The effect of operation type, operand-order and problem-size on processing simple arithmetic

Rebecca Webb

Supervised by: Dr Philippe Chassy

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Abstract

Once learned, simple arithmetic facts are thought to be represented in a dedicated long term memory store, known as an arithmetic retrieval network. Although the existence of this network is generally agreed upon, the organisation and accessibility of it is not. The aim of the current study was thus to test the effect of operation type, presentation format and problem-size on the processing of simple arithmetic. In two separate experiments participants were presented with a series of single digit addition, multiplication and 'wrong' problems. In experiment one they indicated which operation sign was used in each problem and in experiment two they indicated what the correct answer to each problem should be. In both experiments, the participants responded to large multiplication problems more slowly than addition and small multiplication problems, but more quickly than wrong problems. Participants were equally fast in responding to small multiplication and addition problems in experiment one but faster to respond to small multiplication problems than addition problems in experiment two. In addition to this, there was an operand-order effect in experiment two but not experiment one. The results challenge two main assumptions of the identical elements model but lend support to the interacting neighbours model. A new model is proposed to account for some of the short comings of previous models.

KEY WORDS	NUMERICAL COGNITION	OPERAND-ORDER EFFECT	IDENTICAL ELEMENTS MODEL	PROBLEM-SIZE EFFECT	INTERACTING NEIGHBOURS MODEL

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Introduction

Simple arithmetic problems can be defined as single- digit equations with only two operands, each of which are no larger than 10. Many models of arithmetic agree on the assumption that, once learned, certain simple arithmetic equations become stored in an arithmetic retrieval network (Ashcraft, 1992; Campbell, 1996; Rickard, Healy, & Bourne, 1994; Verguts & Fias, 2005). However, there is little agreement on how this retrieval network is accessed, how it operates or how it is organised. The aim of the current study is thus to investigate the effect of the operation type, presentation format and problem-size on the direct retrieval of simple arithmetic equations from memory. In doing so, this could give important insight into how simple arithmetic is processed, which could lead to more valid ways of testing arithmetic in future studies, and eventually help the education sector develop more appropriate methods of teaching arithmetic in schools.

Overview of the theories

Associative models of simple arithmetic (Ashcraft, 1992; Campbell, 1996; Rickard, et al., 1994; Verguts & Fias, 2005) assume that, within the memory network, there are memory nodes representing mathematical problem sets, such as |4x2| (with **|** representing the start and end of the problem set/memory representation) and memory nodes representing the answers to those problems, such as |8| (these will be referred to as problem nodes and answer nodes, respectively). During retrieval the problem node becomes activated, which triggers the activation of the associated answer node. According to the identical elements (IE) model (Rickard et al., 1994), the only way any of these problem nodes can become activated is if the three key elements of information in a presented problem match that of the problem node. These elements include the two operands (the two digits on either side of the operation sign) and the operation type (whether it is addition, multiplication, division or subtraction). For multiplication problem nodes, the order in which the operands are presented does not matter. However, for division problems, there are two separate problem nodes for possible order, and thus the order in which these problems are presented is essential for the arithmetic retrieval of division problems (see Figure 1) (Rickard, et al., 1994).

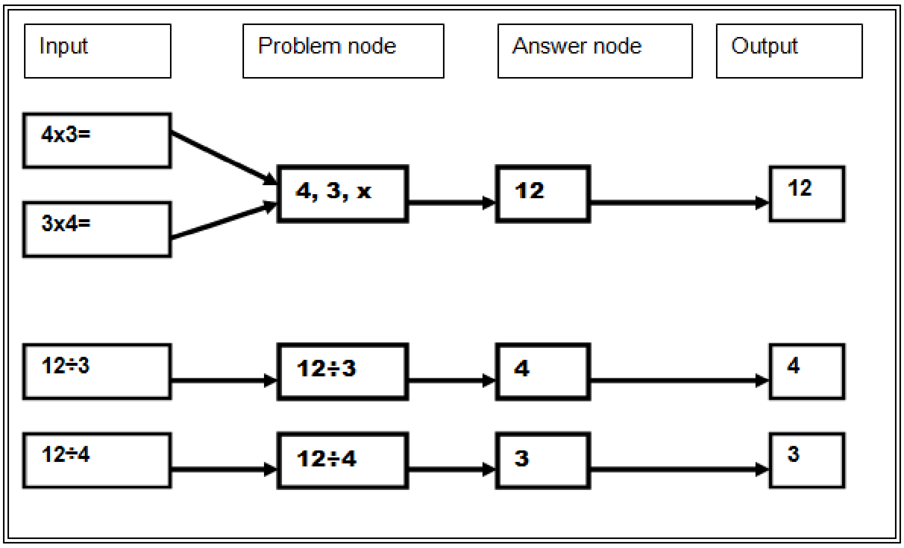


Figure 1. A model to exemplify the process of arithmetic fact retrieval for simple multiplication and division problems (Rickard, et al., 1994).

In contrast to the identical elements model (Rickard, et al., 1994), as well as other models of arithmetic (Ashcraft, 1992; Campbell, 1996), the interacting neighbours (IN) model (Verguts & Fias, 2005) assumes that the retrieval network only stores multiplication facts. Because of this, the operation sign is unnecessary for the activation of the problems nodes. A further assumption of this model is that before the problem node can be accessed, activation must pass through two input nodes (one for each operand), which are arranged in a preferred operand-order. If this preferred order is organised in a min-max fashion, the presentation of a problem, such as |2x6|, will allow for an easy access of that problem node. However, if the node is presented as |6x2|, the operand-order must be mentally switched round before the corresponding problem node can become activated. Another assumption of this model is that, before a holistic answer node is activated, separate decade and unit output nodes are activated. The final assumption is that when the correct problem node becomes activated, neighbouring nodes also become activated. If the value in the decade output node of the neighbouring problem is different to the target answer, retrieval is delayed (see Figure 2).

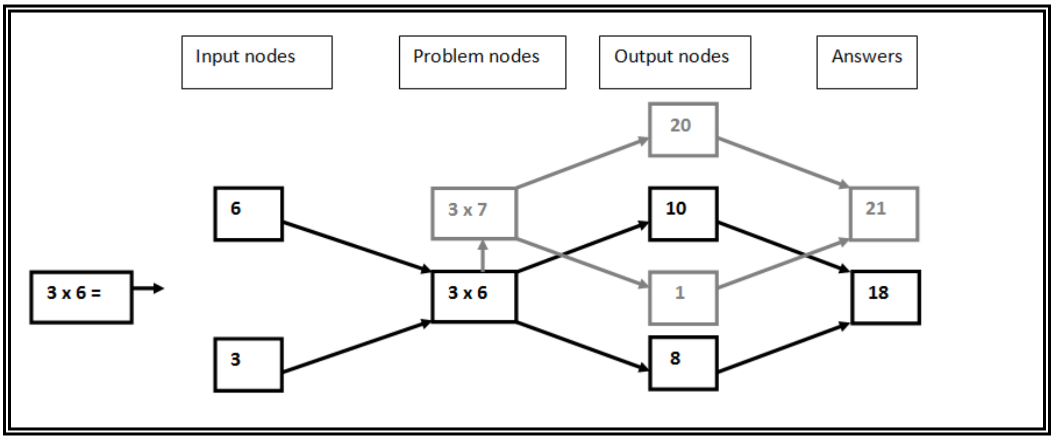


Figure 2. A model to exemplify the retrieval process of multiplication fact retrieval, as assumed by the interacting neighbours model (Verguts & Fias, 2005), with grey representing the activation of neighbouring nodes.

The problem-size effect

The main assumption that distinguishes the IN model from other models of arithmetic is its explanation of the problem-size effect. This refers to a large response time (RT) for large multiplication problems and a smaller RT for small problems, defined as problems with a product of above 25 and below 25, respectively (Zhou & Dong, 2003). The majority of arithmetic processing models agree with the early theory that the problem-size effect occurs because small problems are retrieved and large problems are calculated (LeFevre et al., 1996). However, research by Brauwer, Verguts and Fias (2006) has demonstrated that the problem-size effect can decrease with practice, which may suggest that it is possible to develop new memory nodes for larger problems and thus that the use of computation for larger problems is not necessarily the case. The IN model (Verguts & Fias, 2005) assumes an alternative account of the problem-size effect. According to this model, large multiplication problems are retrieved directly from memory and the problem-size effect is caused by a larger amount of inconsistent neighbours (output nodes with a different value to the target answer) for larger problems compared to smaller problems.

Evidence to support this model can be found in research by Domahs et al. (2007), which showed that the correlation between problem-size and accuracy in a multiplication task vanished when the amount of neighbourhood consistencies was included in the analysis. However, it is important to note that neighbourhood consistency also correlates with problem-size, which may mean that both correlations in this study tested the relationship between problem-size and RT but in an alternative way. More useful support for this theory can be found in research by Campbell, Dowd, Frick, McCallum and Metcalfe (2011), which showed that learning a novel set of equations, whereby neighbourhood consistency was not correlated with problem-size, was harder when the equations had more inconsistent neighbours. This suggests that interference between inconsistent neighbours inhibited learning and that this was not due to the size of the problem.

Still, the theory that the problem-size effect is caused by the use of computation for larger problems, as opposed to retrieval for small problems should not be ruled out completely. Self-report studies have shown that people do rely more on procedural methods to solve maths equations that are large and retrieval when they are small (LeFevre et al., 1996; LeFevre, Sadesky, & Bisanz, 1996) and, although the validity of self-report methods have been criticised (Kirk & Ashcraft, 2001), there is recent fMRI evidence to confirm these findings. For example, Jost, Khader, Burke, Bien, and Rösler (2009) found increased activation in the cingulate gyrus for large multiplication problems relative to small ones, suggesting that small and large problems are processed differently in the brain. Furthermore, an EEG study by Jost, Hennighausen, and Rösler (2004), revealed more activity in the intraparietal sulcus (IPS). This, together with research showing that the IPS is a region of the brain involved in processing numerosity, which is needed for calculation (Dormal, Andres, & Pesenti, 2012) further supports the theory that large problems are calculated.

