Geometry, subjectivity and the seduction of language: the regulation of spatial perception

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Abstract: Following Husserl's speculations on how geometry originated, we suggest that spatial perception is *seduced* by language as a result of human attempts to capture, signify and share its concepts. And this language traps geometry and humans themselves in to the forms that have guided and regulated past practices, thereby obscuring possibilities for cultural growth and adjustments to new conditions. Some body movement exercises reveal student teachers' spatial orientations. The paper proposes that the very evolution of geometry, and the ontological status of its objects, relate to their representation in cultural forms referenced to human self-image. It is further argued that learning crucially relates to evolving mathematical or pedagogical understandings of spatial phenomena.

Key words: geometry, subjectivity, language, history, unity, embodiment, Husserl

1. Introduction

In geometrical study we are seemingly confronted with ideal mathematical objects that are also a function of their cultural heritage. Yet these supposedly ideal objects typically derive from human constructions, made with respect to configurations observed in the physical world by humans at a particular stage in their own evolution with their given natural perceptual apparatus (eyes, fingers, bodies, etc) and technological supplements (telescopes, cameras, computers, sensors etc). Badiou (2009) has argued that these objects are products of particular culturally and historically situated frames of knowledge. Any particular form of knowledge relates to what he calls a *world*, a particular set of circumstances, where any analytical apparatus has definite limits of applicability.

Newtonian physics draws on Euclidean geometry in defining such a world. This analytical apparatus that had earlier seemed to be universally applicable, does not apply to the very large, such as in deep space, or to the very small, as in quantum mechanics. The apparatus would also probably not convince dolphins with their sensual apparatus and their very different ways of moving in space. There is clearly an outside to the world of Newtonian physics. The geometric objects that Euclid described are "ideal" only within a very specific human apprehension of the world and can only ever be accessed through technology or perceptual filters that are both time and culture specific.

Yet our very selves have been created according to a physical organisation and an analytical heritage consequential to a long history of spatial awareness. We sit on chairs, climb stairs, wash round dishes, swim with fishes, ride on Ferris wheels, remove orange peels, travel on trains and fly in planes. Our bodies have learnt to function and know themselves in physical environments that result from culturally embedded conceptions of geometry. We fit in to the social/physical world through participation in shared ways of organising, apprehending and constructing. Our perceptions of the physical environment are inevitably processed through aspects of this symbolic heritage. But so too are our *perceptions* of ourselves. We make sense of who we are through using the same vocabulary. Who we are is a function of the stories we tell of ourselves in this language. We cannot be geometrically naïve insofar as our sense of who we are results from identifications with this shared heritage. Our physical experiences are processed and understood through that vocabulary of set moves and analytical strategies. We have learnt some of these things in school, or through everyday life experiences. Yet in a fundamental sense they are also part of us, contributory as they were to our very formation, as we have learnt to move our bodies in a specific physical world, partially created by our ancestors who had similar bodies.

An earlier paper offered examples: of a child being directed to move around precisely in a seemingly haphazard banana plantation in Uganda, an equatorial country where consistent patterns of daylight assisted orientation, and of students in school in the same country trying to apprehend geometric configurations derived from Western culture (Bradford & Brown, 2005). That paper sought to show how mathematical ideas and conceptions of the students' task derived from cultural parameters. This paper broadens the scope of analysis by offering further theoretical and practical reflection on how we signify geometry and make it part of our lives, and in so doing actively participate in a shared cultural heritage that continues to evolve. It is argued that what it is to be human, and what it is to be geometrical, evolve together, but where previous models of each police and thereby hamper adjustment to new conditions.

The traditions of continental philosophy that inform the arguments of this paper and the theorists it cites begin with human apprehensions of the world. These apprehensions are functions of the phenomena we see and the past illusions that have led to our seeing things in these ways. For example, in his phenomenological exploration of the *Origin of geometry*, Husserl (1936, p. 173) sought to understand how the perceptual filters favoured by humans have evolved to produce successive conceptions of geometrical knowledge. And he argued that "to understand geometry or any given cultural fact is to be conscious of its historicity, albeit 'implicitly'". Those technologies or filters display some historical continuity, revelatory of how they emerged from earlier manifestations.

In keeping with this theoretical backdrop the paper will be guided by contemporary conceptions of the human subject. This subject is not understood primarily as a cognitive entity, but rather constructed with respect to the symbolic apparatus we use to make sense of the world. Brown (2008a, b, c) has discussed such notions in detail through Lacanian psychoanalysis in relation to the challenges we face in mathematics education. For example, a school child might be understood in terms of being able to do basic arithmetic or not, or according to her fee paying status, her responsiveness to certain stimuli, or according to her classroom behaviour. Similarly, the child will see her task in terms of the demands (codes, rules, preferences) she perceives being made of her. But this image of self is contingent on circumstances.

