Truth and the renewal of knowledge: the case of mathematics education

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Abstract: Mathematics education research must enable adjustment to new conditions. Yet such research is often conducted within familiar conceptualisations of teaching, of learning and of mathematics. It may be necessary to express ourselves in new ways if we are to successfully change our practices. And potential changes can be understood in many alternative, sometimes conflicting, ways. The paper argues that our entrapment in specific pedagogic forms of mathematical knowledge and the styles of teaching that go with them can constrain students' engagement with processes of cultural renewal and changes in the ways in which mathematics may be framed for new purposes. But there are some mathematical truths that survive the changing circumstances that require us to update our understandings of teaching and learning the subject. In meeting this challenge Radford encountered a difficulty in framing notions of mathematical objectivity and truth commensurate with a cultural-historical perspective. Following Badiou, this paper distinguishes between objectivity, which is seen necessarily as a product of culturally generated knowledge, and truth, as glimpsed beyond the on going attempt to fit a new language that never finally settles. Through this route it is shown how Badiou's differentiation of knowledge and truth enables us to conjure more futuristic conceptions of mathematics education.

Keywords Truth . Objectivity . Knowledge . Culture . Badiou . Lacan . Radford

Introduction

Do we conceptualise our task as mathematics educators in terms of initiating our students into existing knowledge? Or might our task be seen more radically as troubling the certainties of that knowledge, or to explore the limits of those certainties, to keep open the prospect of our students accessing a truth that transcends the parameters of our own teaching? Alternative philosophical traditions handle human relationships to knowledge and the language in which they are framed differently. Gallagher's (1992) survey of some of these traditions shows how the various options support alternative conceptions of teacher-student relationships and hence alternative priorities for the task of education. His radical options are defined in terms of students reaching beyond the frameworks that their teachers offer to produce a new future governed by structures unavailable or inconceivable in the present. Such an attitude is encapsulated in the work of Foucault (quoted in Patton and Meaghan 1979, p. 115).

> All my books … are if you like, little tool boxes. If people want to open them, or to use this sentence or that idea as a screwdriver or spanner to short-circuit, discredit

> or smash systems of power, including eventually those from which my books have emerged … so much the better.

What could this look like within mathematics education? Whole Class Interactive Teaching (Harrington, 2002) provides an example where the teacher's task is to facilitate a learning environment that maximises opportunities for students to contribute to an evolving group account of a mathematical situation. Having posed a question or stimulus to the class the teacher then seeks to ensure that a significant number of students are included as she guides them in crafting a composite story out of the diverse comments that they offer. The end point is not necessarily agreement or an outcome that the teacher had in mind in advance. The challenge is to build an understanding of the analytical apparatus that mathematics provides rather than to see this analysis as being shaped around a set of pre-existing ideas that suggest particular outcomes to the discursive process. This discursive generation provides students and teacher with a frame within which they can begin to share ways of talking in relation to mathematical stimuli. Here, rich discussion is seen as evidence of shared mathematical construction where a common objective for teachers working within this approach is to work towards extended mathematically oriented conversations. Whilst familiar mathematical concepts are touched on within such discussions the teacher seeks to promote the students' own mathematical constructions. This enables students to conjure their own mathematical objects from encapsulations of certain aspects. As a brief example from my own teaching, I ask students to imagine a path, "any path at all that you like", connecting two specified but random points on a 4x4 grid lattice. Having collected multiple suggestions from the class that normally reveal the infinity of possibilities I then ask for suggestions for rules that future paths must obey. I then set challenges such as "Can we fix a set of rules such that there are between 15 and 20 possible paths?" This enables students to explore how parameters might be variously set to produce alternative results. Here the pedagogical emphasis is as much on the adjustment of parameters as on the production of results. Such options require that teachers forego a comprehensive understanding of what their students should be able to achieve. Such student achievements, which can offer fresh perspectives, may not be in the teacher's register. They may also be in the future and not be foreseen. The lessons of school can take many years to settle and take us by surprise when we least expect it (Britzman, 2003).

How then might we understand mathematics education against such a radical perspective? A persistent challenge has been located in dealing with a split at the very core of the term "mathematics education" that hampers conceptual advance. Research can relate awkwardly to the two constituent terms that tenuously wave to each other from disparate conceptual domains. Mathematics often continues to be conceptualised as a discipline beyond social discourses where its objectivity is a prized possession. Or perhaps, the supposed universe of mathematics has been *universalised* through processes of power that mark out the territory in particular ways that drag on moves to see things differently. Education is, meanwhile, notionally a social science susceptible to interpretive analysis. Yet it does not sit easily in the broader social sciences and the analytical resources those sciences provide. Partly as a consequence, education as an idea and practice finds itself increasingly susceptible to an array of external definition and regulation. Curriculum decisions are split and shared between various groups that do not necessarily see eye to eye resulting in potential disjunctions between policy setting, implementation by teachers and the conceptualisations made of such implementations by researchers (Saunders, 2007; Whitty, 2007). This has in some countries led to teaching practices being defined according to narrow conceptions of mathematics, with the result that research is commissioned to deliver those limited ambitions. For such reasons current conceptions of mathematics and education are variously restricted and not conducive to their individual evolution. And the definition of the domain predicated by the composite term "mathematics education" is held in place by a variety of culturally bound assumptions. Culture here is seen as the intellectual dimension of civilisation where humans set practices and objects.