There is also behavioural research to support the theory that larger multiplication problems are calculated. For example, it has been shown that more interference occurs as a result of practicing small multiplication problems and answering addition problems, than between practicing large multiplication problems and answering addition problems (Campbell & Timm, 2000). This indicates that the retrieval network is specific to small multiplication problems, and thus that larger multiplication problems must be calculated. It is important to note however, that this evidence is based on the assumption that multiplication and addition facts share the same retrieval network, a theory that has also been argued by the IN model (Verguts & Fias, 2005) and one that will be discussed in the following section.

A multiplication-specific network

Firstly, there is evidence to suggest a multiplication specific network. For example, research by Zhou (2011), has found there to be more neural activity in the left anterior region of the brain in response to multiplication problems, as opposed to addition problems. This, together with the finding that the left anterior regions are involved in arithmetic fact retrieval (Emerson & Cantlon, 2012), suggests that multiplication problems may be processed within the retrieval network, whereas addition problems are processed elsewhere in the brain.

However, an alternative research study has also indicated the involvement of left anterior regions of the brain during the processing of addition problems (Cho, Ryali, Geary, & Menon, 2011), suggesting that it is possible to store and retrieve addition problems directly from memory. One explanation for this is that, because the subjects in this study were children, the arithmetic retrieval networks had not yet developed in to the multiplication-specific network proposed by the IN model. Indeed, a later fMRI study by Zhou et al. (2007) indirectly indicated a dissociation in regional activity for processing simple multiplication and addition problems in adults. Specifically, multiplication problems elicited more activity in language related areas, whereas addition problems elicited more right hemisphere activation in the IPS. Furthermore, Foyal and Thevenot (2012) and Sohn and Carlson (1998) found that priming participants with the operation sign helped them produce the answers to addition problems but not multiplication problems, indicating the pre-activation of an abstract procedural method for addition problems but not multiplication, suggesting that addition problems must be calculated. However, this does not explain why alternative research has shown an interference effect between practicing multiplication problems and answering addition problems (Campbell & Phenix, 2009; Campbell & Thompson, 2012).

It is thus clear that there remains a lack of consensus regarding whether or not the retrieval network is specific to multiplication facts and because of this, confusion also remains over the cause of the problem-size effect. Another, more indirect way of investigating whether or not the network is specific to multiplication facts is by looking at whether the operation sign is an essential element in arithmetic retrieval. If the network is specific to multiplication, the operation sign should not be needed to retrieve multiplication facts. This leads the reader to the next section in the literature review, which will discuss the assumption of the IE model that the operation sign is needed for the retrieval of arithmetic facts (Rickard & Bourne, 1996).

Identical elements

As outlined earlier, the identical elements model suggests that all three elements of an arithmetic problem must be present if the corresponding answer of that problem is to be retrieved from memory. Early evidence to support this theory can be found in a study by Rickard and Bourne (1996), which showed that practicing answering multiplication equations did not facilitate the test performance on the corresponding multiplication problem. For example |3x4=12| during practice did not facilitate test performance for |12/4=3|. Furthermore, Galen and Reitsma (2010) found that test performance on a simple answer production test was not improved when participants practiced arithmetic equations with a missing essential element, suggesting that the absence of a key element made retrieval impossible during practice.

In contrast to this, previous research has demonstrated cross-operational facilitation effects, even when the operation sign at practice was different to that at test (Campbell & Phenix, 2009; LeFevre & Morris, 1999; Mauro, LeFevre, & Morris, 2003). This suggests that the three elements needed to activate a problem node can include the operands and the answer, rather than the two operands and the operation. However, there is a problem with this suggestion. The participants in the study by LeFevre and Morris (1999) reported recasting the division problems into their multiplication counterparts during practice, so |24/4| was practiced as |4x6=24|. This suggests that the practice-test facilitation effect was actually caused by practicing the same operation type, rather than a different operation type. This confirms the theory that there are three key elements essential to the retrieval of arithmetic facts (Rickard, et al., 1994). That being said, it does not give a full account of how the retrieval network is accessed, as it ignores the possibility of a third element; the physical order in which the operands of the problem are presented, another assumption of the IN model (Verguts & Fias, 2005).

Operand-order effect

Support for this theory can be derived from research by Zhang, Si, and Zhu (2012), which showed different RTs in a simple arithmetic task depending on the order in which the operands in a problems were presented. Further support for this can be found in a study by Zhou, Zhao, Chen, and Zhou (2012), who measured the neural activity elicited by eyemovements of participants whilst they completed a simple arithmetic task. They found that, when problems were presented with the larger operand first, participants looked to the left operand during the visual processing stage but to the right operand during the retrieval stage, suggesting that the smaller operand was required to access the arithmetic retrieval network.

In spite of this, the theory of an operand-order organisation of the retrieval network should not be ruled out completely, as there is evidence of an operand-order effect in a Western population. For example, Arbuthnott and Campbell (1996) found, in a Canadian sample, that the interference was larger when the operand value and order of the problems at practice matched that at test, as opposed to when they just matched the operand value. Another study, also using a Canadian sample, found an operand-order effect for auditory stimuli (Kiefer & Dehaene, 1997). However, it is important to note that the researchers interpreted this as evidence to suggest that there is an essential operand-order organisation of the retrieval network, and thus that problems that do not match the order and cannot be automatically switched to match, must be calculated. This disagrees somewhat with the IN model (Verguts & Fias, 2005), which assumes only a preferred operand-order organisation of the network. More importantly, there is also evidence to challenge the operand-order effect altogether. For example, Robert and Campbell (2008) found no RT difference depending on the order in which the operands were presented and Rickard and Bourne (1996) found that problems presented in a specific operand-order facilitated test performance for the same problems presented in the opposite order, suggesting a common memory node for both multiplication facts. Zhou et al. (2007) have also provided evidence to suggest that the operand–order effect was a product of learning the multiplication Table in only one order, a method unique to the Chinese education system.

Conclusions

It is clear that there remains a lack of consensus regarding many of the assumptions and theories about the arithmetic retrieval network. There is strong, contrasting evidence to support each account of the problem-size effect, resulting in a conflict that is yet to be resolved. This is also true to the debate regarding whether or not the operation sign is essential for arithmetic fact retrieval. The research directly used to support the IE model leans towards the conclusion that the operation sign is needed for arithmetic fact retrieval. However, the research indicating that the retrieval network is specific to multiplication facts suggests otherwise. Finally, although evidence of an operand order effect is lacking, this may be attributed to the lack of research in Western samples. It thus calls for further investigation in to the operand-order effect in a Western sample.

The present study

Experiment one

The first experiment tested participants' speed and accuracy in indicating the missing operation sign in a series of mathematical equations, as indicated by a blank space between the two operands. For example |2 4=8|. This tested their ability to check the answer they retrieved/calculated against that in the example and then recall which operation type they used to do this. The main aim of this experiment was to test whether the lack of operation sign caused participants to use direct retrieval or calculation. If they use calculation, addition will be faster than multiplication because it presumably takes less time to transform (add together) two numbers into an answer for addition than it does to add together several times for multiplication problems. This effect should remain when large problems are removed but it should be smaller, as less calculation is needed to work out small multiplication problems than large ones. If there an effect of operand-order it should only be in the multiplication condition and it should be large because to transform |2x9| takes nine steps (2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2) whereas |9x2| takes only two (9 x 9).

Experiment two

The second experiment tested participants' speed and accuracy in retrieving the correct answer to simple arithmetic equations. For this experiment participants were presented visually with a simple arithmetic equation whereby the product was replaced with a blank space, for example |2x4= |. This was to indicate that the aim of the task was to indicate the product of the equation. It is assumed that for this task, the presence of the operation sign allowed for direct fact retrieval. The aim of this task was thus to test the effect the operation type and problem-size had on retrieval. Based on the assumption that the retrieval network is specific to multiplication facts, participants should be quicker and more accurate in response to small multiplication problems than addition problems. Based on the theory of a neighbourhood-consistency effect, participants should be faster to respond to small multiplication problems than they are to large ones but wrong problems should take longer than any other problem type. Finally, based on the assumption of a network organised in terms of operand-order, there will be an effect of operand-order, but it is likely to be small.

Experiment three

The final experiment merged experiment one and two together to compare the performance in each task. The aim of this was to directly test the effect of including the operation sign on the processing of simple arithmetic. In accordance with the IE model, responses to small multiplication problems should be faster and more accurate overall in task two than in task one. Based on the assumption of a multiplication-specific network, addition and wrong problems should be responded to just as slowly in the second task as they were in the first. If there is an effect of operand-order it should be larger in task one than in task two, due to the differential use of calculation and retrieval.

Experiment one

Method

Design

A 2 x 3 within-subjects ANOVA, with operand-order (left bigger vs right bigger) and operation (addition vs. multiplication vs. wrong) as independent variables and response time the dependent variable design was used for the RT data. Due to the lack of normality in the distribution of the data, A Friedman's test, with operation (addition vs. multiplication vs. wrong) as the independent variable and accuracy as the dependent variable, was used for the accuracy data. To control for the problem-size effect (Brauwer, et al., 2006; Domahs, Delazer, & Nuerk, 2006; Jost, et al., 2004; LeFevre, Sadesky, et al., 1996; Smedt, Holloway, & Ansari, 2011), these analyses were then repeated with large multiplication problems excluded (suggested by Zhou and Dong in 2003 to be equations with a sum of 25 or more). Large addition problems were not controlled for because research suggests that there is no difference in processing large and small addition problems (Zhou, et al., 2006; Xilin & Dong, 2003). Two additional 2 x 2 repeated measures ANOVAs with operation (addition vs. multiplication) and order (left bigger vs. right bigger) were carried out on the RT data, one of which included large problems and one that excluded large problems.

