Towards a hermeneutical understanding of mathematics and mathematical learning

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Introduction

Pursuit of the notion of mathematical meaning has dominated much of the discussion to do with the philosophy of mathematics education. The flavour of current discussion generally seems opposed to notions of absolute meaning towards seeing meaning as a socially constructed phenomena. Such a move seems consistent with theoretical shifts in other academic fields but it still retains a tendency towards seeing meaning as in some ways independent of time and context, a notion associated with concepts rather than with conceiving, a fixity to which the learner converges. In particular, any attempt to assign a socially conventional meaning remains problematic since any account of what this might be is mediated by some symbolic system subject to the interpretation of any individual user.

Hermeneutics, the theory and practice of interpretation, attends to the process through which we develop an understanding of the world. Unlike post-structuralism which asserts ‘a multiple play of meaning held in language’ (Urmson & Ree, 1989), hermeneutic understanding is more governed by a belief that whilst the world may exist independently of humans, it cannot present itself directly to the human gaze. The hermeneutic task can then be seen as an uncovering of meaning, but a meaning dependent on the media and experiences through which it is observed.

The principal task of this chapter is to explore the consequences of asserting that mathematics is an essentially interpretive activity, comprising a system of symbols that is only activated within individual human acts. By seeing mathematical expressions as being used by humans in particular situations, rather than as things with inherent meaning, emphasis is placed on seeing mathematical activity as a subset of social activity, and as such, is subject to the methodologies of the social sciences. Firstly, I shall outline some issues arising through seeing mathematical expressions as being necessarily contained in action, resulting in meanings that transcend mathematical symbolism. This is followed by a discussion of how the emergence of mathematical phenomena in human understanding is a consequence of a linguistic process of classifying. After outlining certain ideas within hermeneutics, the method is discussed as an approach to describing mathematical learning and assessment of this.

Speaking and writing as acting

In Philosophical Investigations, Wittgenstein (1958) suggested that the meaning of a word might be seen as its usage in language. Here, the notion of meaning as a phenomena subsides as a greater emphasis is placed on what an individual sees as being held in a given symbolic form in a particular time-dependent context. Within mathematics education this can facilitate a move away from seeing mathematics as something with an independent existence but rather as something that arises in the context of some particular social event; a style of activity rather than something to be learnt. A necessary characteristic of any social event however, is a style of symbolising facilitating a system of exchange. In a situation concerned with mathematical learning this symbolising will take many forms, using both inherited and original symbols in ways that endeavour to enable communication between participants. So viewed mathematical utterances, both spoken and written, can assume many attributes associated with any linguistic utterance in a social space. For mathematics, it is the symbols themselves that are the material reality since mathematical phenomena do not have a tangible existence outside of symbolisation. However, the symbols are human creations that arise in speech or writing and are a subset of other human expressive acts and as such are subject to the methodology of social scientific enquiry. In particular, any issuing of a mathematical utterance into a social space needs to be seen as an action with a meaning that transcends any purely literal enquiry.

In his work on word usage Austin (1962) identifies three levels of effect arising from any sentence being uttered in a social context. Firstly, the locutionary effect is that of the literal meaning where the words are taken at 'face value'. Secondly, the illocutionary effect is that done <u>in</u> the utterance. For example, if I say 'I name this ship', the action is in the saying. Thirdly, the perlocutionary is that done <u>by</u> the sentence, for example, the effect on the atmosphere in a classroom as a consequence of the teacher saying something in a loud, disapproving tone. Ricoeur (1981) has suggested that these three effects are in a hierarchy according to the degree of interpretation required. The interlocutor can resolve the meaning of the locutionary effect by consulting a dictionary. However, to understand the perlocutionary effect one needs to have experience of being in the appropriate language using community where sentences are used alongside physical actions in a material environment.