As mathematics education researchers we are clearly interested in the interface of mathematics and humans. Yet at a philosophical level, there are many ways of understanding what mathematics is, and many ways of understanding what it is to be human. These alternatives have major implications for how we understand practice. Philosophies of mathematics are often centred in positivistic notions of mathematical truth, objectivity and stable meaning. These philosophies are not typically disposed to the predominant twentieth century philosophies centred on what has been called a "linguistic turn". Badiou (2007) argues that the linguistic orientation is privileged in all three mainstream contemporary philosophical traditions of the twentieth century, hermeneutics (Heidegger, Gadamer), analytic philosophy (Wittgenstein, Carnap) and postmodernism (Lyotard, Derrida, Foucault). In each of these three cases, truth, he suggests, insofar as it is entertained, is processed through language to produce knowledge. Knowledge emerges through the operation of discursive systems. And knowledge houses tendencies that are not always in the business of portraying a world defined by consensual harmony or where final answers might be available.

The three traditions each provide alternative frames for understanding mathematics and for understanding human subjects. Perhaps analytic philosophy comes closest to reaching out to mathematics as a logical system centred, as the tradition is, on logical and grammatical analysis to demarcate which utterances have meaning and those that do not. Such approaches combined with cognitive psychology have been predominant influences. Yet in analytic philosophy the notion of meaning underwent substantial revision when Wittgenstein (1983, p. 20) equated the meaning of a word with its *use* in language. Attention to usage of language and by implication social practices necessarily brings with it interpretations that project us beyond the strict categories of logical analysis. And it is this sort of manoeuvre that muddies the water between logical frameworks such as mathematics and how those frameworks are encountered by human minds and the meanings and practices they bring to them. We are necessarily confronted by some profound questions: In which sense can those logical or reality frameworks exist independently of the communities that created them? How might we understand mathematical truth in its encounter with humans? Does mathematics depict truth or does it merely work as analytical apparatus in some instances? Mathematics itself has been susceptible to cultural and historical turbulence in its very formation. Disputes within philosophy sometimes focus on whether there is a greater truth to be found beneath the operation of the multiple and perhaps conflicting discursive systems humans use to make sense of their world.

Objectification and cultural understandings of mathematics

These themes have been explored in mathematics education research. Radford, Bardini and Sabena (2007, p. 2) question conceptions of learning where the learning is understood as "something mental, as something intrinsically subjective, taking place in the head". There is a prevalent tradition in mathematics education research in which mathematical objectivity is seen as transcending cultural specificities. Radford (e.g. 2007) questions this tradition. Nevertheless Radford's work is not entirely uncontroversial with its recasting of objectivity as something more contextually bound. And such concerns have led Radford to be more precise in defining the term "objectivity".

In one paper, Radford (2006) focuses specifically on mathematical objects and how they are apprehended[1]. He outlines the classic opposition between a "time honored tradition that meaning is the real and objective description of the intrinsic properties of objects or states of affairs" and conceptions of meaning as a subjective construct, based on "intentions that we want to convey" (p. 40). The former is well known in scientific traditions. The latter privileges human construction. Meanwhile, Radford is not entirely convinced by newer discursive approaches and suggests: "that mathematical knowledge may still claim some sort of objectivity" (p. 41). The paper is motivated by the question: "if, in one way or another, knowledge rests on the intrinsically subjective intentions and deeds of the individual, how can the objectivity of conceptual mathematical entities be guaranteed?" (p. 39) It provides a philosophical account of mathematical objects as understood alternatively in relation to the theories of Peirce and Husserl each seen as offering a potential solution to this loss of objectivity – but with mixed results.

For Peirce "reality influences our thoughts but is not created by them". And "semiotic activity yields knowledge" (p. 42). "Peirce advocated a view according to which we inhabit a world whose objects, laws and state of affairs are intelligible and semiotically knowable, even if to know them we have to go through an unlimited process of semiosis. Truth, indeed, is the ultimate point of this process" (p. 46). The strict separation of subject and object proposed by Peirce allows the possibility of ideal objects, counter to Radford's historical conception of mathematical entities. So viewed individual subjects would take little part in constructing the mathematical objects, reduced as they are to "transforming these *cultural concepts* embodied in texts, artifacts language, and beliefs into *objects of consciousness*" (p. 60, my emphasis). This restrictive conception of the learner is at variance with both constructivist conceptions of learning within mathematics education research *and* broader understandings of subjectivity throughout cultural studies, where human subjects are seen as *effects* of discourse. So Peirce fails on two key counts; a limited conception of the human subject; an unsatisfactory account of mathematics' objectivity.

Husserl is presented as a potential saviour of objectivity through another route. His Idealism was centred on "the role played by intentions in our apprehension of things" (p. 47). Yet Radford is concerned that this conception of the subject is divorced from social

[1] Radford's paper appeared in a special issue of *Educational Studies in Mathematics* that I analysed with respect to how the terms "teachers", "students" and "mathematics" were variously understood (Brown, 2008b).

influences: “By removing the contextual and cultural factors surrounding intentionality, Husserl’s account ends up portraying a theory of truth and meaning that is universal and beyond culture and time” (p. 51).

This discussion of Peirce and Husserl provides Radford with a platform for discussing meaning in mathematics education. His core conclusion is that objectivity, whether understood by Peirce’s Realism or by Husserl’s Idealism, is an untenable enterprise. Radford (p. 39) thus opts for a notion of “contextual objectivity”, which “gives up transcendentalism” in dealing with this tension within mathematical learning. Yet Radford has paid a high price in which both Truth *and* objectivity are shown the door in much the same breath:

> Here we abandon the idea of Truth in the essentialist metaphysical tradition, according to which Truth is that which remains once all that is ephemeral has been removed - an idea that goes back to Plato’s aristocratic ontology … We also abandon the idea of objectivity as an uncompromised access to transcendental entities (p. 60).