Participants

Thirty-six undergraduate psychology students (seven males and twenty nine females) from Liverpool Hope University participated in this experiment. Thirteen of the students participated in partial fulfilment of their first year psychology course and received course credits for their participation. No participants reported any eyesight or learning disabilities that could have caused distress or interfered with their ability to take part in the study. During data collection, six of the participants dropped out of the study for various reasons. This left a sample of 30 students, 13 of which were male and 17 of which were female. Participants ranged in age from 18 to 23 years (Mean = 20.04, SD = 1.96).

Materials and apparatus

Participants received an information sheet stating the aims and instructions of the experiment (appendix A). They also received a consent form (appendix C) and a short questionnaire (appendix D). TrueBasic was used for the programming of the experiments and SPSS was used to analyse the data. Both programs were executed on an Asus (model K54C) laptop, (screen resolution 1024x768). Participants' responses were inputted into the application program via the laptop keyboard. Latencies and errors were recorded automatically by the program installed on the laptop.

Stimuli and procedure

Stimuli included 240 arithmetic problem sets (80 addition problems, 80 multiplication and 80 wrong problems), the operands used in these problems ranged from 1-9. All the numbers were presented visually in Arabic form only. Cross modalities of numbers were avoided due to the fact that they are processed differently (Priftis, Albanese, Meneghello, & Pitteri, 2013). Each presented problem contained the answer but not the operation sign, so |2x3=6| was presented as |2 3=6|. Every problem was presented once for each possible operand-order combination. All conditions were randomised across trials and tie problems containing numbers of two were removed due to an incompatibility with the program.

Participants were fully informed of the aims of the experiment and were given instructions on how to complete the trials. The experiment took place in a quiet, single study booth without the presence of the researcher. Pressing the spacebar started the experiment. Once the experiment started, participants were required to press a key on the keyboard to indicate which operation sign they thought the arithmetic problem used. If it was addition they pressed A, if it was multiplication they pressed X and if it was neither they pressed W. Each trial occurred immediately after the other with no break in between. The participants were asked to complete the trials as quickly and accurately as possible. Timing began with stimulus onset and ended when a key was pressed. The participants were thanked for their participation at the end.

Results

Analysis one (large problems included)

Overall differences

This experiment tested the effect of operation type, problem-size and operand-order on the RT and accuracy in a simple arithmetic task. The data were pre-processed so that all responses with an RT of less than 200ms were removed, all problems with a RT of more than three standard deviations above the means were removed, it was transformed in to logarithms and all trials with errors were removed. Errors were computed by calculating the percentage of correct answers in each task and then converting it to a decimal Figure. This data were also pre-processed so that all responses with a RT of less than 200ms were removed and responses with a RT of more than three standard deviations above the means were removed. The overall means and standard deviations for RTs and accuracy after pre-processing are presented in Table1.

Table 1. The overall means and standard deviations for RTs (in seconds) and accuracy in all conditions.

	Response time						Accuracy				
	Left			Right			Left			Right	
Operation	M	SD		M	SD		M	SD		M	SD
Add	2.45	0.70		2.43	0.75		0.95	0.06		0.96	0.05
Multiply	2.99	1.16		2.98	1.20		0.95	0.06		0.93	0.06
Wrong	3.27	1.13		3.22	0.16		0.91	0.10		0.92	0.09

Note. M: Mean, SD: Standard deviation

Differences in reaction time

A 2 x 3 repeated measures ANOVA, with operand-order (left bigger vs. right bigger) and operation (addition vs. multiplication vs. wrong) as the independent variables and RT as the dependent variable, was carried out on the data. The results revealed a significant main effect of operation ($F(1, 29) = 45.8$, $p < .001$, $\eta\rho^2 = .6$). There was no significant main effect of operand-order and there was no interaction between operand-order and operation. Post-hoc comparisons with adjustments for multiple comparisons (Bonferroni) revealed a significant difference between addition and multiplication ($p < .01$, effect size $r = .27$), multiplication and wrong problems ($p < .01$, effect size $r = .11$) and between addition and wrong problems ($p < .01$, effect size $r = .39$) (see Table 2 for means and standard deviations).

Table 2. The mean values and standard deviations for RTs (in seconds) and accuracy for each operation type.

	Response time			Accuracy	
Operation	**M**	**SD**		**M**	**SD**
Add	2.44	0.72		0.95	0.05
Multiply	2.98	1.17		0.94	0.06
Wrong	3.24	1.13		0.92	0.09

Note. M: Mean, SD: Standard deviation

Differences in accuracy

A Kolmogorov-smirnov test for normality revealed that the accuracy data were significantly skewed and thus that the data violated the normal distribution assumption needed to run an ANOVA. As a result, a Friedman's test, with addition, multiplication and wrong operation types as the three independent variables and accuracy as the dependent variable was run on the data. This revealed a significant main effect of operation type ($p<.05$). To test this further, post-hoc comparisons were run (Wilcoxon's signed ranked test). The only significant difference was between addition and wrong problems ($Z(29)=2.50$, $p=.01$) (see Table 2 for means and standard deviations). A separate Wilcoxon's test was then carried out to test the difference between left-bigger and right-bigger operands. This revealed no significant difference. It should be mentioned here that, due to the post-hoc nature of nonparametric tests, the accuracy results reported here should be interpreted as exploratory rather than definite.

Analysis two (large problems excluded)

Overall differences

This analysis was exactly the same as the first analysis but problems with a result superior to 25 were excluded. The overall means and standard deviations of each conditions are presented in Table 3.

Table 3. The mean values and standard deviations for the RTs (in seconds) and accuracy percentages for each possible operand-order combination for each operation type.

	Response time				Accuracy			
	Left		Right		Left		Right	
Operation	M	SD	M	SD	M	SD	M	SD
Add	2.45	0.70	2.42	0.75	0.96	0.05	0.96	0.05
Multiply	2.44	0.83	2.40	0.81	0.95	0.05	0.93	0.07
Wrong	3.00	0.98	2.94	0.89	0.91	0.11	0.92	0.11

Note. M: Mean, SD: Standard deviation

Differences in reaction times

A 2 x 3 repeated measures ANOVA, with operand order (left bigger vs. right bigger) and operation type (addition vs. multiplication vs. wrong) as the independent variables and RT as the dependent variable, was run on the data. The only significant main effect observed was for operation type ($F(1, 29)=48.15$, $p<.05$, $\eta\rho^2=.62$). Post-hoc comparisons with an adjusted p-value to account for multiple comparisons (Bonferonni) revealed a significant difference between multiplication and wrong problems ($p<.001$, effect size $r= .31$), and between addition and wrong problems ($p<.001$, effect size $r=.31$) (see Table 4 for means and standard deviations).

Table 4. The main values and standard deviations for the RTs (in seconds) and accuracy data for each operation type.

	Response time			Accuracy	
Operation	**M**	**SD**		**M**	**SD**
Add	2.44	0.70		0.96	0.05
Multiply	2.42	0.78		0.95	0.06
Wrong	2.97	0.90		0.91	0.10

Note. M: Mean, SD: Standard deviation

Analysis three (wrong problems excluded)

Overall differences

This analysis was exactly the same as the first analysis but wrong problems were excluded. The overall means and standard deviations of each conditions are presented in Table 5.

Table 5. The overall mean response times (in seconds) for each condition.

Operand order	Response times				
	Addition			Multiplication	
	M	**SD**		**M**	**SD**
Left bigger	1.45	0.70		3.00	1.16
Right bigger	2.43	0.75		3.00	2.00

Note. M: Mean, SD: Standard deviation

Inferential statistics

A 2 x 3 repeated measures ANOVA with order (left bigger vs. right bigger) and operation type (addition vs. multiplication) revealed no significant effect of operand-order or interaction between operation type and operand-order.

Analysis four (large and wrong problems excluded)

Overall differences

This analysis was exactly the same as the second analysis but wrong problems were also excluded. The overall means and standard deviations of each condition are presented in Table 6.

Table 6. The mean values (in seconds) with standard deviations for all conditions.

Operand order	**Response times**				
	Addition			**Multiplication**	
	M	**SD**		**M**	**SD**
Left bigger	2.47	0.70		2.45	0.84
Right bigger	2.45	0.75		2.41	0.82

Note. M: Mean, SD: Standard deviation.

Inferential statistics

A 2 x 3 repeated measures ANOVA with order (left bigger vs. right bigger) and operation type (addition vs. multiplication) revealed no significant effect of operand-order or interaction between operation type and operand-order.

Discussion

The main aim of this experiment was to see if the lack of operation sign caused participants to calculate the answers (and check them against the example) or whether the participants retrieved the facts from memory (and checked them against the example). Due to the lack of statistical power of nonparametric tests and lack of space, this discussion will focus on the RT data only. In confirmation of the first hypothesis, the results showed that multiplication problems were responded to with the same accuracy but significantly more slowly than addition problems. However, contrary to the second hypothesis, the RTs for multiplication were reduced to the same speed as addition when large problems were removed from the analysis, meaning that in this case we must accept the null hypothesis. An unpredicted finding was that responses to wrong problems were significantly slower than any other operation type regardless of whether large problems were included and there was no effect of operand-order.

The problem-size effect

Identical elements

The different response times depending on the inclusion of large multiplication problems demonstrates a commonly reported problem-size effect (Brauwer, et al., 2006; Domahs, et al., 2006; Jost, et al., 2004; Smedt, et al., 2011; Zhou, et al., 2006). As problem-size is related to working memory (WM) capacity (Imbo & Vandierendonck, 2006), and WM is relied upon when calculating equations (Fürst & Hitch, 2000), it may be suggested that the current problem-size effect was caused by the differential strains calculating small and large problems placed on WM (to calculate 9 x 4 you would need four calculations; 9 + 9 + 9 + 9 but for 9 x 2 you would need only one; 9 + 9). This leaves open the possibility that a lack of operation sign in the current study made retrieval impossible, and thus may potentially lend support to the IE model (Rickard & Bourne, 1996; Rickard, et al., 1994).

