Experiencing the space we share: rethinking subjectivity and objectivity.

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Abstract Mathematics in schools exists substantially as pedagogical material crafted for supposed modes of apprehension. But of course such apprehension depends on how we understand mathematical objects and how we understand human subjects. The apprehension of mathematical objects is examined through sessions with student teachers researching their own spatial awareness from a pedagogical point of view. The paper is guided by the recent work of Alain Badiou whose philosophical model develops a Lacanian conception of human subjectivity and defines a new conception of objectivity. In this model the conception of subjectivity comprises a refusal to allow humans to settle on certain self-images that have fuelled psychology and set the ways in which humans are seen as apprehending the mathematically defined world. The assertion of an object, meanwhile, is associated with finding a place for it in a given supposed world, and it may reconfigure that world. The composite model understands learning as shared participation in renewal where there is a mutual dependency between the growth of human subjects and of mathematical objects. Renewal is referenced to a diversity of ever shifting discursive parameters that enable learning through negotiating the spaces within which we operate and the objects those spaces allow. Learning to teach then comprises developing sensitivity towards the discursive spaces that allow others to build objects. The paper provides examples from teacher education activities centred in addressing these concerns.

Keywords *Mathematics, objectivity, subjectivity, spatial awareness, Badiou*

1. Introduction

This paper sets out with the assumption that our conceptions of mathematical objects are functions of how we conceptualise the human subjects apprehending them. Mathematical objects, such as those presented in schools, are positioned within ever changing forms of life that constantly reposition or reconstruct those objects, and the meanings conferred on them. Meanwhile, human subjects are understood in relation to evolving or shifting discursive backdrops that can change *who they are*. The paper then is centred on an interest in understanding how life, and in particular pedagogical activity, produce and confer meanings on mathematical objects, and on human subjects, rather than supposing that those objects and subjects precede the turbulence of life. The paper proposes that we need to think of mathematical thinking not just as a field of knowledge where experts are assumed to be finding out about the gaps. The life that we lead prevents knowledge from being stable except in localised ways. Any assertion of such localities restricts our capacity to conceptualise and occupy new ways of being in worlds hitherto unthought.

Our positioning as teachers and students confronting mathematics responds to newly emerging manifestations of mathematics. For example, machines occupy spaces previously held by human operatives (e.g. cash tills totalling purchases, programmed automatic market trading, robotive factory procedures, medical technology, computer centred mathematics and calculators). Barad (2007) has argued that the materiality of the human reach needs to be understood as incorporating such apparatus. Palmer (2011) shows how Barad's notion of apparatus underlies the very structure and apparatus of schooling that supports mathematical learning (e.g. school building formats, squared exercise books, registration requirements, pedagogical models, curriculum frameworks). Hoyles, Noss, Kent, and Bakker (2010) have shown how humans in the workplace now need to think systemically, not so much engage in detailed mathematical operations. Calder (2012) meanwhile indicates how perceptions of mathematical spaces as approached within mathematical classrooms might be managed in new ways through the facility of computer packages. Meanwhile, initiatives such as curriculum implementation in education and associated assessment impact on how a particular community builds its wider public understanding of mathematics and of associated technology/apparatus in ever-changing circumstances (Brown & Clarke, 2012). Those pedagogical practices ultimately come to define that community's conceptions of mathematics, and how that community expresses its demands on educational processes, and hence on teachers, in those areas.

This paper draws on some contemporary philosophy, in particular, the recent work of Alain Badiou, where conceptions of object and subject are brought into a new relation. Badiou (2007, 2009, 2011) rejects erstwhile distinctions between analytic and continental philosophies through embracing both the technologies of the former and the more temperamental conceptions of subjectivity associated with the latter. On the one hand Badiou builds a new conception of "object" that results from fitting new models to newly supposed worlds. He alerts us to the contingency of hitherto supposed worlds and the objects that they support. Meanwhile, Badiou invokes Lacan's psychoanalytic theory. Here subjectivity is predicated on a more collective conception of the subject, where an individual is understood with respect to his or her collective participation in the name of some wider adjustment. Specifically, as an example to be explored here, one can adopt a range of attitudes or identifications to supposed mathematical correctness in pedagogical situations. Such situations are built around suppositions as to how mathematics provides an analytical frame through which to contemplate our lives, and around alternative pedagogical assumptions as to how ideas are constructed and shared. These different modes of subjective identification display alternative pedagogical attitudes but also result in mathematical objects being produced differently in notionally shared situations. There is a challenge to understand how emergent mathematical thinking can be activated and approached through pedagogical interests. Specifically, teachers will not be adequately prepared for future teaching with past versions of knowledge. They need to be responsive to new ways of thinking that will locate mathematics in new relations with life. We shall specifically counter the idea that a teacher needs to understand new challenges in advance of her students.