In negotiating the split perspective Radford ultimately chooses the side of education rather than of mathematics. Yet the terrain can be depicted differently so that a choice is not required in quite the same way.

In a well-known anthropological example Levi-Strauss describes an aboriginal South American village where inhabitants are divided into two subgroups. This is discussed by Žižek (2007, p. 242).

> When we ask an individual to draw on a piece of paper or on sand the ground plan of his/ her village (the spatial disposition of cottages), we obtain two quite different answers, according to whether he or she belongs to one of the other subgroup: a member of the first sub-group perceived the ground plan of the village as circular – a ring of houses more of less symmetrically disposed around the central temple, whereas a member of the second sub-group perceived his/ her village as two distinct heaps of houses separated by an invisible frontier.

Žižek sees Levi-Strauss’ point as going beyond mere “cultural relativism in which the perception of social space depends on the observer’s group belonging”, or, where a helicopter could fly over to capture the “actual” disposition of the buildings. Žižek argues that there is “a fundamental antagonism that the inhabitants of the village were not able to symbolise, an imbalance in social relations that prevented the community from stabilising itself into a harmonious whole” (p. 243). Žižek supplements Levi-Strauss’ example with some others that may make the story more accessible to a contemporary audience: masculine/ feminine; right/ left, where understanding of the terms and the terrain that locates them are defined according to whether you are one or the other, where no objective mediation could be possible within a consistent form of language. In some respects the phenomenological experience of “objectivity” is different for each mathematician. In another article I wrote about subjective experiences of grappling with the term “circle” (Bradford and Brown, 2005). Yet in writing about it I felt haunted by disapproving mathematicians from another tribe who I imagined would see “circle” as a mathematical notion, untainted by such personal constructs thereby discrediting my analysis. This might be understood as a similar antagonism according to whether one is able or not to dispense with one's active subjective engagement to allow purely abstract entities. Mathematicians who see mathematics as an entirely abstract domain are a different breed to those attentive to its historical evolution and hence its potential immersion within the social sciences. To move from one domain to another requires a major switch in modes of thinking, from one conception of life to another.

Radford's (2006) task was conceptualised as an attempt to articulate such a divide on a grander scale. His paper was constructed for a mathematically oriented audience protective of meaning being seen as objective. With this audience in mind he argued the case against transcendental objectivity and Truth. More commonly however, Radford's analysis in other papers (e.g. Radford et al, 2007) is addressed to an audience of teachers or teacher educators, governed primarily by educational concerns and centred on changing their own practices in line with cultural norms. Besides Radford's own evolving perspective through time these contrasting formulations result in Radford offering two perspectives, which are not easy to reconcile. Each perspective, I suggest, risks presenting a clipped version of subjectivity, culture and hence of mathematical learning that does not account sufficiently for the diversity of educational or mathematical interest. By insisting that it becomes a choice neither side comes out of it very well.

The truth of Badiou

The work of Alain Badiou (e.g. 2006, 2007, 2009) conceptualises a Truth that transcends such alternative subjective modes. His work has begun to occupy the space vacated by the coterie of French philosophers who dominated intellectual life in the second half of the twentieth century (Derrida, Foucault, Lacan, Deleuze, Levi Strauss, Lyotard). Badiou's lineage can also be traced through Bachelard, Lakatos and Althusser who each saw science as a practice marked by the production of *new* objects of knowledge (Feltham, 2008, pp. 20-21). Badiou's canvas extends into the territory of potential futures, creating a framework against which all three twentieth century philosophical traditions that he mentions can be read. His new book builds on his contention that these traditions were excessively centred on contemporary conceptions of the unit of the human, organised according to language-centred analyses. Badiou contends that truth is left out of this analytical mode. For Badiou scientific truth concerns the *invention* of theoretical parameters. "Truth can only be reached only through a process that breaks decisively with all established criteria for judging (or interpreting) the validity (or profundity) of opinions (or understandings) … access to truth can be achieved only by going against the grain of the world and against the current of history" (Hallward, 2003, pp. xxiii-xxiv, see also pp. 209-221). Thus Truth cannot be substantiated or represented in culturally derived media. "Truths have no *substantial* existence" (Badiou, 2009, p. 5, his emphasis). Any attempt to pinpoint truth ultimately disappoints us. So in short Badiou's quest is to understand how alternative forms of knowledge are shaped and evolve around a Truth that is experienced but never finally represented.

This can be illustrated through a classroom example: *Modular doubling* entails number bracelets where successive numbers are doubled in different modulo. Modulos 11 and 13 produce simple bracelets 1,2,4,8,5,10,9,7,3,6,1… 1,2,4,8,3,6,12,11,9,5,10,7,1… Having introduced these examples students are invited to explore other modulo, which can produce rather more complex results (Figure 1).