Calculation vs. retrieval

However, if the lack of operation sign caused all problems to be calculated, we would also expect larger RTs for small multiplication problems in relation to addition problems, as more transformational steps are required to calculate multiplication problems (9 x 8 takes 8 transformational steps, whereas 9 + 8 takes just one). Therefore, as there were no RT differences between addition and small multiplication problems, we can infer that calculation was not used for all the problems and thus that the problem-size effect was not caused by calculation. This conclusion leaves us with the question of what did cause the current problem-size effect. Based on previous research, which has demonstrated that small and large multiplication problems are processed differently in the brain, it may be suggested that the current problem-size effect was instead caused by the use of retrieval for small multiplication problems and calculation for large problems (Campbell & Timm, 2000; Jost, et al., 2004; Jost, et al., 2009; LeFevre, et al., 1996; LeFevre, Sadesky, et al., 1996) (see Figure 3 for an example).

Although this is the most straightforward explanation of the current results, its simplicity may also mean inaccuracy. This is highlighted in the study by Brauwer, et al. (2006), which was concluded in the introduction to suggest that it is possible to develop new memory nodes for larger multiplication problems. Furthermore, the fact that there was a RT difference between large multiplication problems and wrong problems, together with the suggestion that it is highly unlikely that the participants had the wrong problems stored in LTM, suggests that this may have been caused by the differential use of retrieval for large multiplication and calculation for wrong problems. If the difference between small and large multiplication problems was due to retrieval vs. calculation, we would thus expect the size of the difference to be comparable the difference between large multiplication and wrong problems (11%). Therefore, the finding of a 27% difference in RT for small and large multiplication problems, suggests that the problem-size effect was caused by something other than the alternative use of retrieval and calculation to complete the task. This calls for an alternative explanation of the problem-size effect.

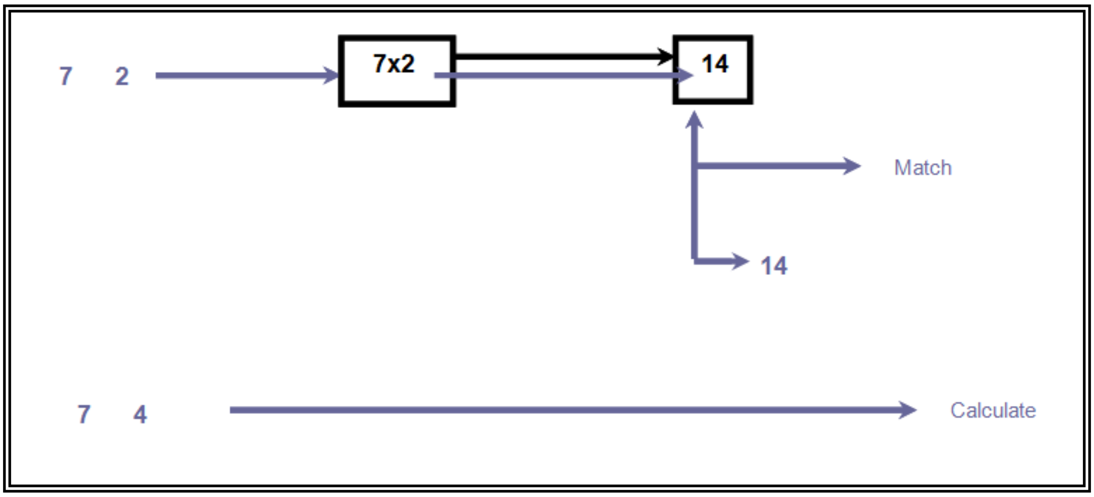


Figure 3. This Figure exemplifies how the retrieval network may have been operating when participants processed small (top) and large (bottom) multiplication facts. The black represented an oversimplified structure of the network and the blue represents the flow of activity.

Neighbourhood interference

Instead it may be that all multiplication problems were retrieved but that the higher amount of inconsistent neighbours for larger multiplication problems (Campbell, et al., 2011; Domahs, et al., 2006), resulted in the activation of more competing output nodes, which caused more interference when checking the answer against the example, and thus a delayed response (see Figure 4 for an example).

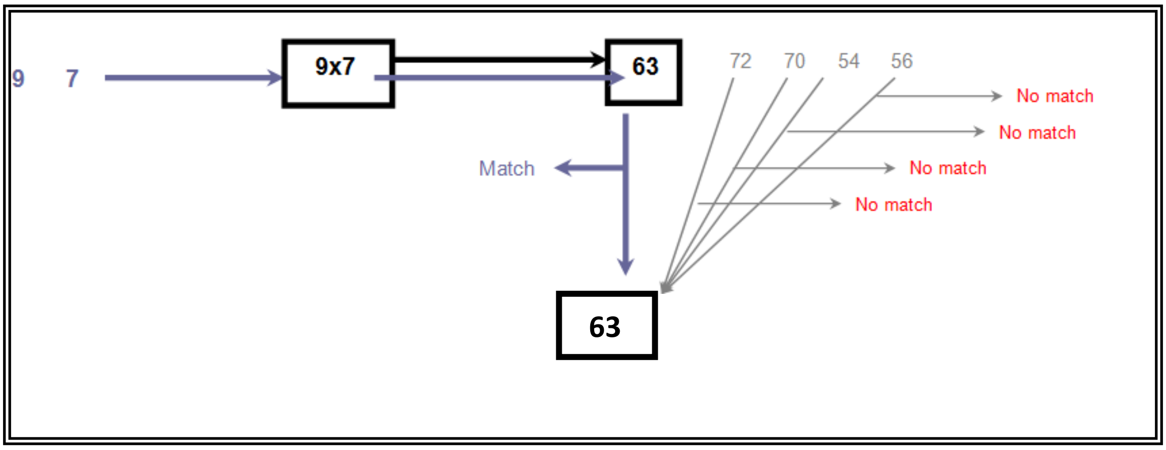


Figure 4. This Figure exemplifies the retrieval process of large multiplication facts. The black represented an oversimplified structure of the network, the blue represents the flow of activity and the grey represents the automatic activation of neighbouring nodes.

If this was the case, it challenges previous research by Jost, et al. (2009), discussed earlier, which suggests that a dissociated neural activity in response to large and small multiplication problems indicates the use of retrieval for small problems and calculation for large problems. It thus calls for an alternative explanation of why participants in this study showed dissociated neural activity in response to large and small multiplication problems Instead, it may be that the area showing increased activity in response to larger multiplication problems (the cingulate gyrus) was a result of a higher need to inhibit more inconsistent neighbours for larger problems. This is consistent with research showing that there is more activation in the cingulate gyrus during the inhibition of unwanted responses (Ansari, Fugelsang, Dhital, & Venkatraman, 2006; Menon, Adleman, White, Glover, & Reiss, 2001). In support of this, a later study found activity within the left angular gyrus, an area associated with arithmetic retrieval (Grabner, Ansari, Koschutnig, Reishofer, & Ebner, 2011; Grabner, Ansari, Koschutnig, Reishofer, & Neuper, 2009), in response to large multiplication problems, which confirms the theory that they are directly retrieved from memory, and thus supports the assumption of the IN model that the problem-size effect is caused by neighbourhood-inconsistency.

A shared network for multiplication and addition facts

The second important finding in this experiment was that multiplication problems were responded to equally as fast as addition problems. This does not support the second hypothesis, which was that there should still be a small difference between addition and multiplication problems when larger problems are excluded. As the data so far provides evidence to suggest that multiplication problems are retrieved directly from memory, the comparable RT between addition and multiplication problems suggests that addition problems were also retrieved directly from memory. These findings are consistent with previous research evidence used to support the theory of a shared multiplication and addition network (Campbell & Phenix, 2009; Campbell & Thompson, 2012; Cho, et al., 2011). As a result, they challenge the assumption of the IN model that the network is specific to multiplication facts (Verguts & Fias, 2005), and that the dissociated neural activity previously reported in response to addition and multiplication facts found by Zhou et al. (2007) may have been a result of different processing at the encoding stage, rather than the differential use of retrieval and calculation strategies.

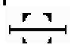
Conclusions

Based on the conclusions made so far in this discussion, it appears reasonable to suggest the that participants completed the task by retrieving addition and multiplication facts from an interrelated memory network and then recalling which route in the network arrived at the matching answer. This challenges the theory proposed by the IE model that the lack of operation sign makes direct fact retrieval impossible, as well as the assumption of the IN model that the retrieval network is specific to multiplication facts. However, the current findings do lend support to the assumption of the IN model that the problem-size effect is caused by more inconsistent neighbours for larger multiplication problems. One problem with the current study is that there was no operand-order effect, suggested to affect direct fact retrieval. Although it is possible the IN model was wrong in assuming that the retrieval network was organised in this way, it could be that the lack of operation sign caused the equations to be processed differently by the retrieval network, and thus over looks an aspect of the model, which is of interest in the current paper. This will therefore be explored further in the following study.

Experiment two

Method

Design, participants, materials and stimuli

The experimental design, participants, materials and stimuli in this experiment were the same as in experiment one, with some exceptions. Firstly, all trials included an operation sign, one of which included a novel operation sign to represent ‘wrong’ as the operation type which looked like this: . Secondly, the trials included three different choices of answers, for example |2x4= | was displayed in the centre of the screen and |8   6   12| was displayed beneath it. Thirdly, a different participant information form was used to explain the aims and instructions of the task (appendix B).