Likewise, in this paper, we shall not take the supposedly ideal objects of geometry at face value. Rather we shall see such objects as consequences of historical processes. They derive from particular conceptions of being human interacting with particular conceptions of the spatial environment. Yet past conceptions of geometry and of what it is to be a human have encountered new circumstances that require the new generation to think again. Newtonian physics proved to be a model that only worked in an empirically defined domain. The Euclidean objects contained therein underwent a shift of status, from being universal objects to being objects that are a function of a domain experienced and described in a culturally specific way. Suppositions that we are dealing with ideal objects or essential human characteristics can mask trajectories that may better enable us to adjust to new conditions. Consequently this paper's principal task is to pinpoint alternative conceptions of processes of change that better align learning to participation in these processes of change.

The paper commences with a consideration of how geometrical study has been conceptualised according to discrete mark-ups of continuous spatial terrain. This is followed by some accounts of students endeavouring to account for bodily movement exercises in descriptive terms. These provide a point of reference in considering how humans relate to the physical environment through geometric constructions. The ensuing discussion considers the broader conceptual challenges of seeing learning in terms of participation in processes of historical change with respect to how humans and how spatial phenomena are conceptualised. The concept of the circle is taken as a specific example of a mathematical object towards better understanding how cultures appropriate mathematical objects and in so doing change what they are.

2. The algebraisation of geometry

Schubring (2008, p. 140) has argued "that the processes of algebraicisation are among the most marked characteristics of the historical evolution of mathematics". Mathematics is initially experienced intuitively prior to its later encapsulation in symbolic form, where there is also some later evolution of the symbolic forms. For example, Spyrou, Moutsios-Rentzos and Triantafyllou (2009) discuss some experimental work with 14 year old children where "embodied verticality" was linked through gravity with "perpendicularity", which led "to the conquest of the 'first level of objectification' (through numbers) of the Pythagorean Theorem, showing also evidence of appropriate 'fore-conceptions' of the second level of objectification' (through proof) of the theorem" (cf. Radford, 2003).

Gattegno (e.g. 1988) argued that geometrical experience is transformed, perhaps compromised, by an insistence on it being converted to symbolic form. At a seminar that Tony attended in 1979 Gattegno spoke about a baby pointing to a fly walking across the ceiling. Each (discrete) arm position signified a fly position on a continuous path. Gattegno was using this as an example of how mathematics is processed through the body to become part of oneself. In this case a simple relationship was being established between arm movement and fly position (and eyes). And it was through this sort of argument that Gattegno asserted that algebra preceded arithmetic in a child's understanding of mathematics, since he saw algebraic relationship as more basic than counting. One can imagine a young child gradually building experiences of her body moving in *relation* to the world and through such processes establishing understandings of her self and of the world. I can walk through most doorways without banging my head, or throw a ball so that my partner can catch it.

Gattegno was concerned that in school, geometrical experience generally gets converted into algebraic formality too readily and that this results in a loss. Such symbolic exchange is key to sharing and being part of a language using community. Yet this sharing has a cost in terms of the need to comply with a way of life that requires some compromise from the individual. Gattegno advocated a school education more concerned with educating the "whole brain". He wanted to foreground experiences of the continuity of geometry in the classroom to stave off premature conformity, around specific ways of organising the experience of continuity. This sense of geometry being compromised through its "algebraicisation" will underpin the discussion that follows. The continuity of life is modified through our need to symbolise it according to a discrete mark up and that process of symbolisation can import other dimensions of life such as inequitable power relations or conformity to past norms. But more mundanely this need can normalise arbitrary choices as to how that baby's future education is understood.

3. Shaping up

Tony teaches a group of first year undergraduates preparing to be teachers of mathematics in secondary schools. In one session the students walked the loci of selected geometric objects according to various instructions, and later at home drew the figures they had walked, or created computer representations using standard packages: *Walk so that you are equidistant from your stationary partner*. *Walk so that you are equidistant from your partner and a wall* (Figure 1. parabola). *Walk so that you remain equidistant from two stationary partners*. *Walk so that you can still touch a piece of loose string held firmly at each end by these two partners* (Figure 2). Acting out shapes and feeling them preceded recognising them as more or less familiar. Yet the shapes were being understood differently given the novelty of the approach. And the specific qualities of any given object could not be apprehended in an instant.