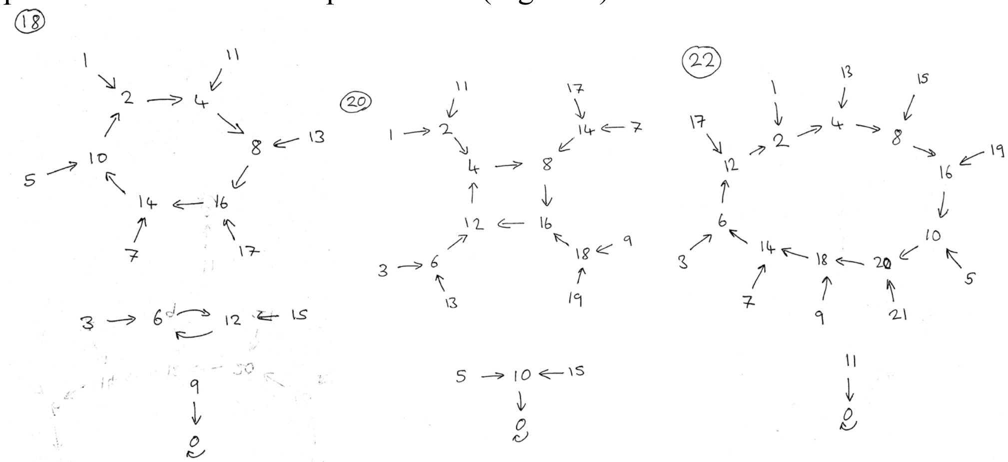

Figure 1. Bracelets for 18, 20 and 22.

Subsequent work initiated by a student included investigation of length of bracelets for odd modulo, leading to statements of generalisation such as: "All odd number modulo produce simple bracelets" or "For any n, that is prime and in the form $n= 2^k-1/m$ (where $k, m \in N$): No. of cycles = $n-1/\log_2(nm+1)$. Size of cycles = $\log_2(nm+1)$". The activity is characterised by experimentation and critique. The task for the student is not so much about checking the correctness of particular configurations as being about deciding upon productive avenues to follow. The student here was guided more by some sense of aesthetic completeness or *qualitative unity* (Sinclair, 2006) than by some pre-ordained model of correctness foreseen by her teacher. He was motivated by encountering new ways of seeing, both within and beyond the situation in question. The purpose was not to learn about doubling modulo as such but rather to learn about the production of generalisations or to become more adept at predicting productive avenues of enquiry. It was about researching how mathematical forms can function as analytical apparatus to produce insights more than supposing that there are specific existing mathematical ideas to be learnt. In Badiou's terms, the results can be cashed in as knowledge. The quest however is motivated by truth.

This attitude to investigation provides an example of why Badiou insists that Truth is to be distinguished from knowledge. For Badiou knowledge relates to a diverse range of domains or "situations". "If what we call the world or the universe is some kind of totality, then we must agree it is primarily a totality made up of subsets, of domains of objects" (Gabriel and Žižek, 2009, p. 15). Badiou includes subsets, such as "words, gestures, violences, silences, expressions, groupings, corpuscles, stars etc" (quoted by Hallward, 2003, p. 94). Each such domain provokes a form or system of knowledge, within which we can say that statements are verifiable, but we are unable to say that they are true. Importantly, in Badiou's account, these alternative forms of knowledge cannot be harmonised into one overarching frame. Truth always exceeds the sum of all the systems of knowledge. Whilst there have been many attempts at different times to capture Truth, successive generations invariably rewrite these attempts. So many failed attempts, but perhaps we learn to fail better. The idea of truth in mathematics has proved to be inadequate. Euclidean geometry, as presented in Euclid Elements, was considered an embodiment of truth. It was believed that the Elements contained truths about space. The creation of Non-Euclidean geometry showed that it was not the case and that it was inadequate to continue speaking about truth in mathematics[2]. In Badiou's formulation Truth stimulates the generation of knowledge but Truth cannot be captured *as* knowledge.

It is Lacan's revitalisation of the term "truth" that motivates Badiou. According to Lacan (2008/1967, p. 17): "truth is always new, and if it is to be true, it has to be new. So you have to believe that what truth says is not said in quite the same way when everyday discourse repeats it". This quote might be best understood in relation to Lacan's psychoanalytic therapy, which entails interrogating the stories the client has constructed about who they are. Such stories have often become fixed in unhelpful ways. And this rigidity can prevent movement to a new story that might suit the client better in new circumstances. Here "truth" is not served by the ways in which these stories have settled in to common sense. We can all sometimes ignore the fact that stories that we tell of ourselves in the world are failing us, as perhaps we have not yet learnt other stories with which to replace them. Psychoanalysis is premised on re-writing the storyline of our lives, such as organising our pasts differently, marking out events in different ways, to open up different ways of understanding possible futures. Some writers extend this in to our capacities to collectively rewrite history to highlight alternative historical trajectories (Pavon, 2010). Earlier accounts could restrict the avenues open to us and that there is now an imperative to replace these accounts. But initially at least this imperative provokes a more vital or experimental approach to the generation of accounts of life revealing to some extent the operation of language itself and the realties this provokes prior to a new settling as a commonsensical construct. But the client's attempt to tell a better story also always ultimately fails as any sort of final version. Yet these failures are informative since the attempts provide greater insight into the *truth* that guides our knowledge. In Lacan's model, truth always slips away as we try to grasp it. He is persuaded that there is a truth that keeps us alive beyond the reach of settled forms of knowledge. This analytical approach can provide a paradigm for the onward march of mathematical discovery that results in earlier work finding new meanings alongside later work. For example, Einstein's work on Relativity enabled us to see in a new light Newton's work on gravity. Likewise, human modes of engagement with mathematics are necessarily processed through specific cultural manifestations that can distort access to any notional mathematical truth beyond. For example, curriculum definitions of mathematics in schools might be shaped more by the supposed needs of good employees in the current economic model, rather than according to more humanist aspirations shaped by intellectual endeavour. For the student depicted above, an excessive concern with curriculum demands may hamper his investigation since he would be less in control of deciding which aspects of his work would be regarded as important.