Procedure

To control for the effect of fatigue or loss of concentration, this study took place at least three days after experiment one. Participants were fully informed of the aims of the experiment and were given instructions on how to complete the trials. The experiment took place in a quiet, single study booth without the presence of the researcher. Firstly the participants were given a practice trial. For this, participants were required to press X A or W on the keyboard to indicate whether each symbol presented was multiplication, addition or wrong. This was so that they could familiarise themselves with the novel operation sign before the start of the task. Next, participants started the experiment. For this, participants were required to press a key on the keyboard to indicate which of the three options given was the correct answer to the equation, if they thought it was the option on the left they were required to press J, if it was in the middle they were required to press K and if they thought it was the option on the right they pressed L. However, if the problem included an operation sign that was neither addition or multiplication they were required to press a key on the keyboard (J, K or L) to indicate which option they thought was the wrong answer (out of two possible right answers and one wrong answer). Each trial occurred immediately after the previous, with no break in between. The participants were asked to complete the trials as quickly and accurately as they could. Timing began with stimulus onset and ended when a keyboard key was pressed. When the experiment was over participants were thanked for their participation.

Results

Analysis one (large problems included)

Overall differences

The data was preprocessed in the same way as experiment one. The overall mean and standard deviations for RTs and accuracy after pre-processing are presented in Table 7.

Table 7. The means and standard deviations (in seconds) for all conditions when large problems were included.

	Response time						**Accuracy**				
	Left			**Right**			**Left**			**Right**	
Operation	**M**	**SD**		**M**	**SD**		**M**	**SD**		**M**	**SD**
Add	2.24	0.66		2.28	0.60		0.93	0.06		0.94	0.06
Multiply	2.67	1.05		2.7	0.94		0.94	0.06		0.95	0.04
Wrong	4.65	2.05		4.25	1.8		0.82	0.23		0.82	0.24

Note. M: Mean, SD: Standard deviation.

Differences in reaction time

A 2 x 3 repeated measure ANOVA, with operand-order (left bigger vs. right bigger) and operation (addition vs. multiplication vs. wrong) as the independent variables and RT as the dependent variable, was carried out on the on the data. The results revealed a significant effect of operation ($F(1, 29) = 215.06$, $p < .001$, $\eta\rho^2 = .88$). Post-hoc comparisons with corrections for multiple comparisons (Bonferroni) revealed a significant difference between addition and multiplication ($p < .001$, effect size $r = .26$), between multiplication and wrong ($p < .001$, effect size $r = .54$), and between addition and wrong problems ($p < .001$, effect size $r = .65$) (see Table 8 for means and standard deviations). However, there was no main effect of operand-order or interaction between operand-order and operation type.

Table 8. The mean RTs and accuracy percentages (with standard deviations) for each operation type for analysis 1.

	Response time			Accuracy	
Operation	**M**	**SD**		**M**	**SD**
Add	2.26	0.60		0.94	0.50
Multiply	2.69	0.97		0.95	0.40
Wrong	4.45	1.69		0.82	0.20

Note. M: Mean, SD: Standard deviation.

Differences in accuracy

As the data was not normally distributed (as assessed using a Kolmogorov-Smirnov test for normality), a Friedman's test was run with addition, multiplication and wrong as the three independent variables and accuracy as the dependent variable. This revealed a main effect of operation type ($p<.05$). post-hoc comparisons (Wilcoxon tests) revealed a significant difference between addition and wrong problems ($p> .001$) and between multiplication and wrong problems ($p<.001$). A separate Wilcoxon's Test assessing the difference between problems with a left bigger operand and a right bigger operand showed no effect.

Analysis two (Large problems excluded)

Overall differences

This analysis was the same as analysis one but with large problems excluded. The overall means and standard deviations for each condition are presented in Table 9.

Table 9. The mean RTs and accuracy percentages with standard deviations (in seconds) for each possible operand-order for each operation in analysis two.

	Response time				**Accuracy**			
	Left		**Right**		**Left**		**Right**	
Operation	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
Add	2.24	0.66	2.28	0.6	0.93	0.06	0.94	0.06
Multiply	2.1	0.65	2.2	0.8	0.94	0.06	0.95	0.05
Wrong	4.31	2.18	3.93	1.66	0.83	0.24	0.83	0.24

Note. M: Mean, SD: Standard deviation

Differences in RT

A 2 x 3 ANOVA with operand-order (left bigger vs. right bigger) and operation type (addition vs. multiplication vs. wrong) and RT as the dependent variable was carried out on the data. This revealed a significant main effect of operation only ($F(2, 28)= 132.35$, $p<.00.1$, $\eta\rho^2=.82$). Post-hoc comparisons revealed a significant difference between addition and multiplication ($p<.05$, effect size $r=0.08$), multiplication and wrong ($p<.001$, effect size $r=0.60$) and addition and wrong problems ($p<.001$, effect size $r= .58$) (see Table 10 for means and standard deviations).

Table 10. The mean values (with standard deviations) for the reaction times and accuracy in each condition when large problems are excluded.

	Response time			Accuracy	
Operation	**M**	**SD**		**M**	**SD**
Add	2.26	0.61		0.94	0.05
Multiply	2.15	0.7		0.95	0.05
Wrong	4.12	1.7		0.82	0.24

Note. M: Mean, SD: Standard deviation.

Differences in accuracy

As the data was not normally distributed, a Friedman's test with addition and multiplication as the three independent variables was carried out on the accuracy data. This revealed a significant main effect of operation ($p<.001$). Post-hoc comparisons (Wilcoxon's test) revealed a significant difference between multiplication and wrong problems only ($p<.001$) (see Table 10 for means and standard deviations).

Analysis three (large problems included, wrong problems excluded)

Overall differences

As can be seen from the means, the differences between left bigger and right bigger wrong problems were in the opposite direction to addition and multiplication, suggesting that it may have confounded the experiment. As the aim of the experiment was to test an operand-order effect for addition and multiplication problems, wrong problems were thus excluded from the experiment. The overall means are presented in Table 11.

Table 11. The mean values (in seconds) for each operand-order combination for each operation type.

Operand order	Response times				
	Operation type				
	Addition			Multiplication	
	M	SD		M	SD
Left bigger	2.24	0.66		2.1	0.65
Right bigger	2.28	0.6		2.2	0.8

Note. M: Mean, SD: Standard deviation

Inferential statistics

A 2 x 2 repeated measures ANOVA with operation (addition vs. multiplication) and operand-order (left bigger vs right bigger) revealed a significant main effect of operation (F(1, 29), =20.72, p<.001). The effect of this difference was small (effect size r= 0.28) (see Table 12 for means) and of operand-order (F(1, 29)=4.262, p<.05), also with a small effect size (effect size r = 0.7) (see Table 13 for means and standard deviations).

Table 12. The mean values (in seconds) for each operation type with wrong problems excluded.

Operation	Response times	
	M	**SD**
Add	2.26	0.6
Multiply	2.7	0.9

Note. M: Mean, SD: Standard deviation.

Table 13. Means values with standard deviations (in seconds) for each operand-order combination with wrong problems excluded.

	Response times	
Operand order	**M**	**SD**
Left bigger	2.39	0.76
Right bigger	2.5	0.73

Note. M: Mean, SD: Standard deviation.

Analysis four (large and wrong problems excluded)

Overall differences

The overall means for all conditions are presented in Table 14.

Table 14. The mean values (in seconds) with standard deviations) for all conditions.

	Response times				
	Addition			**Multiplication**	
Operand order	**M**	**SD**		**M**	**SD**
Left bigger	2.24	0.66		2.1	0.65
Right bigger	2.28	0.6		2.2	0.8

Note. M: Mean, SD: Standard deviation

Inferential statistics

A 2 x 2 repeated measures ANOVA with operation (addition vs. multiplication) and operand-order (left bigger vs right bigger) revealed a significant main effect of operation ($F(1, 29)$, $=6.6$, $p<.05$) with a small (effect size $r= 0.08$) (see Table 15 for means) and of operand-order ($F(1, 29)=4.7$, $p<.05$) with a small effect (effect size $r=0.04$) (see Table 16 for means).

Table 15. The mean values (in seconds) with standard deviations for each operation type.

	Response times	
Operation	**M**	**SD**
Add	2.26	0.61
Multiply	2.15	0.7

Note. M: Mean, SD: Standard deviation

Table 16. The mean values (in seconds) with standard deviations for each operand-order problem with wrong and large problems excluded.

	Response time	
Operand order	**M**	**SD**
Left bigger	2.2	0.63
Right bigger	2.24	0.14

Note. M: Mean, SD: Standard deviation.

Discussion

Overview of the results

Based on the assumption that the inclusion of the operation sign allowed for direct retrieval, the main aim of this experiment was to see which types of arithmetic facts are stored in the retrieval network. As with experiment one, the results showed that multiplication problems were responded to with the same accuracy but significantly more slowly than addition problems when large problems were included but not when they were excluded, demonstrating a problem-size effect, which allows us to reject the null for the second hypothesis for this study. Because this has already been discussed in experiment one, it will not be talked about here. Instead, the current study will focus on explaining the finding that addition problems were responded to more slowly that multiplication problems, and the finding of an operand-order effect. Due to the lack of statistical power of nonparametric tests and the lack of space, this discussion will focus on the RT data only. But please keep in mind that the accuracy data mainly supported the RT data.