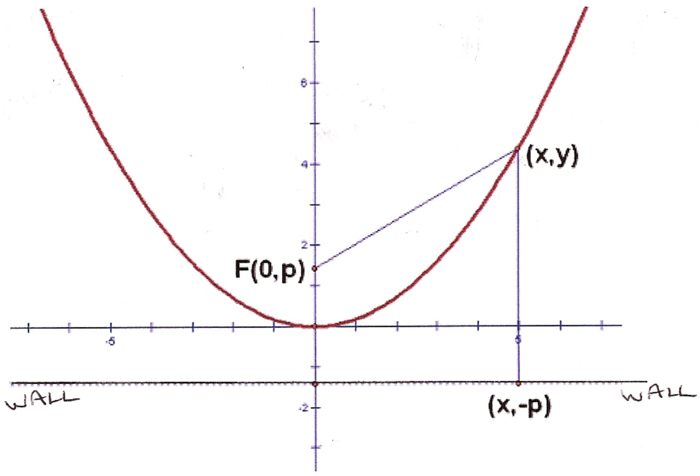
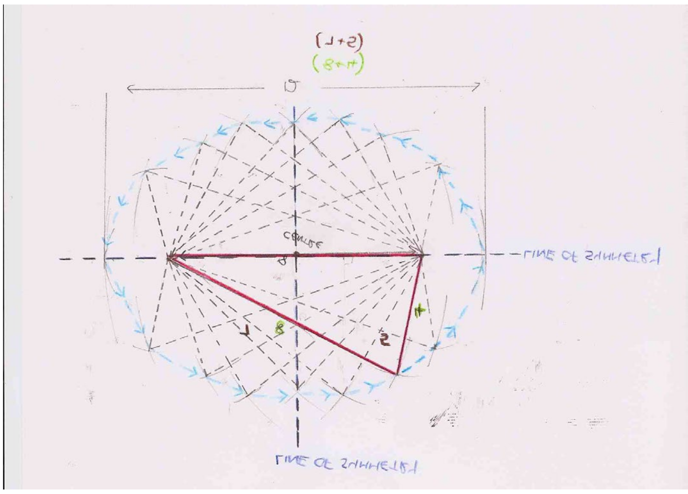


Figure 1.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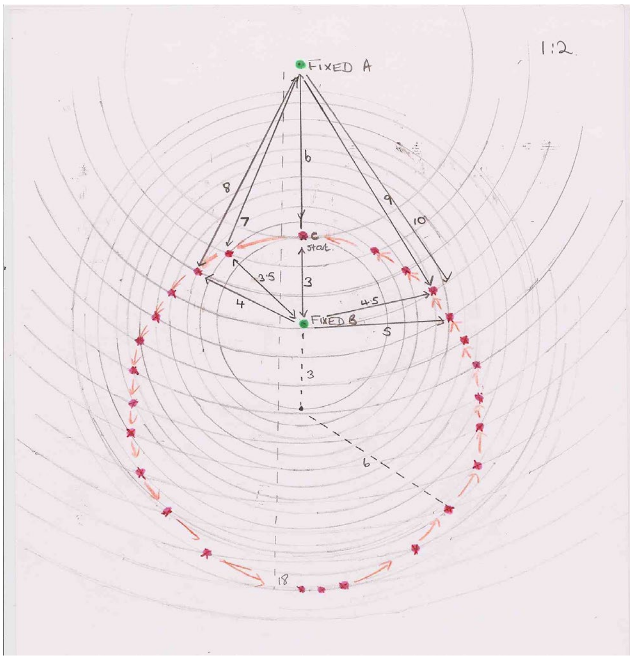


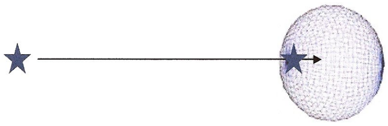
Figure 3.

In steering a particular course a student had to stay twice the distance from one partner as she was from the other. It became apparent that a curve was being produced. Yet the relative imprecision of the body movements resisted anyone achieving complete certainty as to whether it was closed and if so if its regularity suggested a circle or an ellipse (Figure 3). Through extensive discussions we all experienced glimpses of possibilities but remained unsure if our conjectures could be confirmed without more sustained analysis using drawings or calculations. A conceptual layer was needed to confirm intuitive assessments. But these initial moments provided exciting insights into emergent understandings, all the more intense for the person walking, experiencing the mathematical rules through actual bodily movements. For others there was the challenge of assuming some specific perspective on the emerging locus.

In a later interpretation a moving partner decided to stand on a chair and then on a table between her two partners (Figure 4). A third dimension was brought in to play. This departure led to an unexpected exploration for all of the other erstwhile two-dimensional shapes.

Figure 4.

Together such activities provided the students with experiences of moving in space according to more or less precise instructions, more or less drawing on conventional geometrical terminology, such that continuous movement, unlike for the baby above, was associated with a sequence of discrete instructions. The mathematical objects were generally familiar once encapsulated but the routes to them made them seem somehow new, as though they were being encountered in a fresh way that made them seem different. And following the ascent of the chair and table, lines became walls, circles became balls (Figure 5), ellipses became eggs (Figure 6) and various bowls and saddles of infinite dimension and curious orientation also emerged (e.g. Figure 7). And in certain circumstances eggs could become balls or even walls.



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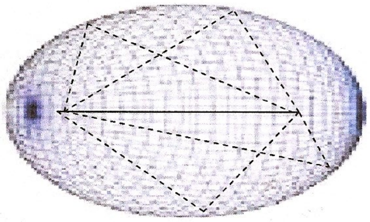

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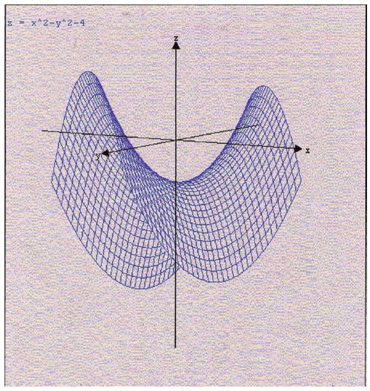

Figure 7.