The proposal for this Special Issue argued that a characteristic feature of much research in mathematics teacher education is that it is conducted within one of the three relatively distinct fields of, teachers' knowledge (e.g. Hill, Rowan & Ball; Rowland & Ruthven, 2011), teachers' beliefs (or affect more generally) (e.g. Zan, Brown, Evans, & Hannula, 2006; Hannula, 2012), and teacher identity (e.g. Black, Mendick & Solomon, 2009; Walls, 2009; Walshaw, 2010). In some more recent instances of research affect is understood in terms of how the trainees experience the demands to participate in emergent professional patterns of discursive activity (Brown & McNamara, 2011; Frade, Roesken & Hannula, 2010; Walshaw & Brown, 2012). The meta-discussion proposed in this present paper relates to these recent approaches to affect by embracing each of the three fields in relation to Badiou's model. The discussion reconfigures knowledge as compliance with particular models of mathematics and of mathematical learning. Affect is understood in terms of resonance or dissonance between the individual's sense of self and the model to which that individual feels obliged to conform. Identity is recast as successful or unsuccessful *identifications* with particular discursive formulations. The cognition/affect interface (McLeod, 1992) is displaced, in crude terms, by subjectivity being referenced to identification with such narrative accounts shifting through time, rather than on the functioning of individual brains in a given situation. The meta-discussion links the trainee teachers' mathematical experimentation to their participation in a permanent state of adjusting to new conditions, where neither brains nor mathematics precede life. There may be affective consequences, or plain awkwardness, in adjusting to new forms of knowledge. Yet such is life. The awkwardness is not something to be abolished. Rather, new conceptions of mathematical knowledge, such as pedagogic framings introduced through new curriculum initiatives, or schematic approaches popularised through work or leisure activities, feed into a collective working through of these conceptions, which make qualitative adjustments to that mathematical knowledge. It is for the new generation of teachers to work out what those new conceptions mean for them personally as they negotiate their path into teaching, and subsequently how they might tap those new conceptions as pedagogical opportunities with their future students.

The paper commences with a brief outline of the university classroom situation in which these themes are explored. A sketch of some Lacanian theory is then provided as a prelude to two sections, which in turn introduce conceptions of subjectivity and objectivity derived from Badiou's work and referenced to the classroom activity. We then provide some examples of research data centred on the negotiation of mathematical objects. This data is discussed from the point of view of how the depicted participants are variously positioned in relation to the mathematical models in question. This provokes a question as to how new teachers might conceptualise the objects that they will eventually teach, as objects to reproduce, or as objects to renew.

2. Setting and aims

The central theme of the paper concerns how participants variously identify with particular conceptions of mathematics and how those identifications support teacher education ambitions. The authors have collected data over five years with successive groups of first year undergraduate students training to be mathematics teachers in British secondary schools. The product of this activity provided data on how students conceptualised mathematical objects. In each of the years one or more of us have been teaching twenty 3-hour sessions to each group. The sessions are each designed to broaden the student teachers' conceptions of mathematics through carrying out a variety of mathematical investigations, to see mathematics from a broad range of unfamiliar perspectives. Through such activity the students were encouraged to explore themes independently, pose and answer their own questions, and reach mathematical generalisations where possible. The agenda of the sessions is set out as being centred on all participants (students and staff) researching together how people build mathematical understanding. We want to know how people learn mathematics. What might our shared learning (as mathematicians, as pedagogues, as researchers) tell us about this? How might the students' analytical approaches be developed and transferred to their work in schools. This enables the production of data (such as, reflective writing produced during and after the sessions, multiple sound and video recordings, alternative approaches to the mathematical work, etc). The students are encouraged to submit a file outlining their research for their end of year assessment. Two or three of the sessions each year are devoted to the apprehension of geometric entities through exercises centred on the students' own bodily movement (Brown & Heywood, 2011, Brown, 2011). One of these sessions includes work on planetary movement as an embodiment of geometrical configurations[1]. The exercises become a prelude to the students formulating mathematical models of the configurations they had encountered in these physical exercises as part of thinking through how mathematical entities come into being for themselves and potentially for other students.

3. Lacan's psychoanalytical theory

We now turn to a consideration of how identification might be understood. Humans progressively work between the physical world that they apprehend in everyday life, and conceptions of that world derived from more socialised ways of making sense of that world. We are especially interested in those ways pertaining to more mathematical accounts of the world, as defined through the symbolic apparatus typically utilised in such accounts, and in turn with how mathematical objects are used to support those accounts. Brown (2011) has suggested that such mathematical accounts presuppose ways of looking, and in this sense shape the parameters of what it is to be a human subject. He develops this idea in Lacanian terms where individuals have a common sense view of the world, and of themselves, through which they apprehend objects and their own relationships to them. That is, individuals initially understand the world, and themselves, through this common sense view. Yet, acceptance in the shared world requires a negotiation of the symbolic networks (such as pedagogical apparatus) that have been produced, by those who have preceded us to make sense of the physical world. The scientifically defined universe contingently defines worlds, and the human's place within them (Lacan, 2008). It may, however, be that the individual is not especially comfortable with these assigned places and that there are consequences to these perceived failures of fit. For example, psychology has a preference for defining individuals in terms of various physical or responsive attributes, which may bypass the affective sense of self possessed by the individual herself. Or alternatively, the individual human might too compliantly accept this external designation. Lacan's model locates life as a negotiation in which the individual works through

[1] On the occasion being described the team comprised the two regular teachers, a science teacher educator initiating the specific activity, an experienced teacher conducting PhD research and a video operator.

successive accounts of the world, each of which points to a place for the individual. He mocks the failure of scientific constructs to keep up to date, consigned as they are to the need for regular renewal, whilst in his view the human always survives.