Multiplication-specific network

The finding that addition problems were answered more slowly than small multiplication problems confirms the first hypothesis of this experiment, which was that the arithmetic retrieval network may be specific to multiplication problems (we can therefore reject the null hypothesis). In doing so, it supports the assumption of the IN model (Verguts & Fias, 2005), as well as previous research (Butterworth, Zorzi, Girelli, & Jonckheere, 2001; Foyal & Thevenot, 2012; Sohn & Carlson, 1998) that the retrieval network may be specific to multiplication facts. However, if this is correct, it contradicts the findings of experiment one, as well as previous research that suggests there is a shared memory network for both multiplication and addition problems (Campbell & Phenix, 2009; Campbell & Thompson, 2012; Cho, et al., 2011). As a result, confusion remains over whether or not there is a multiplication-specific retrieval network or whether it is shared by simple addition problems. This conflict may be resolved by a closer look at the results of the current study. The difference between small multiplication and addition problems was small (8%). If the increased RT was caused by the use of calculation, we would expect the difference between addition and small multiplication problems to be larger than it was, as it is likely that calculation takes much longer than direct retrieval. As a result of this, it may be concluded that both addition and multiplication problems were directly retrieved from memory, and thus supports the conclusion made in experiment one that there is a shared network for multiplication and addition facts.

The operand-order effect

In support of the final hypothesis, and the focus of this section of the discussion, there was an operand-order effect for multiplication and addition problems, even when large problems were included in the analysis. However, it was in the opposite direction to expected, with right-bigger problems being answered more slowly than left-bigger problems. Consistent with the hypothesis proposed by Kiefer and Dehaene (1997), the operand-order effect shown here may appear to have been caused by the use of calculation for problems presented in an order that did not match that originally stored in memory. This is because there are more transformational steps required for equations with a bigger right operand as opposed to the reverse (3x9 requires nine transformational steps; 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 whereas 9x3 takes just three steps; 9 + 9 + 9). However, if this was the case we would expect at least a moderate effect of operand-order. In fact, the size of the difference should have been comparable to the difference between large multiplication and wrong problems (60%), concluded in experiment one to indicate the alternative use of retrieval and calculation. Therefore, as there was only a small difference (8%) in RT between left-bigger and right-bigger problems, it seems highly unlikely that an unmatched operand-order elicited the use of calculation to produce the answer. It seems more likely that, as hypothesised, the order in which the operands were presented affected direct arithmetic fact retrieval. This converges with previous findings (Arbuthnott & Campbell, 1996; Kiefer & Dehaene, 1997) to support the assumption of the IN model that there is a preferred operand-order of the retrieval network (Verguts & Fias, 2005). It also indicates that the operand-order effect is not specific to the Chinese population and thus increases the population validity of the IN model (Verguts & Fias, 2005), and opens up a new factor of the model that has been dismissed in recent years (Robert & Campbell, 2008) to investigation in future studies.

This leaves the question of why Rickard and Bourne (1996) found no such effect. They showed that test performance was facilitated when the problems tested were presented in a different order to that practiced. In this study, it may be that problems presented with an operand-order that did not match the original order stored in the network were switched at every encounter during practice, meaning that the participants practiced switching the operands several times. As a result, this may have become an automatic process at the test phase, resulting in no extra time spent manipulating the order of the operands during test, and thus no operand-order effect. This leads to the conclusion that the operand-order effect may be caused by the extra time spent actively manipulating the operand-order to match that stored in memory.

This may also account for why Kiefer and Dehaene (1997) found an operand-order effect for acoustically presented stimuli but not visually presented stimuli. Because children are explicitly taught to switch the operands round for visually presented equations but not acoustically presented equations (Kiefer & Dehaene, 1997), it suggests the switching process was automatic for visually presented equations, resulting in no operand-order effect. The finding that the order-effect was in the opposite direction to expected (English schools teach multiplication with the left right operand bigger, such as 2x3, 2x4, 2x5 etc) may be accounted for by the lack of mathematical expertise reported by the participants, suggesting that memories from larger times Tables (7s, 8s and 9s) had weakened and were thus more easily retrieved by switching the operands in to a fact from a smaller times Table so that that they could retrieve the answer (3x6 means they have to retrieve an equation from the six times Tables, whereas 6x3 means they can retrieve an equation from the two times Tables, which they do have stored in memory).

Conclusions

The current results suggest that there is a shared network for multiplication and addition facts, but that the retrieval of these facts are somehow slower for addition problems that they are for small multiplication problems, an effect that was not found in experiment one, and one that will be explained in the final discussion. Although this challenges the assumption of the IN model that there is a multiplication specific network, the results of a small operand-order effect do support the IN model, in that of a preferred-operand-order organisation of the retrieval network. However, it is important to note that the small effect also means that this finding should be taken with caution, as it may be the case that a type I error occurred, in which case it would criticise the validity of the IN model. However, it is most likely that the small effect was caused by the limited amount of trials for each problem type, resulting in a lack of statistical power. This calls for future research investigating the operand-order effect in a Western sample and with more trials.

Experiment three

Method

Participants, stimuli, material and procedure

The participants were exactly the same as experiment one and two. The stimuli, materials and procedure, in the checking task were the same as experiment one, which tested participants' speed and accuracy in indicating the missing operation sign in a series of mathematical equations, as indicated by a blank space between the two operands. The stimuli, materials and procedure used in the computing task were the same as experiment two, which tested participants' speed and accuracy in indicating the missing answers in a series of mathematical equations.

Design

The design was a 2 x 3 repeated measures ANOVA with Task (checking vs computing) and operation (add vs multiply vs wrong) as the independent variables and RT as the dependent variable. As the accuracy data was not normally distributed, the design for this analysis was a Friedman's test with addition, multiplication and wrong as the independent variables, and accuracy as the dependent variable.

Results

Overall results

The data were preprocessed in the same way as the previous experiment. The descriptive statistics are presented in Table 17.

Table 17. The mean RTs and accuracy percentages (in seconds) for each operation type in each task.

	Response time				Accuracy			
	Task 1		**Task 2**		**Task 1**		**Task 2**	
Operation	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
Add	2.44	0.7	2.26	0.61	0.96	0.05	0.94	0.05
Multiply	2.42	0.8	2.15	0.71	0.95	0.06	0.95	0.05
Wrong	2.97	0.91	4.12	1.7	0.91	0.01	0.83	0.02

Note. M: Mean, SD: Standard deviation.

Differences in response times

A 2 x 3 ANOVA with task (Checking vs. retrieving) and operation (addition vs. multiplication vs. wrong) and as the independent variables, was carried out on the RT data. There was no main effect of task. However, there was a main effect of operation type ($F(2, 28)= 142.8$, $p<.001$, Ƞρ^2 = .83) (see Table 18 for means and standard deviations). There was also a significant interaction between task and operation type ($F(2, 28)=55.1$,$p<.001$, Ƞρ^2= .66) (see Table 17 for means and standard deviations). Post-hoc comparisons with a corrected p-value to control for multiple comparisons (Bonferroni) revealed a significant difference between addition and wrong problems ($p<.001$, effect size r .54=) and between multiplication and wrong problems ($p<.001$, effect size r =.55).

Table 18. The overall mean RTs (in seconds) with for each operation type.

Operation	Response time	
	M	SD
Add	2.35	0.57
Multiply	2.28	0.65
Wrong	3.54	1.18

Note. M: Mean, SD: Standard deviation.

Differences in accuracy

A Wilcoxon's signed ranks test with task one and task two as the independent variables and accuracy as the dependent variable, revealed no significant overall difference in accuracy between the tasks and so this was not analysed further.

Discussion

Overview of the results

The aim of experiment three was to directly test the effect of including/not including the operation sign on processing simple addition and small simple multiplication problems. It did this by comparing the performance in task one, whereby the operation sign was not presented with the equation, with task two, whereby the operation sign was presented with the equation. As experiment three links in directly with experiment one and two, what follows is a brief discussion and then a more general discussion of all three experiments.

The organisation of the network

In support of the first hypothesis proposed for experiment three, multiplication problems were responded to more quickly in the checking task than in the computing task. However, in challenge to the second hypothesis, addition problems were also responded to more quickly in the computing task than the checking task. Consistent with the identical elements model (Rickard, et al., 1994), and research to support it (Galen & Reitsma, 2010; Rickard & Bourne, 1996), it may be suggested that the longer RT in task one was caused by the fact that the missing operation sign made retrieval impossible, resulting in the use of a lengthy calculation process to complete the task. However, if all problems were calculated in task one, it does not explain why addition and small multiplication problems were responded to equally as fast as one another. If both were calculated, multiplication should have taken longer than addition due to the fact that several sub-calculations are required to calculate multiplication problems and only one is required to calculate an addition. It also contradicts previous findings that show that the retrieval of arithmetic facts can occur (Galfano, Penolazzi, Vervaeck, Angrilli, & Umiltá, 2009), or at least be anticipated, without the presentation of the operation sign (Zhou, 2011). This casts doubt upon the assumption of the IE model that the operation sign is essential for arithmetic fact retrieval (Rickard & Bourne, 1994) and calls for an alternative explanation of why the participants in the current study responded more slowly in the checking task than in the computing task in the current study. Instead, it may be that multiplication and addition problems were retrieved in both tasks but that the lack of the operation sign some how delayed this process in the checking task. An explanation of how the retrieval network is organised and how it may have operated in each task is as follows; There are two routes in the retrieval network, one for retrieving multiplication and one for retrieving addition facts. In task two, the presentation of the operation sign acted as an indicator of which route to activate, resulting in a speedy retrieval of multiplication and addition facts (see Figure 5).