Later Tony asked the group members to each write answers to the question, *What is a circle*?

- A circle is a 2D shape, which starts and finishes at some point. It is a continuous curve and has 360 degrees. Clockwise from the centre point to the curve is called the radius and the radius is the same distance to the curve all the way around the circle. We use the radius to calculate the area and the diameter, which is twice the radius, gives us the circumference when multiplied by π.
- A circle is a regular 2D shape, which has no straight sides. Every point on the circle is an equal distance from its centre point. This distance is called its radius. The distance around the outside (circumference) is known from the formula $2\pi r$ and the area from πr^2

4. Planetary movements

Dave worked with an equivalent group where the focus was on the students' capacity to relate mathematical models of the solar system with their own empirical experiences of space on a grander scale (cf. Parker & Heywood, 1998; Heywood & Parker, 2010). Students were asked to explore in discussion, by drawing, and then by acting out, how they imagined night and day, or the seasons, being explained by planetary movements. Three students working together considered how the earth moved in relation to the sun, using a globe to represent the earth. Fingers pointed to where the sun was imagined to be. The earth was spinning on its axis and rotating about the sun. England was rotating around the Earth's axis. Such words pointed to the continuity of experience. Spinning and rotation do not stop. The continuous cycle of yearly-lived experience on both a daily and seasonal basis is couched in terms of circular and cyclical movement, spin and orbit. In explaining the phenomena however, the students introduced discrete elements where the continuous was interrupted. With the task of explaining the seasons, words such as winter and summer appeared, as in "this side is in winter, and that side is in summer". This cut introduced phases in to words like spin and rotation, such that some parts of the spin, were night or day, or some parts of the rotation were summer or winter. Yet such pairings, night/day, winter/summer, dark/light soon proved to be inadequate when the task moved on to explaining how some periods of daylight were longer than others, or how some days were colder. There was also some discussion as to whether the earth was rotating clockwise or anticlockwise, backwards or forwards, and how that was related to the order of seasons, length of day, time of day, etc. Was a discrete ordering of winter, spring, summer and autumn linked with a continuous clockwise or anti-clockwise rotation? And did this relate to where one was positioned?

The students moved through a range of perspectives. They imagined themselves to be positioned on England on the globe tracking where the sun moved. Here they made hand gestures around the globe, spun the globe, lifted the globe in to different positions, moved around a stationary globe in different ways, or adopted the perspective of a space ship in a fixed location watching the earth spin before it (Figure 8, 9). The yearly cycle, broken down into winter and summer, was also explained in terms of specific mathematical shapes circular, elliptical, eccentric and oval orbits ("Does the earth go round the sun in a perfect circle?" "*I know Pluto goes round in an ellipse. The rest are closer. It just depends on what the tilt is*". "It depends on how close to the sun". "*It depends on the eccentricity of its orbit. I know that Pluto has the most eccentric orbit. Therefore it's the most elliptical.*"). This was captured by drawing sun and earth (in a number of positions), in a fixed plane, on a piece of paper.

Figure 8

Figure 9

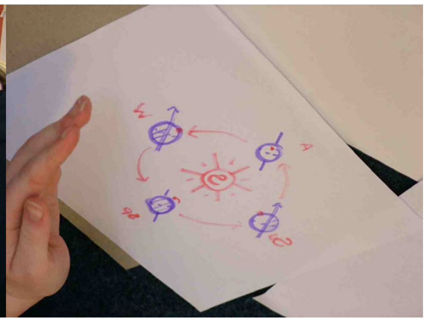

Figure 10

Subsequent explanations included increasing incidence of mathematical framing such as orbital plane being the same in each of the discrete phases ("I think I saw something where all the planets are in the same plane".). Having produced the drawing of planetary positions they then went through a phase of gesturing with their hands around the drawing, looking at the drawing from different perspectives (Figure 10) and returning to the globe to check various things.

> *You get less daylight in winter because the axis is pointing away. Imagine there's an arrow at the top of the axis. Yeah?* Yeah. *In the winter the axis is pointing away from the sun, so therefore the southern part is pointing more towards the sun. So because it's close to the top of the earth… This is the sun. This is winter. So this bit is getting more sun. So imagine it's starting to get sun now. It's dawn, middle of the day, and it's sunset. And round the middle it's constantly getting sun. Getting very little darkness. Imagine in winter it's light for short period of time. In summer start getting here daylight here – it's pointing closer... Whereas in autumn it's in the middle, in the autumn you're getting daylight starts from here… until it's got to that line. That's about half the time.*

Finally they played the part of planets, featuring a fixed human sun with a human earth rotating around it, but not only rotating in a circle (or ellipse) around the sun but also spinning and leaning whilst rotating (Figure 11).

Figure 11

5. The seduction of language

The classroom situations enabled the students to encounter mathematical phenomena in novel ways. The phenomena were "known" in advance, but they were coming to be known in new ways through the activities. How do these physical activities signify more widely known mathematical phenomena? And do the phenomena themselves change as a result of the activities? That is, do these localised encounters provide participation in evolving conceptions of mathematics across populations that ultimately change what mathematics is? This paper's answer to the second question is in the affirmative. And for this reason the paper is about conceptualising learning as active participation in this evolution. In short we will be arguing that mathematical objects do change as pedagogical objects since their cultural housing changes. And given that we have already claimed that ideal objects are culturally specific, the ontological status of such objects is further troubled.