Badiou's philosophical and political ambitions promote insurgency at root, modelled as they are on such psychoanalytic processes, characterised by Lacan as being about the detection of "holes in discourse" (Lacan, 2008, p. 27). And it is the hole that Lacan's anti-philosophy punctures in philosophy and culture more generally that motivates Badiou's own more systematically philosophical pursuit. For Badiou, yesterday's stories

[2] I thank an anonymous referee for this example.

never quite live up to the truth. He sees the task of philosophy as being to challenge the consensual status quo. Any linguistic or symbolic form will, after a while, settle in to a particular way of making sense that will serve some people better than others. Euclid's model does not represent deep space. But it might be in the interests of curriculum authorities to reduce geometrical thinking to Euclidean principles to make school mathematics more accountable through available assessment procedures. That way teachers and students can be told what to do in precise terms. But in regulating mathematical thinking it is being compromised according to a partisan cultural agenda. In Badiou's view an important task of philosophy is to locate holes in the functioning of everyday language to challenge this imbalance and the attendant disquiet from within. Euclidean geometry is not a complete picture and the specific assertion of Euclid's geometrical world distorts mathematics as a whole. Any subset of geometry is knowledge, not truth. In this understanding philosophy is necessarily subversive, out of synch with the discourses that fuel cultural life. The detection of a hole results in attempts to reshape our engagement with life, perhaps through a more experimental attitude to language that achieves alternative experiences of truth. That is, senses of "how things are" are disrupted forcing adaptation to a new understanding of reality governed by radically different parameters, invisible to the eye of someone immersed in the previous reality. Philosophy is thus the on-going attempt to fit a new language that never finally settles. But truth is what it seeks.

Within Badiou's taxonomy Radford's analysis of children's learning in classrooms would fall squarely in a hermeneutic tradition, but with some discomfort resulting from analytical hecklers. Radford focuses on students' expressions of their growing awareness of mathematical attributes as they pass through a succession of perspectives, rather than supposing that objects have an *a priori* positivistic existence; "mathematical objects *are fixed patterns of reflexive human activity incrusted in the ever-changing world of social practice mediated by artifacts* (Radford, 2008, p. 8, his emphasis). As we shall see shortly, mathematical generalisations achieved through processes of updating and fitting new stories to newly articulated situations would in Badiou's theory be seen as operations through which specific unities (such as generalisations within mathematical stories) are asserted. According to Badiou, we experience Truths with great frequency. Everyone including students can experience Truth. Radford's difficulty, however, may be a result of linking Truth and objectivity so directly. Within Badiou's formulation Radford's notion of "contextual objectivity" comprises knowledge referenced to specific forms of culture. So too are the ideal objects of analytic philosophy. Badiou conceptualises a truth beyond existing cultural arrangements where neither mathematical ideas nor human subjects have settled, and can never settle.

What could this look like in the context of mathematical learning? Whole Class Interactive Teaching perhaps provides one conception of teaching in which outcomes are not defined in terms of content knowledge acquisition. The *Doubling modulo* activity outlined above provided an alternative attitude to generating and framing mathematical ideas. Mathematical investigations such as this gained prominence in a number of locations some years ago (e.g. Banwell, Saunders and Tahta, 1972; Association of Teachers of Mathematics, 1977) although their subsequent wider impact in England has declined since a more content-oriented curriculum was legislated. Investigational work was squeezed into ever more remote corners of the new curriculum as this style of work could not be pinned down according to either clear mathematical content objectives or organisational parameters, which made regulative assessment of both teachers and children more difficult in the new climate. Mathematics became ever more policed by prescriptive curriculum defining checklists of skills and procedures that in an important sense asserted a particular and powerful social conception of mathematics (Brown and McNamara, 2005). Investigations had been more or less related to specific mathematical areas and could be interpreted on a number of conceptual levels. They centred on mathematical exploration where the teacher did not necessarily have particular strategies or outcomes in mind. Unlike much work with the socio-cultural tradition, the student, working alone or in groups, was encouraged to take the task in their own direction, to *mathematise* in new ways, rather than to rediscover existing mathematical forms. That is, they were being encouraged to reach beyond *fixed patterns of reflexive human activity.* The limited direction by the teacher was further reduced as students gained more experience with such tasks. This promoted confidence in students posing and addressing their own questions. The initial task specification does not reveal the traditional mathematical area being addressed, and in an important sense the area of mathematics is not the point. The principal purpose is for the students to introduce mathematical structure in their own way and to make generalisations. (See Brown, 2001, pp. 88-100, where I provide an extended discussion of ten-year old children working on an investigation).

I offer another example here to unfold further issues relating to the emphases presented within such paradigms. *Snooker* comprises imagining a snooker table with pockets just at each corner where a ball is projected at 45° from the bottom left corner (cf. Ollerton, 2009). I demonstrated this to a class by drawing a 4x3 “table” and the trajectory of the ball in that case. The task was to predict by which pocket the ball left the table and to understand other aspects of the route that it followed for tables of different dimensions. At this point I absented myself to circulate and observe. For those in the early stages of the task the work entailed careful drawing of the routes on different sized tables. Yet very quickly patterns emerged that led to analysis of structure with the introduction of algebraic or visual relationships of varying complexity. The students decided the work sequence, the layout and the modes of classification, such as colour coding, abbreviations, etc (Figures 2, 3).