In a social situation being directed towards mathematical learning, a variety of linguistic forms will be used within a broad communicative environment. Some of this language will be specifically associated with conventional mathematical ideas, but much will be less precise, supporting other aspects of the exchange. The exactitude often associated with mathematical symbolism does not characterise the communication taking place in many educational situations. Rather, in most learning situations we are concerned with activity taking place over periods of time where learning requires making sense of engagement in this activity and cannot be seen as the composite of getting to grips with a number of clearly defined 'mathematical' concepts. The placing of any written or verbal utterance into this influences communication in the way furniture might influence movement around a room. In this sense, Kaput (1991) who has explored the possibility of reconciling the use of inherited notations within a radical constructivist philosophy, has suggested that in this respect the placing of Cuisenaire rods in a particular arrangement might be seen as a form of writing.

Indeed there are many forms of mathematical discourses each flavoured by their particular social usage. For example, a university lecturer might speak as a platonist, where utterances are made as if they were extracts from a transcendental world of mathematics. Rather different would be the speech of a representative of the National Curriculum Council seeing mathematics in terms of how it might be partitioned for the purposes of testing. Different again would be someone who regards mathematics as a subject whose prestige is consequential its ability to be applied in 'real life' situations. Richards (1991) offers other examples. Each of these people would enter into a dialogue placing varying stresses on their mathematically inclined utterances according to some ideological tainting. A string of symbols, no matter how neutral it may appear, cannot be seen independently of the context into which it is being issued. The selection of this string presupposes a selecting by someone with a particular purpose.

Mathematics as a language

Can mathematics be seen as existing outside of the language that describes it? The question is related to the more general concern of how the world can be seen outside the language that describes it. In describing perception Peirce identifies three ascending levels; quality (the initial sense), fact (the identification of objects), law (the relations between objects). Perceiving the world requires categorisation of it which involves differentiating and relating aspects of this world. It is towards this end we use language. Mathematical phenomena, those aspects of the world seen as pertaining to magnitude and number and the relations between them, emerge in this process, consequential to the way in which the world is sub-divided into categories. However, this very categorisation mediates and thus organises our way of seeing.

The symbols and classifications of mathematics are historically determined. They are arbitrary in the sense that the symbols and classifications of any language are arbitrary. As such it can be viewed phenomenologically in that these symbols and classifications have particular meanings for any individual derived from that individual's experience of their usage. The field of mathematics only comes into being in its classification in language. The sense of this field can only be perceived retroactively and its existence presupposes a language. For someone learning mathematics there is a similarity with learning a language in that there is a need to grapple with an inherited mode of symbolisation and classification, arbitrarily associated with some preexisting world.

The style of describing mathematics is necessarily interpretive according to some mode of signification; a particular way of fitting words and symbols to experience. Barthes talks of certain styles of signification becoming naturalised in the sense that they become the culturally conventional way that seems to be entirely neutral, in the way that a realist painting might be regarded as the most straightforward representational mode. This is discussed by Coward and Ellis (1977). However, in learning mathematics, using alternatives might be seen as a way of initiating productive tensions for forcing awareness towards re-evaluating this supposed natural way. The linguistic overlay given to a situation can be seen as a way of introducing an interplay between the describing and that described. By asserting mathematical activity as essentially interpretive in nature, the production of meaning in this activity can be seen as deriving from a dialogue in a continuous process of introducing linguistic and symbolic form into the socially active space.

As an example, consider the flavours that can be given to a 4x3 rectangular lattice in the context of a particular activity, described in Brown (1990).

a)
* drawn on squared paper

b)
* made out of plastic squares

c)
* drawn on the chalkboard

d) Captured in writing or in spoken words:
e.g. 'A rectangular garden lawn surrounded by a path comprising ten square paving stones.'
'Two green squares side by side surrounded by red squares.'
' Four squares in the top row, four in the bottom and one at each end of the lawn.'

e) Located on a table

garden	1	**2**	3	4	5	6	n
lawn	1	**2**	3	4	5	6	n
path	8	**10**	12	14	16	18	2n+6

f) Pictured in the imagination

Each of these metaphorical representations open up a form of describing such lattices as they change dimensions. The play arising from making (metaphorical) leaps between such forms and (metonymic) moves within such forms results in successive acts of fitting and associating forms. By seeing equivalences between forms we can choose the form most suitable for our current purposes. For example, if we see a 153x3 rectangle as the 151st garden we do not need to build it with plastic squares and may prefer to deduce the information we need (e.g. how many red squares are needed for the path?) algebraically.