This section commences by considering how evolving historical perspectives affect our apprehension of mathematical objects. *Circle* is taken as an example towards considering how the ontological status of that object relates to cultural apparatus. We discuss how sensory and physical experiences provide so much cultural cladding of the notional/erstwhile ideal objects. Perceptions of planetary movement are used to consider how cultural supplements locate mathematical phenomena in modelling exercises. We then formalise this process with reference to Badiou who sees the creation of objects as an operation. This operative move is then considered as a paradigm for wider advances in the physical sciences. The paper concludes by emphasising how the crafting of mathematics and of people in the language we use to describe them enables this advance whilst simultaneously policing these advances in line with older conceptualisations.

5.1 Historical perspective

How could we today engage with Husserl's quest to understand the evolution of geometrical configurations? Where and when could we possibly start? We could envisage extending the search to other mathematical objects, or indeed any empirically derived scientific object. Such an attempt would alert us to the cultural nature of each and every mathematical idea encountered in our educational quest, and of the cultural derivation of the frameworks that produce those ideas. Or do we, in any sense, encounter situations in which some mathematicians suppose they can identify mathematical objectivity beyond culture and its history? And if we do encounter such situations how would they impact on our understandings of how humans apprehend mathematical phenomena? Could one possibly suppose a clear historical perspective on such concerns?

History itself and our collective understandings of time are both linguistic constructions. History is not singular. There are many ways of re-writing history to produce new accounts of who we are now. Or more radically: "History does not exist. There are only disparate presents whose radiance is measured by their power to unfold a past worthy of them" (Badiou, 2009, p. 509). Time, Ricoeur (2006) argues, is a function of the stories we tell about it. But those stories are a function of our sense of temporal existence and how we experience life unfolding. He defines a temporality that defies phenomenology except at the level of narrative. Yet the processes of history cannot be fully captured in the stories about them. Ricoeur (1984) argues that we cannot agree on the existence of key characters, places or events, let alone the relationships between them. And people in earlier times did not understand history better than we do today. During a visit to Venice Tony's then seven-year old daughter Imogen was rather taken aback by Tintoretto's 16th century painting *Creation of the animals*: "Where are the dinosaurs?" Her youthful awareness of cultural history could detect the limits of Tintoretto's worldview. Dinosaurs, a twentieth-century human construction, were unknown to our earlier ancestors. Her brother Elliot, meanwhile, had not realised that God was a man. Cultural narratives have been revised since the painting was created and altered how individuals understand themselves fitting in to the world we inhabit.

Our narratives define who we are and hold us in place. Similarly, Husserl's enquiry into how geometry came into being concluded that without the anchorage of words, or other culturally specific technology, it was quite difficult to conceptualise.

> It is easy to see that even in [ordinary] human life, and first of all in every individual life from childhood up to maturity, the originally intuitive life, which creates its originally self-evident structures through activities on the basis of sense experience very quickly and in increasing measure falls victim to the *seduction of language*. Greater and greater segments of life lapse into a kind of talking and reading that is dominated purely by association; and often enough, in respect to the validities arrived at in this way, it is disappointed by subsequent experience (Husserl, 1936, p. 165, his emphasis).

Our narratives seduce us. They draw us in to their grasp. But this is at some cost to the experiences we seek to capture. Subsequent experience disappoints us. The narratives never quite fit, deriving as they do from past values or earlier ways of making sense.

Such narratives *mythologise* certain expressions or points of reference which contribute to socially constructed *phenomenologies* which serve as anchorages or frameworks for given communities. At any stage the signifier and signified can get jarred into a fixed relation to produce objects in a common sense social construction that is currently being lived (Barthes, 1972; Gabriel & Žižek, 2009, pp. 50-81). The potential meanings and actual usage of such expressions change through time for the individual but not necessarily in the way that the person immediately detects or monitors. Words may start off as placeholders for a particular conception and then go through a phase of being a useful working definition. Later, however, the term may be discarded as it becomes too much of a cliché without functionality. But in this fluid existence, the use of the word collides with other words being used. Words are combined in sentences and impact on each other's meaning. The introduction of any new word activates strains and stresses throughout the whole discursive framework and results in the meaning of all words and symbols being challenged in some sense (Ricoeur, 1984). But in many important respects the words and symbols that had predicated objects were all that had held the objects in place.

Husserl saw geometrical understanding as being linked to an implicit awareness of its historicity. We understand who we are through the narratives we use to explain spatial connections. The sum total of cultural knowledge about geometry remains incomplete, but "the infinite totality of possible experiences in space in general" (Derrida, 1989, p. 52) could never be completed. Yet this "infinite totality", insofar as it is imagined or experienced, is processed through geometrical knowledge as a field of ideas held in place by the forms that it has taken. A perceptual architecture supplements any supposed ideal objects with a necessarily cultural layer. This provides access for those learning the subject. Derrida (2005, p. 127) characterises Husserl as saying that "objectivist naïveté … is produced by the very progress of the sciences and by the production of ideal objects, which, … cover over or consign to forgetting their historical and subjective origin". That is, the objective reality of knowledge conceals its own history – its non-objectivity. Objects have been put there by someone. They derive from our narratives within a reality frame that is not as complete as we had thought.