What does this look like in the specific example that we plan to address here? Humans experience geometric objects in orienting wider spatial awareness, and that empirical site enables individuals to produce or share mathematical objects. Empirical reality here, however, is understood as being produced through particular interpretive procedures derived from specific understandings of human subjects and how they frame their sensual experience. That is, empirical reality is just one version of events that fixes life in a particular way. For example, humans start, inevitably, with naïve ways of apprehending the moon, the sun and the stars. They progress through a more intuitive sense of how things work – *the moon moves during the night*. Then perhaps they encounter a mathematical frame of reference for a more shared human knowledge – *the moon encircles the earth but we only see part of that circular move*. This shared human knowledge takes different forms in different educational locations, according to priorities, level and so forth. Mathematical knowledge, for example, depends on research funds motivated by current utilitarian agenda and more immediately in schools on decisions to include selected aspects in the curriculum as processed through particular pedagogical modes.

4. Subjectivity

Lacan's (2006) approach emphasises the societal demands that shape the individual human subject. The subject derives from the stories that are told about him or her, or from the stories that are told about people, or classes of people more generally. The individual may or may not like the way in which they are being classified. In the framework that we are following the mathematics teacher's identity is a function of how the teacher is understood in a given location or time, perhaps according to the skills, competencies and practices seen as normal. Learning here might be understood more as being about an experience through time rather than being about apprehending mathematical ideas located in a fixed conception of space. Education comprises the formation of objects/events in time/space rather than being about an encounter with ready-made entities. Mathematical ideas cannot necessarily be apprehended in an instant. They may have a time dimension, as a conceptual process (Teissier, 2012), or through their location in an unfolding historical development (Corfield, 2012). The apprehension of an idea may result from a gradual assimilation of the idea's components and qualities and how these are combined in its formation. I may compare new sets with a selection of previously known sets. I may contrast the operation of a newly located function with more familiar functions. The progressive apprehension of the supposed idea becomes part of the story of my life, a part of getting to understand who I am and how I fit in to a supposed world or how I might make that world otherwise. That is, this progressive apprehension builds a story around the abstract entities being located, a qualitative layer in which any learner is fully implicated since it was integral to their very own constitution. The individual's actions comprise part of a collective response to such situations. This collective response might result from mathematics being viewed differently more generally, for example, as a result of a curriculum change, through mathematics being seen differently in popular mythology, in changes to the demands on mathematical capabilities, and so forth.

Badiou draws on Lacan's conception of subjectivity. The subject, rather than being seen primarily as a biologically framed cognitive entity, is understood through a reflection of a broader symbolic universe. Roth (2012) and Brown (2011, 2012) provide alternative accounts of how Lacan relates to more mainstream Vygotskian accounts of psychology within mathematics education research. Lacan's concept of human formation is triggered by a transformation that takes place when a young child assumes a *discrete* image of herself. Lacan's iconic example is that she looks in to the mirror and recognises herself. This allows her to postulate a series of equivalences, samenesses, identities, between herself and the objects of the surrounding world (the equivalence of my movement on the floor, to the drawing on paper, to the image in my mind, seen as continuous movement, or as a configuration of points).

For example, student teacher Imogen carried out a body movement exercise in which she tried to maintain equidistance from her body to a fixed point and to a straight wall. She commenced by being positioned halfway between one of her friends and a nearby wall. The first part of the activity comprised attempts to physically move from one point to another maintaining the equidistance. This challenge was shared with three peers all of whom had different views on how Imogen might achieve this (or not). After much discussion and walking around, a set of points was marked out on the ground using screwed up pieces of paper. The whole episode was videoed for later analysis. A rough drawing was created in which the points were joined. At home Imogen extended her notes. After further rough drawings and calculations, she eventually drew a graph featuring a point and the wall on to squared paper (with the fixed point being the origin and the line y=10 as the wall) and used Pythagoras' rule to generate positions that met the criteria. The second point was located by drawing a triangle (0, 0), (4,0) and (x, 4).

> From this we can pull out a triangle in the hope it will help us calculate what the x coordinate would have to be in order for the distance from the wall to the origin to remain equal. Using Pythagoras theorem we know that:
>
> $a^2=b^2+c^2$
> $6^2=4^2+x^2$
> $x^2=6^2-4^2$
> $x^2=36-16$
> $x^2=20$
> $x=\sqrt{20}$
>
> We can now see that the co-ordinate for the new point should be at ($\sqrt{20}$, 4) for the distance from this point to the wall and this point to the origin to be six. We also know that the points will be symmetrical in the y-axis. So now another point will be ($-\sqrt{20}$, 4)

Imogen then plotted those two points on a graph. This was followed by her finding the x ordinate for the points at a distance of three, generating the points ($\sqrt{40}$, 3) and ($-\sqrt{40}$, 3).

> Carrying on this method I continued altering the y co-ordinates so that the distance from the wall changed which in turn changed the length of the hypotenuse and also the height of the triangle, this gave us many more different co-ordinates where the moveable point could be so that it was equidistant between the fixed point and the wall. However, I did think that once the moveable point passed the x-axis then there wouldn't be a point that would be of equal distant to the wall and the origin however after drawing a diagram and extending the graph a little bit more I came to realize that it was possible for it to be below the fixed point as it was just that, a point, however we could not have a point above the wall as the wall continued on for eternity so the moveable point would always be closer to the wall.