Saussure's influential work in linguistics, carried out at the turn of the century, was directed at the structure of the layer mediating experience. For him the signifier is the mental image or sound of a word or symbol. The signified is the mental concept with which the individual associates it. Together, the signifier and signified form the sign, a wholly mental phenomena constructed by the individual. Two forms of arbitrariness are implicit here. Firstly, if we take the signifier 'square', the word itself is quite arbitrary and is in fact different according to the language you are speaking (e.g. it is 'kwadrat' in Polish). Secondly, a square is a special sort of rectangle, or a type of regular hexagon, or a type of rhombus. It is an arbitrary category and does not need a name of its own. We only introduce a name for convenience since in the way in which we operate in the world we use it a lot. Such a category may not be so crucial for an aborigine.

Saussure did little in investigating how such signs are associated with the real world, but rather saw meaning as being derived purely from the play of differences between signs. In this way, mathematics as a language, held in symbols, can be seen as independent of the real world, a mediating layer rather than a quality endemic in the physical world. So viewed categories of mathematics are cultural rather than transcendental, arbitrary rather than implicit. The play of meaning is consequential to sets of words being combined by humans in individual speech acts. In seeing mathematics as a language, as educators, we are not so much concerned with its qualities as a system (langue) but rather with its realisation as discourse in the social environment (parole), i.e. with the way in which it is being used to signify, towards producing meaning.

The phenomenology of acting and meaning

By seeing mathematics as something arising in social activity and the framing of mathematical statements as social actions, we are permitted the possibility of employing social scientific techniques towards establishing mathematical understanding. Much recent work in the human sciences has worked from the premise that the individual human subject perceives the world phenomenologically, that is, s/he sees the world comprising phenomena having particular meanings to him or her in particular contexts. Here individual objects are not seen as having meaning in themselves but only take on a meaning in the gaze of the individual who sees them from his or her own particular position and according to his or her current interest.

Underlying this view are specialised uses of the terms objective and subjective. Whilst there may be an independent material reality it only comes into a meaningful 'objective' reality when classified within the language of an individual human subject. I might talk about the situation I see myself in, as if it were independent of me, but I can only do this only after experiencing myself as part of it. In this way object and subject are in some sense part of each other. An object can only present itself to the gaze of the individual subject with his or her own particular phenomenology. The world of material objects only comes into being retroactively through being captured in language. An individual's consciousness is always a consciousness of... and is always intentional insofar as it seeks to make sense of that within its gaze according to some schema. A consciousness is always of an object and an object only presents itself to a consciousness. The nature of the objective is dependent on the way in which it is captured and accounted for in language by the subject. The mediating layer through which language is derived seems inescapable, brought into existence by consciousness itself and its need to organise that which it perceives. These ideas, generally accredited to Husserl, are discussed at length by Ricoeur (1966).

In making a mathematical statement I express certain intentions but am unable to guarantee that I communicate the meaning I myself attach to this statement. Such an action and its meaning to me are consequential to my categorisation of the world in which I see myself as part. Whilst I may attempt to predict how my action might be read by others the meaning of my action cannot be seen only in terms of my intention since it cannot be seen independently of the social environment into which it is issued. In addressing this Ricoeur (1966) differentiates between the 'voluntary' and 'involuntary' components of any action. The individual subject can assert him or herself through the voluntary component of an action, i.e. that which he or she intends. The meaning of this action, however, only emerges as the resistances to this action take shape around it. These resistances, the involuntary component of the action, have no meaning in themselves, but rather are the contextual framing activated by the voluntary component. This implies a hermeneutic process where the subject voluntarily acts in the world he or she supposes it to be, but this in turn gives rise to (involuntary) resistances which are always at some distance from those anticipated. In order to act, however, there is a need for the subject to suspend doubt and act as if his or her reading is correct. This has been discussed in more depth by Brown (in press).