5.2 The circle

How might we understand the circle's formation as a mathematical object? How have apprehensions of circles evolved? Circles have acquired so much baggage as they have been progressively used in building our stories. Some curious perspectives are apparent in the definitions of "circle" above. They use words or ideas derivative of circles. Indeed the examples both use the word "circle" in their definitions. How might we imagine circles without this linguistic apparatus that is seemingly consequential to the supposed existence of circles? Inevitably contemporary conceptions get in the way of any such attempt.

School experiences of "circle" were often centred in the construction with a pair of compasses. Things could go wrong by the pencil slipping. The circle came to be understood through the control one had in generating it. The experience of circle would be different in LOGO or *Cabri Geometre*, or through the exercises described above. Mathematical terms are situated in shared and in individual histories, and the terms' meanings derive from their relations with other terms. The terms do not have meanings in themselves. The uptake of geometrical terms would be different across peoples according to how the terms intervened in everyday living or were included in intellectual life. The natural environment of our rural Ugandan would not provide many instances of squares or triangles to which Western educations refer. As different aggregations of such objects shape our wider apprehensions of life the formative impact of "circle" continues to evolve and operate in diverse ways. Yet increasingly such usage conceals its original historical contingency as an arbitrary construction from the past, more or less motivated by empirical observation, against which we could perhaps understand our spatial environment in a different way.

Yet residues of previous eras, and earlier conceptions of those eras, remain locked in to the later formulations. Circles are a function of contemporary thinking (and vice versa). We have also changed as humans, such that those earlier humans could not have known circles in contemporary terms, and those earlier humans and their apprehensions could not be processed in contemporary terms. So many other mathematical constructs would have histories and meanings rooted in different, more or less recent, intellectual circumstances. But, most people can immediately apprehend a circle. It is a widely recognised cultural object. Yet there could be a considerable variety of meanings brought to it as indicated. Other mathematical entities would also have been generated, signified or encountered through physical embodiment and embellished in similar ways (relation, straightness, counting, iteration). But many entities require rather more specialist training to even apprehend their existence, let alone their finer qualities.

5.3 Circularity

Observed cyclical events, such as night and day, the phases of the moon, seasonal variation and planetary motion provide alternative spatial perspectives on following a cyclical or circular path. Our senses of self are shaped in relation to repeated cycles, marked events, rites of passage, the working week, annual school plays, harvest festival, Christmas, birthdays, the taking of exams, starting university, entering a profession and so on. And at a micro level these time phases are experienced through the beat of a heart, the tick of a clock, the beat of music or a sequence of TV programmes. The self is mapped and linked to cyclical changes, which are part of lived experience. The models of explanation abstracted from these observed events inevitably impact on the way in which we read and internalise the experiences we have of them, whether that be following a closed loop in space as on a Merry-go-round, or following the numbers on a clock in modulo 12.

These experiences not only relate to personal experience, they also underpin wider contemporary scientific revolutions (Kuhn, 1985). From Copernicus through to Newton and beyond, understandings of the physical world have impacted on the spatial awareness of successive generations and hence the evolution of the mathematical and geometrical constructs that are conjured. The circle, or cycle, is a multi-faceted tool used in modelling many such situations. And in relation to planetary movements we as humans encounter the phenomenon from many diverse perspectives that are not easy to assimilate into the organisation of our empirical experience (such as being on a fixed point on an imaginary circle on a very real earth that rotates every day, observing a moon following a closed circuit too big for us to grasp for most of the day, travelling on a route around the sun completed every 365 and a quarter days.)

Qualitative features of varying *intensity* (Badiou, 2009) such as light, warmth, growth of living things, length of day, time of day, position of sun as seen from earth and length of shadow, dressed and structured the students' apprehensions of those cycles/circles. Those apprehensions were linked to personal experience, but more fundamentally to that person's constitution in a specific location on earth's surface. Continuous understanding or experience (of spin, rotation, of changing temperatures or degrees of light) is mapped into discrete categories, "Things are getting light so they have enough energy to grow". Cultural apparatus was introduced to hold on to and orient the supposed ideal objects. But that immersion and mark up now makes the objects what they are.