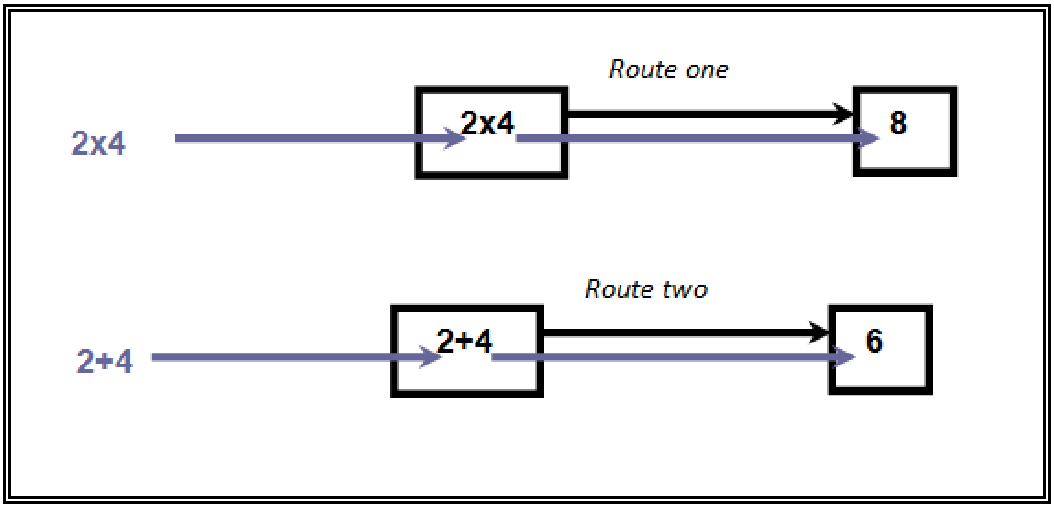


Figure 5. An oversimplified version of the model to exemplify the structure of the network (in black) and the retrieval process during task two (in blue).

However, in task one, this indicator was missing. As a result, the network had no indication of which route should be activated and therefore, against the natural operation of the network, both routes became activated simultaneously. This allowed addition and multiplication problems to be simultaneously activated and checked against the answers in the experiment. As a result, it caused an added strain to the retrieval process, resulting in a delay in response to multiplication and addition problems in task one compared to task two (see Figure 6).

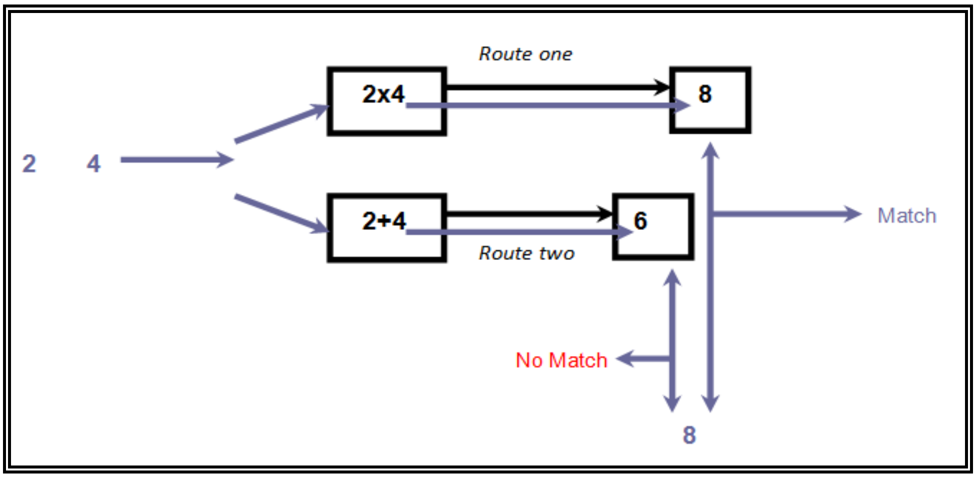


Figure 6. This Figure exemplifies how the retrieval network may operate when the operation sign is missing.

General Discussion

Although the problem-size effect was the same in each experiment, there were some contrasting findings between experiment one and two, which results in difficulty making a conclusion of the exact organisation of the retrieval network. The following discussion will discuss the results in relation to previous research and the model proposed in the previous discussion (Figure 6).

A multiplication-specific network

One of the main conflicting findings between experiment one and two was that, in experiment one, there were no differences in RT between addition and small multiplication problems, suggesting a shared retrieval network for addition and multiplication facts. However, in experiment two, small multiplication problems were responded to more quickly than addition problems, suggesting a multiplication-specific network, a conflict of which resembles the conflicting findings in the current literature (Campbell & Phenix, 2009; Campbell & Thompson, 2012; Cho, et al., 2011; Zhou, 2011). To account for the finding that addition problems were responded to more slowly than small multiplication problems within experiment two, it may be suggested that the multiplication route is dominant in the network, and thus that when switching between the retrieval of alternative problem types, as was done in experiment two, the addition route became weakened and retrieval more difficult. This hypothesis is consistent with previous research, which has shown that practicing multiplication equations causes problems for the retrieval of addition facts (Arbuthnott & Campbell, 1996; Campbell & Phenix, 2009; Campbell & Thompson, 2012) and thus challenges the assumption of the IN model that there is a multiplication-specific retrieval network (Verguts & Fias, 2005).

However, if the multiplication route is dominant in the network, it would suggest that the lack of operation sign in task one would have activated multiplication retrieval by default. This leaves the problem of explaining why addition equations were retrieved just as quickly as multiplication equations in task one. It may be that, as the network received the knowledge that the operation sign was going to be missing and that it had to decide which route should become activated, this elicited an overall increased amount of inhibition to the multiplication route, so that this default setting was altered to allow for a better chance at completing the task. As a result, the task started with an overall equal amount of activation for each route. This hypothesis may be used to explain why Foyal and Thevenot (2012) found the pre-presentation of the addition sign to facilitate arithmetic performance. Rather than it causing the pre-activation of procedural methods for addition problems, it may have acted as an indicator for the inhibition of the more dominant multiplication route, which allowed for an easier retrieval of addition facts.

An operand-order effect

Another contradictory finding between experiment one and two was that there was an operand-order effect for multiplication and addition problems in experiment two but not experiment one. As the existence of this effect was concluded earlier to suggest a preferred-operand-order organisation of the retrieval network, it causes confusion as to why no such effect was found in experiment one, which also tested direct retrieval. To account for this, it may be that the operand-order switching process, discussed earlier in experiment two, was affected by the presentation of the operation sign.

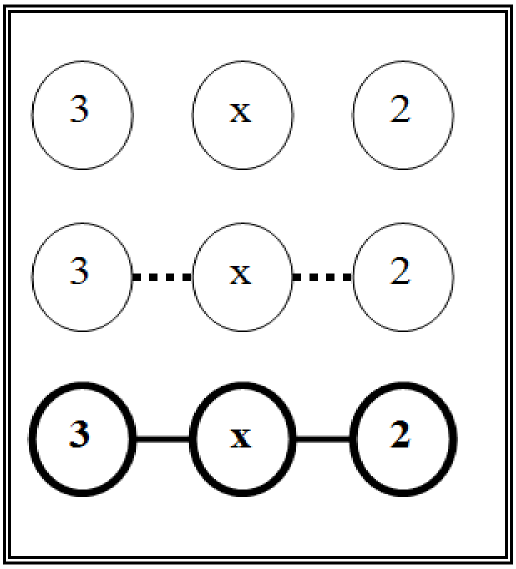


Figure 7. A Figure showing the Hebb-learning process of arithmetic facts.

Instead, the process may have been a result of Hebb learning. According to this principle, and explained in an oversimplified way; the neurons that code for the operation sign and the two operands fire separately at first but, as these stimuli are seen repeatedly together, the neurons that become active in response to each of the stimulus do so in synchrony, which eventually creates a new single memory chunk (Hebb, 1949) (Figure 7). Accordingly, it may be that the operand-order effect in experiment two was caused by the fact that participants coded |3x2| as a single memory chunk, which made it impossible to automatically switch the order of the operands. In experiment one, the lack of an essential element of this memory chunk meant that the participants coded the operands separately, allowing for automatic switching. This may also be extended to account for the unexpected finding that wrong problems were retrieved more slowly and less accurately in experiment two compared to experiment one. It may be that during visual encoding, the wrong sign acted as a substitute for the operation sign within the holistic memory node. As a result it may have wrongly activated the retrieval of arithmetic facts from memory, causing an added delay on top of the lengthy calculation process that followed.

Conclusions

It appears plausible to suggest that there is an effect of operation type, operand-order and problem-size on the processing of simple arithmetic. In challenge to the IN model, it is likely that the arithmetic retrieval network is not specific to multiplication facts but that is also represents simple addition facts, albeit to a weaker extent. However, in support of the IN model, the theory that the problem-size effect was caused by an increased amount of inconsistent neighbours for large problems was confirmed by the findings in both experiment one and two. The results showing an operand-order effect also lend support to IN model, as they suggest that there is some sort of operand-order organisation of the retrieval network and that this is not unique to the Chinese population. More importantly, this effect only occurred when the operation sign was presented within the equations. This is an important finding, as it suggests that any tasks that include presenting the operation sign separately from the equation, as many of the studies reported within this paper did, are highly unlikely to find an effect of operand-order. This may be why the research to support this effect is so limited and calls future research using a more appropriate task to test for an operand-order effect in a Western sample. Finally, although this shows that there was an effect of the presence of the operation sign on processing arithmetic, it is most likely that this merely slows the retrieval process rather than makes it impossible, it thus fails to directly support the IE model yet agrees with the idea that the operation sign does affect arithmetic retrieval somehow. It may therefore be concluded that the current findings lend more support towards the IN model and thus that this model better explains the cognitive processes of arithmetic fact retrieval. As a result, it should be investigated further in future studies so that our understanding of these processes can be developed.