In later work, Ricoeur (1981) talks of the 'meaningful effect' of an action as being its 'objectification'; *the mark it leaves on time*. Thompson (1981), Ricoeur's translator, sees this as being related to the way in which the action might be described in retrospect, as if in some historical account. Ricoeur explores this in terms of an analogy with the objectification speech goes through in being committed in writing. By pursuing the paradigm of text interpretation he sees acting as analogous with writing and interpretation of this action as reading. It is through this sort of fixation that we can employ techniques of interpretation for both tasks of understanding (learning through signs) and of explanation (learning through facts), by seeing such tasks as necessarily inter-twined. It is this relation between understanding and explanation that hermeneutic enquiry seeks to unfold.

Hermeneutic understanding

Hermeneutics was originally developed and employed in the analysis of biblical texts but was extended, largely by Dilthey working at the turn of the century, to cover the whole of human existence. Leading modern exponents are Ricoeur and Gadamer who have developed it within phenomenology. Hermeneutics, whilst acknowledging that some interpretations are better than others recognises that none is ever final. Hermeneutical understanding never arrives at its object directly; one's approach is always conditioned by the interpretations explored on the way. While one's understanding may become 'fixed' in an explanation for the time being such fixity is always contingent. In choosing to act as if my explanation is correct, the world may resist my actions in a slightly unexpected way, giving rise to a new understanding, resulting in a revised explanation, providing a new context for acting and so on. This circularity between explanation and understanding, termed the hermeneutic circle, is central to hermeneutic method.

Hermeneutics resists distinctions frequently made between the explanations of the natural sciences (knowing through facts) and the understanding of the human sciences (knowing through signs), preferring to see them both as subject to an interpretive framework. Within history, for example, whilst it may be possible to continue offering ever more interpretations of 'what happened?' if we are to act in the light of this knowledge we have to suspend doubt for the time being and assume a certain position towards get things done. Conversely, taking mathematics, as an example from the other end of this scale, while we may have statements that 'on the surface' seem entirely incontrovertible, it is still necessary for an individual human to decide how such statements will be used in the social space or how they have been used. This is discussed in relation to mathematics teaching in Brown (1991).

In speaking of mathematics I cannot simply quote, in a neutral way, expressions as if from some platonic formulation. I am necessarily acting in time - whereas platonic mathematics is outside of time. Further, in doing this I refer, by implication (through the perspective I reveal), to myself, to the world I see, and to the person(s) to whom I am talking. Ricoeur (1981) emphasises these discursive qualities of language usage in distinguishing langue and parole. In doing this he combines Saussure's linguistics with the speech acts described by Austin (1962) and Searle (1969). Mathematics is only shareable in discourse and the act of realising mathematics in discourse brings to it much beyond the bare symbols of a platonic formulation of mathematics. The mathematics I intend to communicate is always mediated by the explanatory procedures of such a social event. My interlocutor is obliged to interpret my speech, reconciling parts with the whole, stressing and ignoring as he or she sees fit. The distinction between knowing through facts and knowing through signs becomes blurred in this process since the facts of mathematics are immersed in the usage of them. The expressions of mathematics are only arising within actions in social events

Notions of hermeneutic understanding as applied to mathematics then require a shift in emphasis from the learner focussing on mathematics as an externally created body of knowledge to be learnt, to this learner engaging in mathematical activity taking place over time. Such a shift locates the learner within any account of learning that he or she offers, thus softening any notion of a human subject confronting an independent object.

In this way positivistic descriptions that draw hard distinctions between process and content of learning mathematics are avoided since there is no end point as such but rather successive gatherings-together of the process so far, seen from the learner's perspective. In such an educative space, characterised by the communication of mathematical thinking, the introduction of different interpretations gives rise to the possibility of a productive tension between mathematical activity and accounts of it, enabling the very hermeneutic process of coming to know through juxtaposing varying perspectives as in the example above.

