapple with new meanings in an as yet unfamiliar domain. In tackling the equation $8x + 30 = 5x + 9$ she is quite able to use the model in simplifying the equation to $3x + 30 = 9$ but then falters in reaching the final solution despite her previous proficiency in handling equations of this form. It has to be suggested to her that she abandon the geometric model and return to her old methods before she can complete the task. This might be seen as exploratory work with new forms (mimesis 2). Thirdly, the model that had assisted this transition becomes increasingly marginalised and we hope eventually the new realm is operated in more confidently as the new home base (mimesis 3). Filloy and Rojano (op cit, p. 23) provide an example of a child breaking away from the geometric model, as she transfers the "operativity on the coefficients to the terms containing the unknown". In being confronted with $16x - 15x$ she can quickly conclude that "There should be one times x". They indicate however (ibid) that in their study the children varied in their level of persistence with any particular model, even when they displayed similar levels of pre-algebra proficiency. Some children persisted with the geometric model even when this required very complicated modelling procedures. Meanwhile, others had already moved to proficiency without needing the model. The course through these three phases is individualised and not analogous to cognitive stages in that the choice of approach made is not directly related to cognitive ability. Rather, it is more down to individual preference of working method, contextual awareness, perceived objective and the narratives these produce. Ricoeur's assertion that time is dependent on and conditioned by narratives offered within it seems closely analogous to the suggestion that mathematical development in students is dependent on and conditioned by the narratives embedded in the teaching and learning process. Some of these narratives are derived from students', teachers' and researchers' respective storying backdrops, built through their familiarity with particular situations and their approach to orienting themselves within them. Others are introduced explicitly for a particular purpose, such as teaching devices designed to enable students to negotiate certain transitions. Such devices necessarily introduce a metaphorical impertinence whereby familiar apparatuses are utilised in unfamiliar ways to introduce a new noticing not available within familiar styles of expression. Here the distancing of the story from the mathematics is an inevitable aspect of the metaphorical adjustment to existing mathematical sense, necessary to enable new mathematical thinking. However, in taking such a move into this modified context familiar styles of expression also begin to mean something else, dislodging to some degree, probably temporarily as work cited suggests, meanings that were previously held securely.

CONCLUDING COMMENTS

Our attempt here has been to a) draw attention to suppositions and assumptions inherent in the ways we describe mathematical learning (i.e. that these accounts are developed within particular discourses), and b) offer an approach to depicting mathematical understanding that explicitly incorporates a more sophisticated notion of time drawing on Ricoeur's hermeneutics. We have suggested that teaching devices (e.g. geometric or balance models as aids in understanding linear equations), which derive from alternative accounts of teaching and learning, can be understood as contributing to the necessary and inevitable temporal dimension of the constitution of the ideas we seek to address in our teaching. That is, they can be seen as emplotments that highlight or analogise particular features and then organise and sequence them. However, it may be that these strategies temporarily draw the students a little away from the mathematical objectives being ultimately sought. So for a child seeking to negotiate a perceived boundary one might understand the need for a plot that sees them across, connecting old emplotments, which have lost some of their old meanings, with new emplotments residing in the extended domain. It is this sort of process through which the metaphorical sense of any mathematical form is challenged to open up new ways of seeing. Mathematics is mediated and articulated through such teaching devices. These devices however should not be seen merely as a means to an end, since such embedding is crucial to the constitution of the ideas being studied within "school mathematics". Such constructions of mathematics however also result in associated constructions of the students working through mathematics construed in this way. That is, the student is seen as "high" or "low" ability, at a particular "developmental stage", "ready" for a particular style of teaching, "mathematically intuitive", an "interpreter" or a "doer", etc. These terms predicate particular learning theories or evaluation strategies, and the particular characteristics they value. Nevertheless, proficiency with concretisations is integral to the broader proficiency of moving between concrete and abstract domains, a proficiency which lies at the heart of mathematical endeavours (at least in schools). Indeed, one might suggest that for many students and many teachers proficiency in specific concretisations forms the backbone and principal motivation of activity pursued within the classroom.

POSTSCRIPT: THE FUNCTION OF RESEARCH

"for nothing is more necessary today than to renounce the arrogance of critique and carry on with patience the endless work of distancing and renewing our historical substance." (Ricoeur, 1981, p. 246)

What is the main function of learning theories and how are they associated with revisions of practice? In the United Kingdom, as in many other countries, "mathematical performance" in school, conceived as a social construct, is closely related to how it is described in government documentation, with all the associated paraphernalia of curriculum descriptions, testing instruments, "competencies" for teachers, etc. Our intention in this paper is not to suggest ways of improving the teaching of algebra. We have preferred to use a very specific aspect of algebra as an example of how mathematical transition can be conceptualised, given that social context is so integral to mathematical formation in the school setting. Thus the consequence of reading this paper may not be so much a task for the teacher of adjusting teaching techniques but rather to become more sensitive to the enterprises particular approaches favour. We thus diverge from cognitive stage theories whose analysis is centred around students progressing through successively more difficult mathematical concepts. Our analysis has sought to emphasise the inter-relation, within a socially constructed domain, of mathematical competencies, mathematical understandings, teaching methods, learning theories, curriculum frameworks, research perspectives, etc.

We also need to take care in understanding the ways in which we perceive shifts in the conceptions of the mathematics education research community. We conjecture that it is unhelpful to understand research as enabling us to move towards an ideal whereby we make iterative steps towards a better way of doing things. Change itself is the characteristic we seek. We need to understand research as a mechanism present within this process of change through which we distance and historicise ourselves. Ricoeur's analysis suggests that narratives might be seen as always imperfect accounts of time but of a time that depends on these very narratives. Thus, Ricoeur still seeks to give a higher status to narratives in the construction of time. He also demonstrates the rather fine dividing line between historical and fictional constructions (Ricoeur, 1985). Teaching strategies are often seen by researchers as subordinate to the mathematical conceptions they seek to engender. Here we have asserted here that the teaching devices of school mathematics need to be understood more as constructed and implicit components of the mathematical ideas we wish our students to encounter. Similarly, research discourses inevitably create the analytical frames we use which in turn create the objects we research; objects that grow whether we acknowledge this growth or not.

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Manchester Metropolitan University,
799 Wilmslow Road,
Didsbury,
Manchester,
M20 2RR,
UK.