Limitations

There are some limitations to the current study. Firstly, the participants reported varying skills in mathematics, which was reflected in the large standard deviations around the means. This may be problematic, as previous research has shown that individual math skill and practice affects the type of strategy used to produce the answers to maths sums (Bailey, Littlefield, & Geary, 2012; Imbo & Vandierendonck, 2007). Because of this, that large multiplication problems are retrieved directly from memory may be questioned, as the higher skilled participants may have used calculation for large problems, whereas others used retrieval. It may also be that the simultaneous activation of both routes during task one may have been faster for those with higher expertise. This calls for future research comparing the effects of mathematical expertise on each task. A further problem with this study is the small effect of operand-order. Although this has been explained and accounted for in relation to previous theories, the results showing an operand-order effect should be taken with caution and needs to be tested further with a larger sample to ensure that this result was not only significant due to a statistical error. Finally, the model proposed to account for the increase RT in the checking task vs. the computing task was based on the assumption that the network operated in a feed forward process and thus that participants could not retrieve arithmetic facts by looking at the answer and then searching backwards. Rickard (2005) has suggested that arithmetic facts can be also be retrieved by means of factoring. Although it is unlikely that this was the case in experiment one because addition problems cannot be factored, it may explain why multiplication problems were responded to more quickly than addition problems in experiment two. Future research could repeat this experiment using an eye tracker to ensure that this was not the case for the current study.

References

Ansari, D., Fugelsang, J., Dhital, B., & Venkatraman, V. (2006). Dissociating response conflict from numerical magnitude processing in the brain: An event-related fMRI study. NeuroImage, 32, 799-805.

Arbuthnott, K., & Campbell, J. (1996). Effects of operand order and problem repetition on error priming in cognitive arithmetic. Canadian Journal of Experimental Psychology, 50, 182-195.

Ashcraft, M. (1992). Cognitive arithmetic: A review of data and theory. Cognition, 44, 75-106.

Bailey, D., Littlefield, A., & Geary, D. (2012). The codevelopment of skill at and preference for use of retrieval-based processes for solving addition problems: Individual and sex differences from first to sixth grades. Journal of Experimental Child Psychology, 113, 78-92.

Brauwer, J. D., Verguts, T., & Fias, W. (2006). The representation of multiplication facts: Developmental changes in the problem size, five, and tie effects. Journal of Experimental Child Psychology, 94, 43-56.

Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A. (2001). Storage and retrieval of addition facts: The role of number comparison. The Quarterly Journal of Experimental Psychology, 54, 1005-1029.

Campbell, J. (1996). Mechanisms of simple addition and multiplication: A modified network-interference theory and simulation. Mathematical Cognition, 1, 121-164.

Campbell, J., Dowd, R., Frick, J., McCallum, K., & Metcalfe, A. (2011). Neighbourhood consistency and memory for number facts. Memory & Cognition, 39, 884-893.

Campbell, J., & Phenix, T. (2009). Target strength and retrieval-induced forgetting in semantic recall. Memory & Cognition, 37, 65-72.

Campbell, J., & Thompson, V. (2012). Retrieval-induced forgetting of arithmetic facts. Journal of Experimental Psycholog: Learning, Memory and Cognition, 38, 118-129.

Campbell, J., & Timm, J. (2000). Adults' strategy choice for simple addition: Effects of retrieval interference. Psychonomic Bulletin & Review, 7, 692-699.

Cho, S., Ryali, S., Geary, D., & Menon, V. (2011). How does a child solve 7 + 8? Decoding brain activity patterns associated with counting and retrieval strategies. Developmental Science, 14, 989-1001.

Domahs, F., Delazer, M., & Nuerk, H.-C. (2006). What makes multiplication facts difficult: Problem size or neighborhood consistency? Experimental Psychology, 53, 275-282.

Domahs, F., Domahs, U., Schlesewsky, M., Ratinckx, E., Verguts, T., Willmes, K… & Neurk, H-C. (2007). Neighborhood consistency in mental arithmetic: Behavioral and ERP evidence. Behavioural and Brain Functions, 3, 66-79.

Dormal, V., Andres, M., & Pesenti, M. (2012). Contribution of the right intraparietal sulcus to numerosity and length processing: An fMRI-guided TMS study. Cortex, 48.

Emerson, R., & Cantlon, J. (2012). Early math achievement and functional connectivity in the fronto-parietal network. Developmental Cognitive Neuroscience, 2, 139-151.

Foyal, M., & Thevenot, C. (2012). The use of procedural knowledge in simple addition and subtraction problems. Cognition, 123, 392-403.

Fürst, A., & Hitch, G. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. Memory & Cognition, 28, 77-782.

Galen, M. v., & Reitsma, P. (2010). Learning basic addition facts from choosing between alternative answers. Learning and Instruction, 20, 47-60.

Galfano, G., Penolazzi, B., Vervaeck, I., Angrilli, A., & Umiltá, C. (2009). Event-related brain potentials uncover activation dynamics in the lexicon of multiplication facts. Cortex, 45, 1167-1177.

Grabner, R., Ansari, D., Koschutnig, K., Reishofer, G., & Ebner, F. (2011). The function of the left angular gyrus in mental arithmetic: Evidence from the associative confusion effect. Human Brain Mapping, 00, 000-000.

Grabner, R., Ansari, D., Koschutnig, K., Reishofer, G., & Neuper, F. E. C. (2009). To retrieve or to calculate? Left angular gyrus mediates the retrieval of arithmetic facts during problem solving. Neuropsychologia, 47, 604-608.

Imbo, I., & Vandierendonck, A. (2006). The development of strategy use in elementary school children: Working memory and individual differences. Journal of Experimental Child Psychology, 96, 284-309.

Imbo, I., & Vandierendonck, A. (2007). Do multiplication and division strategies rely on executive and phonological working memory resources? Memory & Cognition, 35, 1759-1771.

Jost, K., Hennighausen, E., & Rösler, F. (2004). Comparing arithmetic and semantic fact retrieval: Effects of problem size and sentence constraint on event-related brain potentials. Psychophysiology 41, 46-59.

Jost, K., Khader, P., Burke, M., Bien, S., & Rösler, F. (2009). Dissociating the solution processes of small, large, and zero multiplications by means of fMRI. NeuroImage, 46, 308-318.

Kiefer, M., & Dehaene, S. (1997). The time course of parietal activation in single-digit multiplication: Evidence from event-related potentials. Mathematical Cognition, 3, 1-30.

Kirk, E., & Ashcraft, M. (2001). Telling stories: The perils and promise of using verbal reports to study maths strategies. Journal of Experimental Child Psychology: Learning, Memeroy& Cognition, 27, 157-175.

LeFevre, J., Bisanz, J., Daley, K., Buffone, L., Greenham, S., & Sadesky, G. (1996). Multiple routes to solution of single-digit multiplication problems. Journal of Experimental Psychology, Learning, Memory and Cognition, 126, 284-306.

LeFevre, J., & Morris, J. (1999). More on the relation between division and multiplication in simple arithmetic: Evidence for mediation of division solutions via multiplication. Memory & Cognition, 27, 803-812.

LeFevre, J., Sadesky, G., & Bisanz, J. (1996). Selection of procedures in mental addition: reassessing the problem size effect in adults. Journal of Experimental Psychology, Learning, Memory and Cognition, 22, 216-230.

Mauro, D., LeFevre, J.-a., & Morris, J. (2003). Effects of problem format on division and manipulation performance: Division facts are mediated via multiplication-based representations. Journal of Experimental Psychology: Human Learning and Memory, 29, 163-170.

Menon, V., Adleman, N., White, C., Glover, G., & Reiss, A. (2001). Error-related brain activation during a go/no go response inhibition task. Human Brain Mapping, 12, 131-143.

Priftis, K., Albanese, S., Meneghello, F., & Pitteri, M. (2013). Pure left neglect for Arabic numerals. Brain and Cognition, 81, 118-123.

Rickard, T. (2005). A revised Identical Elements Model of arithmetic fact representation. Journal of Experimental Psycholog: Human Learning, and Cognition, 31, 250-257.

Rickard, T., & Bourne, L. (1996). Some tests of an Identical Elements Model of basic arithmetic skills. Journal of Experimental Psychology, 22, 1281-1295.

Rickard, T., Healy, A., & Bourne, L. (1994). On the cognitive structure of basic arithmetic skills: Operation, order, and symbol transfer effect. Journal of Experimental Psychology, 20, 1139-1153.

Robert, N., & Campbell, J. (2008). Simple addition and multiplication: No comparison. European Journal of Cognitive Psychology, 20, 123-138.

Smedt, B. D., Holloway, I., & Ansari, D. (2011). Effects of problem size and arithmetic operation on brain activation during calculation in children with varying levels of arithmetical fluency. NeuroImage, 57, 771-781.

Sohn, M.-H., & Carlson, R. (1998). Procedural frameworks for simple arithmetic skills. Journal of Experimental Psychology, 24, 1052-1067.

Verguts, T., & Fias, W. (2005). Interacting neighbors: A connectionist model of retrieval in single-digit multiplication. Memory & Cognition, 33, 1-16.

Zhang, M., Si, J., & Zhu, X. (2012). The reversed neighborhood effects in mental spoken Madarin number words. Psychology, 3, 57-61.

Zhou, F., Zhao, Q., Chen, C., & Zhou, X. (2012). Mental representations of arithmetic facts: Evidence from eye movement recordings supports the preferred operand-order-specific representation hypothesis. The Quarterly Journal of Experimental Psychology, 65, 661-674.

Zhou, X. (2011). Operation-specific encoding in single-digit arithmetic. Brain and Cognition, 76, 400-406.

Zhou, X., Chen, C., Dong, Q., Zhang, H., Zhou, R., Zhao, H… & Gong, Q. (2006). Event-related potentials of single-digit addition, subtraction, and multiplication. Neuropsychologia, 44, 2500-2507.

Zhou, X., Chen, C., Zang, Y., Dong, Q., Chen, C., Qiao, S… & Gong, Q. (2007). Dissociated brain organization for single-digit addition and multiplication. NeuroImage, 35, 871-880.

Zhou, X., Chen, C., Zhang, H., Chen, C., Zhou, R., & Dong, Q. (2007). The operand-order effect in single-digit multiplication: An ERP study of Chinese adults. Neuroscience Letters, 414, 41-44.

Zhou, X., & Dong, Q. (2003). The representation of addition and multiplication. Acta Psychologica Sinica, 35, 345-351.