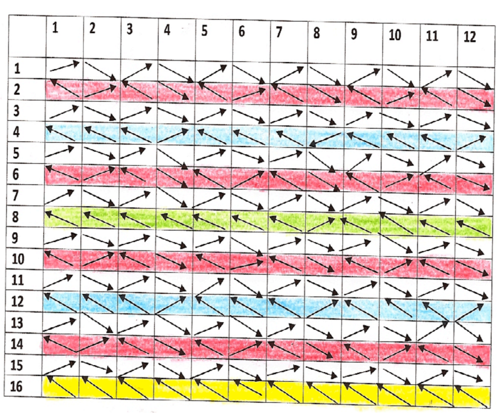

Figure 2. Exit points for different tables.

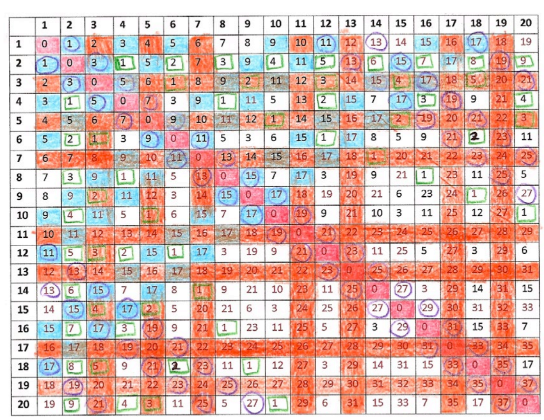

Figure 3. Number of bounces.

This work led to generalisations such as:

- I feel we can safely say that this will continue with all the numbers in the 16 times table but not in the 32 times table.
- the negative diagonal is all zeros and is also a line of reflection.

The outcomes were not fixed in advance. Students could become better able to approach complex or unfamiliar situations with an intelligent attitude marking out new territory. Many opportunities arose for the teacher to engage with the students' work, to expand its scope and reach out to more powerful realms for both student and teacher. The student could be asked to prove her result rigorously rather than merely "safely say", or check the consistency of her work with other students, or simply convince other people that she was correct.

This style of work supplements the task of learning as depicted by Radford in this significant paragraph from the paper discussed above:

> Within this semiotic-cultural approach, an important distinction has to be made between learning and the production of new knowledge. While new cultural concepts arise from communal, reflective, mediated activities in the zone of proximal development of the culture, school learning is the process of activity and creatively transforming these cultural concepts embodied in texts, artefacts language, and beliefs into objects of consciousness. This process, in which subject and object modify each other, is the process of meaning, the process where subjective knowing and objective knowledge merge. (Radford, 2006, p. 60)

Firstly, the paper distinguishes between learning and the production of new knowledge or cultural growth. Secondly, the Vygotskian formulation sees learning as the process by which cultural concepts are transformed in to objects of (individual) consciousness. These assumptions are centred on seeing the primary goal of education as being to bring students into existing practices. I am arguing, however, that learning, by individuals, or across communities, can also be conceptualised alternatively as being implicated in the production of new knowledge, where new forms are constructed against new backdrops. This echoes Radford's later work (e.g. 2007, p. 1790), where "learning is not merely appropriating something or assimilating something; rather, it is the very process by which our human capacities are formed"[3].

Yet Badiou's conception of objects is novel. Badiou (2009, pp. 10-16) provides the example of number theory derived from the ancient Greeks. Here conceptions of infinity are fully present but where the statement "*there are as many prime numbers as numbers* is for an ancient Greek, even for a mathematician, to speak an entirely unintelligible jargon" (p. 11). Yet after Cantor's radical re-depiction of infinity in the twentieth century the statement is permissible. So "'prime numbers' does not have the same meaning in Euclid's Greek language as it does in ours, since ancient Greek could not even comprehend what is said about prime numbers in the modern language" (p. 12). We have a multitude of mathematical statements that are each *verifiable with respect to some domain of knowledge* but the grounds of mathematics or of the domains that house mathematics are constantly on the march. Any new discovery eventually loses its original zest as later discoveries re-contextualise earlier findings. The work of the ancient Greeks

[3] Badiou's conception of subject and of human formation is a radical reconfiguration of Lacan's notion and quite different to the Vygotskian premises underpinning Radford's work (see Brown, 2008a).

still points to truth, in a sense, but its present coexistence with other contemporary mathematics redefines its relationship to mathematical universality, and how we understand it fitting in as it were, and how we ourselves relate to it. Lacan (2008, p. 74) suggests that after Gödel's results, which show that the price of consistency is incompleteness, "even arithmetic turns out to be a basket … there are lots and lots of holes in the bottom." The universal set of mathematical knowledge is always in the process of being expanded, which has an impact on the status and meaning of ideas previously included, not least in terms of how humans relate to the scientific knowledge. This on-going process of re-conceptualisation in mathematics impacts on the signification of specific mathematical forms in any given social context. And hence stable conceptions of objectivity, or of objects, cannot be secured.

How then do objects come into existence? Badiou develops a systematic conception of ontology shaped on Cantorian set theory. In this conception of how things exist, mathematics *is* ontology, the very state of being. Here "being" "is the sheer multiplicity of the world, a plurality of stuff (facts, states of affairs, etc.) that cannot be reduced to any single organising principle" (Critchley, 2008). In this Platonic orientation mathematics is a model for all linguistic construction where "being is to be ultimately explained by mathematics" (ibid). Badiou's theory is entirely formal, rather than a specific determination of action, or concept of life. Somewhat radically, "*there are no* mathematical objects. Strictly speaking mathematics *presents nothing*" (Badiou, 2007, p.7, his emphasis). Mathematics is a *grammar* organising how we are and how we see, rather than a body of knowledge comprising entities. There is no mathematical *knowledge* outside of specific cultural forms. And Badiou's immersion is in a mathematics caught in an evolution that impacts on the very being and becoming of the entities it allows. Such is Badiou's ontology. But it is also the case when we are talking about students producing mathematical ideas within classrooms. Mathematics is a way of structuring life to produce entities (or generalisations) in a variety of ways. "Unity is the effect of structuration – and not a ground, origin, or end" (Clemens and Feltham, introduction to Badiou, 2006, p. 8)). *Objects* are produced by "counting as one" a subset of elements of wider multiplicities.