The calculations were combined into a table. As she developed more summative results over time writing of this sort provided a narrative spanning twelve pages of notes, calculations and diagrams that documented her shifting perspectives from enacting physical movements on the floor to creating more formal diagrams and equations. This work thus provided a narrative of the student teacher's journey of learning during which the curve came into being for her. In the perspective that we are pursuing, such narratives document human subjects and mathematical objects coming into being. By creating such narratives in this and other sessions the student teachers become more adept at accounting for their own learning process, making sense of who they are and how they fit in. The narratives on the process of emergent understanding provided excellent material for discussing and comparing learning experiences in our group sessions. The discussions enabled more refined use of mathematical terms but more importantly the discussions provided a forum for considering more generic pedagogical terminology, such as "generalisation", "conjecture", "logical sequence" and "proof". Consequently, the student teachers became better able to report on the learning of their own students in a more refined language when they tried out similar activities in schools

More theoretically, according to Lacan (2006), an image of self fixes an egocentric image of the world shaped around that image of self. That is, the assumption of a self results in a supposed relation to a world and a partial fixing of the entities she perceives to be within the world. The self is understood through being gauged against this supposed world. Initially, in our case, Imogen builds a sense of such relations by moving herself around the physical space. Imogen's sense of herself is referenced to instructions that have guided her movement. In due course these relations become implicated in more overtly mathematical phenomena that underpin her more formal approach. This shift of perspective comprises reflective awareness of symbolised relationships, such as how specific bodily positioning responds to a coded spatial environment. These objects are linked to "mathematical knowledge" and become relatively fixed with consequential restrictions on how relations between people and geometry can be understood. Imogen's assumption of self comprises a collation of a set of characteristics, attributes, organs, positions, etc. that make up that self. This set of characteristics is "counted as one" person. Lacan, however, cautions that we should be wary of this image, since it is illusory. It is a snap shot that never quite works. It never fully captures the real me as it were, rather like the production of a formula not fully capturing the experience of moving according to the locus of a curve. In Lacan's model the limits of our "real" self are never fully visible to us.

5. Objectivity: counting as one

Badiou commences his analysis with a sheer multiplicity of elements in a pure state of being. His set theoretic approach locates a mode of organisation with no empirical reference. Here, there is no over-arching unity, such as the Oneness sometimes celebrated in theology. In this state the elements are not anywhere but can be combined in subsets of that multiplicity to create or define unities. Badiou's assertion is that any such unity, or object, derives from an *operation* of "counting as one". That is, an *object* is produced by the operation of counting a set of elements, within a *supposed* world, as one object. These elements could be atoms, blood cells, GPS coordinates, emotions, humans or items on a mathematics curriculum. This operation brings the object (kettle, mouse, Swaziland, schizophrenia, the Manchester United football team, mathematics curriculum) into existence within a *world* (kitchen implements, rodents, Africa, health conditions, the Premier League, schooling). And in a sense it also brings the world into being. The assertion of an object asserts the world that is the outside of that object, a world that has perhaps been changed a little by the specific noticing of the object. The *world* is itself a result of a wider "counting as one" (of the total elements of that *world*). In this formulation any element can itself be a set and a potential member of other sets. And within any assertion of a set, yet further possibilities are created, resulting from the construction of subsets or power sets producing yet more new entities. This very proliferation itself defies any final stability in the universe. For this reason there can be no settling or convergence in the meaning of the constituent terms. Badiou contemplates a partially managed multi-dimensional infinity. Yet forms of knowledge are predicated on a world, comprising specific sets of terms within this world. Such forms of knowledge might be disrupted as they readjust around the ever-expanding set of sets being counted as one. The advance of mathematics can be seen as the practice of producing its objects. For example, Badiou cites the introduction of i as a disruption to the conception of number. Such expansion reveals objects not previously identified within earlier overarching multiplicities.

How might this approach support the exploration of learning, or more generally the human apprehension of mathematical objects? Mathematical thinking can generally be understood through the pursuit of noticing or asserting a generality, a notion resonant with "counting as one". The construction of a model results from an *operation* that apprehends, or perhaps creates, a set of elements as a unity. To continue our example from the last section, after nine pages of calculation, further maps and reflective writing, Imogen convinced herself that she could carry on producing points. Finally, she plotted the points and joined them to produce a curve. That is, the points (a-i in Figure 1) were *counted as one* set, which Imogen finally concluded marked out the course of a parabola with an equation of the form $y=5-x^2$.