5.4 The unity of an object

We have speculated on how notions of the circle are developed, transmitted *and transformed* through the need to traverse alternative perspectives. The objectivity of this concept was shown to be far from stable, although it would be difficult to achieve clear consensus on how mathematical objectivity is understood. Our depictions of bodily movement meanwhile occupy similar territory to a variety of incommensurable work on gestures and embodiment. Lakoff & Núñez (2000) aspire to a scientific understanding of mathematics grounded in processes common to all human cognition. Radford (2004, p. 18) suggests that we "consider mathematical objects as fixed patterns of activity in the always changing realm of reflective and mediated social practice". Nemirovsky & Ferrara, (2008, p. 4) frame their analysis in terms of "perceptuo-motor-imaginary activity" that is "fully embedded in the body". Brown (2008a) shows how Lacanian conceptions of the human subject contrast with Piaget's developmental account of the child. Meanwhile Roth and Thom (2008, p. 2) suggest: "Both Kant and Piaget … conceive of mathematics generally and of geometry particularly as paradigmatic examples of knowledge that is independent of sensual experience, though always given in the form of representations that can be related to the things that we come to know through sensory experiences". They contrast this constructivist epistemology with the model of van Hiele: "In the Piagetian model, the human mind necessarily develops to specific endpoints given by classical logic, whereas in the van Hiele model, emphasis is placed on the learning processes that - mediated by language - are specific to the historical period". Consequently, they argue: "A conception always pertains to the activation of the traces previous experiences have left in the body, and therefore, reflexively, is always an embodied conception" (p. 13). But even for Kant an "object", or conception, is "that which represents *a unity* of representation *in experience*" (Badiou, 2009, p. 231, our emphases). And experience is necessarily continuous through time but variously partitioned in its apprehension.

Consensus or reconciliation between such perspectives is unlikely. We chose to follow Badiou (2007) who asserts that an object results from an operation of "counting as one" (see Brown, 2010). The term circle entails an operation to "count as one" the objects of a given set. For example, the set of points on the rim of a bowl may be "counted as one" and given a name, circle. Or the moon and the sun might be seen as displaying a "shape" also occurring in naturally occurring objects, such as, berries, oranges, eyes, etc. The group of objects so classified may be given a name, such as "circular shapes", or "spherical shapes". But thereafter the term can become a member of other sets of objects such as "regular two-dimensional shapes" {pentagons, ellipses, squares, circles etc}) seen as making up a world and utilised in organising our apprehension of the world. Algebraisation comprises a similar operation of "counting as one" (e.g. identifying the set of points obeying the relation $x^2+y^2=1$). The objects get to be there, in a world, as a result of the operation. But they need that prior construction, of a world, to *be there.*

And in this sense learning can be seen as putting things *there*. Learning comprises the placing of an object in a world. With regard to the student moving around according to geometric loci the task is to apprehend continuous movement as a sequence of points, which are then aggregated to count as one object, understood in terms of this mode of aggregation. Retroactively the students can recognise the shape they have walked against a new register and declare “that’s it”.

Likewise, in Lacan’s concept of human formation a transformation takes place when a young child assumes a *discrete* image of herself. This allows her to postulate a series of equivalences, samenesses, identities, between herself and the objects of the surrounding world (the equivalence of my movement on the floor, to the drawing on paper, to the image in my mind, seen as continuous movement, or as a configuration of points). The image of self, as characterised by a name, fixes an egocentric image of the world shaped around that image of self. That is, the assumption of a self (a “that’s me”) results in a supposed relation to the world and a partial fixing of the entities she perceives to be within the world, that the “me” has been gauged against. In due course these relations become implicated in more overtly mathematical phenomena that underpin the child’s formal mathematical education (Brown, 2008a, forthcoming). Unlike Gattegno’s baby the older student can become aware of symbolised mathematical relationships or of how specific bodily positioning responds to a coded spatial environment. And notions of humans and of geometrical objects become relatively fixed in such images with consequential restrictions on how relations between people and geometry can be understood. A “that’s it” encounters a “that’s me” and a relation between these two entities may be asserted. Yet Lacan cautions that we should be wary of this image, since it is illusory. “Lacan also notes that scientific truth is only attained at the price of completely forsaking perceptual information, and therefore everything that would connect the world to the organs of the body” (Badiou, 2009, p. 477). Our real self is not fully visible to us. The image is crafted retroactively within the limits of the apparatus we have available. And this apparatus has a track record of being changed on a frequent basis. The operation of “count as one” can always be performed differently according to new circumstances.

5.6 The search for unity

Galileo’s provisional account of the universe being heliocentric preceded contemporary conceptions. But the coexistence of his account with other contemporary astronomy redefines the relationship his ideas have to the entirety of human knowledge, and how we understand his ideas fitting in, and how we ourselves relate to them. Black holes or black stars may have been a shock to Galileo (Barcelo, Liberati, Sonego & Visser, 2009). Yet Galileo in his time was surely formalising, through telescopic observations and deductions, much that had previously been known intuitively. He could not have been the first person to notice the phenomena that he described, but perhaps his encapsulation enabled alternative modes of noticing, that shaped later thought. Any supposed universality of earlier conceptions would be disrupted, or localised, by later developments.