The exact expressions conventionally associated with mathematics only ever find expression in such activity, within the context of many other sorts of expression. Such statements are always, in a sense, offered as part of a distillation process; a 'looking back' concerned with pinning down key points of the event. The reflective dimension inherent in this results in the active generation of mathematical expressions through time being part of the reality described. Similarly, the intention to learn is always associated with some presupposition about that to be learnt and learning is in a sense revisiting that already presupposed. This continual projecting forwards and backwards affirms an essential time dimension to mathematical understanding that can never be brought to a close by an arrival at a 'concept', since the very framing of that concept modifies the space being described.

Where then lie conventional notions of mathematical understanding? This issue seems problematic in that within hermeneutics understanding does not pertain to concepts with fixed meanings. Understanding is a process rather than a state. This clearly runs in conflict with sublime notions of understanding that suggest a state beyond that accountable in words. A more humble notion of mathematical understanding may be that it is simply the ability to tell a package of convincing stories generated by the learner himself or borrowed from the teacher. Further, this understanding is only proven if the learner can make use of certain aspects of the conventional, inherited system of exchange.

Describing and assessing mathematical learning

As educators we are often not so much concerned with learning as with giving an account of learning. This might be the reproduction of a famous result, the application of a method in a particular real life situation or some verbal explanation of some work completed. Such an account is necessarily in some symbolic order associated with some over-arching system of exchange. For the student engaging in a mathematical activity there are a variety of ways of reporting back on the experience. The nature of any understanding demonstrated in this report, however, is always conditioned by the method of reporting chosen. But what can be captured in such a report? Is it the mathematics, the understanding, the activity? None of these can be described purely within the realm of mathematics, whatever that is. Some perspective of the describer is required and this depends on his position, his biographically defined background and his current motives.

By seeing the assessment of mathematics being directed towards the student making sense of his mathematical activity we overtly move in to the realm of interpretations. Assessment by the teacher can be directed towards participation in a dialogue involving the student in generating linguistic forms in respect of his view of the activity. A two-tiered interpretation is implied here; the student capturing his experience in symbolic form, and the teacher assessing this symbolic product as evidence of understanding. What is not implied here is any notion of a universal meaning to which both teacher and student converge, but rather '..objectivity is achieved through the coincidence of interpreting, that is, agreeing' (Brookes, 1977)).

One might legitimately protest that there is a certain power relation here that creates a somewhat asymmetrical sort of agreeing, where the teacher, as representative of the conventional way of talking about things, sees her task as introducing this. Whilst the child may have the opportunity of offering some account of their understanding, within their own mode of signifying, the teacher, in entering any discussion, introduces a more conventional mode. The communication being sought in such an exchange brings into play some symbolic medium, comprising symbols, actions and words. But such is the power of the conventional mode of discourse that the quest for the learner may be to believe that he is joining the teacher in using the inherited language. This highlights a particular aspect of the teacher's power, consequential to the linguistic overlay she brings to the situation. The teacher's style of looking is accustomed to spotting concepts which are, after all, merely culturally conventional labellings. In this way the teacher's way of making sense of a student's work involves classifying this work as if looking to tick off categories on a National Curriculum checklist. The student's access to any notional transcendental mathematics is always mediated by a social pressure to capture this in the categories arbitrarily assigned by our ancestors. I would argue that much investigational work, such as that described in Brown (1990), permits the student to develop their style of signification more fully, prior to interception by the teacher introducing more conventional ways of describing the product, than might be possible in more traditional approaches.

Concluding comments

By accepting a hermeneutic view of mathematical understanding we give primacy to the linguistic qualities of mathematical learning and so soften the distinction between mathematics and other disciplines. Mathematics becomes something held in the expressions of participants in mathematical activity, who are asserting their view of, and their relation to, some supposed mathematics. The reality of any transcendental mathematics relies on people acting as if it is there. The assertion that mathematics has no existence outside the material symbols that describe it echoes the 'lack' that Lacan describes as emerging after the layers of description are peeled away (Brown, Hardy and Wilson, 1993). Whilst this may be too extreme for many professional mathematicians, the transcendental mathematical truth that might be uncovered by hermeneutic enquiry cannot escape some flavouring from the process through which it is reached by the individual.

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