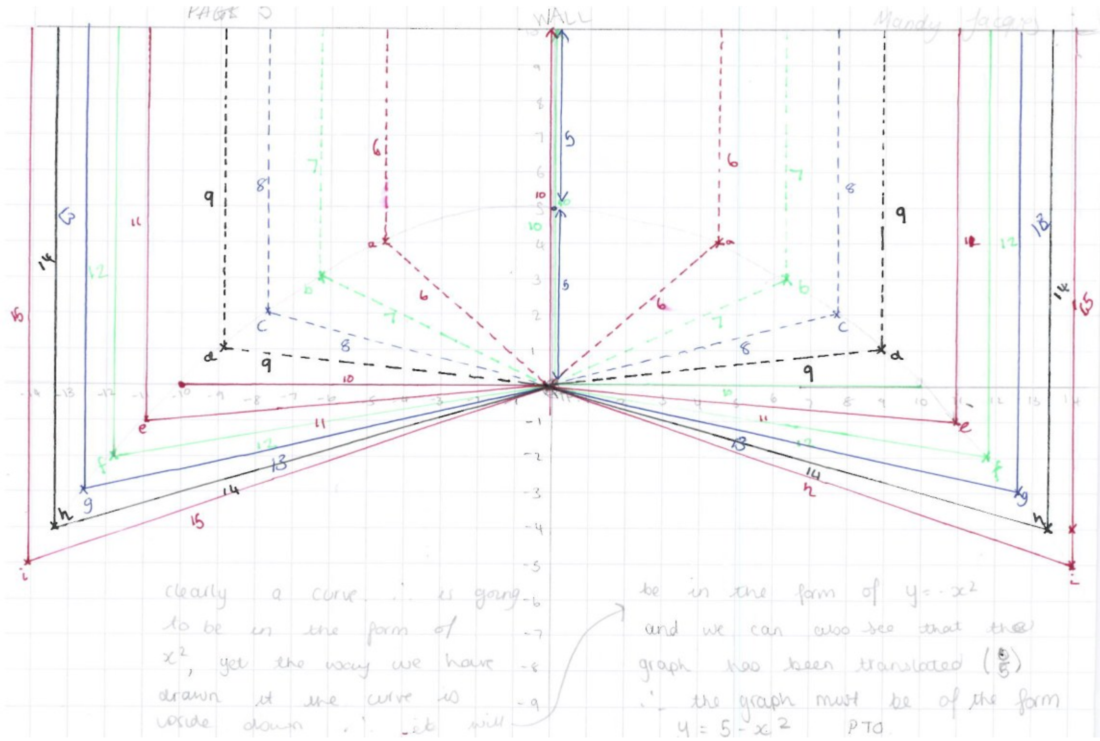

Figure 1.

As she puts it, rather speculatively: “clearly a curve, ∴ is going to be in the form of x^2, yet the way we have drawn it the curve is upside down, ∴ it will be in the form of $y=-x^2$ and we can also see that the graph has been translate, ∴ the graph must be of the form $y=5-x^2$”. Her remaining pages of notes include an attempt at giving the general formula $y=a/2-x^2/2a$.

Badiou’s notion of “counting as one” works whether we are considering students encountering socially known ideas for the first time, such as a parabola, or new innovations by researchers. A “counting as one” seen as the acquisition of a new model could be understood in either of these two situations in relation to a newly extended situation. The assertion of a given entity entails an operation to “count as one” the objects of a given set. But thereafter the term can become a member of other sets of objects such as “conics”, e.g. parabolas, ellipses, circles, etc, seen as making up a world and utilised in organising our apprehension of the world. Algebraization comprises a similar operation of “counting as one” (e.g. identifying the set of points obeying the relation $y=5-x^2$). The objects get to be there, in a world, as a result of the operation. But they need that prior (or simultaneous) construction, of a world (in this instance two-dimensional space, structured according to some rules), to *be there.* The existence of an object requires a place for it to exist. Badiou distinguishes between mathematics as a domain of truth and mathematical knowledge pertaining to a specific conception of a world. For example, geometry is knowledge if it is predicated on a Newtonian, or human, construction of the physical space in which we reside. Truth is eternal (but not static) whilst knowledge is forever being updated to newly perceived conditions but at any point in time frames our perception of the world, as we know it.

Learning comprises the formulation and positioning of an object in a *world*. This requires the assertion of an object, and an assertion of a world. Object and world imply each other. With regard to the students moving around according to geometric loci the task is to apprehend continuous movement as a sequence of points. These points are then aggregated to “count as one” object, understood in terms of this mode of aggregation. Retroactively the students can recognise the shape they have walked against a new register and see it as an object.

In short an individual human (a set of attributes counted as one) confronts an object (made of elements that have been counted as one). These two entities come into relation in the given supposed world, for now. Yet the operation of “count as one” can always be performed differently according to new circumstances. The relation is contingent on a world that is always changing, and needs to move on. This “moving on” underlies the concept of pedagogy as participation in the adjustments to life being addressed in this paper.

6. Apprehending mathematical objects in planetary movement

To recap, a central tenet of Badiou's work is that an object must *be* in a world for it to *exist*. Brown (2011) has discussed in more detail Badiou's set-theoretic distinction between *being* and *existence* in relation to school mathematics. In this approach, the objects of school mathematics are functions of implied worlds, whether those worlds are "real life situations", or "mathematical domains" with their specific modes of functioning and inclusions. In this sense, *all* school mathematics is embodied. Brown and Clarke (2012) have shown how school mathematics is a function of institutional contexts and regulated as such. Barad (2007) has shown how scientific phenomena more generally are functions of the inspection apparatus through which they are viewed. Shulman (1986) famously made a distinction between *subject knowledge* and *pedagogical content knowledge*, whilst other writers questioned whether this distinction was valid since all subject knowledge is itself a form of representation (e.g. McNamara, 1991). A key argument of this paper is that pedagogical contexts (SK and PCK) define their objects. Indeed Badiou (2007, p. 7) takes the extreme view that "*there are no* mathematical objects. Strictly speaking mathematics *presents nothing*". This surely applies to Badiou's use of set theory. The growth of geometry however has been shaped around empirically motivated objects, such as a circle. It is not easy to sort mathematics entities according to whether they are empirically referenced or not in their historical formation. In this section we shall consider a specific embodiment of mathematical phenomena towards considering how these phenomena might be experienced and thought.