The student work above was surfing on the fringes of school-oriented mathematical knowledge, marking out new territory against which findings could be understood and, in Badiou's sense, incorporating new objects. This is indicative of a potential dimension of school mathematics where the student is concerned with generating new stories, of defining new generalisations, with counting new entities as one, (pursuing processes of *objectification*, cf. Radford, 2008), and with mapping out the domain of mathematics in novel and unexpected ways. *Learning is not understood primarily as growing alignment with more or less familiar cultural forms, or with fixed patterns of activity.* The statements "the negative diagonal is all zeros and is also a line of reflection" and "No. of cycles = $n\text{-}1/\log_2(nm+1)$" are both new mathematical "objects", consequential to the students' novel ways of structuring the activities. The set of numbers on the diagonal and the set of cycles of a particular configuration are each "counted as one" to produce these "objects". They are verifiable with respect to particular domains of knowledge. Such assertions set a new topography such that all the "objects" in the space are seen, or created, slightly differently. Yet as a consequence perceptions of the particular space and of mathematical spaces more generally are unsettled. Truth may be glimpsed beyond the horizons afforded by these new perspectives.

Conclusion

Much mathematics in schools and elsewhere exists as pedagogical material crafted for supposed modes of apprehension. But of course our comprehension of such apprehension depends on how we understand mathematical objects and how we understand the people thinking about them. Specifically much mathematics education research rests on supposed cognitive models in which the human being is understood in particular ways with pedagogical models/ apparatus shaped accordingly. Yet learning can be productively viewed as an experience through time where there are changes in both the human subject and the objects they apprehend. Lacan's model of subjectivity is a function of the symbolic universe that survives and re-positions the culturally specific dimensions of Piaget's or Vygotsky's conceptions of humans (Brown, 2008a). Lacan claims that Piaget neglects the societal demands on child development, demands expressed according to that society's self image. Meanwhile, Vygotsky points to a rather compliant social assimilation. I have argued that the prominence of Piaget and Vygotsky in our research has overly restricted analytical opportunities (ibid). Badiou's Lacanian model actively seeks to break with "historical constituted cultural forms of thinking and being" (Radford, 2008, p. 16).

The refusal to settle on any given story, and on the objects anchoring past stories, underpins this paper's plea for a greater alignment between mathematical learning and cultural renewal. Echoing the spirit of Foucault's words above, learning is the task of resisting and revising current cultural models. Mathematics is held in place by its culturally developed apparatus as a *universalised* field of knowledge. Mathematics in schools, for example, is normally presented in specific pedagogical forms that vary from country to country, place to place, time to time. It can be informative to use psychoanalytic metaphors in our attempts to break through the cover stories that take the form of pedagogical apparatus that can obscure as well as provoke our insights (Brown, 2008a, 2008b). We can become stuck in particular pedagogical formulations that may not have adjusted as well as they might have done to new circumstances or new challenges. For this reason it is not enough to see the goal of education as being solely to bring students into existing practices or current conceptions of mathematics. Many factors can dampen the responsiveness of pedagogical apparatus to new conditions, such as the prevalent structural definitions of "teacher", "student" and "mathematics" (Brown, 2008b). For example, a particular curriculum may emphasise skills or factual acquisition rather than facility with problem solving, or some areas of mathematics rather than others, or only certain forms of mathematics as being suitable for less able students. Such preferences may shape teaching styles accordingly. Learning mathematics needs to include getting beyond these structural rigidities. I have argued here that we must trouble pedagogical forms (and the objects they reinforce) that have become over-familiar towards better releasing the analytical capabilities that mathematical thinking can offer.

Yet understandings of mathematics itself and of how people learn about it are necessarily subject to temporal disturbances more generally. For example, in Husserl's

(1989/1936) account, geometrical understanding is linked to an implicit awareness of its historicity. Our bodies have learnt to function and know themselves in physical environments that result from culturally situated conceptions of geometry and this feeds our conceptions of geometric objects. Derrida (2005, p. 127) characterises Husserl as saying: “objectivist naiveté is no mere accident. It is produced by the very process of the sciences and the production of ideal objects, which, … cover over or consign to forgetting their historical and subjective origin”. That is, the objective reality of knowledge conceals its own history – its non-objectivity. In this perspective objects are always continuously a function of their (on-going) historical formation, as referenced to the wider symbolic network and the way in which we talk about the physical world. (This is also true of human subjects.) The sum total of cultural knowledge about geometry remains incomplete, but the infinite totality of possible experiences in space could never be completed. I have argued here against seeing mathematics as staged checklists requiring conformity from students and teachers according to a centralised register. Both teachers and students need to creatively confront the horizons that they encounter.