Michael Green (a string theorist now occupying the chair previously held by Newton and Hawking) has speculated on how new frontiers might be presently understood against past discoveries that had unified earlier work:

> The whole history of physics, for centuries, has been one of unifying things. In the 19th century electricity and magnetism were considered to be two completely disconnected phenomena and then it was realised that they were different aspects of the same thing. And that was a great breakthrough in understanding. And of course more recently, with Einstein, there was an understanding of the implication of ideas about space-time and gravity. I guess in the biological sciences things work completely differently – although, of course, a great model, the most glamorous thing you can possibly imagine happening, is the work of Crick and Watson, who had no right to believe that there was a simple elegant solution for how animals, how entire biological systems procreate – but they understood the structure of DNA, and with that understanding came along the understanding of how it all worked. And so in a

completely different context, and obviously in a very different way, that's the kind of thing that we are looking for (quoted in Edemariam, 2009, p. 34).

The very act of unifying or of "counting as one" redefines the parameters that govern those new entities, since the world *as a whole* and the elements within it are understood differently.

This opens the wider question of how we define the constitution of mathematics when it is clearly an infinite realm. How does the assumption of any particular frame result in an adjustment to the meaning of the constituent terms? It would indeed be difficult to achieve consensus on how such limits could be drawn. Mathematical meaning requires clarity about its axioms and the worlds that give them meaning (Badiou, 2009). Yet meaning also depends on how it is apprehended. People are diverse in character and any individual can be understood through a variety of social filters to produce alternative subjective modes (e.g. cognitions, subjects, bodies, reflective practitioners). They can identify or be identified with different ways of making sense of the world. It is not just a case of what you see, but from where you see it, and who *you* are. We of course cannot make a final decision as even mathematicians refuse to reach consensus on the philosophical and social terrain, or the ontological status of mathematics: "Realists cannot explain how mathematical perception works, formalists cannot explain why meaningless mathematical statements apply so conveniently to physical reality, and intuitionists cannot explain why so much of classical mathematics seems reliable and coherent" (Hallward, 2003, p. 74).

6. Implications

Mathematics is constructed, preserved and signified through apparatus that is necessarily cultural and hence temporal. New and existing mathematical phenomena derive their meanings from how they now relate to an ever-expanding mathematical knowledge base. Or more mundanely, school knowledge derives from administrations trying to administer populations of teachers and children with more or less predictable results against a register of externally defined standards (Brown & McNamara, in press). Meanwhile, for individuals, mathematical constructions are held in place by incomplete accounts of school learning. And teachers in schools working with children will, like all of us, have specific and restricted historical and mathematical conceptions in some areas of their knowledge. On the one hand mathematical ideas are cropped to meet the needs of restrictive curriculums. On the other hand they are cladded with cultural paraphernalia that perhaps make them more identifiable.

Geometry has sometimes been depicted as a field comprising ideal objects held in place by the technologies that have been developed to provide access. "But this technology is culture- and time-dependent implying a two fold task for students - learning the present cultures of mathematics for social participation in that era *and* also access to Truth through experimentation and critique" (Brown, 2010). School mathematics teaching is often in the business of enabling students to better apprehend and use socially derived mathematical apparatus (Brown & McNamara, in press). And that can drive mathematics into forms more easily managed in the educational contexts concerned, and accountable within the regulative apparatus that doubles to formally assess understanding of the field and student conformity with social norms. That is, in the *world* of teaching situations, mathematical objects are recast as pedagogical and assessment objects that result in the erstwhile mathematical definitions becoming implicated in socially governed processes. Within educational contexts the meanings of mathematical objects are necessarily a function of the relationships within such social settings. That has always been the case. The currency in education comprises pedagogically or socially defined objects, not so much mathematical objects understood in a more Platonic sense (Radford, 2008). In the specific case being considered, geometry has been converted into particular linguistic forms for accountancy purposes or formal recognition, such as tests/exams.

This can compromise aspects of geometrical learning in the way Gattegno highlighted, such as where continuous experience of certain geometric forms is prematurely seen in terms of discrete categorisation, which may obscure or close off potential apprehensions of spatial phenomena.

And teachers and students also find themselves understood in terms of discrete categories with respect to their engagement with mathematical phenomena. Their actions are partitioned according to a discrete mark up of the mathematical terrain. Teachers are not teachers in themselves but teachers subject to particular cultural specifications. They need to be employed in a job with certain social expectations and responsibilities that restrict how others read their actions and indeed how they assess their own practice. Specifically they work to curriculums that mark out the field of mathematics in particular ways. And student engagement with mathematics is assessed according to how recognisable it is against this frame. The "that's me" is forced into alignment with the "that's it" within an externally defined register that defines "learners", "teachers", "mathematics" and the relations between them (Brown, 2008b, forthcoming). We are not so much concerned here as to whether particular teaching strategies were achieving good pedagogical results in a particular context. Such a call depends on the specific educational regime and the way in which it frames mathematical objects. Rather the issue is to do with how mathematical objects were located in these activities, and how those locations suggest interfaces with humans and their respective tasks/roles more generally.

Mathematics education research itself meanwhile seeks to inform the social interactive processes that locate but also transform the objects concerned. This task can be understood from a range of perspectives that can mark out various operational levers, not just changes to teacher practice. And as researchers we need to be aware of how our work is governed and formatted by a range of agencies, from employers allowing limited space between other duties, to funding agencies being specific about the perspectives they want, to research assessment exercises or journals defining what is of value to the research community.

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