Some of the bodily movement exercises conducted in our sessions involved acting out the relative configurations of earth, moon and sun and how these configurations might be seen from alternative perspectives; from deep space, from the surface of the earth, etc. (Heywood & Parker, 2010). These configurations were enacted firstly with a globe for the earth, a small white sphere for the moon and a torch for the sun (Figure 2). Later individuals took the place of the moon then spinning in relation to the earth (Figure 3). The purpose of the sessions was to enable students to share their attempts to apprehend variously located mathematical objects, experienced as if navigating and orienting themselves inside big versions of the shapes concerned. That is, they told progressive stories of themselves, as apprehenders of the variously perceived spatial environments, developing technologies through which specific orientations could be achieved.

Figure 2.

Figure 3.

We now offer some pieces of data as examples. These derive from the research orientation of the sessions more generally. Everyone, students, tutors and visitors alike, kept extensive records of their activity during each activity in an attempt to understand how the learning of mathematics happened. An extensive catalogue of video clips and photographs were collated, which were later used to build the written records that were produced. In the subsequent discussion we suggest alternative approaches to framing school mathematical objects arose. From the teacher education perspective being taken we seek to show how alternative subjective positions can be productive of important qualitative aspects of the mathematical phenomena being portrayed. That is, these qualitative features, specific to the *world* in which the ideas were encountered, provided markers for observing and orienting the mathematical ideas being approached. We are making the assumption that school mathematics is typically encountered through qualitative features of the pedagogical worlds being entered (e.g. needing to make an argument to peers or tutors, representing ideas in different forms, reference to standard ways of depicting ideas for examination settings, etc). Learning about the mode of embedding and working within it is part of the necessary learning required in many instances, especially those directed at supporting utilitarian agenda, e.g. conceptualising the moon that we can see as being on a circular orbit.

i) The first piece of data comprises an extract from our written records collected during the activities as part of our own research:

> Kelly had brought some data with her, such as the exact duration of the day and the year, and it was apparent that her preparation for the activity was systematic, mentioning terms such as 'aphelion' ("which is the point of orbit furthest from the sun; … which is going to be our winter"). As soon as the activity started Imogen said 'the sun shines and the earth spins and when you don't have the sun on you it's night time'. Kelly pointed out that the length of a day is exactly 23.97 hours, reading it from the data that she had brought with her. … Imogen replied immediately that 'there is noon when the sun is at its highest, when you are closest to the sun'. She gave an example by choosing Saudi Arabia on the globe and turning it: 'if we look to Saudi Arabia, it is noon in Saudi Arabia, as it moves away the sun is sinking again and then it goes to night time and then this is the midnight, and then it gets early, the sun is rising, the sun is rising, it gets to the noon, the sun is at its highest point'.

ii) Another piece of data comprises extracts of reflections from an experienced mathematics teacher within the team researching how mathematical objects result from pedagogical exchanges as part of his doctoral studies. During the session he was observing the students but occasionally found himself drawn into discussions as the students had known that he was quite good at mathematics. In the reflections the teacher is exploring the consequences of these unexpected interventions from a pedagogical point of view. The extracts refer to the sequence above. They were chosen with view to showing how the teacher's reflections are revelatory of his own identification with particular conceptions of pedagogy and of scientific discourse.

The following comments indicate his pedagogic orientation:

- I was kind of prepared for it
- I don't want to "spoil the fun of discovery"
- I responded with an expression of approval
- I pretended to agree
- I instinctively tried to break the rhythm, so I said something that wouldn't be much of a clue
- I repeated what Kelly said, trying to sustain Kelly's conclusion as a base for the subsequent investigation
- Without realizing it I was entangled in the group discourse in the way that I was initially trying to avoid
- I fully understand that [was] my old reflex as a teacher
- I had fallen in the trap of influencing the group, as I could not disengage myself from its activity and as I interrupted the group's interaction to some extent; I became a victim of my own devices

These comments however point to a "correct way" of seeing things:

- she was not using 'aphelion' *the correct way*[2]
- using 'aphelion' and 'perihelion' *the right way*
- trying *to keep the level of the group discourse as advanced as possible*

iii) In the final extract Kelly, Imogen and Rebecca (Figure 3) share their apprehensions of how the moon moves in relation to the earth. They experience difficulty in communicating these apprehensions in words. Finally, they enacted the orbit of the moon through bodily movements

[2] The teacher reports that at one stage: "Kelly mentioned the summer time, introducing the term 'aphelion'".

that seemed remarkably coordinated, with all three students moving in the same trajectory around a suspended sphere (the earth), where they each maintained a constant orientation to the earth throughout. Successive attempts interrupt each other:

> K: Because we are on an angle of let's say this way I am looking at it… as we come round if we keep on that angle we only ever see my face, you never see the back of my head.
> I: It doesn't matter whereabouts.
> K: Yeah you split…
> R: Kelly's focus stays on that ball so her body might be turning but she is still looking…
> K. So you only ever see…
> R: So, if someone is stood on there, they would only ever see my face, not the back of my head, otherwise I'd be going…
> I: We must be right because we are all on the same wavelength. We all agree.
> K: If I could spin myself like this ...
> I: The moon's just on an angle. That's what it is. Spin round double ... see it's worked… best logic I've thought of.