Nevertheless mathematical curriculums are full of objects presented as though they are in settled forms, often associated with pedagogical apparatus or preferences suggesting naturalised modes of access. This artificially fixes mathematics and its pedagogy in relation to the evolving world they seek to support. In seeing mathematical understanding as being linked to an awareness of our historical formation we need to critically connect our educational challenges with trajectories from the past into the future that reveal turbulence in the present and our linguistic strategies that conceal this turbulence. We cannot know how much current mathematical knowledge will support new discoveries or circumstances. And we do not know how much school knowledge as currently understood prepares the pupil for later life. Mathematics as a field of ideas is held in place by its cultural technology which doubles as a mode of access for those learning the subject. But this technology is culture- and time- dependent implying a two fold task for students - learning the present cultures of mathematics for social participation in that era *and* also access to Truth through experimentation and critique. By stressing the former potentially at the expense of the latter we may be insisting that Truth be reached primarily through the filter of compliance with existing cultural forms, a route rejected by Badiou. This may not be fast enough and as teachers we may be distracting our students from seeing options that we are too old to see. New cultural awareness cannot be attributed to the intentional quest of individual frontier mathematicians or teachers taking a lead for others to follow. For the individual human, learning should be about seeing and experiencing mathematics coming into being, as part of oneself, a self that evolves in the process. The human subject is caught reflexively in an ever-changing world and needs to be aware of this. The human is no more stable than the ideas she seeks to acquire. Each is dependent on, or an effect of, the ever-changing symbolic or discursive terrain that defines them. Here education would be implicated in the formation of objects, of events, of generalisations in time and space, rather than being an encounter with ready-made objects or cultures or selves or patterns of activity. One might think more productively of cultural renewal being consequential to a more widespread innovation in curriculum or mode of governance being introduced to a community with more or less predictable results. Such innovation activates new, perhaps unexpected, modes of mathematical engagement or educative encounters across that community. Mathematical objects and modes of subjectivity never finally settle in relation to each other. I have argued in this paper that the task of education is to ensure that people do not think that they should settle.

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References

Association of Teachers of Mathematics (1977). *Points of departure.* Derby, UK: Association of Teachers of Mathematics.

Badiou, A. (2006). *Infinite thought.* London: Continuum.

Badiou, A. (2007). *Being and event*. London: Continuum.

Badiou, A. (2009). *Logics of worlds.* London: Continuum.

Banwell, C., Saunders, K. & Tahta, D. (1972). *Starting points.* Oxford: Oxford University Press.

Bradford, K. & Brown, T. (2005). Ceci n'est pas un circle, *For the Learning of Mathematics, 25*(1), 16-19.

Britzman, D. (2003). *After education.* Albany: State University of New York Press.

Brown, T. (2001). *Mathematics education and language: interpreting hermeneutics and post-structuralism. Revised second edition.* Dordrecht: Kluwer.

Brown, T. (2008a). Lacan, subjectivity and the task of mathematics education research, *Educational Studies in Mathematics, 68*, 227-245.

Brown, T. (2008b). Signifying "learner", "teacher" and "mathematics": a response to a special issue, *Educational Studies in Mathematics, 69*(3) 249-263.

Brown, T., & McNamara, O. (2005). *New teacher identity and regulative government*: *The discursive formation of primary mathematics teacher education*. Springer: New York.

Critchley, S. (2008). *Infinitely demanding*. London: Verso.

Derrida, J. (2005). *Rogues: Two essays on reason.* Stanford: Stanford University Press.

Feltham, O. (2008). *Alain Badiou: Live theory*. London: Continuum.

Gabriel, M. & Žižek, S. (2009). *Mythology, madness and laughter. Subjectivity in German idealism.* London: Continuum.

Gallagher, S. (1992). *Hermeneutics and education.* Albany: State University of New York Press.

Habermas, J. (1991). *Communication and the evolution of society*. London: Polity.

Hallward, P. (2003). *Badiou: a subject to truth*. Minneapolis: University of Minnesota Press.

Harrington, A. (2002). *Whole class interactive teaching in mathematics*. London: General Teacher Council.

Husserl, E. (1989). The origin of geometry. In Derrida, J. *Edmund Husserl's Origin of geometry: An introduction*. Lincoln (USA): University of Nebraska Press.

Lacan, J. (2008). *My teaching.* London: Verso.

Ollerton, M. (2009). *The mathematics teacher's handbook*, London: Continuum.

Patton, P. & Meaghan, M. (1979). *Michael Foucault: Power, truth, strategy*. Sydney: Feral Publications.

Pavon, D. (2010). *From the conscious interior to an exterior unconscious: Lacan, discourse analysis and social psychology.* London: Karnac Books.

Radford, L. (2006). The anthropology of meaning, *Educational Studies in Mathematics, 61*(1-2), 39-65.

Radford, L. (2007). Towards a cultural theory of learning. In D. P. Pantazi & G. Philippu (Eds.), Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education (CERME 5) (pp. 1782-1795). Lacarna: University of Cyprus.

Radford, L. (2008). The ethics of being and knowing: Towards a cultural theory of learning. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in mathematics education: Epistemology, history, classroom, and culture*. Rotterdam: Sense.

Radford, L., Bardini, C., & Sabena, C. (2007). Perceiving the general: The multisemiotic dimension of students' algebraic activity, *Journal for Research in Mathematics Education*, *38*(5), 507-530.

Saunders, L. (2007). *Educational research and policy-making: Exploring the border country between*

research and policy. London: Routledge.
Sinclair, N. (2006). *Mathematics and beauty: Aesthetic approaches to teaching children.* New York: Teachers College Press.
Whitty, G. (2007). Education(al) research and education policy-making: is conflict inevitable? In L. Saunders (2007). *Educational research and policy-making: Exploring the border country between research and policy*. London: Routledge.
Wittgenstein, L. (1983). *Philosophical investigations*. Oxford: Blackwell.
Žižek, S. (2007). *Interrogating the real*. London: Continuum.