The three examples comprise individuals displaying a range of pedagogical interests and attitudes towards notional mathematical objects. We have trainee teachers who oscillate between an unsteady grasp of the terminology and a more symbiotic immersion in the evolving world to which this terminology attempts to cling. This terminology is included in their own learning narratives within which meanings evolve. We have a teacher referencing his own interventions to established parameters. We have teacher educators in the background managing an activity towards influencing certain pedagogical results. We have researchers adopting more theoretical perspectives on how mathematical ideas are being framed. These alternative perspectives link to alternative conceptions of learning (discovery approaches, gravitation to correct understandings, creation of fresh perspectives, etc.) that variously construct and position mathematical objects, and shape the apprehension of more or less familiar forms of knowledge. The enquiry in this paper is specifically focused on how the participants variously identify with particular conceptions and how those identifications supports teacher education ambitions, specifically those relating to building narrative around learning experiences. We cannot assume any sort of correct overview of the activity that took place, nor be representative of the multitude of insider perspectives.

In the first extract, Kelly's experimental introduction of specific terminology is depicted as the consequence of advance preparation at home, preceding a more settled understanding of the parameters that framed the terms that she used. "Aphelion", as an embodiment of, or subjective perspective on, an ellipse, for example, was occasionally asserted as being linked to a position on an orbit closest to the sun, rather than furthest. Yet the bigger point is that the *world* that would host this term within a more secure scientific discourse was apparently not yet in place for her. Neither the host space nor the objects it allowed had been established. The technical term "aphelion" for Kelly was dislocated, floating in space as it were - its home had not yet been fully conceptualised as a point within an overarching spatial structure. Yet clearly she was introducing the term to provisionally mark out the territory that she was seeking to better understand.

In the second piece we view the events through brief extracts from the teacher's extensive reflective writing where he indicates his own unexpected participation. The extracts point to a specific mediation and more or less obliquely depict his involvement in the activity. The teacher's supposition of the task in hand is at least partially centred in a particular conception of the knowledge to be apprehended. Yet this interest is obscured by his own concern that he be an observer rather than a teacher. This is against the backdrop of Imogen, Kelly and Rebecca playing out alternative approaches to the task where they have prepared for the task differently and get to be convinced differently. The teacher has an ambivalent role focusing primarily on understanding how the others are apprehending the task, where his own involvement in the proceedings is nevertheless having some impact. The ideas in question are manifested differently through the thoughts, action and speech of the people in question, in relation to a set of activities designed with certain pedagogical ambitions in mind. But the issue for this paper is not with the relative merits of the perspectives achieved but with how mathematical objects derive from alternative subjective positions or modes of identification.

In the final example, the mathematical object in question is a circle (or ellipse) but where many qualitative dimensions of the pedagogical world supplement the students' experience. The perspectives assumed of this circle obscure its appearance as a clear cut geometric entity. The task was centred on being able to apprehend an orbit from various given perspectives, such that the students were challenged to situate themselves *within* and experience mathematically conceived space. The question of moving around this ellipse whilst maintaining the correct orientation further complicates the sharing of perceptions in words. The keenly felt perception of being on the "same wavelength" within shared movement, however, somehow reduced the need for a clear set of words. Indeed, the desire for a correct set of words seemed destined to fail as the power of shared movement became far more evocative of the entity in question. De Freitas & Sinclair (2012) have discussed how gestures and diagrams provide alternative evocations of mathematical ideas. The students are identifying with an experience that defies final capture in a symbolic form, but it also defies final capture of the students themselves in finished form. Their subjectivity is referenced to a lived experience, with no fixed relation of object (an elliptical orbit sought through a succession of fragmented sentences) to subjects (held by names and relations to other subjects). In a "real life" context the affectivity of the space teaches the students to recognise their position in time and space through sensual clues, (e.g. shadows, direction of moon, darkness, temperature, reciprocities of sharing space with others). Their emergent spatial and temporal awareness, marked by these qualitative features, occurs as part of a layering of complex systems of relationships and spaces within constantly changing circumstances.

7. The ontology of mathematical objects

The set of people present are each assessing the domain according to their own respective perceptual capacities, and according to the demands being made. They are each apprehending objects in potentially different ways, more or less from a pedagogic perspective. But to what extent is it meaningful to speak of them as sharing mathematical entities in some absolute sense? There is an experience through time within the episode depicted that is unique for each individual yet clearly there is some orientation around supposed points of sharing. In our example, Imogen developed her conception of a parabola, without ever naming it as such, through discussion and shared activity with some peers. But how might we understand such sharing? Ricoeur (e.g. 1984) argues that the passage of time does not lend itself to being described as a sequence of events, features or stages but instead needs to be understood as being mediated by narrative accounts of such transitions, relying on interpretations, which at a very basic level cannot be seen as comprising phenomenological features. The perceptual or phenomenological mark-up is different for each person. Each has a story to tell. The mark-up is a function of the individual's specific identification with the wider discourse. A book edited by Doxiadis and Mazur (2012) brings together a set of papers each concerned with how mathematical experience might be understood through narrative where a time dimension to mathematical conceptualisation is highlighted.

How might we resolve the ontology of mathematical ideas in a school context? Is it possible to think of school mathematical objects as being outside of some sort of agenda? For example, the conceptions of students in England doing Advanced Level examinations at 18+ are rather constrained by the way in which questions typically frame conics, such as a parabola. In school mathematics geometrical entities are normally presented as if from an objective perspective within a limited set of frames. The idea of subjective perspective, seeing a shape as if from being inside of it and moving around in it, would be rather peculiar in this setting. Perspectives are regulated. The mathematical entities are required to assume specific modes of existence for assessment purposes. As we have emphasised, Badiou's approach to ontology resists notions of primordial unity in favour of multiplicity, comprising elements in a pure state of *being*. This multiplicity precedes any notion of primordial relations, or objects. To *exist* these elements must be conceptualised within a "world" in which relationships between elements can be understood, and objects can exist. The name "parabola", if it is known, can be assigned to a walked path or to a pencil line on a sheet of paper. Each world has a logic but our immersion in any one world is always uncertain, or a holding position that will surely reach the limits of its validity. Any specified domain of knowledge could be such a world. Importantly, Badiou introduces contingency to any relational structure keeping open the possibility of the currently dominant world fading into obscurity in favour of some new configuration of this multiplicity, linked again to an ontology unhampered by erstwhile conceptions of objects, relationships or priorities.

So for example, our conception of our entire number system can be disturbed by the introduction of a new element, *i*, the square root of -1, or by Cantorian set theory conceptualising infinite sets as objects. Our examples above point to a powerful status quo that asserts traditionally understood ideas with a fixed set of relationships between them. Those ideas and relationships, however, are a function of a given world. The world of formal relations may or may not help students to enhance their more intuitive spatial awareness. Their locality might also be understood in terms of their positioning within a pedagogic world where the spatial landscape can be depicted in many diverse ways to reveal alternative configurations of objects, relationships and pedagogical priorities. For example, Williams (2012) reports on a national approach to teaching mathematics influenced by testing demands that resulted in a narrow conception of learning ill suited to more advanced study and a reduced disposition to subsequently learning the subject. This version of mathematics, centred on mechanical application, filtered out more nuanced relationships in mathematical learning defining the interface of humans and mathematics, such as “understanding” or, as other examples, mathematical intuition, imaginative problem solving capability, geometric awareness within bodily movement exercises, computer mediated conceptions of mathematical fields, and so forth.

For Badiou subjectivity is not centred in individual humans *qua* humans. An internet connected human, for example, defies all attempts to draw limits around her receptive or expressive capabilities, or the control she has over them. Her individuality may be subsumed as part of a trend. Badiou sees subjectivity in terms of “fidelity” to *events*. For Badiou events comprise new ways of being in a somehow expanded multiplicity (the inclusion of *i* in number system, recognising atonal music *as* music, votes for women, or an anti-slavery movement working to include more people as humans). Badiou (2009) posits alternative modes of identification with such events: One can go with it (*faithful*), deny it (*obscure*) or describe it in the terms of the old way (*reactive*). Such events disrupt the status quo triggering a wider adjustment to new conditions, consequential to a disturbance from within. De Freitas (under review) relates Badiou’s notion of event to her experience of a mathematical problem that “became a problem only when it shook my cherished assumptions and set my mathematical discourse trembling with indeterminacy”. The students immersed in reliving an elliptical orbit are perhaps more involved in self-reflexively exploring the apparatus through which they apprehend their spatial environment rather than the environment itself (Barad, 2007). This might be seen as developing sensitivity to how space is apprehended rather than supposing that there is a correct way of doing this. Education then is not reproduction of knowledge. It is predicated on perpetual renewal, where objects, relationships and priorities persistently adjust to new conditions, and to new subjectivities. Ultimately, in teaching and teacher education we are motivated by pedagogy and productive interaction, knowing that we can never finally represent the subjects that we want to teach and educational encounters will always be about negotiating those representations.

In the reflective writing presented in the second example, the teacher’s perspective might be seen as being referenced to a settled discourse, but that very settlement presupposes specific human relations to any given objects, such that words like “correct” or “systematic” can be stated, social roles can be assessed, and “approval” can be granted. There is more at stake than the mere sharing of objects. It is not just reproduction of the objects but the reproduction of the world that is pre-supposed by their existence. The objects are linked to a conception of the wider world where social roles are set through the make up of the world being assumed. Yet at the same time his attempt at refusing to supply the direct answers that are sought keeps open an experimental attitude in which the final constitution of the objects and the relationships between them are postponed. After all in this instance the exact meaning of certain terms is educationally less important than the preservation of rich social interaction shaped around the shared formation of notional objects and relationships.

The central players in this paper have been trainee teachers. Their main task has been to build a language for describing mathematical experience. As teacher educators we have resisted seeing our objective as being about securing standard understandings of certain concepts for onward transmission to pupils. The challenge for us as teacher educators has been to enable the student teachers to make up their own minds, to exercise critical capability as an attitude to mathematical learning. The latter entails their being able to articulate learning through time, and to provide narratives of how ideas come into being, emphasising the experience of mathematics rather than static mathematical knowledge. In becoming teachers the reflective engagement with how people share mathematical construction remains central, motivated as it is by the pending demands of sharing constructions with future pupils. The research of our students was centred on learning about possible relationships to mathematics in which mathematical objects and relationships were brought in to *existence* rather than it being about sharing found objects, towards better understanding the educational effects that might be produced. In *becoming* teachers they are participating in the *becoming* of mathematics. This becoming is centred on building a sense of how social interaction might work with their future students to enable the shared production of mathematical objects. Through conversation, through shared bodily movement exercises, through producing shared mathematics and reflections, the regulative discourse of the dominant order was being held at bay, until a more lively attitude had been developed enough to tolerate its arrival.

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